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Accretion induced crust screening for the magnetic field decay of neutron stars

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Abstract. We propose that accretion can induce a neutron star magnetic field decay because the ferromagnetic accreted matter screens the magnetic field of neutron star. Our model results suggest that: (i) the surface magnetic field decay is inversely correlated with the accretion mass in the X-ray binary accretion phase as $\Delta M^{-0.4}$, (ii) the 'Bottom Field Strength ' of about 10^8 G can occur when the accreted matter has completely replaced the crust mass of neutron star, (iii) most of the original magnetic field is compressed into the beneath of the accreted matter, so the internal field strength is larger than the original field strength and (iv) the field-period relation is obtained which is consistent with the evolutionary track of the binary X-ray sources.

Key words: stars: neutron – stars: magnetic fields – stars: binaries: close – X-rays: stars

1. Introduction

The magnetic field of neutron star has long been a very complex issue and not yet to be solved since its discovery (Bhattacharya & Van den Heuvel 1991; Chanmugam 1992; Phinney & Kulkarni 1994; Bhattacharya & Srinivasan 1995; Cheng & Dai 1997). It was first believed that the original magnetic field 10^{12} G of the neutron star decaying with a time scale 5×10^6 yr (Ostriker & Gunn 1969; Gunn & Ostriker 1970) results from the Ohmic dissipation in the crust. But recent calculations of Ohmic dissipation suggest that isolated neutron star magnetic field may not decay significantly (Sang & Chanmugam 1987). If the field occupies the entire crust, then it can remain large for the Hubble time. Nevertheless, recent statistical analyses of the isolated radio pulsars still cannot resolve the debate on the field decay time scales (Narayan & Ostriker 1990; Bhattacharya et al 1992; Lorimer et al 1997). Although many physical mechanisms on the generation and evolution of the neutron star magnetic field are proposed (Blandford et al 1983; Blondin & Freese 1986; Ruderman 1991a,b,c; Ding et al. 1993; Young & Chanmugam 1995), there has not yet been a commonly accepted idea on such issue (for a review see Bhattacharya and Van den Heuvel 1991).

However, the companion of the binary radio pulsars, the isolated millisecond pulsars in the globular cluster and these pulsars in the B versus P diagram strongly support the idea that they are very old neutron stars spun up by the accretion. Their magnetic field, generally 10^9 G, had already undergone substantial decay (Bailes 1989; Backer & Kulkarni 1989; Bhattacharya and Van den Heuvel 1991). In addition, there is evidence that the magnetic field of these neutron stars is correlated with the duration of the mass accretion phase or the total amount of matter accreted (Taam & Van den Heuvel 1986; Van den Heuvel et al 1986; Lewin et al 1993). In fact, Taam and Van den Heuvel have already discovered a possible inverse correlation between the magnetic field and the estimated total mass of accreted matter for the binary X-ray sources. According to this, Shibazaki et al. (1989) have presented a phenomenological formula for the decay of magnetic field as a function of accretion mass, which seems to reproduce the observed field-period relations of the recycled pulsars quite well.

For explaining the accretion induced field decay, some suggestions and models have been proposed (Bisnovati-Kogan and Komberg 1974; Romani 1990; Zhang et al. 1994; Urpin & Geppert 1995). However more recently, Van den Heuvel and Bitzaraki (1995a, 1995b), from the statistical analysis of 24 binary radio pulsars with nearly circular orbits and low mass companions, discovered a clear correlation between spin period and orbital period, as well as the magnetic field and orbital period. These relations strongly suggest that the amount of mass accreted is inversely correlated to the decay of the magnetic field, and the 'bottom' field strength of about 10^8 G is also implied.

Based on the idea by Bisnovati-Kogan and Komberg (1974) that the accreted matter may bury the original field of neutron star, Zhang et al (1994) proposed a crust screen model to account for the inverse correlation between the magnetic field and the estimated amount of mass of the recycled neutron star. In this paper, we developed the magnetic crust screen model (Zhang et al 1994) through considering the possible detail of the mag-

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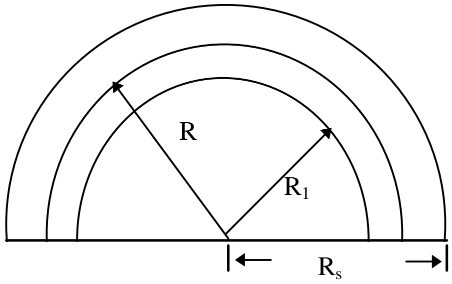

Fig. 1. The cross section of the accreted neutron star. The magnetic moment is generated in the crust region. The radius of the accreted neutron star $R_s = R + \Delta R$.

netic properties of the accreted neutron star crust, by which the "bottom" surface fied strength of about 10^8 Gauss is expected to be reached when the accreted material completely replaces the original star crust mass $\sim 0.4 M_{\odot}$.

