Astron. Astrophys. 330, 10–13 (1998)

ASTRONOMY AND ASTROPHYSICS

Late formation of quark stars and their possible survival

Ashok Goyal[1,2] **and Deepak Chandra**[1]

[1] Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India (e-mail: agoyal@ducos.ernet.in)
[2] Inter-University, Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune, 411 007, India

Received 30 January 1997 / Accepted 25 August 1997

Abstract. We have studied the late formation of quark stars at a temperature much lower than the quark-hadron phase transition temperature by calculating the rate of formation of hadron bubbles for interacting quark gluon plasma and hadron gas and by incorporating the effect of curvature energy. This is essential to resolve the horizon problem as well as the survivability of these objects. We find that a small change in surface tension results in a large variation in the size and mass of these objects and a mass of the order of 10^{-5} to one solar mass requires a rather large value of the surface tension.

Key words: dense matter – elementary particles – early Universe

1. Introduction

It is now widely believed that a phase transition from quark gluon plasma to confined hadronic matter must have occured at some point in the evolution of the early Universe. This leads to an exciting possibility of the formation of quark nuggets through the cosmic seperation of phases (Witten 1984; Farhi & Jaffe 1984; Applegate & Hogan 1985). In this scenario, it is believed that as the Universe cooled through some critical temperature T_c during expansion, the deconfined quark-gluon phase existed in pressure and chemical equilibrium with a dense hot gas of hadrons. This quark-hadron phase transition proceeds through the bubble nucleation of the hadron phase which does not become large until there is sufficient supercooling. Bubble nucleation of the hadronic phase is governed by the formation rate of critical bubbles, i.e. bubbles bigger than the critical size start to expand and coalesce till the entire Universe is filled with the hadronic phase. As the hadronic bubbles expand, they drive the deconfined quark phase into small regions of space and it may happen that the process stops after the quarks reach sufficiently high density to provide enough pressure to balance the surface tension and the pressure of the hadronic phase. The quark matter trapped in these regions constitute the quark nuggets. The quark nuggets formed in the hot environment around the critical temperature $T_c \simeq 100\,\mathrm{MeV}$ are however, susceptible to evaporation from the surface (Alcock & Farhi 1985) and to boiling through subsequent hadronic bubble nucleation (Alcock & Olinto 1989; Frieman et al. 1989) on account of the fact that at these temperatures the hadronic phase is thermodynamically the more favoured phase and has lower free energy. Alcock & Farhi (1985) showed that the quark nuggets with baryon number $\leq 10^{52}$ and mass $\leq 10^{-5} M_{\odot}$ are unlikely to survive evaporation of hadrons from the surface. Boiling of nuggets through spontaneous nucleation of hadronic bubbles was shown to be even more efficient mechanism of nugget destruction by Alcock & Olinto (1989). These authors thus concluded that these nuggets, since formed at around 100 MeV, cannot have more baryons than are present in the horizon which in the standard model of cosmology is $\sim 10^{49}(T/100\,\mathrm{MeV})^{-3}$ and would not have survived till the present epoch. These results were somewhat modified by Madsen et al. (1986) by taking into account the effect of flavour equilibrium near the nugget surface for the case of evaporation and by considering the effect of interactions in the hadronic gas in a relativistic mean field model descrived by Walecka for the case of boiling (Madsen & Olesen 1991). They suggested that the quark nuggets with large baryon number allowed by the causality limit may after all be able to survive from the early Universe.

There have been recent observations by gravitational microlensing (Alcock et al. 1993; Aubourg et al. 1993) of dark objects in our galactic halo having masses of about 0.01–1 solar mass. If these objects have to be identified with quark nuggets, they could only have been formed at a time later than the time when the Universe cooled through T_c i.e. later than $\sim 10^{-4} s$ after big bang. Thus, if there is significant supercooling and late nucleation of the bubbles, the horizon can in principle have enough baryons to produce star size nuggets. The survival of these star sized nuggets called quark stars is then facilitated by the fact that they are produced in a much cooled environment. Just such a possibility of the nuggets forming at a temperature $\sim 1\,\mathrm{MeV}$, implying a high degree of supercooling and strongly first order phase transition, was recently investigated by Cottingham et al. (1994) in an effective model of QCD, the Lee-Wick model (Lee & Wick 1974; Friedberg & Lee 1977). Their investigation showed that the time of formation and the baryon content of these nuggets are essentially determined by the rate at which the hadronic bubbles nucleate.

