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Estimate of solar radius from f-mode frequencies

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Abstract. Frequency and rotational splittings of the solar f-modes are estimated from the GONG data. Contrary to earlier observations the frequencies of f-modes are found to be close to the theoretically computed values for a standard solar model. The f-mode being essentially a surface mode is a valuable diagnostic probe of the properties of the solar surface, and also provides an independent measure of solar radius. The estimated solar radius is found to be about 0.03% less than what is traditionally used in construction of standard solar models. If this decrease in solar radius is confirmed then the current solar models as well as inversion results will need to be revised. The rotational splittings of the f-modes yield an independent measure of the rotation rate near the solar surface, which is compared with other measurements.

Key words: Sun: oscillations – Sun: rotation

1. Introduction

The frequencies and splittings of the solar p-modes have been extensively and profitably used in helioseismic analysis to infer conditions in the solar interior. The f-mode which is essentially a surface mode has also attracted attention because of the reported difference in the frequency between the observed value and that computed for a solar model (Libbrecht, Woodard & Kaufman 1990; Bachmann et al. 1995). Since frequencies of the f-mode are essentially independent of the stratification in the solar interior, they can provide a diagnostic of flows and magnetic fields etc. present in the near surface regions (Murawaski & Roberts 1993; Rosenthal & Gough 1994; Ghosh, Chitre & Antia 1995; Rosenthal & Christensen-Dalsgaard 1995). These frequencies can also provide an accurate measure of solar radius.

The amplitudes of f-modes are very low, and consequently, the frequencies have so far been measured only at high degree where there is sufficient power. These frequencies probably suffer from systematic errors (Antia 1996), presumably because of ridge fitting techniques adopted in data reduction. It may also be noted that for the f-mode, horizontal and vertical components of velocity are comparable in magnitude and the usual assumption in spatial filtering about velocity being predominantly vertical is untenable. Systematic errors introduced because of this assumption have perhaps not been estimated. However, with the good quality data now available from the GONG network (Hill et al. 1996) and the MDI project (Kosovichev and Schou 1997), it is possible to detect the fundamental mode down to approximately $\ell = 100$, where ℓ is the degree of the mode. The advantage with the GONG and MDI data is that they provide information about individual modes and hence ridge fitting is not involved. Thus we can expect the systematic errors in the estimated frequencies to be much less.

Rest of the paper is organized as follows: In Sect. 2, we describe the results obtained for the frequencies of the f-mode from the GONG data including the estimate of solar radius, while Sect. 3 describes the results for splitting coefficients for the f-modes and the surface rotation rate as inferred from them. Finally, Sect. 4 summarizes the main conclusions from this study.

2. The f-mode frequencies

To determine the frequencies of f-modes from GONG power spectra it is best to use the rotationally corrected m-averaged power spectra; because of addition of spectra for all value of m the signal to noise ratio is improved and it is possible to identify the f-mode peaks without much difficulty for $\ell > 100$. We use the GONG month 4 power spectra because these were available for ℓ = 0–250. From these m-averaged spectra the frequencies have been found using the standard peak-finding technique in the GONG pipeline (Anderson, Duvall & Jefferies 1990).

The results for modes with radial harmonic number n = 0–3 are shown in Fig. 1. Although in this work we are mainly interested in the f-mode ($n = 0$), results for other n's are also included to show the gradual variation in results with n. These figure shows the difference between the observed frequencies and those of a standard solar model. The standard solar model has been constructed by including the diffusion of helium and heavy elements and uses the OPAL opacities (Iglesias & Rogers 1996) and OPAL equation of state (Rogers, Swenson & Iglesias 1996). The convective flux in the model is calculated using the formulation of Canuto & Mazzitelli (1991). In order to estimate systematic errors between different observations we have