2. Description and calculation of the model

The structure of the accreted neutron star is depicted in Fig. 1. We assume that the original magnetic field comes from a uniform magnetization **M** generated in the crust region, which has the magnetic moment $m = 4\pi(R^3 - R_1^3)/3 \sim 10^{30} Gcm^3$. The accreted overburden matter is piled up on the top of the magnetic moment domain, and the thickness of the accreted matter $\Delta R = R_s - R$ depends on the amount of matter accreted. A uniform ferromagnetic permeability μ of the accreted layer is assumed. For such simplified geometry structure of the neutron star, we solve the magnetostatics boundary value problem. Equivalently, the problems are simplified to solve the Laplace equation of scalar potential in spherical coordinate system with magnetic boundary conditions (Jackson 1975).

For the different regions depicted in Fig. 1., the magnetic scalar potential should have the following forms,

$$\Phi_1 = \sum_{n=0}^{\infty} (a_n + b_n/r^{n+1}) P_n(cos\theta), \quad R_1 \le r \le R \tag{1}$$

$$\Phi_2 = \sum_{n=0}^{\infty} (c_n + d_n/r^{n+1}) P_n(cos\theta), \quad R \le r \le R_s \tag{2}$$

$$\Phi_3 = \sum_{n=0}^{\infty} (f_n/r^{n+1}) P_n(cos\theta), \quad R_s \le r \tag{3}$$

The potentials above satisfy the following magnetostatic equations respectively.

$$\mathbf{B_1} = \mathbf{H_1} + 4\pi\mathbf{M}, \quad R_1 \le r \le R \tag{4}$$

$$\mathbf{B_2} = \mu\mathbf{H_2}, \quad R \le r \le R_s \tag{5}$$

$$\mathbf{B_3} = \mathbf{H_3}, \quad R_s \le r \tag{6}$$

$$\mathbf{H_{1,2,3}} = \nabla\Phi_{1,2,3} \tag{7}$$

where $\mathbf{B_1}$, $\mathbf{B_2}$, $\mathbf{B_3}$ are the induced magnetic field strength in different regions, $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$ are the magnetic field strength and **M** the magnetization which is assumed to be uniform along Z-axis.

The boundary conditions at $r = R$ and $r = R_s$ are H_θ and B_r to be continuous. Since the core of neutron star is type I superconductor (Pines 1981), the Meissner effect requires that the magnetic continuous condition at $r = R_1$ is $B_r = 0$ (On the other hand, the superconducting core could be type II which allows quantized flux tubes penetrate the core. However, Ding, Cheng & Chaw (1993) show that the strong coupling between quantized flux tubes and quantized vortex lines results in the rapid decay of core magnetic field). Due to boundary conditions above, the nonvanishing coefficients satisfy the following equations. For the simplicity, we neglet the lower index of the coefficients,

$$a - 2b/R_1^3 = 4\pi M \tag{8}$$

$$a - 2b/R^3 - \mu(c - 2d/R^3) = 4\pi M \tag{9}$$

$$a + b/R^3 - c - d/R^3 = 0 \tag{10}$$

$$\mu(c - 2d/R_s^3) + 2f/R_s^3 = 0 \tag{11}$$

$$c + d/R_s^3 - f/R_s^3 = 0 \tag{12}$$

The solutions for these coefficients are given in Appendix. Now, the most interested issue is the surface magnetic field strength. From Appendix, we obtain,

$$f = \frac{m}{S}, \; m = 4\pi(R^3 - R_1^3)/3 \tag{13}$$

$$S = 1 + (\mu - 1)(1 - \alpha)[(2k + 1)\mu - 2k + 2]/9k\mu, \tag{14}$$

$$k = (\frac{R}{R_1})^3, \; \alpha = (\frac{R}{R_s})^3 \tag{15}$$

The polar magnetic field on the surface of the star B_s is, or $B_s = B_3(r = R_s, \theta = 0)$

