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Correction of the galactic luminosity function for opacity effect

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Abstract. If the disk of the spiral galaxies has a moderated value of the mean optical depth, the observed luminosity function should be corrected for the combined effect of the inclination and the opacity of the galactic disk. We obtain the corrected luminosity function for current models of opaque matter distribution.

Key words: ISM: dust,extinction – galaxies: luminosity function

1. Introduction

There is a controversy about how opaque spiral galactic disks are to blue light. An analysis of the optical properties of the ESO-LV catalogue made Valentijn (Valentijn 1990) conclude that spiral galaxies, and dwarf Sc spirals, present an obscuring component with a mean optical depth $\tau = 1.3$. This unexpectedly high value for τ has been refuted by some authors (Huizinga & Albada, 1992). Recently, Peletier et al (1995) have presented a study of the radial surface brightness profiles in B and K for a sample of 37 galaxies, statistically determining the extinction at various places in the galaxies. They have fitted quite well the observations assuming the presence of an obscuring component with the optical depth decaying exponentially $\tau = \tau_c \exp[-\frac{r}{\alpha_d}]$, with high values for the central optical depth ($\tau_c > 1.5$) , and with a scale length superior to that of the star distribution $\alpha_d > \alpha_*$. In this paper we study the influence that a given distribution of opaque matter through the galactic disk has on the observed luminosity function (LF). If the galaxies were completely transparent, the observed LF would be independent of the inclination of the disk, but if their mean opacity is not negligible, the observed LF contains a contribution from the inclination of the galactic disk that should be estimated. The LF plays an important role in cosmology: the estimation of the mean luminosity density, the selection function , the number counts in redshift, or in magnitude, are based on the LF (Binggeli et al 1988). In a recent paper (Leroy & Portilla 1996) we demonstrated a relation between the opacity of the disk and the excess number counts of faint-blue galaxies. The conclusion was derived from a corrected LF which was obtained under three hypothesis: i) the opaque matter was formed at some recent redshift z_d , ii) the opacity of the disk is infinite, iii) there is a universal LF represented by the Schechter function. In the present paper, we develop a method for correcting the observed LF which is valid for more realistic distributions of opaque matter, with finite mean opacity. This is done in Sect. 3 in two steps. We obtain the face on LF firstly (i.e., the observed LF corrected for effects of inclination), and then, we correct for face on extinction obtaining the true LF. The choice of the observed LF is not a trivial issue. According to Binggeli et al (1988) the existence of a universal LF is questionable. The LF of spiral galaxies has a maximum, all the Virgo spirals can be modeled by a gaussian. Therefore, we should consider each spiral type indpendently, because probably each one of them will have a different mean opacity. Unfortunately we do not have enough information about this subject. The LF of the irregular galaxies has a maximum too, shifted to the faint end. These galaxies, like the spirals, have a disk, and if they had opaque matter, they would contribute to the modification of the true LF. In this paper we shall illustrate the method, developed in Sect. (3), with two examples of LF. We shall consider a gaussian and a Schechter function. The justification of the first case is obvious. As for the second, we have two reasons. One is that we want to compare the case of finite opacity considered in this paper with the case of infinite opacity treated in the previous one. The other one is that if we consider all the spiral and the irregular galaxies jointly, the summed LF becomes flatter at the faint extrem, and a Schechter type function could be considered as a good approximation.

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2. The observed and the true luminosity function