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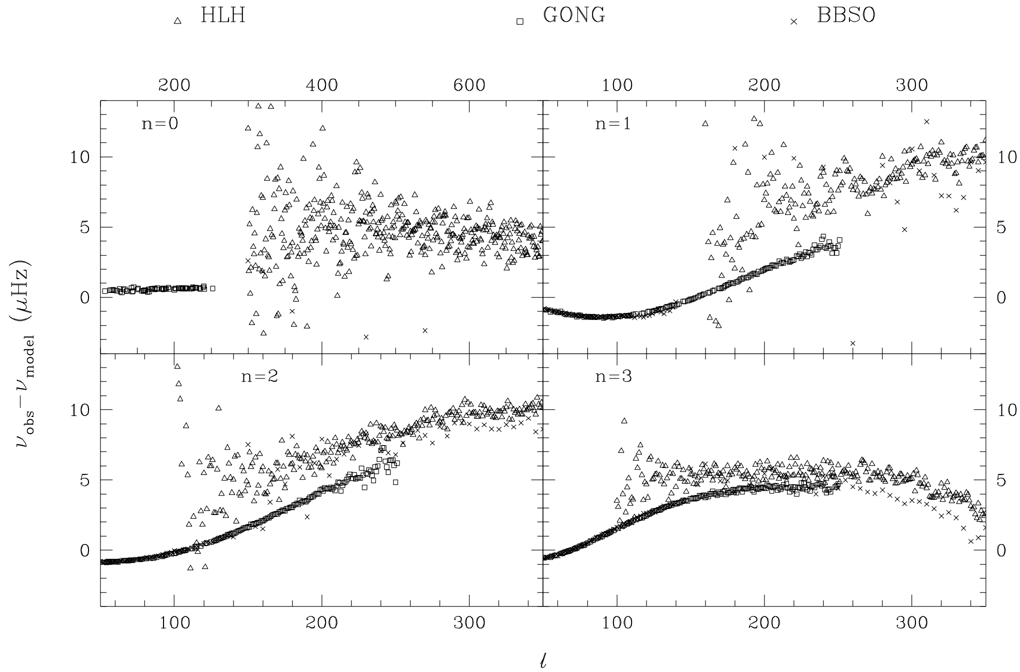

Fig. 1. Difference between various observed frequencies and those of a standard solar model for n = 0–3 as a function of degree ℓ.

included observed frequencies from the BBSO data (Libbrecht, Woodard & Kaufman 1990) as well as the HLH data (Bachmann et al. 1995). The BBSO data falls in two categories one for $\ell \leq 140$ where the frequencies have been determined by fitting individual peaks, similar to what is done for the GONG data, and the second set for higher ℓ where the frequencies have been computed by a ridge fitting technique. The HLH frequencies have all been computed using ridge fitting. It is clear from the figure that for n = 0 and 1, there are significant differences between the frequencies computed by fitting individual modes and those from ridge fitting. Further, the frequencies from GONG and BBSO ($\ell \leq 140$) data are very close to those of the standard solar model, while those from ridge fitting techniques are systematically different. The difference between different data sets reduces as one goes to higher values of n. The thickness of the ridges in these figures should give an estimate of statistical errors in observed frequencies and it is clear that the errors in GONG data are much less than those in HLH data.

The f-mode frequencies from GONG data are close to those computed with a standard solar model and various mechanisms invoked to explain the reported differences between the observed and computed frequencies may not be necessary. Of course, we still do not have reliable results at high degree and only better data from MDI or a reanalysis of HLH data would be able to resolve the question whether there is indeed any significant difference between the observed and computed frequencies. From Fig. 1, it appears that there is still a small difference of the order of 0.5 μHz between the observed and computed frequencies.

From the behavior of systematic error with n it appears that the neglect of horizontal component of velocity in spatial filtering is a possible cause for the systematic differences. From the computed eigenfunction we can obtain the ratio of the horizontal to vertical component of velocity at the photosphere. This ratio is unity for n = 0, between 0.4–0.5 for n = 1, between 0.24–0.32 for n = 2 and between 0.15–0.25 for n = 3. The influence of horizontal component of velocity on ridge fitting techniques needs to be further studied. It may be noted that for the p-modes the ratio of horizontal to vertical velocity will be a function of frequency and as such spatial filtering could introduce asymmetry in the peaks thus causing a shift in the frequency when symmetric profiles are fitted (Kosovichev et al. 1997).

