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Solar activity cycle frequency shifts of low-degree p-modes

S.J. Jiménez-Reyes, C. Régulo, P.L. Pallé, and T. Roca Cortés

Instituto de Astrofísica de Canarias, E-38205 La Laguna, Tenerife, Spain

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Abstract. We report on an extensive analysis of the low-degree p-mode frequency shifts over Solar Cycle n$^{\underline{o}}$ 22 (1984–1995) based on continuous data taken at the Observatorio del Teide (Tenerife). Aside from the known good correlation between frequency shifts and solar activity indices, we have also investigated on short and long time-scales correlations (from 2 months to 1.5 years and from 1.5 to 11 years) showing different behavior. In addition, by using smoothed data for both, frequency shifts and solar activity indices, a *"hysteresis"* phenomenon is observed. This implies different behavior of both quantities in the ascending and descending parts of the cycle, while saturation effects exists at extreme phases. Finally, a degree dependence of this behavior is also noticed when analyzing separately the shifts for the even (0, 2) and odd (1, 3) mode groups. These results show that the p-mode frequency shifts are very sensitive to structural changes taking place in the Sun as the solar activity cycle proceeds; either in their upper layers, as activity migrates towards the equator, or, in its interior with a phenomenon that slowly progresses outwards, or both. A much more complete picture should emerge when data obtained from the new operational helioseismic projects over more than one solar cycle come into existence.

Key words: Sun: activity – Sun: general – Sun: interior – Sun: oscillations

1. Introduction

After the first detection of the shift in the frequencies of low-degree solar p-modes (Woodard & Noyes 1985), many authors (e.g., Régulo et al. 1994 and references therein) have, to different extents, confirmed this result. At present, the existence of a $\sim$0.45 μHz, peak-to-peak, shift positively correlated with the solar activity cycle n$^{\underline{o}}$ 22 is well established. For intermediate-degree modes ($5 \leq \ell \leq 100$), the observed frequency shifts (Libbrecht & Woodard 1990a, 1990b) are comparable to the low-degree ones, although the observations have a much poorer time coverage.

Send offprint requests to: P.L. Pallé

At present, the study of the frequency variation of the solar acoustic p-modes is a hot topic of debate in helioseismology. From the observational point of view, the main questions are: whether such a solar-cycle correlation still holds for short time-scale variations ($\sim$weeks); the possible dependence on the mode degree ℓ, and how these frequency changes correlate with other activity indices ones, which can give clues to the understanding of the build-up of activity in the Sun.

In this paper, a continuous Doppler velocity data set obtained at the Observatorio del Teide (Tenerife) from 1984 to 1995 have been analyzed, revealing new features in the low-degree ($\ell \leq 3$) solar p-modes frequency shifts.

2. Observations, analysis and data sets

The raw data were obtained from full-disk resonant scattering spectrometric measurements at the KI769.9 nm solar line. The instrument, described in detail in Brookes et al. (1978), has been sited at the Observatorio del Teide (Tenerife) since 1975 and, after some hardware updates, it has been running continuously since 1984. The data are reduced and analyzed to yield radial velocity determinations of the Sun, as is extensively explained elsewhere (e.g., van der Raay et al. 1986, Pallé 1986, Pallé et al. 1993). Briefly, the data are daily calibrated by fitting the already known solar system velocity components and corrected of the observed solar line shape, to obtain the velocity residuals. Further, these daily residuals are joined into longer time series of 36 days; this time span is a compromise between a satisfactory resolution in the power spectrum of each series (0.32 μHz) and the lifetime of the modes, which is only slightly longer than the solar rotational period (Elsworth et al. 1990). An overall total of 112 series, starting on 1984 April 18 up to 1995 June 29 has been obtained. The mean duty cycle of each series is $\sim$30%, being higher in summer when the days are longer and the weather more stable; however, 21% of the series have duty cycles lower than 12% and have not been used for further analysis.

