

# Semi-analytical radiative transfer in plane-parallel geometry: application to accretion disk coronae

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**Abstract.** A simplified frequency-integrated radiative transfer equation is solved to study Compton scatterings in the corona of the disk by using numerical iterating method. We find that the vertical thickness of the corona cannot be used as the typical length to measure the optical depth of the corona. A semi-analytical approach is proposed to calculate the energy dissipations in the corona of the disk. We demonstrate that our approach can reproduce the numerical solutions to an accuracy of  $< 2\%$ .

**Key words:** accretion, accretion disks – corona, galaxies: nuclei

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## 1. Introduction

The so-called UV bump and the X-ray power law spectra are the most important features in the spectra of Active Galactic Nuclei (AGN). It is well believed that the luminosities of the AGN are produced by accretion of matter through disks onto massive black holes. Shields (1978) suggested that the UV bump observed in AGN could be attributed to the thermal radiation from the disk. Malkan (1983) showed that the UV bump in several quasars could be fitted with predicted spectra of optically thick accretion disks. The observed spectrum in the medium X-ray range is close to a power law, with a small dispersion in the values of the spectral index, whose average for Seyfert galaxies is  $\sim 0.7$  (Mushotzky 1984; Turner & Pounds 1989). In fact, the average X-ray spectral index becomes  $\sim 0.9$  after subtracting the component of X-rays reflected by cold matter (Pounds et al. 1990).

The standard geometrically thin, optically thick accretion disk model (Shakura & Sunyaev 1973) can be employed to explain the UV bump (Malkan 1983), but the standard disk is too cold to produce the X-ray radiation. Another solution for the inner parts of accretion disks around black holes is the optically thin, hot disk model, which was first proposed by Shapiro, Lightman & Eardley (1976). Electron temperature of the disk

in this model becomes  $\sim 10^9$  K and it can explain observed X-ray emissions. However, it fails to reproduce the observed UV bump. Haardt & Maraschi (1991) proposed a two-phase thermal disk-corona model. In this model, most of the thermal soft photons emitted from the optically thick, cold disk pass the optically thin corona without being scattered and are observed as UV bump. Only a small fraction of them is Compton upscattered to the X-ray range by the high temperature thermal electrons in the corona. About half of the Compton scattered photons are directed toward the disk and are reprocessed to emerge as black-body radiation, while the remaining half are directed upwards and are observed as power law X-ray spectrum. In this model, the gravitational energy of the accreting matter is mainly released in the corona of the disk. Along the line of this model, some workers investigated the structure of the disk-corona system with different assumptions (Nakamura & Osaki 1993; Kusunose & Mineshige 1994; Svensson & Zdziarski 1994). The X-ray spectra emitted by the corona are studied in detail by using different approaches, especially the Monte Carlo methods (Haardt 1993; Haardt & Maraschi 1993; Titarchuk 1994; Poutanen & Svensson 1996). Nakamura & Osaki (1993) found in their numerical simulations that the effective optical depth of photons traveling through the corona may be larger than the optical depth in the normal direction of the disk.

In this work, we solve the integro-differential radiative transfer equation by using numerical iterating method. The basic equations describing the problem are listed in Sect. 2. Sect. 3 contains the results and a semi-analytical approach proposed to reproduce the numerical results. The last section is the discussion of the results.

## 2. Basic equations

The frequency-integrated radiative transfer equation is

$$\mu \frac{\partial I(\tau, \mu)}{\partial \tau} = I(\tau, \mu) - S(\tau), \quad (1)$$

where  $\tau$  is the optical depth measured in the normal direction of the disk,  $I(\tau, \mu)$  and  $S(\tau)$  are the frequency-integrated specific

intensity and source function,  $\mu = \cos \theta$ , describes the direction of  $I(\tau, \mu)$  with respect to the normal direction.

Here we only consider pure Compton scatterings in the corona and assume the Compton scatterings to be isotropic. Thus, the source function  $S(\tau)$  could be written as

$$S(\tau) = \frac{1+A}{4\pi} \int I(\tau, \mu) d\Omega = \frac{1+A}{2} \int_{-1}^1 I(\tau, \mu) d\mu, \quad (2)$$

where  $A = 4\Theta + 16\Theta^2$ , is the mean fraction of the energy change of the photon through single scattering. The dimensionless electron temperature  $\Theta = kT_e/m_e c^2$ , where  $k$ ,  $m_e$  and  $c$  are the Boltzman constant, rest mass of the electron and the light speed. Strictly speaking, the Compton scattering in the corona is anisotropic (Ghisellini et al. 1991; Haardt 1993). Here we only intend to investigate the total energy transferred from the mild relativistic electrons in the corona to the scattered photons. In the first attempt, we employ Eq. (2) to describe the source function in the radiative transfer equation for the sake of simplicity. We believe this will not affect the main results of our present investigation.