$$B_s = \frac{2f}{R_s^3} = \frac{B_o}{S}, \; B_o = \frac{2m}{R_s^3} \tag{16}$$

where B_o is the original field strength at radius R_s in the case of no accreted matter. For the reason of simplicity, the following approximations are taken into account. Here, we have assumed that R_s is roughly unchanged during accretion and $\frac{\Delta R}{R} \ll 1$ for most neutron star models. Generally, the ferromagnetic permeability μ is in the range of 10^2 to 10^6 (Jackson 1975; Cullity 1972), So $\mu \sim 10^4 \gg 1$ is approximately used. Then from Eqs. (14),(16)

$$B_s = \frac{B_o}{1 + \mu\frac{\Delta R}{R_s}} \tag{17}$$

The surface magnetic field strength is inversely correlated to the depth ΔR and permeability μ of the ferromagnetic medium. The depth of the accreted matter can be determined by means of the state equation of the accreted matter, which has little difference with the normal neutron star crust matter (Haensel 1995). Approximately, the state equation of the crust matter can be mainly described by the nonrelativistic neutrons, which can be ascribed to a simple equation of state in the polytropic form (Shapiro and Teukolsky 1983). The equation of state for the pure neutron gas has no tremendous deviation from BBP equation of state in the case of the mass density $5 \times 10^{12} < \rho < 10^{15} gcm^{-3}$ when dealing with crust mass and crust depth of neutron star (Shapiro and Teukolsky 1983, Baym et al 1971).

$$P = K\rho^{\Gamma},\ \Gamma = \frac{5}{3},\ K = 5.38 \times 10^{9},\ c.g.s. \tag{18}$$

If we neglect the general relativity effect in calculating the crust gravitational potential, the pressure gradient equation and gravitational pressure can be expressed as the following forms,

$$\frac{dP}{dh} = \rho g,\ g = \frac{GM_*}{R^2} \tag{19}$$

$$P = \frac{\Delta M g}{4\pi R^2} \tag{20}$$

where P the pressure, ρ the mass density, G the gravitational constant, g the gravitational acceleration of neutron satr, h the depth of the accreted matter from the surface of the star, M_* the neutron star mass and ΔM the accreted mass. From the equations above we give the expression for accretion depth related to the accreted mass,

$$\Delta R = 2.76 \times 10^{5}(\frac{\Delta M}{0.1M_{\odot}})^{2/5},\ (cm) \tag{21}$$

Substituting Eq. (20) into Eq. (16), we obtain the surface field decay formula related to the accretion mass,

$$B_s = \frac{B_o}{1 + 0.2\mu(\frac{\Delta M}{0.1M_{\odot}})^{0.4}} = \frac{B_o}{1 + 0.2\mu(\frac{\dot{M}t}{0.1M_{\odot}})^{0.4}} \tag{22}$$

Where $\dot{M}$ the mass accretion rate.

3. Results and discussions

3.1. Inverse correlation between field and accretion

From Eq. (22), we find that the surface field of the neutron star experienced binary accretion phase is inversly related to the accreted mass as $B_s \propto \Delta M^{-0.4}$. It should be noted that this formula is only valid for the accreted mass larger than $5 \times 10^{-4} M_{\odot}$ which corresponds to the mass density $\rho > 5 \times 10^{12}$ at where the pure neutron gas matches BBP equation of state. For the initial field strength $B = 3 \times 10^{12}$ Gauss, $\mu = 10^4$, the logarithm of Eq. (22) gives approximately,

$$logB_s = 9.1 - 0.4log\frac{\Delta M}{0.1M_{\odot}} \tag{23}$$

Eq. (23) fits quite well for the plotted line of surface dipole magnetic field versus the estimated total amount of mass accreted of the binary X-ray sources, which induces the conclusion of inverse correlation between B_s and ΔM by Taam and Van den Heuvel (1986).

In fact, the observed relation diagram between field and accreted mass by Taam and Van den Heuvel (1986) can also be fitted by an assumed simple field decay versus accreted mass formula proposed by Shibazaki et al (1989) as,

$$B_s = \frac{B_o}{1 + \frac{\Delta M}{m_B}} \tag{24}$$

where $m_B \sim 10^{-4} M_{\odot}$ the empirical mass constant for field decay. This assumed formula can be deduced from our field decay version if the accreted matter is homogeneously distributed above the progenitor neutron star (Zhang et al 1994).