The thermodynamic work required to form critical size hadronic bubbles is given by $W_c \simeq \frac{16\pi}{3}\frac{\sigma^3}{(P_h-P_q)^2}$, where P_q is the pressure of QGP, P_h the pressure in the hadronic bubble and σ is the surface tension. In Cottingham et al. (1994) the pressure $P_h - P_q$ is given by the difference in energy density of the two minima which is equal to the bag pressure exerted by massless non-interacting quark-antiquark pairs.

Studies of quark-hadron phase transitions in the early Universe, in heavy ion collisions and in high density nuclear matter point, to the importance of interactions in the two phases (Kapusta 1979; Shuryak 1979; Cleymans & Suhonen 1987; Kuono & Takagi 1989; Kapusta & Olive 1983; Walecka 1974; Serot & Walecka 1986; Campbell et al. 1990; Venugopalan & Prakash 1992; Kapusta et al. 1995; Glendenning 1985; Goyal et al. 1995). It has been shown that unless hadronic interactions are taken into account, the hadronic phase again becomes thermodynamically the preferred phase at high T and that QCD interactions upto order g^3 in the coupling constant are important in the QGP phase (Singh et al. 1994) and the critical parameters depend on the interactions. Recently it has been pointed out (Mardor & Svetitsky 1991) that in situations in which nucleation takes place, the curvature energy $8\pi\gamma R$, a term in the free energy in addition to the surface energy term $4\pi R^2\sigma$ (where σ and γ are the surface and curvature energy densities respectively) also plays an important role and should be kept in the calculations of the nucleation rate.

In this paper we calculate the rate of formation of the hadronic bubbles for interacting quark gluon plasma and hadronic gas. For this purpose QGP is treated as a gas of massless u, d quarks, massive s, c quarks and massless gluons. QCD interactions are treated perturbatively to order g^3, and the long range confinement effects are parameterized by the bag pressure B. For the hadron gas we use the known spectrum of low lying baryon and meson resonances and the repulsive interactions between them by incorporating the hard core excluded volume effects (Cleymans & Suhonen 1987; Kuono & Takagi 1989). We also keep the curvature term in the calculation of the nucleation rate.

2. Bubble nucleation

When the early Universe as a quark-gluon thermodynamic system cools through the critical temperature T_c, the new hadron phase becomes preferred. But energetically the new phase remains unfavourable as there is free energy associated with the surface of separation between the phases. Creation of small volumes of the new phase are thus unfavourable and all nucleated bubbles with radii less than the critical radius collapse and die out, but those with radii greater than the critical radius expand until they coalesce with each other (so supercooling occurs before the new phase actually appears and takes over, and then reheating takes place due to release of latent heat). The critical radius is obtained by maximizing the thermodynamic work expended to create a bubble

$$W \simeq -\frac{4\pi}{3}r^3(P_h - P_q) + 4\pi r^2 S - 8\pi(\gamma_q - \gamma_h)r, \tag{1}$$

where P_h and P_q are the pressures in the hadronic and the quark phase respectively and $\gamma = (\gamma_q - \gamma_h)$ is the curvature coefficient and has been estimated by Mardor & Svetitsky (1991) in the MIT bag model. The critical radius r_c is obtained by putting $\frac{\partial W}{\partial r} = 0$ and we get

$$r_c = \frac{S}{\Delta P}(1 \pm \sqrt{1-\beta}). \tag{2}$$

where $\Delta P = P_h - P_q$ and $\beta = \frac{2\Delta P}{S^2} \mid \gamma \mid$. Since β is positive definite, a real solution exists only if $\beta \leq 1$, the critical radius is actually a maximum and is given by r_+.

$$W_c = W(r_+) = \frac{4\pi S^3}{3\Delta P^2}\{2 + 2(1-\beta)^{3/2} - 3\beta\}. \tag{3}$$

β decides the effect of the curvature term and decreases the critical radius, thereby enhancing nucleation of the hadron phase.