The power spectrum of each series is calculated using an iterative sine-wave fitting procedure (Pallé 1986) at the interval 2500 to 3700 μHz, with steps of 0.1 μHz. All the spectra show the peaks of the low-degree p-modes, corresponding to $\ell \leq 3$, and their sideband structure due to the observing window. Each

Table 1. Results from an iterative sine-wave fitting applied to the different data sets studied.

Data Set	Amplitude Peak-to-Peak (μHz)	Phase at 0.077 year^{-1} (radian)
$\delta\nu$	0.42±0.04	−1.52±0.08
$\delta\nu_{1,3}$	0.42±0.01	−1.45±0.02
$\delta\nu_{0,2}$	0.34±0.03	−1.61±0.07
$\delta\nu_0$	0.35±0.05	−1.40±0.13
$\delta\nu_2$	0.39±0.08	−1.34±0.19

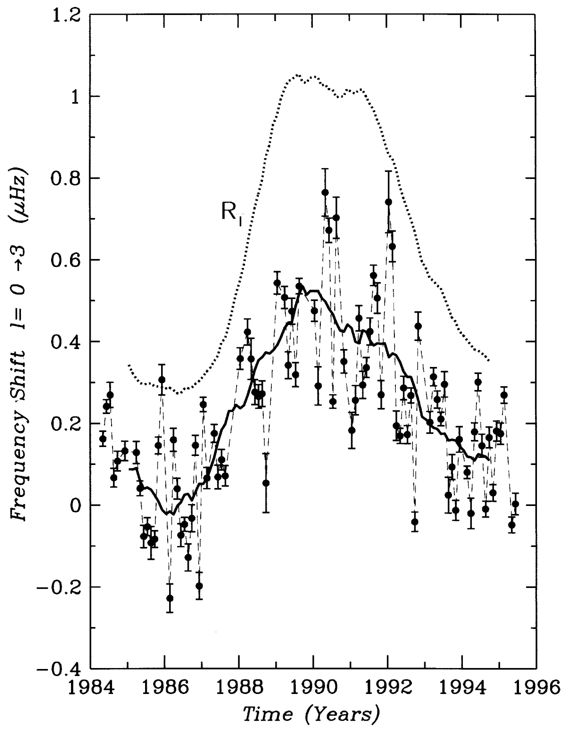

Fig. 1. Time evolution of the frequency shift of the low-ℓ p-modes (circles) and of the Solar Cycle n$^{\underline{o}}$ 22, as measured by means of the International Sunspot Number, R$_I$ (dotted line). The solid line shows the values of the frequency shifts smoothed over 1.5 years.

of the spectra are further cleaned up for background noise, as described elsewhere (Régulo et al. 1994).

With these power spectra in hand, we look for mean frequency variations of the peaks in each one. The technique employed consists in determining the position of the peak of the cross-correlation functions of each spectrum with the reference one, calculated as the mean of the best four spectra corresponding to the solar activity minimum period (Régulo et al. 1994). This technique results to be the most appropiate one to carefully study the global p-mode frequency variations with time because, firstly, it makes use of all the information contained in the spectrum and, secondly it is an *"objective"* technique, e.g., does not require and physical hipothesis or modeling on the p-mode excitacion and damping mechanisms.The alternative technique (individual peak fitting of all modes in the spectra and then compute the differences), applied to the time series of 36 days length, provides much higher errors in the final measured frequency differences, so the real time variation is severely reduced and, in some cases, could even be completely masked. Longer time series could be used in order to reduce the errors, in which case the information on eventual short time variations will be lost. An extensive comparison of the use and results obtained with both techniques when applied to similar data sets, could be found in Pallé (1995), where it is also pointed out the main advantage of the peak fitting technique: the possibility to analyze the frecuency dependence of the frequency shifts. In the present work, cross-correlation functions of the spectra were calculated at intervals ±5 μHz around zero lag; these functions look like Gaussians (Pallé et al. 1989). To calculate the position of the cross-correlation peak, the maximum of a fitted second-order polynomial to the logarithm of the cross-correlation function in an interval $\pm\sigma$ around the peak (where σ is the second-order moment) was taken. This parameter is calculated starting at the appropiate lag for which the function is symmetric, and this lag is obtained calculating the third-order moment. This procedure directly provides a value of the mean frequency shift of the $\ell \leq 3$ p-modes relative to the corresponding values at minimum solar activity. This time series, together with one of the solar cycle indicators, is plotted in Fig. 1 and will hereafter be named $\delta\nu$.