Moreover, in the initial phase of X-ray sources, especially for the high mass X-ray binary (HMXB), a little mass is accreted, such as $\Delta M < 10^{-4} M_{\odot}$, the equation of state of the accreted layer is mainly dominated by the degenerate electrons, which is similar to the situation of normal neutron star surface. The magnetic property of the material on the surface of the star may be diamagnetic or low ferromagnetic, which should has little influence on the crust magnetic field, otherwise the normal neutron star magnetic field strength should present a modest value which contradicts with the observational distribution of the field strength of pulsars. Further, most observed X-ray pulsars (HMXB) present strong field strength, which are believed to be accreted little mass in its history. All in all, the magnetic properties of the neutron star matter are still an open questions.

3.2. The bottom magnetic field strength

The existence of the bottom magnetic field of pulsars has been suggested by many authors with the discovery of the long lived modest strong magnetic neutron star, pulsar in the white dwarf companion binary system (Kulkarni 1986), which is thought unchanged compared with the assumed Ohmic decay field of the normal radio pulsar. The conception of bottom field, the possibly arrived minimum field of accreted neutron star, was first evidently predicted by Van den Heuvel and Bitzaraki (1995a,1995b) from the analysis of the millisecond pulsar magnetic field versus the estimated accreted mass, who also present a persuasible explanation of the physical reason for the existence of bottom field.

In our field decay model, the surface magnetic field is determined by the ferromagnetic screen of the buried material from accretion. If the ferromagnetic permeability decreases to nothing at some critical mass density, the total efficiency of screening will remain constant after that point. As we know, in the domain with mass density $\rho < \rho_{nuc} = 2.8 \times 10^{14} gcm^{-3}$, the matter is composed of nuclei, electrons, and free neutrons. The nuclei disappear at the density larger than ρ_{nuc} because their binding energy decreases with increasing density. The nuclei dissolve into neutron liguid, so the phase transition appear at point $\rho = \rho_{nuc}$. The mass of the solid crust is about $\sim 0.4M_{\odot}$, which also partly depends on the detail equation of state depicted in the core region. Our present problem is that when the

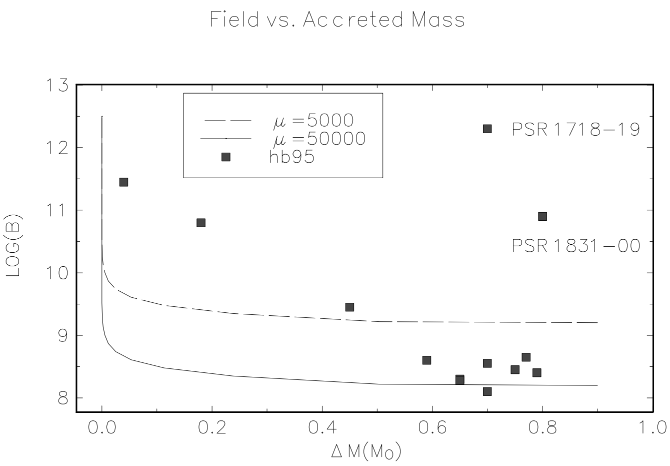

Fig. 2. The magnetic field versus accreted mass diagram with various permeability. The initial field strength $B_0 = 3 \times 10^{12}$ G. The blacken boxes are the samples from van den Heuvel and Bitzaraki 1995 (HB95).

accreted mass matches the mass of the crust, the accreted mass will replace the crust of the neutron star, and the equation of state of the accreted crust has little difference from that of the normal neutron star (Heansel et al 1995). The new added accreted matter will induce the beneath accreted crust to immerge into the neutron liquid domain, where the phase transition will take place. From the ferromagnetic theory, the ferromagnetic property is originated from the spin exchange force of the electron bound around nuclei (Cullity 1972). As an extension of the idea, the transition from nuclei into free neutron should result in the transition from ferromagnetic into diamagnetic. If this phenomenon occurs, one interesting physical effect will appear. If the accreted mass exceeds over the crust mass, the extra mass dissolved into the neutron liquid domain may lose its ferromagnetic screening effect. In such case, the accreted crust screening will saturate after accreted mass reaches the crust mass. Therefore, the field has no further decay whether how much extra mass exceeding over crust mass is accreted. The minimum field strength of accreted neuton star will be related to the crust mass, about 10^8 Gauss, which may account for the existence of the bottom field proposed by Van den Heuvel and Bitzaraki (1995).