If the galactic disk is considered as a mixture of luminous and absorbent matter one has a relation between the apparent luminosity l and the inclination i of the galactic plane

$$l = l_0 g(\mu) \tag{1}$$

where $\mu = cos\ i$, and l_0 is the apparent luminosity when the galaxy is seen face on, with $i = 0$. The function $g(\mu)$ depends on how luminous and absorbent matter are distributed throughout the disk. This is considered in Sect. (3.1). Absolute luminosity is determined from the apparent luminosity and the distance to the galaxy, therefore, a similar relation stands for the absolute luminosity $L = L_0 g(\mu)$. If we knew the inclination of the galactic disc we could use this to determine the face on absolute luminosity L_0 from that obtained directly from the apparent luminosity. As this is not the case, we can proceed alternatively considering L , L_0 and μ as random variables with probability densities $\Phi(L)$, $\Phi_0(L_0)$, $f_\mu(\mu)$ respectively. The first one will be identified with the observed luminosity function, the second one is the face-on luminosity function, the probability of the random variable μ will be considered below). These probability functions are related by an integral equation (Papoulis 1965)

$$\Phi(L) = \int_0^1 \frac{f_\mu(\mu)}{g(\mu)} \Phi_0 \left(\frac{L}{g(\mu)}\right) d\mu \tag{2}$$

We shall take two functions for representing the observed LF (see the Introduction). One is a gaussian with mean of $M = -18.4$ and a dispersion of $\sigma = 1.5$ mag, and the other one a Schechter function

$$\Phi(L) = \phi^* (\frac{L}{L^*})^\alpha exp(-\frac{L}{L_*})\frac{dL}{L_*} \tag{3}$$

with parameters : $\alpha = -1$, $\phi^* = 0.02 h^3 Mpc^{-}3$, $M^* = -19.5$. Or, in terms of absolute magnitudes

$$\begin{aligned} \Phi(M)dM &= C \exp(-10^{0.4(M^* - M)})dM \\ C &= 0.14 ln(10)\phi^* 10^{0.4(\alpha+1)(M^* - M)} \end{aligned} \tag{4}$$

The function $\Phi(L)$ is knwon in a range of values of L. A cutoff (or an inflection point) for low luminosities is necessary to make the integral of $\Phi(L)$ finite. Once the integral Eq. (2) has been solved, and the face-on LF Φ_0 has been determined, one must correct for face on extinction, $M_0 = M_A + 2.5 log E_x$, to determine the true absolute magnitude M_A. So, the probability density function of the true absolute magnitude will be

$$\Phi_A(M_A) = \Phi_0(M_A + 2.5 log E_x) \tag{5}$$

3. The corrected luminosity function

In this section we shall solve the integral equation (2) for different values of the mean opacity : $\tau = 0.4$, 0.6 , 1.2. Let us write (2) in the form $\Phi = A_\tau \Phi_0$, where $A_\tau : L^1 \rightarrow L^1$ is the a linear operator defined on the space of the integrable functions $\phi \in L^1$

$$(A_\tau \phi)(L) = \int_0^1 \frac{f_\mu(\mu)}{g(\mu)} \phi \left(\frac{L}{g(\mu)}\right) d\mu \tag{6}$$

We can write the solution to the Eq. (2) in the form $\Phi_0 = A_\tau^{-1}\Phi$. To find the inverse of the operator A_τ we have used different approximations for the cases $\tau < 1$, and $\tau > 1$. Using Eq. (5) we get the true luminosity function, but first we need to choose a probability density for the r.v. μ , and also determine the extinction E_x. Both depend on the amount and distribution of opaque matter in the galactic disk.

3.1. Distribution of matter in the galactic disk

Let us obtain the function $g(\mu)$ and the extinction E_x. We shall consider two models for the distribution of matter. In the first one, luminous and absorbent matter are uniformly distributed on a thin slab. In this case one gets

$$g(\mu) = \mu \frac{1 - \exp(-\frac{\tau}{\mu})}{1 - \exp(-\tau)} \tag{7}$$

$$E_x = \frac{\tau}{1 - \exp(-\tau)} \tag{8}$$

In the second model, the optical depth and the emission coefficient are exponentially distributed on a thin disk: $\tau = \tau_c \exp(-r/\alpha_d)$, $j = j_o \exp(-r/\alpha_*)$. In this case a simple calculation gives

$$g(\mu) = \mu \frac{I(\tau_c, b, \mu)}{I(\tau_c, b, 1)} \quad , \quad E_x = \frac{I(\tau_c, b, 1)}{\tau_c} \tag{9}$$