In order to separate different p-mode contributions in such shifts, we go back to the spectrum of each series and divide each one into two spectra. As predicted by the asymptotic theory, the low-ℓ p-modes are equally spaced in frequency, and modes with odd degree (1 and 3) have frequencies separated only by ∼15 μHz, the modes with even degree are separeted by only ∼9 μHz; further, these later sets are spaced from the former ones by ∼67 μHz. This structure allows the selection of either even-degree p-modes groups (ℓ=0 and ℓ=2), or the odd degree ones (ℓ=1 and ℓ=3) from each spectrum; in both separated spectra, the frequency interval occupied by the removed modes is filled up with generated background white noise of equal mean power and disppersion than the one existing at neighboring frequencies.

These are then subjected to the same cross-correlation analysis described previously and two new frequency shift time series are obtained: one with information of only even-degree modes (thereafter named $\delta\nu_{0,2}$) and the other with only odd-degree modes ($\delta\nu_{1,3}$). Moreover, in this latter case, since the power ratio of a given pair of modes (ℓ=3 to ℓ=1) is ∼0.1, the obtained information from these spectra comes basically from the ℓ=1 modes; however, in the former case (ℓ=0 and 2) this situation is different because their power ratio is nearly unity.

We can go further by trying to separate the contributions of the ℓ=0 from the ℓ=2 modes but it is not possible using the central peaks; this is because the sideband structure of the obtained observing window (from only one observatory) causes the first order sidebands in either ℓ to get mixed up with the central peaks. However, each ℓ=0 and ℓ=2 peak has a "clean" sideband (the higher and lower frequency ones, respectively) with power ∼50% of the central peak, which can be isolated from the spectra. Using this property, therefore, from each spectrum $\delta\nu_{0,2}$ two new ones were created: one by selecting the higher frequency sideband of the ℓ=0 peaks, and the other with the lower-frequency sideband of the ℓ=2 peaks. These newly formed spectra are subjected to the cross-correlation analysis and two new frequency shift time series are obtained: the one with information of only the ℓ=0 modes (hereafter named $\delta\nu_0$) and the other with only the ℓ=2 modes ($\delta\nu_2$).

It is very interesting to see the behavior of odd and even degree mode groups separately as the solar cycle proceeds. In Fig. 2, the smoothed $\delta\nu_{0,2}$ and $\delta\nu_{1,3}$, time series are plotted one against the other with different symbols for the increasing and descending activity phases; as it is clearly seen, the relation $\delta\nu_{0,2}$ and $\delta\nu_{1,3}$ is not linear, thus it could be interpreted as it these modes do not respond simultaneously as would be initially expected. Probably, the migration of activity towards low latitudes as the activity cycle proceeds could have some differential effect (Moreno Insertis 1996).

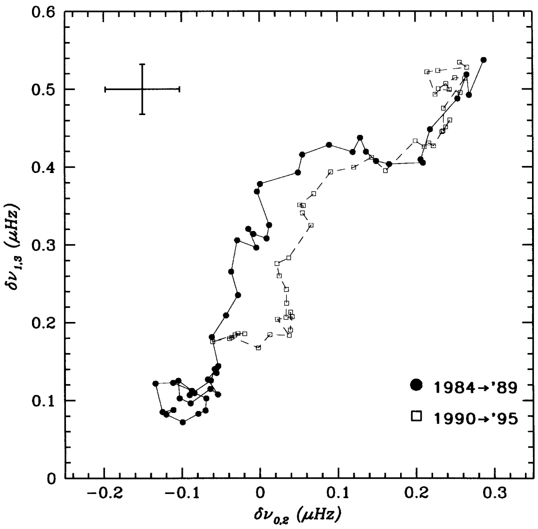

Fig. 2. Frequency shifts of odd-degree p-modes are plotted against even-degree ones, both with short-term variations (1.5 year) filtered out. Solid circles represent the ascending phase of the solar activity cycle (from 1984 to 1989), while open squares stand for the maximum and decreasing phase (from 1990 to 1995). Typical errors in the measurement of both direction are also shown.