However, ferromagnetic of the neutron star crust matter still remain to be developed. In fact, macroscopic ferromagnetic material itself is very complex theoretically and experiemently, which depends on many factors such as, structure, composition, temperature, density and history, etc. (Cullity 1972).

As a summary, considering the different X-ray accretion phases, the completed accretion induced field decay formula is described as follows

$$B_s = \begin{cases} B_o & \Delta M < 5\times10^{-4}M_\odot \\ \frac{B_o}{1+0.2\mu(\frac{\Delta M}{0.1M_\odot})^{0.4}} & 5\times10^{-4}M_\odot \le \Delta M \le 0.4M_\odot \\ 2.9\mu^{-1}B_o & \Delta M > 0.4M_\odot \end{cases} \quad (25)$$

The field accretion mass correlation is plotted in Fig. 2. There are two pulsars, PSR B1831-00 and PSR B1718-19 (van den Heuvel and Bitzaraki 1995), much beyond our theoretical predictions. As argued by Cheng and Dai 1997, PSR B1831-00 is in the globular cluster, and it may be the result of recent capture events and has possibly accreted only very little mass. For PSR B1718-19, van den Heuvel and Bitzaraki (1995) argued that the star should be formed by accretion-induced collapse of white dwarft. Thus little mass should be accreted after the neutron star formed and its field appears strong.

3.3. The internal field strength of the recycled neutron star

From Appendix, the asymptotic solution for the potential coefficients in the different domain are given if the approximation $\mu \gg 1$ is applied,

$$S = (1-\alpha)[(2k+1)\mu - 2k + 2]/9k \propto \mu^{-1} \quad (26)$$

$$a = \frac{\mu(1-\alpha)}{3(k-1)R_1^3}(\frac{m}{S}) \propto m \quad (27)$$

$$b = -\frac{\mu(1-\alpha)}{3(k-1)}(\frac{m}{S}) \propto m \quad (28)$$

$$c = \frac{2}{3R_s^3}(\frac{m}{S}) \propto \mu^{-1}m \quad (29)$$

$$d = \frac{1}{3}(\frac{m}{S}) \propto \mu^{-1}m \quad (30)$$

$$f = (\frac{m}{S}) \propto \mu^{-1}m \quad (31)$$

where m is the magnetic moment. The magnetic field strength is defined by these coefficients as depicted in Eq. (4)-Eq. (7). From these asymptotic solutions, we find that the field strength outside the surface of the accreted star is proportional to $B_3 \propto f \sim \mu^{-1}m$, which means that the effective magnetic moment outside the surface is reduced by a factor of μ. The field strength in the accreted screening domain $B_2 = \mu H_2 \sim \mu c \propto m$, which means that the average screening field strength remain the same magnitude order of the original field strength. But the field strength in the magnetic moment domain $B_1 \propto b \propto m$ is proportional to the magnetization, which will be increased because the original magnetic moment domain is compressed into the higher mass density layer. If the initial magnetization is uniformly distributed in the inner crust and proportional to the density of the matter, $\mathbf{M} \propto \rho$, the maximum magnetization after accretion should be increased a multiple factor, which is estimated as $(1 + \frac{\Delta M}{M_{outcr}})^{3/5}$. where ΔM the accretion mass, $M_{outcr} \sim 0.003M_\odot$ original out crust mass of the neutron star before the onset of accretion. Although the magnetization increases averagely, the compression of the magnetic moment volumn results in the magnetic induction strength to increase a little. All in all, the internal magnetic field strength should be larger than the original value despite the deduction of the surface field on account of the shielding of accreted matter.