$$I(\tau_c, b, \mu) = \int_0^1 [1 - \exp(-\frac{\tau_c}{\mu} x^b)] \ln x dx \tag{10}$$

$$I(\tau_c, b, 1) = \int_0^1 [1 - \exp(-\tau_c x^b)] \ln x dx \tag{11}$$

where b is a simple function of the scale length ratio

$$b = \frac{1}{\frac{\alpha_d}{\alpha_*} - 1} \tag{12}$$

In both models $g(\mu)$ is a monotone and convex function: $dg/d\mu > 0$, $d^2 g/d\mu^2 < 0$, joining the origin to the point $(1, 1)$ of the $(\mu$, $g)$ plane. We shall approximate this function by a polygonal

$$g(\mu) = m\mu \, , \; for \;\; \mu < \mu^* \tag{13}$$

$$g(\mu) = 1 \, , \quad for \;\; \mu > \mu^* \tag{14}$$

where $\mu^* = 1/m$ and $m = g'(0)$ is the slope of the tangent to $g(\mu)$ at the origin. The values of m for each model are easily calculated. For the homogeneous slab model we obtain

$$m = \frac{1}{1 - e^{-\tau}} \tag{15}$$

and for the inhomogeneous model

$$m = \frac{\int_0^1 \ln x dx}{I(\tau_c, b, 1)} \tag{16}$$

With the polygonal approximation we are in fact infraestimating the effect of the inclination, but it makes the resolution of Eq. (2) easier. Taking into account expressions (15) and (16) one obtains a useful result: an inhomogeneous model with parameters τ_c , α_d is equivalent (has the same slope) to an homogeneous slab with mean opacity τ given by the equation

$$1 - e^{-\tau} = \frac{I(\tau_c, b, 1)}{\int_0^1 \ln x dx} \tag{17}$$

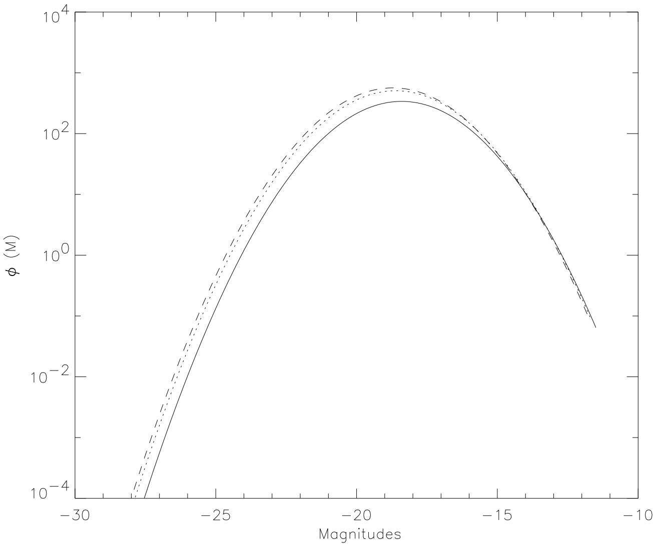

Fig. 1. One shows the corrected LF when the observed LF is a gaussian with a mean of $M = -18.4$ mag . We assume the inclination i uniformly distributed. The dotted line corresponds to a mean opacity τ = 0.4, and the dashed line to τ = 0.6.