In the ridge fitting techniques where the $\ell \pm 1$ leaks in the power spectra are not resolved the systematic errors are found to be of the order of frequency separation ($\nu_{l+1,n,m} - \nu_{l,n,m}$). Thus in the GONG data where the $m \pm 2$ leaks are not resolved we may expect systematic errors of the order of separation between these peaks, which is consistent with the actual difference of the order of 0.5μHz seen between the observed and computed frequencies for a solar model. Thus it is possible that this difference is again due to systematic errors in observed frequencies. However, in that case the frequency difference would be independent of ℓ, but the actual difference is more or less proportional to the frequency. It thus appears unlikely that most of the difference could be accounted for by systematic errors in measured frequencies. Hence in the following section we neglect the possibility of unknown systematic errors and investigate the consequences.

2.1. Estimate of solar radius

The frequencies of f-modes are asymptotically expected to satisfy the simple dispersion relation, $\omega^2 = gk$, where g is the acceleration due to gravity at the surface and $k = \sqrt{\ell(\ell+1)}/r$ is the horizontal wave number. Fig. 2 shows the quantity ω^2/gk for a solar model and for the corresponding GONG frequen-

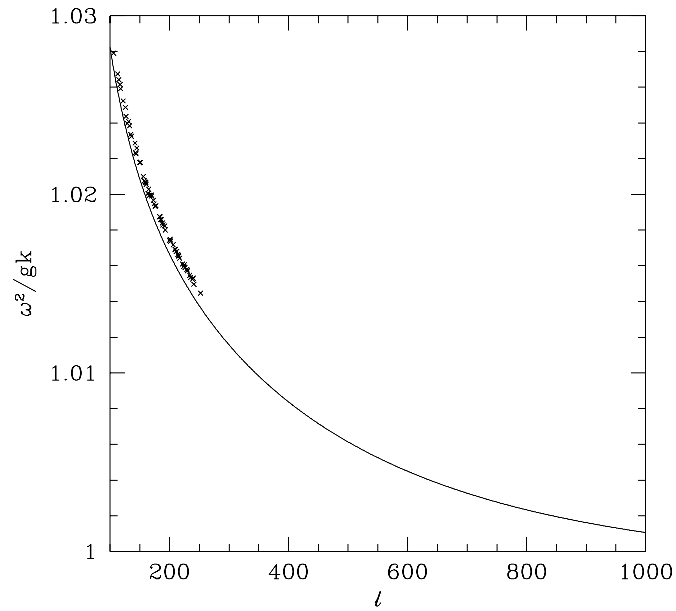

Fig. 2. ω^2/gk for the f-mode in a solar model (solid line) are compared with observed values represented by crosses.

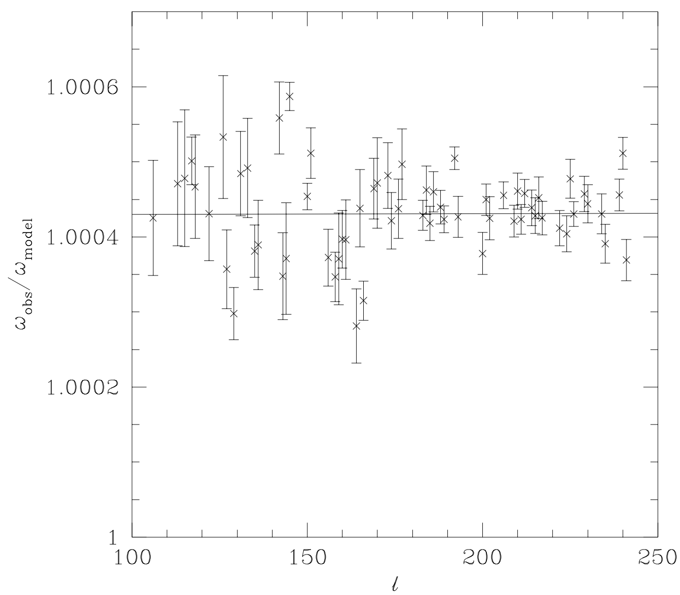

Fig. 3. The ratio of observed and model frequencies for f-modes. The horizontal line defines the average over all modes.