3.4. The field versus period relation

For acquiring the magnetic field versus period relation, we use the formula for the variation of the rotation due to accretion

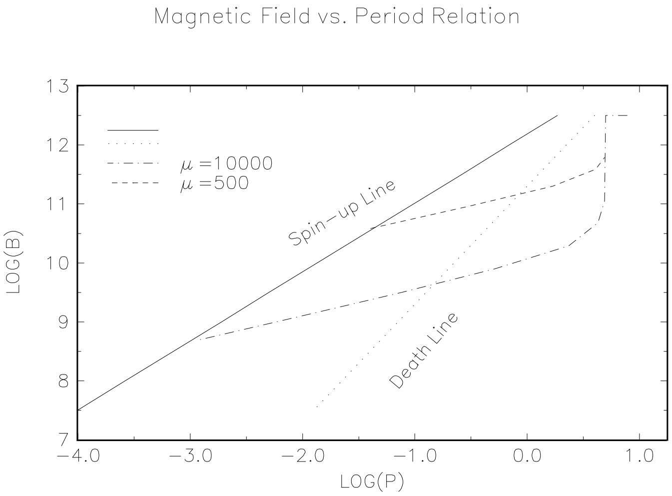

Fig. 3. Magnetic field vs. spin period diagram. The initial magnetic field $B_0 = 3 \times 10^{12}$ G and initial period $P_o = 5$ s, the accretion rate is Eddington limited. The upper solid line is the spin-up line and the lower dotted line is the death line defined as Bhattacharya and Van den Heuvel (1991).

given by Ghosh and Lamb (1979) (review see Shapiro & Teukolsky 1983)

$$-\dot{P} = 5.8 \times 10^{-5}(\frac{M}{M_\odot})^{-10/7} R_6^{-2/7} B_{12}^{2/7} (PL_{37}^{3/7})^2 \ syr^{-1} \quad (32)$$

where B_{12} is the surface field in unit of 10^{12} G, I_{45} is the moment of inertia in units of $10^{45} gcm^2$, L_{37} the X-ray brightness in units of $10^{37} ergs^{-1}$. Solving the Eqs. (22) and (32) with the initial period condition $P_0 = \infty$, we get the field-period relation approximately as,

$$P = 15.1(ms) B_9^{31/14} (\frac{\dot{M}}{\dot{M}_{Ed}})^{1/7} \quad (33)$$

where B_9 is the field strength in unit of 10^9 Gauss and $\dot{M}_{Ed}$ is Eddington limited accretion rate. For a detail presentation, the numerical solution of the field-period relation is plotted in Fig. 3, which shows the evolutionary track curve of the magnetic field and rotation. Onset of the accretion, a little mass transfered, and the neutron star is spun-up from the death valley with longer period, which causes the modest field decay and produces the systems such as PSR0655+64 and PSR1913+16. The binaries with longer-lived accretion phases, low mass X-ray binary (LMXB), will accept sufficient mass from their companions, which yields the substantial field decay and the fast millisecond pulsars such as PSR1953+10 and PSR1620+21.

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Appendix

The coefficient matrices of the linear algebra Eqs. (8)— (12) are defined as follows, which are (S), (A), (B), (C), (D), (F) respectively,

$$\begin{pmatrix} 1 & \frac{-2}{R_1^3} & 0 & 0 & 0 \\ 1 & \frac{-2}{R^3} & -\mu & \frac{-2\mu}{R^3} & 0 \\ 1 & \frac{1}{R^3} & -1 & \frac{-1}{R^3} & 0 \\ 0 & 0 & \mu & \frac{-2\mu}{R_s^3} & \frac{2}{R_s^3} \\ 0 & 0 & 1 & \frac{1}{R_s^3} & \frac{-1}{R_s^3} \end{pmatrix} \begin{pmatrix} 4\pi M & \frac{-2}{R_1^3} & 0 & 0 & 0 \\ 4\pi M & \frac{-2}{R^3} & -\mu & \frac{-2\mu}{R^3} & 0 \\ 0 & \frac{1}{R^3} & -1 & \frac{-1}{R^3} & 0 \\ 0 & 0 & \mu & \frac{-2\mu}{R_s^3} & \frac{2}{R_s^3} \\ 0 & 0 & 1 & \frac{1}{R_s^3} & \frac{-1}{R_s^3} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4\pi M & 0 & 0 & 0 \\ 1 & 4\pi M & -\mu & \frac{-2\mu}{R^3} & 0 \\ 1 & 0 & -1 & \frac{-1}{R^3} & 0 \\ 0 & 0 & \mu & \frac{-2\mu}{R_s^3} & \frac{2}{R_s^3} \\ 0 & 0 & 1 & \frac{1}{R_s^3} & \frac{-1}{R_s^3} \end{pmatrix} \begin{pmatrix} 1 & \frac{-2}{R_1^3} & 4\pi M & 0 & 0 \\ 1 & \frac{-2}{R^3} & 4\pi M & \frac{-2\mu}{R^3} & 0 \\ 1 & \frac{1}{R^3} & 0 & \frac{-1}{R^3} & 0 \\ 0 & 0 & 0 & \frac{-2\mu}{R_s^3} & \frac{2}{R_s^3} \\ 0 & 0 & 0 & \frac{1}{R_s^3} & \frac{-1}{R_s^3} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{-2}{R_1^3} & 0 & 4\pi M & 0 \\ 1 & \frac{-2}{R^3} & -\mu & 4\pi M & 0 \\ 1 & \frac{1}{R^3} & -1 & 0 & 0 \\ 0 & 0 & \mu & 0 & \frac{2}{R_s^3} \\ 0 & 0 & 1 & 0 & \frac{-1}{R_s^3} \end{pmatrix} \begin{pmatrix} 1 & \frac{-2}{R_1^3} & 0 & 0 & 4\pi M \\ 1 & \frac{-2}{R^3} & -\mu & \frac{-2\mu}{R^3} & 4\pi M \\ 1 & \frac{1}{R^3} & -1 & \frac{-1}{R^3} & 0 \\ 0 & 0 & \mu & \frac{-2\mu}{R_s^3} & 0 \\ 0 & 0 & 1 & \frac{1}{R_s^3} & 0 \end{pmatrix}$$