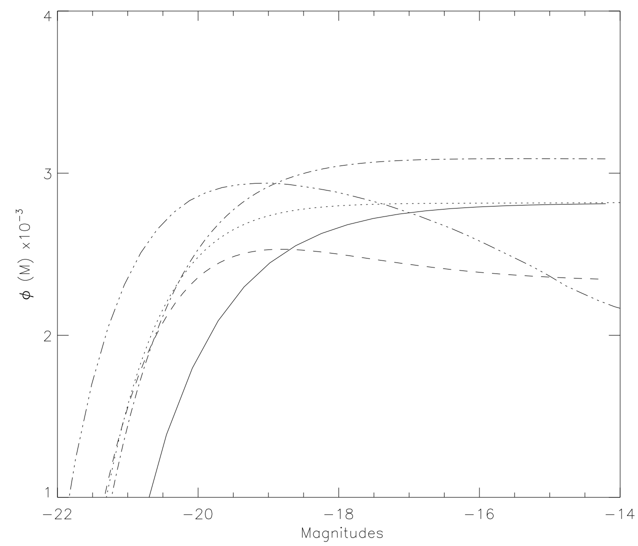

Fig. 2. One shows the corrected LF when the observed LF is a Schechter function, assuming the inclination i of the galactic plane uniformly distributed, for three different values of the mean optical depth. The dash-dot line corresponds to τ = 0.4 , the dashed line to τ = 0.6, and the dash-three dots line to τ = 1.2. The solid line represents the Schechter function, and the dotted line represents the correction for inclination effect in the case of infinite opacity.

For instance, an inhomogeneous model with $\alpha_d = 3\alpha_*$ and τ_c = 1.5 is equivalent to an homogeneous model with mean optical depth τ = 0.6. This value is suggested by the observations made by Peletier et al (1995). An homogeneous model with τ = 1.3, according with the Valentijn's paper (Valentijn 1990), corresponds to an inhomogeneous model with τ_c = 4 , α_d = $3\alpha_*$.

3.2. The probability density of the random variable μ = *cos i*

The probability density of the random variable $\mu = \cos i$ depends on the sample of galaxies used to determine the luminosity function. If the galaxies of the sample have been formed inside a region subtending a very small solid angle, the probability density of the r.v. i will be $f_i(i) = \frac{1}{2}$ sin i, and the r.v. $\mu = cosi$ would be uniformly distributed $f_\mu(\mu) = 1$. But if the galaxies have been formed in a large region of the sky, the r.v. i is more uniformly distributed than in the previous case. We have already shown (Leroy & Portilla 1996) that $f_i(i) = 1/\pi$ is a good approximation in this case. The probability density of the r.v. $\mu = cosi$ is obtained by the standard procedure. We get

$$f_\mu(\mu) = \frac{2}{\pi}\frac{1}{\sqrt{1-\mu^2}} \tag{18}$$

when i is uniformly distributed, and

$$f_\mu(\mu) = 1 \tag{19}$$

when $f_i(i) = \frac{1}{2} sin\ i$. Choosing (19) we could obtain slightly major effects for the same value of the mean opacity τ. With the second option (19) it is possible to reduce the case of infinite opacity to the resolution of an Abel's integral equation, which may be solved analytically. This point is summarized in the next subsection.

3.3. The case of infinite opacity

In the case of infinite opacity the integral Eq. (2) may be solved analytically. Taking the limit $\tau \to \infty$ in Eq. (7) one gets $g = \mu$. The Eq. (2) is then $\Phi = A\Phi_0$, with $A = \lim_{\tau\to\infty} A_\tau$ given by

$$(A\phi)(L) = \int_0^1 \frac{f(\mu)}{\mu}\,\phi\left(\frac{L}{\mu}\right) d\mu \tag{20}$$

Substituting the probability density (18) into (20), the equation $\Phi = A\Phi_0$ is reduced to an Abel's integral equation, which may be solved analytically $\Phi_0 = A^{-1}\Phi$:

$$\Phi_0(L) = (A^{-1}\Phi)(L) = -\frac{d}{dL}\int_L^\infty \frac{u\Phi(u)}{\sqrt{u^2-L^2}}\,du \tag{21}$$

In Fig. (2) we show the graph of Φ_0 in the case where $\Phi(L)$ is a Schechter function. One can prove that $\int_0^\infty \Phi_0 dL = \int_0^\infty \Phi dL$, therefore the graph of Φ_0 should cross the graph of the observed luminosity Φ at some luminosity L_c. In this paper we take Φ equal to the Schechter luminosity function, with a cutoff at some small luminosity. The crossing point of the functions Φ_0 , Φ lays outside the interval shown in the figure.