cies. It is clear that although both follow the same trend, there are some systematic differences between the two. The systematic trend away from unity at lower degree is due to the fact that the peak in kinetic energy density associated with f-mode shifts inwards with decreasing degree and thus these modes are effectively localized somewhat below the solar surface, where gk would be larger. This arises because although the velocity falls off exponentially with increasing depth, the density increases very rapidly just below the solar surface. As a result, the kinetic energy density increases until the density scale height becomes comparable to velocity scale height ($1/k$). Fig. 3 shows the ratio ($\omega_{\rm obs}/\omega_{\rm model}$) and it is clear that this ratio is more or less constant within the expected errors. Moreover it is significantly different from unity, with the average ratio being 1.000437 ± 0.000005. The simplest explanation for this difference in frequencies would be an error in the assumed radius of the solar model. In order to explain the observed discrepancy the solar radius will need to be decreased by about 0.029% or about 203 km, which is perhaps somewhat larger than the quoted uncertainty of 70 km in the radius (Allen 1973). The exact extent of reduction in radius required to match the f-mode frequencies will also depend on the input physics used in constructing the standard solar model and this effect needs to be investigated.

There is a significant variation in measured value of the solar radius, both with time and with different observational techniques (Laclare et al. 1996). Thus a reduction of 203 km in present solar radius cannot be ruled out. Of course, some of the difference could arise from the assumed definition of solar radius. For the present study, the solar model was constructed with a radius of 6.9599×10^5 km, and the radius was defined as the radial distance at which the temperature equals the effective temperature. This point would be about 50 km above the level where the optical depth equals unity. Of course, the exact height of this layer (above the unit optical depth level) will depend on the atmospheric model used, but the difference in height between various atmospheric models is much less than the quoted error of 70 km in the solar radius. However, the definition of radius as used by observers is quite different as they measure the radial distance to the inflection point of the limb intensity profile, which probably occurs at a much lower optical depth. Wittmann (1974) has estimated this point to be 340 km above the level where optical depth is unity. The exact location of this point will depend on the observational technique adopted as well as the model atmosphere used to estimate the height of the corresponding layer. Thus, the reduction by 203 km in solar radius suggested by the f-mode frequencies appears to be roughly consistent with the standard value of radius.

Of course, there could be other sources to explain the difference between observed and model frequencies, (Murawaski & Roberts 1993; Rosenthal & Gough 1994; Ghosh, Chitre & Antia 1995; Rosenthal & Christensen-Dalsgaard 1995) but these will again yield a different behavior of differences with ℓ. Since the observed relative difference is essentially independent of ℓ, even when ℓ varies by more than a factor of two, it appears that a dominant contribution to this difference is coming from the error in radius. Further, the kinetic energy density for the f-modes with ℓ = 100–250 peaks at a depth of about 3-7 Mm below the solar surface, where the mechanisms considered in the above references may not be very effective. With the quality of data presently available it does not appear to be possible to separate out the contributions from various possible sources to the measured frequency differences. In any case before the f-mode frequencies can be used to draw inference on any of these effects it is essential to determine the solar radius correctly.

With better data on the f-mode becoming available, it may be possible to estimate the value of solar radius more accurately as also its possible variation with solar cycle. Since the frequencies of these modes can be determined to a relative accuracy of 10^{-5}, in principle, it would be possible to determine the solar radius to much better accuracy.