The solutions of the coefficients are given by the calculation of the determinants of the coefficient matrix.

$$a = \frac{det(A)}{det(S)}, \quad b = \frac{det(B)}{det(S)}, \quad c = \frac{det(C)}{det(S)}, \quad (34)$$

$$d = \frac{det(D)}{det(S)}, \quad f = \frac{det(F)}{det(S)} \quad (35)$$

$$\begin{aligned} det(S) \ = \ & \frac{2}{R^6 R_s^6 R_1^3} \\ & \times \ (R_1^3 R^3 \mu + 2R_1^3 R^3 - R_1^3 R_s^3 \mu \\ & + \ 2R_1^3 R_s^3 + R_1^3 R^3 \mu^2 - R_1^3 R_s^3 \mu^2 \\ & - \ 4R^6 \mu + 2R^6 - 5R^3 R_s^3 \mu - 2R^3 R_s^3 \\ & + \ 2R^6 \mu^2 - 2R^3 R_s^3 \mu^2) \end{aligned}$$

$$det(S) = \frac{-18\mu}{R^3 R_s^3 R_1^3} \times S \quad (36)$$

$$S = 1 + (\mu - 1)(1 - \alpha)[(2k + 1)\mu - 2k + 2]/9k\mu \quad (37)$$

$$k = (\frac{R}{R_1})^3, \quad \alpha = (\frac{R}{R_s})^3 \quad (38)$$

$$\begin{aligned} det(A) \ = \ & \frac{8\pi M}{R^6 R_s^6 R_1^3} \\ & \times \ [R_1^3 R^3 \mu + 2R_1^3 R^3 - R_1^3 R_s^3 \mu + 2R_1^3 R_s^3 \\ & + \ R_1^3 R^3 \mu^2 - R_1^3 R_s^3 \mu^2 - 2R^6 \mu \\ & + \ 2R^6 - R^3 R_s^3 \mu - 2R^3 R_s^3] \end{aligned}$$

$$det(B) = \frac{8\pi M\mu}{R^3 R_s^6}[R^3\mu - R^3 - R_s^3\mu - 2R_s^3] \quad (39)$$

$$det(C) = \frac{16\pi M(\mu-1)}{R_1^3 R^3 R_s^6}[R_1^3 - R^3] \quad (40)$$

$$det(D) = \frac{8\pi M(\mu+2)}{R_1^3 R^3 R_s^3}[R_1^3 - R^3] \quad (41)$$

$$det(F) = \frac{24\pi M\mu}{R_1^3 R^3 R_s^3}[R_1^3 - R^3] \quad (42)$$

$$a = \frac{1 + (\mu-1)(1-\alpha)[\mu - 2k + 2]/3k\mu}{(k-1)R_1^3}(\frac{m}{S}) \quad (43)$$

$$b = -\frac{\mu(1-\alpha) + 2 + \alpha}{3(k-1)}(\frac{m}{S}) \quad (44)$$

$$c = \frac{2(\mu-1)}{3\mu R_s^3}(\frac{m}{S}) \quad (45)$$

$$d = \frac{\mu+2}{3\mu}(\frac{m}{S}) \quad (46)$$

$$f = (\frac{m}{S}),\ m = 4\pi(R^3 - R_1^3)/3,\ \text{the magnetic moment} \quad (47)$$

References

Backer, D., & Kulkarni, S.R., 1989, Phys. Today, 300, 615.

