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Research Note

On velocity and intensity line asymmetries

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Abstract. We show that, if solar 5 min. oscillations are excited by convection in the upper layers of the convective envelope, it is impossible to explain the opposite line asymmetries observed in the velocity and intensity spectra with assumptions on the dissip

ations which reduce the problem to a second order one. The interpretation of that observation requires to solve the full non-adiabatic problem which is of the fourth or sixth order. We also analyze the causes of line asymmetries in the frame of the gene

ral problem and we show that to locate the source, it is better to study line asymmetries not too far from line centers.

Key words: Sun: oscillations – lines: profiles

1. Introduction

Duvall et al. (1993) have discovered that lines of the solar acoustic power spectrum show asymmetries which have opposite signs in the velocity and the intensity signals. Following Abrams and Kumar (1996), a line will be said to have a positive (negative) asymmetry if it has more (less) power on the high frequency-side than on the low-frequency one. The velocity lines show a negative asymmetry while the intensity ones show a positive one though the amount of asymmetry varies with frequency. This finding has been recently confirmed by MDI which is one of the SOHO experiments.

The line asymmetry was also predicted theoretically (Gabriel 1992, 1993, 1995) and discussed by Kumar (1994), Lou and Fan (1995), Roxburgh and Vorontsov (1995), Abrams and Kumar (1996), Rast and Bogdan (1997a) and Nigam et al. (1997) but in nearly all cases for the velocity signal only.

It is considered that solar p-modes are non-linearly excited by convection in the upper layers of the convective envelope. The theory was originally proposed by Goldreich and Keeley (1987) and further developed by Goldreich and Kumar (1988, 1990) and Goldreich et al. (1994) (see also Osaki (1990) and Musielak (1994)). Recently Rast (1997b, 1997c) has proposed a slightly different mechanism in which excitations are associated with new down-flow plume formation. However,so far, these theories have been unable to predict the location and the width of the excitation zone which must be found from the interpretation of observations. Attempts have been made by Kumar (1994) and Abrams and Kumar (1996).

Until recently, no attempt had been made to use intensity line asymmetries. The reason for this situation is easy to understand. Velocity measurements are much easily connected to theory as even the simple adiabatic theory makes predictions concerning the velocity. In the studies of line asymmetries done so far (with the exception of Kumar 1994), dissipations have however been taken into account but in a very rough way which allows, with the Cowling approximation, to keep a second order boundary value problem which is much easier to handle than the full fourth order non-adiabatic one. Even if we can question the validity of the approximations done to keep a second order problem when it comes to make theoretical predictions accurate enough for the interpretation of observations, such predictions can be done and it is tempting to confront them with observations. However the perturbation of the luminosity does not appear in those simplified problems. Moreover the observations are often made in a frequency interval and it is not obvious whether the intensity fluctuations may be simply associated to the perturbations of the total luminosity obtained by theoretical computations. Even if this is allowed, we have to solve a fourth order problem the theoretical basis of which are poorly understood as it requires to solve the linear interactions between pulsation and convection. It is probably for this reason that attempts trying to explain the opposite line asymmetries in velocity and intensity spectra with second order boundary value problems, have been made only recently after the confirmation of the discovery by MDI. To do that, it is necessary to introduce one more hypothesis which links the intensity fluctuations to either the Lagrangian or the Eulerian temperature (or pressure) perturbations.

In this note we prove that the solution of second order problems leads always the same line asymmetries for the two variables. It is therefore impossible to explain both the velocity and intensity observations with such simplified problems. Their interpretation requires to solve the full non-adiabatic equations.

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2. Equations of the problem

To study the problem of pulsations excited by convection, we have to solve a non-homogeneous linear problem (Gabriel 1993):

$$\frac{d\boldsymbol{Y}}{dr} = A\boldsymbol{Y} + \boldsymbol{F} \tag{1}$$

The homogeneous system is given by the equations of non-adiabatic stellar stability and $\boldsymbol{F}$ is the forcing term produced by convection. A is a matrix, function of the distance to the center and of the oscillation frequency which indeed takes real values only (while the eigenvalues of the homogeneous problem are complex). The solutions of this system have to fulfill the same boundary conditions as the homogeneous one. For radial oscillations and in the Cowling approximation for non-radial ones, the system is of dimension four, while when the perturbation of the potential is taken into account in the non-radial case the problem is of the sixth order.