The bubble nucleation rate (number of bubbles formed per unit time per unit area) at temperature T is given by

$$\Gamma = \left(\frac{W_c}{2\pi T}\right)^{3/2} T^4 e^{-W_c/T}. \tag{4}$$

The light particles (photons, neutrinos and electrons) contribute equally to the pressure in both phases and get subtracted out. As discussed above, the quark matter is described by a plasma of massless u, d quarks, massive s and c (m_s = 150 MeV and $m_c = 1.5 GeV$) quarks and massless gluons interacting peturbatively to order g^3 in the QCD coupling constant. Long range non-perturbative effects are parameterized by the bag constant B. The pressure in the QGP phase can be calculated by using the thermodynamic potential given in Kapusta (1979) and Shuryak (1979).

For the hadronic phase we use the known masses of the low lying 33 baryons and 48 mesons whose masses and degeneracy factors are taken from the Particle Data Group Summary (1994). The hadronic pressure and number densities for non-interacting point baryons and mesons are given by the usual thermodynamic relations. One of the simplest ways to account for the short range repulsive force is by considering the finite volume of the baryons that modify the space available for occupation in a manner akin to that of a Van der Waal's equation. We take baryons and antibaryons to have the same size as protons. The proton volume may be related to the bag volume ($V_P = \frac{m_P}{4B}$) but may not necessarily be equal to the hard core volume. A radius r_P lying between 0.6 fm and 0.8 fm is often used (Cleymans & Suhonen 1987; Kuono & Takagi 1989). The hadronic pressure corrected for finite volume effects is now given by

$$P_h = \frac{\sum_b P_b^{pt}}{1 + \sum_b n_b^{pt} V_p} + \frac{\sum_b P_{\bar{b}}^{pt}}{1 + \sum_{\bar{b}} n_{\bar{b}}^{pt} V_p} + \sum_m P_m^{pt} \tag{5}$$

where b, $\bar{b}$ and m stand for baryons, anti-baryons and mesons, respectively. The pressure equilibrium between the two phases occur at a temperature T_c which is decided by the parameters in the theory. Strictly speaking, we get T_c as a function of μ_B, but in the early Universe $\mu_B \ll T$, so essentially the critical temperature of the phase transition is obtained by setting $P_h = P_q$.

The number density of nucleation sites at temperature T, after taking into account the expansion of the Universe, can now be calculated and is given by

$$N(t) = \int_{t_c}^{t} \Gamma(t') \left\{ \frac{R(t')}{R(t)} \right\}^3 dt', \quad (6)$$

where $\frac{T}{T_c} = \sqrt{\frac{t_c}{t}}$.

As pointed out by Cotingham et al. (1994), this is an overestimate as bubbles will not nucleate in volume already under the new phase. The fraction of the Universe which is unaffected by nucleation during this process should therefore be less than one. however, since $\Gamma(t)$ rises rapidly with t, one can assume that Eq. (6) is a reasonable result. This essentially means that almost all the bubbles are nucleated at the same time.