Bailes, M. 1989, ApJ, 342, 917

Baym, G., Beth, H.A. & Pethick, C.J.1971, Nucl. Phys. A175, 225

Bhattacharya, D., & Van den Heuvel, E.P.J. 1991, Phys. Rep., 203, 1

Bhattacharya, D., Wijers, R.A.M.J., Hartman, J.W., & Verbunt, F. 1992, A&A, 254, 198

Bhattacharya, D., & Srinivasan, G. 1995, in X-ray Binaries, eds. W.H.G. Lewin, J. van Paradijs and E.P.J. van den Heuvel, (Cambridge University Press).

Bisnovati-Kogan, G., & Komberg, B.1974, Soviet, Astron, 18, 217

Blandford, R.D., Applegate, J.H. & Hernquist, L. 1983, MNRAS, 204, 1025

Blondin, J.M., & Freese, K.1986, Nature, 323, 786

Chanmugam, G. 1992, ARA&A, 30, 143

Cheng, K.S., & Dai, Z.G.1997, ApJ, 476, L39

Cullity, B.D., 1972, Introduction to Magnetic Materials. Addison-Wesley, Massachusetts

Ding, K.Y., Cheng, K.S., & Chau, H.F.1993, ApJ, 408, 167

Geppert, U., & Urpin, V.1994, MNRAS, 271, 490

Ghosh, P., Lamb, F.K., 1979, 234, 296

Gunn, J.E., & Ostrikeer, J.P.1970, ApJ, 160, 979

Haensel, P., Salgado, M., & Bonazzola, S. 1995, A&Ap, 296, 745

Jackson, J.D., 1975, Classical Electrodynamics. Wiley, New York

Kulkarni, S.R. 1986, ApJ, 306, l85

Lewin, W.H.G., Van Paradjis, J., & Taam, R.E.1993, Space Sci. Rev., 62, 223

Lorimer, D.R., Bailse, M., & Harrison, P.A. 1997, MNRAS, 289, 592

Lyne, A., & Smith, F.G. 1990, Pulsar Astronomy. Cambridge University Press, Cambridge

Narayan, R., & Ostriker, J.P.1990, ApJ, 352, 222

Ostriker, J.P., & Gunn, J.E.1969, ApJ, 157, 1395

Pines, D. 1980, Science, 207, 597

Phinney, E.S., & Kulkarni, S.R. 1994, Ann. Rev. Astron. Astrophys., 32, 591.

Romani, G.W. 1990, Nature, 347, 741

Ruderman, M. 1991a, ApJ, 366, 261

Ruderman, M. 1991b, ApJ, 382, 576

Ruderman, M. 1991c, ApJ, 382, 587

Sang, Y., & Chanmugam, G.1987, ApJ, 323, L61

Shibazaki, N., Murakami, T., Shaham, J., & Nomoto, K. 1989, Nature, 342, 656

Shapiro, S.L., & Teukolsky, S.A. 19833, Black Holes, White Dwarfs and Neutron Stars. Wiley, New York

Taam, R.E., & Van den Heuvel, E.P.J. 1986, ApJ, 305, 235

Urpin, V., Geppert, U. 1995, MNRAS, 275, 1117

Van den Heuvel, E.P.J., van Paradijs, J.A., & Taam, R.E.1986, Nature, 322, 153

Van den Heuvel, E.P.J., Bizaraki, O. 1995, A&A, 297, L41

Van den Heuvel, E.P.J., Bizaraki, O. 1995, in The Lives of the Neutron Stars, eds. M.A. Alpar, Ü. Kilziloğlu and J. van Paradijs, (Kluwer Academic Publishers, Dordrecht).

Young, E.J., & Chanmugam, G. 1995, ApJ, 442, L53

Zhang, C.M., Wu, X.J., & Yang, G.C. 1994, A&A, 283, 889

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