The solution of that system is given by (Gabriel 1993)

$$\boldsymbol{Y}(\boldsymbol{r}) = \int_0^R G(r, r').\boldsymbol{F}(r')dr' \tag{2}$$

where $G(r, r')$ is the Green function matrix of the homogeneous system.
Let $2N$ be the order of the system. The homogeneous problem has N independent solutions verifying the boundary conditions at the center $\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots \boldsymbol{Y}_N$ and N independent solutions verifying the boundary conditions at the surface $vecY_{N+1}$, $\boldsymbol{Y}_{N+2}, \ldots \boldsymbol{Y}_{2N}$. The corresponding fundamental matrix is $M = (\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_N, \boldsymbol{Y}_{N+1}, \ldots, \boldsymbol{Y}_{2N})$ and the Green function matrix is given by:

$$\begin{aligned} G_{ij}(r, r') &= -\sum_{k=1}^{N} M_{ik}(r) M^{-1}_{kj}(r') \qquad r < r' \\ &= \sum_{k=N+1}^{2N} M_{ik}(r) M^{-1}_{kj}(r') \qquad r > r' \end{aligned} \tag{3}$$

Eq. (2) can also be written as

$$\begin{aligned} \boldsymbol{Y}(r) &= -\sum_{k=1}^{N} \boldsymbol{Y}_k(r) \in t_r^R \frac{|M_k(r')|}{|M(r')|} dr' \\ &+ \sum_{k=N+1}^{2N} \boldsymbol{Y}_k(r) \int_0^r \frac{|M_k(r')|}{|M(r')|} dr' \\ &= \sum_{k=1}^{k=2N} C_k(r) \boldsymbol{Y}_k(r) \end{aligned} \tag{4}$$

where M_k is obtained by replacing the k^{th} column of the fundamental matrix by $\boldsymbol{F}$.

If the oscillations are excited by convection, the source term is different from zero below the observation level only and the coefficients of the solutions regular at the center are zero.

If the first component of $\boldsymbol{Y}$ is $\omega\delta r$ and the fourth one is δL, the power spectra for velocity and intensity fluctuations are given by

$$|Y_i|^2 = \sum_{j=N+1}^{2N} \sum_{k=N+1}^{2N} C_j(r) C_k^*(r) Y_{ij}(r) Y_{ik}^*(r)$$

with i equal to 1 and 4 respectively. Because of the summation, it is possible that the two spectra show different line asymmetries. If the Cowling approximation is used for non-radial oscillations, it is, for instance, possible to choose $\boldsymbol{Y}_3$ such that $\delta r(R) = 0$ and $\boldsymbol{Y}_4$ such that $\delta L(R) = 0$. Then the line asymmetries for the velocity spectrum will be given mainly by the behaviour of C_4, while those of the intensity spectrum will be related mainly to the variation of C_3 with frequency.

If, as discussed above, the system is reduced to the second order, then Eq. (4) reduces to $\boldsymbol{Y} = C_2(r)\boldsymbol{Y}_2$. There is no summation and if one spectrum is associated to each component of $\boldsymbol{Y}$, the two spectra show indeed the same asymmetries. Notice that this result is independent of the explicit form of the A_{ij} and therefore of the hypothesis made to take dissipations into account, provided that a second order problem is obtained.

Since it is necessary to solve the full non-adiabatic problem to explain the different line asymmetries of the two spectra, we can also wonder whether the source depth obtained from the velocity data only and using a second order problem can be trusted.

After discussing the problem of velocity and intensity line asymmetries with a second order problem and failing to explain the opposite asymmetries indeed, Rast and Bogdan (1997a) have suggested that the problem can be solved if the two spectra have different noice levels. This might be the case but their remark mostly challenges observers who ought to remove properly the noice in their data analysis. If after this has been done and that informations concerning the excitation source have been clearly obtained, the two spectra still show opposite line asymmetries then the problem will have to be studied with the full fourth (in the radial case and in the nonradial one with the Cowling approximation) or sixth order system of non-adiabatic stability equations.

3. The causes of line asymmetries

We will analyze the causes of line asymmetries in the Cowling approximation since it is very good for 5 min. p-modes. Then the system (1) is of the fourth order and it is interesting to write Eq. (4) as

$$\begin{aligned} \boldsymbol{y}(r) &= -\boldsymbol{Y}_3(r) \int_0^r \frac{\boldsymbol{Y}_4(r').\boldsymbol{V}(r')}{|M(r')|} dr' \\ &+ \boldsymbol{Y}_4(r) \int_0^r \frac{\boldsymbol{Y}_3(r').\boldsymbol{V}(r')}{|M(r')|} dr' \end{aligned} \tag{5}$$

with $\boldsymbol{V}$ given by:

$$\{(T_{12}F_2 - T_{13}F_3 + T_{14}F_4), (-T_{21}F_1 + T_{23}F_3 - T_{24}F_4), (T_{31}F_1 - T_{32}F_2 + T_{34}F_4), (-T_{41}F_1 + T_{42}F_2 - T_{43}F_3)\}$$

$T_{ij} = T_{ji}$ is the determinant of the two by two matrix obtained from $(\boldsymbol{Y}_1, \boldsymbol{Y}_2)$ after suppressing lines i and j. (we have kept the four components of $\boldsymbol{F}$ though $F_3 = 0$ if the third line of Eq. (1) is the transfer equation.)