3. Time variations of p-modes frequency shifts

In Fig. 1, the time variations of the measured frequency shifts, $\delta\nu$, can be clearly seen, showing a general trend with a period of ~11 years and an overall shift of ~0.5 μHz in phase with the solar cycle, represented by means of one of many solar activity indicators: R_I, the Sunspot International Number. However, some shorter variations can be observed too. Some of these are noise due to the different window functions of each series, which would slightly change the sideband structure of each peak; this problem has been addressed before and is believed to be less than ±100 nHz (Pallé et al. 1989) and to have a yearly period. In order to evaluate the characteristics of the general trend present in the $\delta\nu$ and also in the other data sets ($\delta\nu_{0,2}$, $\delta\nu_{1,3}$, $\delta\nu_0$, $\delta\nu_2$), a sine wave fitting procedure was computed in the low frequency range, where the signature of the solar activity index is expected to be. In Table 1, the results obtained for each set and for the same frequency bin (12.9±2.8 year) are shown.

While it is well stablished that this general eleven- to thirteen-year general trend, matches the solar activity cycle very well (e.g., Régulo et al. 1994 and references therein), no clear explanations for the shorter time variations exist yet. However, Woodard et al. (1991) claim that, for intermediate-degree ($\ell\leq$100) p-modes, the short-time variation was well correlated with surface solar magnetic activity. In trying to clarify this claim, when using our data, long and short time variations have been separated by filtering the data with a 15 point (~18 months) running mean, replacing each data point by its value (long-term) or substracting from it (short-time). Before filtering, the missing data (21%) were interpolated by using a modified linear predicition algorithm (Press et al. 1992), which makes use of the maximum entropy criterium.

The two sets of filtered data (for long- and short-term variations) have been correlated with different solar activity indices: R_I, the International Sunspot Number, F_{10}, the integrated radio flux at 10.7 cm, MPSI, the magnetic plage strength index from the Mount Wilson magnetograms (Ulrich 1991) and KPMI, the magnetic index from Kitt Peak (Harvey 1984). A mean value, together with associated error, was computed for each index over the same intervals corresponding to the individual frequency shifts measurements $\delta\nu_i$, so simultaneous values for solar indices and $\delta\nu_i$ were obtained. Furhter the same filtering procedure was applied. The results of the correlations are shown in Tables 2 and 3; the correlation coefficient r_P, the Spearman rank correlation statistics r_S and the calculated probability of having null correlation P_S are also given. For the long term variation (see Table 2), the correlation coefficients are close to unity, although they diminish slightly for those data sets with less information and higher noise $\delta\nu_0$ and $\delta\nu_2$. The corresponding values for P_S are not shown because in all cases they are smaller than 10^{-15}. Concerning the correlation coefficients for shorter term variations (see Table 3), they are not conclusive, thus reflecting the lower signal-to-noise ratio of this particular data sets. Moreover, if only data from middle of 1988 to middle of 1992 (maximum activity solar cycle) are considered, the correlation increases slightly, always being higher with the magnetic field indices (KPMI and MPSI). However, one should also bear in mind that the correlations amongst the standard solar indices used here for the same period are also low, varying from from 0.6 to 0.8.