3.4. The case of low opacity

To solve the integral equation (2) for finite values of the mean opacity we shall use the polygonal approximation to the function $g(\mu)$. Substituting (13) into (2) one gets

$$\left[1 + \frac{A_{\mu^*}}{\lambda}\right]\Phi_0 = \frac{\Phi}{\lambda} \tag{22}$$

$$\lambda = \int_{\mu^*}^{1} f(\mu) d\mu \tag{23}$$

where A_{μ^*} is the operator

$$\left(A_{\mu^*}\phi\right)(L) = \int_0^{\mu^*} \frac{\mu^* f(\mu)}{\mu} \phi\left(\frac{L\mu^*}{\mu}\right) d\mu \tag{24}$$

Let us denote by ω the norm of the operator $\lambda^{-1}A_{\mu^*}$ in the space of integrable functions. A simple calculation gives

$$\omega = \frac{1-\lambda}{\lambda} \tag{25}$$

Provided that $\omega < 1$ the Eq. (22) has a solution which may be expressed by a convergent series in powers of ω (Kolmogorov& Fomin 1956)

$$\Phi_0 = \Phi_{00} - \omega\Phi_{01} + \omega^2\Phi_{02} \,..\, + (-1)^n \omega^n \Phi_{0n} .. \tag{26}$$
$$\Phi_{00} = \lambda^{-1}\Phi \tag{27}$$
$$\Phi_{01} = \lambda^{-1} A_{\mu^*}\Phi_{00} \tag{28}$$
$$\Phi_{0n} = \lambda^{-1} A_{\mu^*}\Phi_{0(n-1)} \tag{29}$$

The value of λ depends on the choice of the probability density $f(\mu)$. We have $\lambda = 1 - \frac{2}{\pi}$ arcsin μ^* in the case of (18) and $\lambda = 1 - \mu^*$ in the case of (19). The values of μ^* depend on the distribution of opaque matter in the galactic disk. Taking into account (15) and (16) one gets $\mu^* = 1 - e^{-\tau}$ and $\mu^* = I(\tau_c, b, 1)/\int_0^1 \ln x dx$ for the homogeneous and inhomogeneous model respectively.

3.5. *The case of great opacity*

We shall reduce the Eq. (2) to the problem $A\Phi_0 = \Phi$, considered in Sect. (3.3). In this case λ is close to zero (see Eq. (23)). The norm of the operator $\lambda^{-1}A_{\mu^*}$ in Eq. (22) is not smaller than unity and the equation cannot be solved as previously shown (Sect. (3.4). Let us write the Eq. (22) in the form

$$\left[\lambda + A_{\mu^*}\right]\Phi_0 = \Phi \tag{30}$$

The operator A_{μ^*} will be close to A for $\tau > 1$, so, we change A_{μ^*} by αA in Eq. (30) , choosing α such that the operators αA and A_{μ^*} have equal norm. So, we obtain the equation

$$[\lambda + (1-\lambda)A]\,\Phi_0 = \Phi \tag{31}$$

The operator A is invertible, hence, multiplying the previous equation by A^{-1} we get

$$\left[1 + \frac{1}{\omega} A^{-1}\right]\Phi_0 = \frac{1}{1-\lambda} A^{-1}\Phi \tag{32}$$
$$\omega = \frac{1-\lambda}{\lambda} \tag{33}$$

It can be seen that the norm of the operator $\omega^{-1}A^{-1}$ is $1/\omega$, therefore, provided that $\omega > 1$, Eq. (32) may be solved as power series in $1/\omega$

$$\Phi_0 = \Phi_{00} - \frac{1}{\omega}\Phi_{01} + \frac{1}{\omega^2}\Phi_{02} \,..\, + (-1)^n \frac{1}{\omega^n}\Phi_{0n} .. \tag{34}$$
$$\Phi_{00} = \frac{1}{1-\lambda} A^{-1}\Phi \tag{35}$$
$$\Phi_{01} = A^{-1}\Phi_{00} \tag{36}$$
$$\Phi_{0n} = A^{-1}\Phi_{0(n-1)} \tag{37}$$

with A^{-1} given by Eq. (21).