We will first assume that the excitation force is well localized and can be represented by a delta function and afterwards we will generalize the discussion to an extended source.

First, we discuss the denominator $|M(r')|$. It cancels for each eigenvalues. Therefore, in the vicinity of one of them $\sigma = \sigma_R + i\sigma_I$, $|M(r')| = f(r', \sigma)(\omega - \sigma_R - i\sigma_I)$ with $f(r', \sigma) \neq 0$ and for real ω, $|M(r')|^2$ has minima close to the real part of the eigenvalues. Therefore it is nearly a periodic function (with a "period" equal to the frequency separation between two successive line centers) but not exactly, for two reasons:

1. the real parts of the eigenvalues are not exactly equidistant.
2. the extrema of $|M(r')|^2$ show a variation with frequency, especially close and above the cut-off frequency. This behaviour is already seen in simple idealized problems (Gabriel 1992).

Therefore, close to the real part of an eigenvalue, $|M(r')|^2$ is symmetric but over a wider frequency range, it produces some skewness of the line profiles. However the asymmetries introduced by this term will be the same for the velocity and for the intensity spectra.

Let us now consider the numerator.

The two solutions regular at the surface vary slowly with frequency and they have generally been considered as constant in previous discussions. This is a good approximation for narrow lines. However, if line profiles are considered over a frequency range of the order of the eigenvalue separation, these two solutions will also introduce some skewness different for each spectrum. We now discuss the consequences of the variation of $\boldsymbol{V}$ assuming that $\boldsymbol{Y}_3$ and $\boldsymbol{Y}_4$ are constant. Since $\boldsymbol{V}$ varies with the source type, the asymmetries will also be a function of the source type. Different spectra will have the same asymmetries only if $\boldsymbol{V}$ does not change its direction when ω varies, i.e. if $\boldsymbol{V}(r', \omega) = f(\omega)\boldsymbol{V_1}(r')$. This is very unlikely to occur as it requires very special properties for $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$ and $\boldsymbol{F}$ and the velocity and intensity spectra will show different line asymmetries though not necessarily of opposite sign. Because the radial and horizontal displacements of the eigenfunctions have one more node when the order is increased by one, the T_{ij} show also an oscillatory behaviour with a quasi-period in frequency a little larger than twice the frequency separation between successive eigenvalues. However as these terms have to be squared to get the spectrum, they finally lead to a function with a quasi-period in frequency a little larger than the frequency separation between successive eigenvalues. Again, because the real part of the eigenvalues are not exactly equidistant and the extremal values of the T_{ij} vary with frequency, the numerator will produce some asymmetry in the lines if a large enough frequency domain is considered. But more important, the two quasi-periodic functions appearing in the numerator and denominator have different "periods" and they will be out of phase except, may be, for one or two lines. (The number of symmetric lines increases with the source depth and if it is close enough to the surface, there may be no such line.) This will be the main source of asymmetry close to the center of the lines and the only one connected to the source position. Therefore to locate the source, it is better to study the asymmetries in a not too wide frequency domain around the line centers. Also, if a symmetric line is observed, the asymmetry presently under discussion must have opposite signs for lines on each side of the symmetric one.

In the case of an extended source, we must notice that the quasi-period of $\boldsymbol{V}$ changes with the position in the Sun; it increases as the point under consideration moves inward while that of the denominator does not change and therefore the phase lag between the numerator and the denominator changes too. Summing up the contributions of several layers will indeed smooth out the oscillations produced by the numerator and therefore reduce the amount of line asymmetries.

4. Conclusions

In this note, we have proved that the opposite line asymmetries observed in velocity and intensity power spectra cannot be explained by theories which approximate the dissipations in such a way to reduce the problem to a second order one. The solution of that problem requires the resolution of the full non-adiabatic problem. We have also analyzed the causes of line asymmetries in the frame of the general problem. The solution of that problem should be obtained including *all* the interactions between pulsation and convection (i.e. not just the perturbation of the convective luminosity) because they are all important in the upper layers of the convective envelope. There are several causes of asymmetry but only one of them can be related to the properties of the source. To pick up that one it is better to study line asymmetries not too far from line centers.

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