An interesting result appears when the variation in frequency shifts are plotted against a given solar activity index, as show in Fig. 3 for $\delta\nu$ and two indices, KPMI and F_{10}. As it is well evident, the frequency shift does not have a linear dependence with the solar activity indices, the slope (sensitivity) changing at extreme phases of the activity cycle. Moreover, although for F_{10} (and also for R_I) the ascending and descending paths of the cycle are well within errors, this is not the case for the magnetic

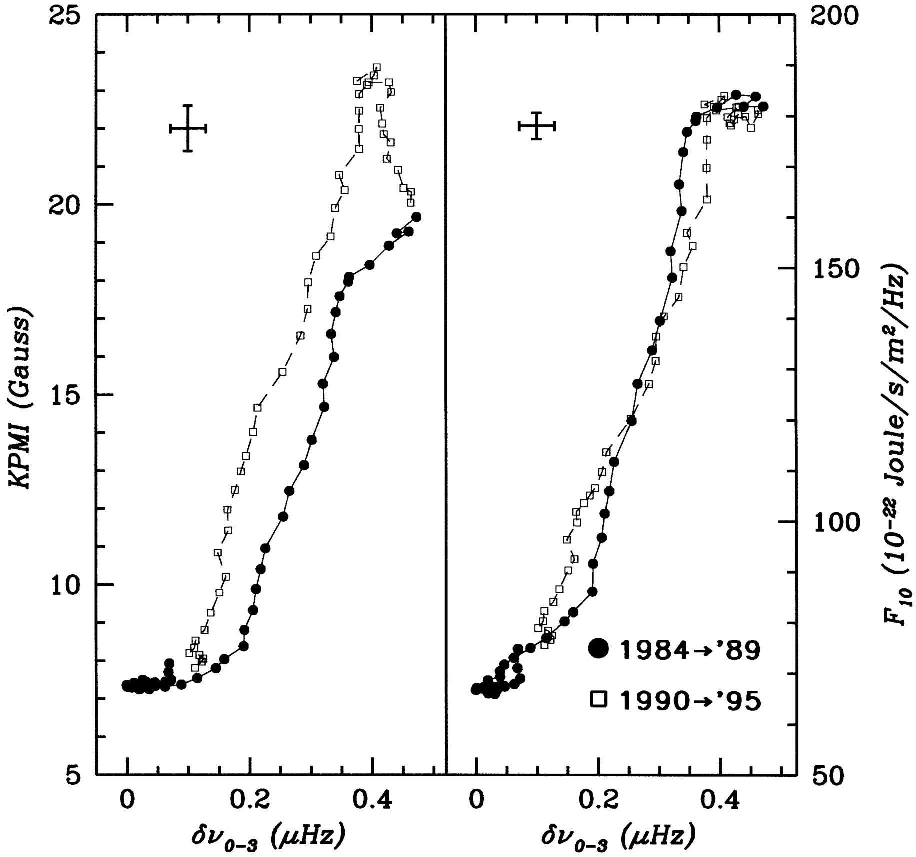

Fig. 3. The two different types of activity indices (spectral irradiance and surface magnetic field) used in this paper, are plotted against the smoothed frequency shifts $\delta\nu$.

Table 2. Results of the correlations between the various solar indices and the long-term variation frequency shifts for all data sets.

Index:		R_I	F_{10}	KPMI	MPSI
$\delta\nu$	r_P	0.98	0.98	0.94	0.97
	r_S	0.98	0.97	0.95	0.96
$\delta\nu_{1,3}$	r_P	0.97	0.96	0.93	0.94
	r_S	0.97	0.96	0.95	0.94
$\delta\nu_{0,2}$	r_P	0.97	0.97	0.95	0.98
	r_S	0.97	0.98	0.96	0.97
$\delta\nu_0$	r_P	0.93	0.93	0.86	0.90
	r_S	0.90	0.90	0.86	0.88
$\delta\nu_2$	r_P	0.86	0.87	0.86	0.87
	r_S	0.82	0.82	0.77	0.78