From Eqs. (3), (4), and (6), the number density of nucleation sites at time t can be written as

$$N(t) = \left(\frac{D}{2\pi T_c}\right)^{3/2} T_c^4 (\frac{t_c}{t})^{3/2} t_c \times \int_1^{\frac{t}{t_c}} \frac{x^{1/4}}{\Delta P^3} exp(-\frac{D\sqrt{x}}{T_c \Delta P^2}) dx \quad (7)$$

where

$$x = \frac{t'}{t_c} \ \ and \ \ D = \frac{4\pi S^3}{3} \{2 + 2(1-\beta)^{3/2} - 3\beta\} \quad (8)$$

Writing this in terms of temperature by using the relation in the radiation dominated era i.e $t = \frac{3}{4}\sqrt{(5/\pi^3 g^*)} m_p T^{-2}$ (where m_p is the Planck mass and g^* is the effective number of degrees of freedom of particles whose mass $m \ll T$), we have

$$N(T) = (\frac{D}{2\pi T_c})^{3/2} (\frac{3}{4}\sqrt{\frac{5}{\pi^3 g^*}} m_p) T^3 T_c^{-1} \times \int_1^{\frac{T_c^2}{T^2}} \frac{x^{1/4}}{\Delta P^3} exp(-\frac{D\sqrt{x}}{T_c \Delta P^2}) dx. \quad (9)$$

$N(T)$ depends mainly on D which is a function of surface tension S and curvature coefficient γ. We have numerically estimated the value of $N(T)$ and find that there is a large contribution essentially at a particular temperature (time) below $T_c(t_c)$.

The seperation distance between the different nucleated sites at a given temperature T is given by $l = \frac{1}{N(t)^{1/3}}$, and the quark number density is given by

$$n_q = \frac{2}{\pi^2} \zeta(3) (\frac{n_q}{n_\gamma}) T^3, \quad (10)$$

where $(\frac{n_q}{n_\gamma})$ is the quark to photon ratio estimated from the abundance of luminous matter in the Universe.

The nucleated bubbles having $r > r_c$ expand and trap the quarks in the plasma phase. Thus the number of quarks trapped in a nugget at time t (temperature T) will be equal to the number of quarks in a volume $\frac{1}{N}$ at time t, and is given by

$$N_q = \frac{n_q}{N(t)}. \quad (11)$$

The temperature at which the nuggets are formed when bubbles coalesce is obtained by finding the time at which the expanding bubble surfaces meet. Assuming (Cottingham et al. 1994) a cubic lattice, this is given by setting

$$r(t) \simeq v \int_{t_n}^{t} \frac{R(t)}{R(t')} dt' = \frac{\sqrt{3}}{2} [N(t)]^{-1/3}, \quad (12)$$

where v is the velocity of the bubble wall and t_n is the time when nucleation mainly occurs, which is a little after t_c. $r(t)$ can be approximated by assuming $t \gg t_n$, i.e., the time when bubble walls meet is much larger than their nucleation time, and we get, using $\frac{R(t)}{R(t')} = (\frac{t}{t'})^{1/2}$, $r(t) \simeq 2tv$. We have numerically solved Eq. (12) by using the expression (7) for $N(t)$ to ascertain the time at which the nuggets are actually formed. Converting into temperature and assuming bubbles expanding relativistically, the temperature at which the nuggets are formed is obtained by solving

$$\sqrt{\frac{15}{\pi g^*}} \frac{m_p}{T^2} = (\frac{1}{N(T)})^{1/3}. \quad (13)$$

For a given value of $\frac{n_q}{n_\gamma}$ and g^*, we can estimate the temperature at which the nuggets are formed and the number of quarks N_q trapped in the nugget for different values of the bag pressure B and the surface tension S.

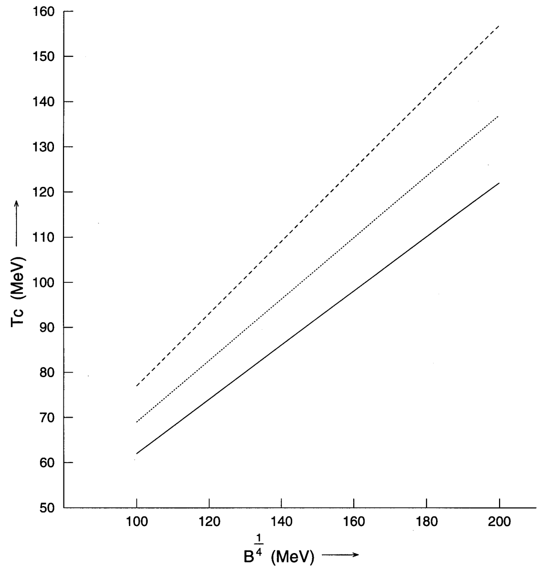

Fig. 1. $B^{1/4}$ versus the quark-hadron transition temperature T_c for $\mu_B = 0$. Effect of QCD interactions upto order g^0, g^2 and g^3 are shown by the dotted, dashed and solid curves ($g^2/4\pi = 0.4$).