We consider the homogeneous isothermal layer in a plane-parallel geometry, which can model the hot corona over an accretion disk. Compton parameter  $y$  is defined by

$$y = \tau_0(4\Theta + 16\Theta^2) \max(1, \tau_0), \quad (3)$$

where  $\tau_0$  is the optical depth of the corona by electron scattering measured in the vertical direction. Now, we can solve Eq. (1) combining Eq. (2) with appropriate boundary conditions, if the electron temperature  $\Theta$  is specified.

In this work, we assume the input soft photons from the cold disk are isotropic, then the boundary conditions for the problem are as follows:

$$I(\tau, \mu) = I_0, \quad \mu > 0, \quad \text{at} \quad \tau = \tau_0 \quad (4)$$

and

$$I(\tau, \mu) = 0, \quad \mu < 0, \quad \text{at} \quad \tau = 0, \quad (5)$$

where  $\tau$  is the electron scattering optical depth measured in the normal direction of the disk. The energy flux of the soft photons from the cold disk is

$$F_s = 2\pi \int_0^1 I(\tau_0, \mu) \mu d\mu = \pi I_0. \quad (6)$$

Most of the soft photons emitted by the cold disk pass the optically thin corona without being scattered. The flux of the soft photons escaped from the system is given by

$$F_{esc} = 2\pi \int_0^1 I(\tau_0, \mu) e^{-\tau_0/\mu} \mu d\mu. \quad (7)$$

A small fraction of soft photons from the cold disk is Compton upscattered in the corona. About half of the scattered photons are directed upwards. The remaining half are directed downwards, part of them are absorbed by the cold disk and part are reflected.

The fluxes of the upward scattered photons and downward scattered photons are

$$F_{uC} = 2\pi \int_0^1 I(0, \mu) \mu d\mu - F_{esc} \quad (8)$$

and

$$F_{dC} = 2\pi \int_{-1}^0 I(\tau_0, \mu) \mu d\mu, \quad (9)$$

respectively.

The fraction  $D$  of the energy dissipated in the hot corona to the input soft photon energy flux from the cold disk is then given by

$$D = \frac{F_{uC} + F_{dC} + F_{esc} - F_s}{F_s}. \quad (10)$$

Finally, the fraction  $\eta$  of Compton upscattered photon energy directed downward to the cold disk is

$$\eta = \frac{F_{dC}}{F_{dC} + F_{uC}}. \quad (11)$$

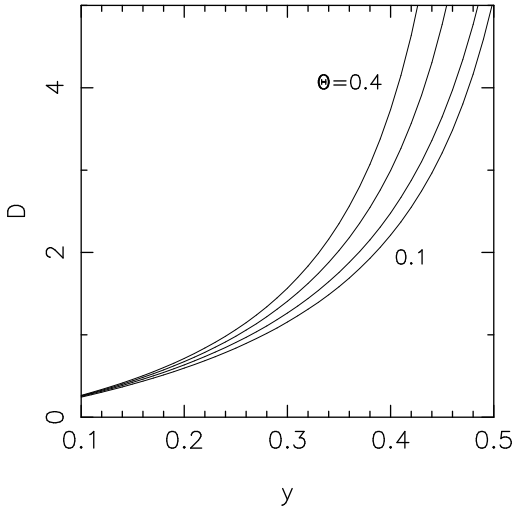
### 3. Results

Eq. (1) is an integro-differential equation and cannot be solved by simply iterating, since the source function is not known a priori and depends on the solution  $I(\tau, \mu)$  at all directions through a given point. In our numerical calculations, we first set the source function  $S(\tau) = 0$  and the test solution  $I_1(\tau, \mu)$  is available. Let  $I(\tau, \mu) = I_1(\tau, \mu)$ , the source function  $S_1$  is obtained from Eq. (2). Then the more accurate solution  $I_2(\tau, \mu)$  could be obtained by iterating Eq. (1), if we let  $S(\tau) = S_1(\tau)$  in Eq. (1). Similar to the first step we have taken, the solution  $I_n(\tau, \mu)$  and the source function  $S_n(\tau)$  will be known by repeatedly iteration. The iteration is terminated until it reaches  $(S_n - S_{n-1})/S_n < 10^{-5}$ , then the solution to Eq. (1) is available.

The numerical results on the fraction  $D$  of the energy dissipated in the hot corona to the input soft photon energy flux from the cold disk are plotted in Fig. 1. It is found that the behaviour of  $D$  varies with the different electron temperatures  $\Theta$ . For the higher electron temperature, the value  $D$  becomes larger even at the same Compton parameter  $y$ .