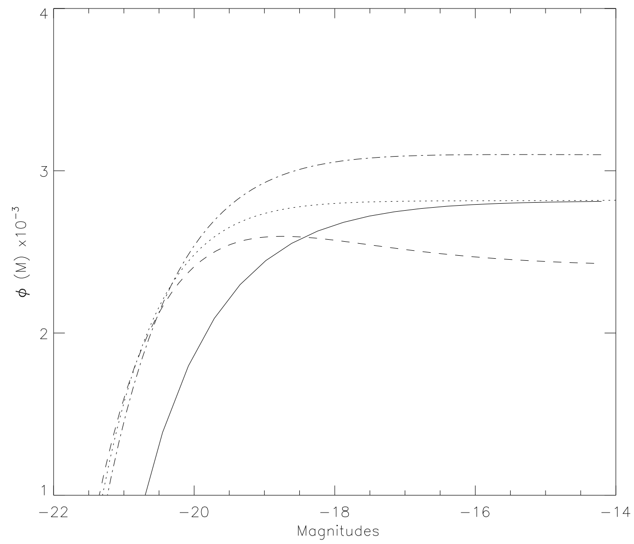

Fig. 3. One shows the corrected luminosity function when the observed LF is a Schechter function, assuming $\mu = \cos i$ uniformly distributed, for two different values of the mean optical depth. The dash-dot line corresponds to $\tau = 0.4$,and the dashed line to $\tau = 0.6$. The solid line and the dotted line represent the same as in Fig. 2.

4. Results and conclusions

We have developed a method to estimate the combined effect of extinction and inclination of the galactic disk on the luminosity function. The magnitude of the effect depends on the amount of opaque matter present in the disk. We have assumed exponential laws for the optical depth and the emission coefficient : $\tau = \tau_c \exp(-r/\alpha_d)$, $j = j_o \exp(-r/\alpha_*)$. We have seen in Sect. (3.1) that an inhomogeneous model with parameters τ_c , α_d is equivalent to an homogeneous slab with mean opacity τ given by the Eq. $1 - e^{-\tau} = I(\tau_c, b, 1)/\int_0^1 \ln x dx$.

In Fig. 1 we show the corrected LF when the observed LF has been modeled by a gaussian with a mean of $M = -18.4$ mag. We have considered two values for the mean optical depth $\tau = 0.4$ and $\tau = 0.6$. One sees a growth in the number of brilliant galaxies. This increment is compensated by a diminution in the faint end, under the inflexion points of the gaussian In Fig. 2 we show the corrected LF when the observed LF is a Schechter function. We have considered three different values of the mean optical depth, $\tau = 0.4$, $\tau = 0.6$, $\tau = 1.2$, and, the case $\tau = \infty$

has also been included in order to compare with the previous paper (Leroy & Portilla 1996).

The extinction effect is manifested as a shift of the Schechter function towards the left of the figure. For small opacities ($\tau < 0.4$) we observe the extinction shift to the left, plus a slight growth in the number of faint galaxies (the opacity is too small to change the number of bright galaxies for the effect of inclination). This changes for $\tau > 0.6$. One recognizes now the inclination effect as an increase in the number of bright galaxies and a decrease in the number of faint ones. In the same figure we show the inclination effect in the case of infinite optical depth. We have obtained similar results (Fig. 3) assuming that the variable μ is uniformly distributed. In a recent paper (Leroy & Portilla 1996) we used the results for infinite optical depth to study the influence of the opacity on the faint galaxy number counts. The results obtained in this work indicate that the combined effect of a finite optical depth plus the extinction correction is similar to the inclination effect in the case of infinite opacity. We are using the results of this paper to estimate the mean luminosity density and the mass luminosity ratio. The selection function may also be influenced by these effects.

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