It may be noted that most of the current standard solar models (e.g., Christensen-Dalsgaard et al. 1996) use the standard value of solar radius with the surface defined at a level where optical depth is between 1 and 1/3 and thus these models need to be revised. If the radius of a theoretical solar model is defined as the radial distance at which the temperature equals the effective temperature, then the optical depth at the surface will depend on the temperature optical depth relation used in constructing the models (Morel et al. 1994) and this value is between 1/3 and 2/3. Helioseismic inversions assume a similar definition of solar radius and will also need to be revised. In order to estimate the possible errors due to uncertainty in radius we have tried helioseismic inversions for the sound speed using the GONG months 4–10 data with different estimates of radius. For this purpose we use a regularized least squares technique (Antia 1996) with two different reference models M0 and M1, using identical physics and identical composition profiles but with radius 695990 and 695780 km respectively. In order to compare the two results the relative difference with respect to the reference models is converted to that with respect to model M1 and the results are shown in Fig. 4. This figure shows the relative difference taken at the same fractional radius. It is clear that the difference caused due to a change of radius by 210 km, is much more than the estimated errors in helioseismic inversions over most of the solar interior. Clearly, we need an accurate measure of the solar radius in order to infer the conditions in solar interior accurately. In actual practice the composition profile in an evolutionary solar model will also be affected by the estimated radius at the present epoch. This effect was not included in our models. However, the inverted sound speed profile will not depend on the composition profile in the reference models and results shown in Fig. 4, will not be affected by this difference in composition profile with estimated radius.

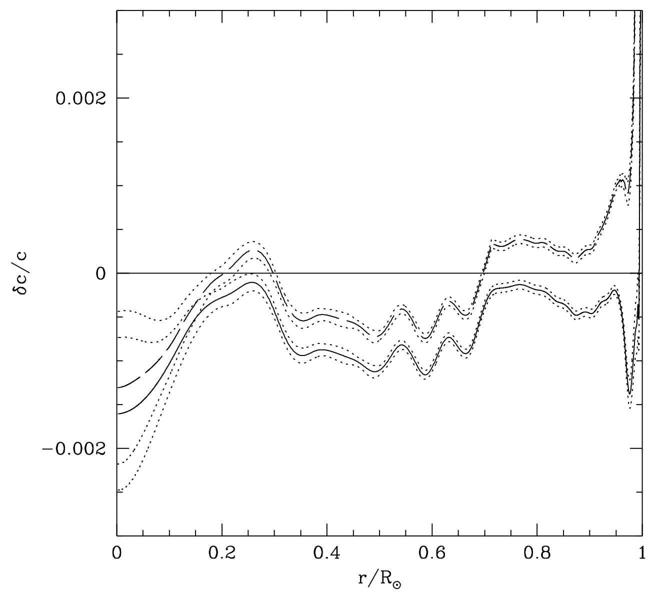

Fig. 4. The relative difference in sound speed between the Sun and model M1 as inferred by helioseismic inversions using two different estimates for radius. The continuous and dashed lines show the result using $R_\odot$ = 695780 and 695990 km respectively. The dotted lines represent the 1σ error limits on these inversions.

Table 1. Mean rotational splitting coefficients for f-modes

	Months 4–7 (nHz)	Months 4–8 (nHz)	Months 4–10 (nHz)
c_1	895.4 ±0.6	895.5±0.5	895.1±0.3
c_2	-0.041±0.010	-0.007±0.010	0.003±0.004
c_3	-18.4±0.4	-18.5±0.4	-18.8±0.2
c_4	-0.009±0.012	-0.003±0.013	0.005±0.006
c_5	-1.8±0.3	-1.0±0.4	-1.7±0.2

3. The f-mode splitting coefficients

Apart from the frequencies it is also possible to estimate the rotational splitting coefficients for the f-modes. For this purpose we have used the results from GONG spectra averaged over several months, which are available up to $\ell = 150$ only. We attempt to calculate the mean frequency and the first five splitting coefficients, using a least squares fit to polynomials of Ritzwoller and Lavely (1991),

$$\nu_{n\ell m} = \nu_{n\ell} + \sum_{i=1}^{5} c_{i,n\ell}\gamma_{i,\ell}(m), \tag{1}$$

where, $\gamma_{i,\ell}(m)$ are the polynomials defined by Ritzwoller & Lavely, $\nu_{n\ell}$ is the mean frequency and $c_{i,n\ell}$ are the splitting coefficients.

Since all the f-modes are restricted to a narrow region just below the surface, we would expect the splitting coefficients to be roughly independent of ℓ. Thus it is possible to take mean of all these values and obtain more accurate splitting coefficients. The average values for the first five coefficients are listed in Table 1, which shows the results obtained using different averaged spectra. It can be seen that the three results are reasonably close to each other.