3. Results and discussion

The pressure equilibrium between the two phases ($P_q = P_h$) gives us the critical temperature T_c of the phase transition. In Fig. 1

we give a plot of $B^{1/4}$ versus T_c obtained by including upto g^3 terms in the computation of the quark pressure and including the volume corrected contribution of low lying baryons and mesons in the computation of hadron pressure. In the same graph we have also shown the curve when QCD interactions are not taken into account (dotted curve) and when terms upto g^2 only (dashed curve) are included in P_q. We see from the graph that the effect of g^3 terms is to lower the critical temperature by roughly as much amount as the increase resulting from including g^2 terms in comparison to the case when perturbative QCD effects are not taken into account. Hadronic interactions can also be accounted for through an average mean field repulsive potential arising from the Reid potential and $\pi - \pi$ effective interactions and parameterised by Kapusta & Olive (1983) and by Campbell et al. (1990). Alternatively, repulsive effects can also be included by modifying the chemical potential as done by Kapusta et al. (1995): a simple method of implementing the mean field theory. Finally, one could use the frame work of relativistic mean field theory itself (Walecka 1979). It has been shown by Goyal et al. (1995) that in the early Universe when the baryonic chemical potential is negligible, all the interaction schemes mentioned above give roughly the same value of the hadronic pressure. there are, however, marked deviations at high T and/or high μ_B regime.

For different values of the bag pressure B and the surface tension S, we have looked at the possibility of forming quark nuggets by assuming n_q/n_γ to be equal to 3×10^{-10} and $g^* = 6$. Late formation of these objects is essential to resolve the horizon problem as well as their survivability. For $B^{1/4} = 100\,\mathrm{MeV}$ and g^3 terms present ($\alpha_c = 0.4$), we find that the number of quarks can be as large as 10^{86} for $S = (118.5 Mev)^3$ giving a formation temperature $T_f \simeq 1.1\,\mathrm{MeV}$ and $T_c \simeq 61.93\,\mathrm{MeV}$. However a small change in the value of S results in a big change in the number of quarks in the nugget because N_q depends exponentially on $\frac{S^3}{\Delta P T_c}$. This large sensitivity to the parameters allows for a large variation in the size and mass of the nuggets $\{ M = (12\pi^2 B)^{1/4} N_q$, and $R = (\frac{9\sqrt{3}}{128\pi})^{1/6} B^{-1/4} N_q^{-1/3} \}$ that can be formed. For example a decrease in S to $(110\,\mathrm{MeV})^3$ results in N_q falling to $\sim (10)^{31}$. QCD interactions are important, in the absence of interactions, T_c rises to 68.86 MeV and N_q reduces to $(10)^{74}$, and the effect of including interactions to the lowest order results in T_c rising to 76.92 MeV and N_q falling to $(10)^{63}$ for $\alpha_c = 0.4$ and $S = (118.5\,\mathrm{MeV})^3$. The results thus show the necessity of taking g^3 terms (arising from ring diagrams) particularly at low baryon densities where they not only negate the g^2 contributions but contribute in the opposite direction roughly by an equal amount. The effect of the curvature term can be estimated by following Mardor & Svetitsky (1991). The curvature energy in the MIT model for massless quarks is given by