field indices KPMI, (and also MPSI), where the descending path always follows a track slightly higher than the ascending one, being separated by more than 2σ. The above picture is quite different depending on the ℓ-value of the modes, as it may be seen in Fig. 4 for the even and odd shifts although one on the main characteristics, the higher significance of the shifts with magnetic indices, seems to be independent of the angular degree of the modes considered. In order to evaluate this diference and also the significance of the results, we have procced to evaluate, for the data shown in each one of the diagrams in Figs. 3 and 4, the parameter $\oint\Delta\nu$: the mean frequency difference between the descending and ascending phases of the solar activity cycle. What is precisely done is, for each case, first to omit the saturation part of the diagrams (at low and high activity) and then calculate the respective integrals (are under each one of the two tracks). The two obtained values are substrated and then divided by the value of the common scanned range of activity (F_{10} or MPSI). Propagating the individual errors of data points thoughout all the process, allows us to obtain $\oint\Delta\nu$ for each diagram together with the associated error. The results are, for the global shift $\delta\nu$, $\oint\Delta\nu^{KPMI}$ = 64.7±9.2nHz and $\oint\Delta\nu^{F_{10}}$ = -6.0±9.6nHz, thus supporting the conclusions drawn above. For the odd degree shifts $\delta\nu_{1,3}$, we obtained $\oint\Delta\nu^{KPMI}$ = 107.9±9.2 nHz and $\oint\Delta\nu^{F_{10}}$ = 29.4 ±8.8nHz, and for the even degree $\delta\nu_{0,2}$, 42.9±12.7nHz and -46.1±14.7nHz respectively.

Table 3. Results of the correlations between the solar activity indices and short-term frequency shifts corresponding to the whole time series (1984→1995), and to the high solar activity period (1988.5→1992.5).

	Total:1984→1995			Max: 1988.5→1992.5		
Index	r_P	r_S	P_S	r_P	r_S	P_S
R_I	0.24	0.21	0.04	0.34	0.36	0.02
F_{10}	0.27	0.23	0.02	0.39	0.39	0.01
KPMI	0.34	0.23	0.02	0.53	0.48	0.002
MPSI	0.30	0.25	0.02	0.46	0.47	0.002

Although hysteresis shapes amongst several activity indices have been found in the past (Bachmann & White, 1994), this