Since the f-modes are confined to layers immediately below the solar surface, we would expect that the splitting coefficients would directly give the corresponding components of the rotation rate. Thus we make the simple assumption that the averaged splitting coefficients given in Table 1, are equal to the corresponding component of rotation rate $w_s(R_\odot)/R_\odot$ as defined by Ritzwoller & Lavely (1991). With this assumption it is straightforward to calculate the surface rotation rate as a function of latitudes. This assumption is not fully justified as the relevant rotation kernels do not exactly integrate to unity and their peak is slightly below the solar surface. However, it turns

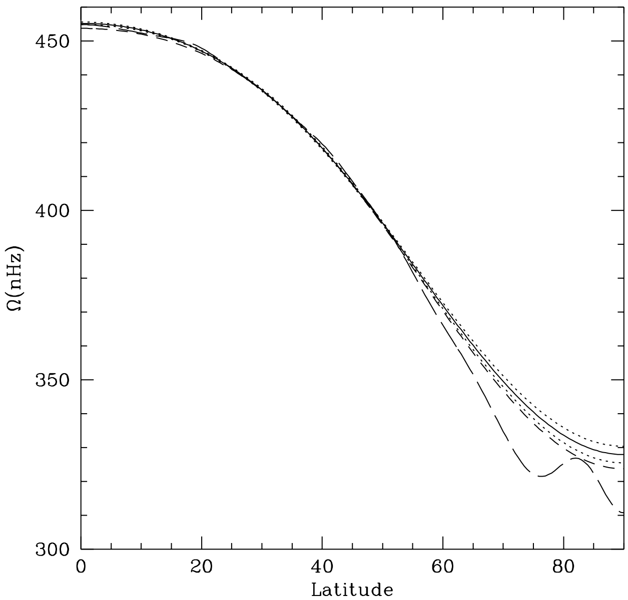

Fig. 5. The solar surface rotation rate as inferred from the f-mode splittings (solid line) is compared with other measurements. The dotted lines represent 1σ errors in the rotation rate, while the short dashed line represents the surface rotation rate as inferred by Doppler shifts (Snodgrass 1992) and the long dashed line represents the surface rotation rate as inferred by inversion of GONG months 4–10 data

out that for the case of solar rotation rate these two effects cancel each other to first order and the simple assumption gives a reasonably good estimate of the surface rotation rate. The results using the coefficients as determined from the GONG months 4–10 data are shown in Fig. 4, which also shows the solar surface rotation rate as inferred from Doppler measurements (Snodgrass 1992). This is consistent with the results obtained from MDI data (Kosovichev & Schou 1997). This figure also shows the surface rotation rate as inferred by proper inversion of all splitting coefficients from the GONG months 4-10 averaged spectra. It can be seen that the inverted rotation rate is fairly close to that inferred directly from the f-mode, and further some of the difference is due to inclusion of higher coefficients in inversion (i.e., $c_7 - c_{35}$). From this figure it is clear that there is a reasonable agreement in surface rotation rate inferred from various techniques, though the differences are possibly larger than estimated errors.

4. Conclusions

The frequencies of the solar f-modes as determined from the GONG power spectra for $100 \le \ell \le 250$ are reasonably close to those of a standard solar model and it appears that a significant fraction of the discrepancy noted in earlier observations is due to systematic errors in estimating the frequencies from the observed power spectra. Large horizontal component of velocity for the f-mode could be a possible source of systematic errors in observed frequencies, which needs to be investigated. There is still some difference between the observed and theoretical frequencies of the f-mode at the level of 0.5 μHz. If these differences are real then the simplest interpretation would be that the solar radius needs to be decreased by about 203 km as compared to the standard value. This error is sufficiently large to affect the standard solar models and the corresponding inversion results at a level which is much larger than the statistical errors in inversions. With availability of better data, f-mode frequencies can be used to provide an independent estimate of the solar radius and its variation with time.

The f-mode splittings have also been determined from the GONG power spectra for $\ell \le 150$. These splittings provide an independent measure of solar surface rotation rate, which appears to be close to that obtained from Doppler measurements.

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