$$E_c = \frac{gr}{3\pi} \int_0^\infty dkk \{1 + exp(k - \mu_B)/T\}^{-1}$$

where g is the statistical weight and r the radius. We get $\gamma_q \simeq \gamma \equiv \frac{-T^2}{12}$, where we have assumed $\gamma_h \ll \gamma_q$ and μ_B to be negligible. This has the effect of increasing the nucleation rate of hadron bubbles which further reduces the quark content of the nuggets by about two orders of magnitude (N_q drops from 10^{86} to 10^{84}). Late formation of quark stars at a temperature much below the transition temperaturew is essential to resolve the horizon problem as well as their survivability. Quark stars formed with $N_q < (10)^{50}$ are prone to boiling and evaporation from the surface (Alcock & Farhi 1985; Alcock & Olinto 1989; Frieman et al. 1989). If the formation temperature however, is $\sim 1\,\mathrm{MeV}$ as is the case here, boiling is unimportant, and the star survives this process. Surface evaporation still requires $N_q \geq (10)^{52}$ for it to survive until the present day.

We thus find that for physically reasonable values of the parameters, the bubble nucleation of the hadronic phase takes place at a temperature below T_c after supercooling. The nuggets are, however, formed quite late, when the bubble walls meet each other and coalesce, which happens at around $T \sim 1\,\mathrm{MeV}$. This is crucial for the subsequent survival of these nuggets. We find, however, that the surface tension needed for solar mass range nuggets is larger than what is believed at present (Berger & Jaffe 1987). In case such objects are indeed formed in a much cooler environment rather than in the transition era, they could contribute significantly to the energy density of the Universe and be candidates for dark matter. However, it is not clear what the dark matter component is and none of the possible candidates have been conclusively determined so far.

Acknowledgements. One of us (AG) would like to thank J. V. Narlikar for providing hospitality at the Inter University Centre for Astronomy and Astrophysics, Pune, where this work was completed.

References

Alcock, C., and Farhi, E., 1985, Phys. Rev. D32, 1273
Alcock, C., and Olinto, A., 1989, Phys. Rev. D39, 1233
Alcock, C., et al., 1993, Nature 365, 621
Applegate, H., and Hogan, C. J., 1985, Phys. Rev. D31, 3037
Aubourg, E., et al., 1993, Nature 365, 623
Berger, M. S., and Jaffe, R. L., 1987, Phys. Rev. C35, 213
Campbell, B. A., et al., 1990, Nucl. Phys. B345, 57
Cleymans, J., and Suhonen, E., 1987, Z. Phys. C37, 51
Cottingham, W. N., et al., 1994, Phys. Rev. Lett. 73, 1328
Farhi, E., and Jaffe, R. L., 1984, Phys. Rev. D30, 2379
Friedberg, R., and Lee, T. D., 1977, Phys. Rev. D15, 1694
Frieman, J. A., et al., 1989, Phys. Rev. D40, 3241
Glendenning, N. K., 1985, ApJ, 293, 470
Goyal, A., et al., 1995, ApJ, 452, 501
Kapusta, J. I., 1979, Nucl. Phys. B148, 461
Kapusta, J. I., and Olive, K. A.,1983, Nucl. Phys. A408,478
Kapusta, J. I., et al., 1995, Phys. Rev. C51, 901
Kuono, H., and Takagi, F. Z., 1989, Z. Phys. C42, 209
Lee, T. D., and Wick, G. C., 1974, Phys. Rev. D9, 2291
Mardor, I., and Svetitsky, B., 1991, Phys. Rev. D44, 878
Madsen, J., et al., 1986, Phys. Rev. D34, 2947
Madsen, J., and Olesen, M. L., 1991, Phys. Rev. D43, 1069
Particle Data Group, 1994, Phys. Rev. D50
Serot, B. D., and Walecka, J. D., 1986, Adv. Nucl. Phys. 16
Singh, C. P., et al., 1994, Phys. Rev. D49, 4023
Shuryak, E. V., 1979, Phys. Rep. 61, 71
Venugopalan, R., and Prakash, M., 1992, Nucl. Phys. A546,718
Walecka, J.D., 1974, Ann. Phys. NY. 83, 491
Witten, E., 1984, Phys. Rev. D30, 272