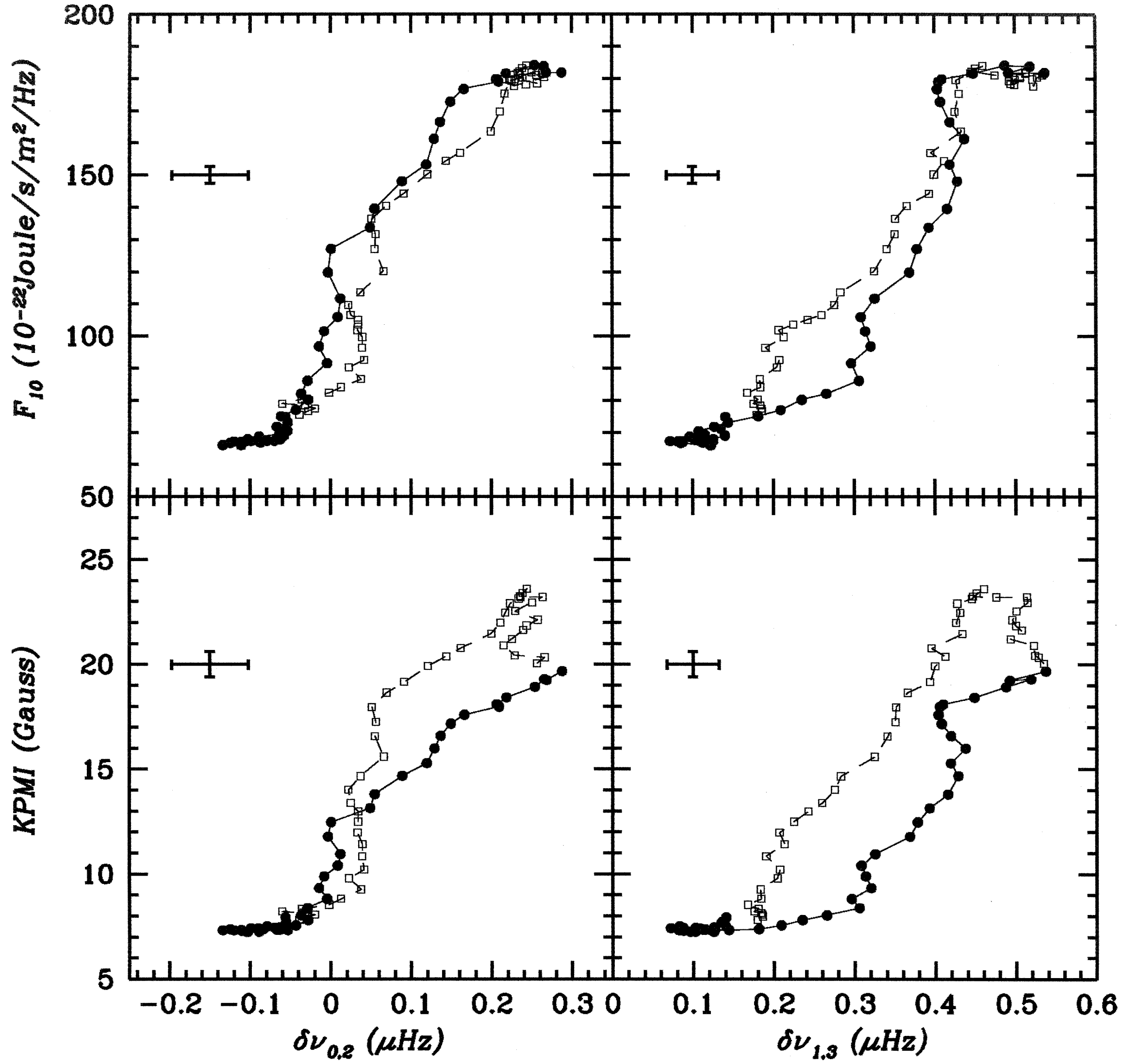

Fig. 4. Same as Fig. 3 but the even $\delta\nu_{0,2}$ and odd $\delta\nu_{1,3}$ frequency shifts.

is the first time that such relations are seen using structural parameters such as p-modes frequencies.

4. Discussion

The first conclusion is that the p-mode frequency shifts are very sensitive to structural changes due to the solar activity cycle. Further, the sensitivity depends slightly on the degree of the modes.

As already pointed out the obtained *"hysteresis"* dependence Fig. 2 could just be a manifestation of surface effect; as sunspots migrate towards the equator along the solar cycle they interact differentially with modes of even and odd degree giving rise to the effect found (Moreno Insertis 1996). On the other hand, the comparison of this plot with those obtained for the activity indices (Figs. 3 and 4) could also suggest different interpretations; the hysteresis shape could be also due to time delayed responses to one single phenomenon which can be located deeper in the Sun. In such a way, the very low ℓ p-mode frequencies react first, because they are also sensitive to changes in the deep solar interior; as this perturbation progresses outwards, the frequencies and the standard activity indices respond together until the process becomes saturated for all indices. When the activity falls, so also do the frequencies of the p-modes; however, the magnetic field indices (KPMI and MPSI) seem to do so rather more slowly than the frequency shift. This scenario could explain the claim of Woodard et al. (1991) of a short time correlation between intermediate-degree p-mode frequency shifts (sensitive to upper solar layers only) and magnetic activity, while, in our case, this correlation is unclear.

Assuming that this process could drive the solar activity cycle, it could be located at several places: a) in the deep core, in which case the frequencies of the very low ℓ modes should correlate better with the neutrino flux variations as suggested by Krauss (1990) and Delache et al. (1993), and additionally, the observed $\delta\nu$ could be related to structural core deformations due to e.g., different core rotation rate with an 11-year periodicity (Jiménez et al. 1994); b) just below the base of the convection zone associated with the "thermal blanketing," which would also cause asphericity variations across the solar disk (Kuhn et al. 1988) or the "undershooting" process that might take place; c) in the convection zone, where the buoyant magnetic tubes originate and rise to form sunspots in the photosphere. These several processes could be checked when data from low-, high- and very high-ℓ p-mode frequencies, sensitive to different layers of the Sun, become available for a complete solar cycle, thanks to operational projects currently under way: IRIS, BiSON, GONG, etc. (Pallé 1997).

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