Considering that the soft photons only suffer single scattering in the corona, we can obtain the mean energy gained by the photons from the hot electrons in the optically thin corona is

$$\Delta E = A \int_0^1 I(\tau_0, \mu) (1 - e^{-\tau_0/\mu}) \mu d\mu = I_0 y \frac{a(\tau_0)}{\tau_0}, \quad (12)$$



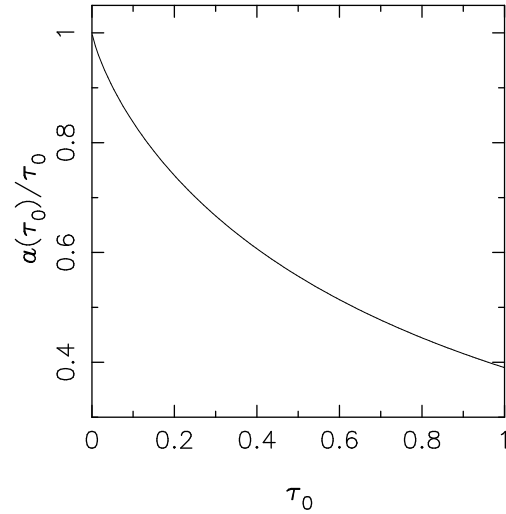
**Fig. 1.** The numerical results of the fraction  $D$  of the energy dissipated in the corona to the input soft photon energy vs. Compton parameter  $y$ , with respect to the electron temperatures  $\Theta = 0.1, 0.2, 0.3, 0.4$ .

where

$$a(\tau_0) = \int_0^1 (1 - e^{-\tau_0/\mu}) \mu d\mu, \quad (13)$$

is the fraction of the photons which suffer single scattering in the corona to the injecting isotropic soft photons. It can be seen in Fig. 2 that the value  $a(\tau_0)/\tau_0$  decreases with the  $\tau_0$ . Eq. (12) indicates that the mean energy gained by soft photons through first scattering depends not only on the Compton parameter  $y$ , but also on the optical depth  $\tau_0$  of the corona. Only when  $y \ll 1$ , the mean energy gained per scattering could be written as  $\propto y$ . It is shown from Eq. (3) that the optical depth becomes smaller for the higher electron temperature of the corona when the value of  $y$  is fixed. From Eqs. (12), (13) and Fig.2, we know that, for the same Compton parameter  $y$ , there is more energy gained by the soft photons through single scattering for the corona with a smaller optical depth  $\tau_0$ , i.e., a higher electron temperature  $\Theta$ . This is the reason why the  $D - y$  behaviour depicted in Fig. 1 varies with the electron temperature of the corona.

The relation  $D \sim 2$  is required in the disk-corona system (Haardt & Maraschi 1991; Nakamura & Osaki 1993). The Compton parameter  $y$  is about 0.6 (Haardt & Matt 1993). Our numerical results show that  $y \sim 0.35 - 0.4$  is more desirable to yield  $D \sim 2$  corresponding to different electron temperatures  $\Theta$ . The difference may be attributed to an ambiguity in measuring the optical depth of the system with slab geometrical configuration. For the photon travels just in the normal direction of the slab and the optical depth  $\tau_0 \ll 1$ , the Compton parameter  $y$  well describes the situation. Nonetheless, most photons pass the slab in the directions other than the vertical direction, these photons will travel a longer distance than the vertical length before they leave the corona since the disk is in slab geometry. The effective optical depth of these photons are larger than  $\tau_0$ . Hence



**Fig. 2.** The function  $a(\tau_0)/\tau_0$  vs. vertical optical depth  $\tau_0$  of the corona.

the Compton parameter  $y$  defined by Eq. (3) cannot describe the corona in slab configuration correctly.

It should be indicated that the present calculations are not effective in the case that saturation of Compton scatterings is important. The typical blackbody radiation temperature of the cold disk in Active Galactic Nuclei is  $\sim 5 - 50$  eV (Haardt & Maraschi 1993). Assuming the temperature of electron in the corona  $\Theta \sim 0.5$ , the mean scattering number of the photon is  $\sim 5$  before the Compton process saturates. For a lower electron temperature  $\Theta$ , the photon suffers more scatterings before it reaches saturation, for example, the mean scattering number is  $\geq 20$  for  $\Theta \sim 0.1$ . Hence, the saturation of the Compton process in the disk should be taken into account at least in some cases with relatively higher electron temperatures. In principle, the recoil of the electron in the scattering and Klein-Nishina electron scattering cross-section should be taken into account in the study of the saturation of the Compton process for the mild relativistic electrons treated here. For simplicity, we do not apply the complicated formalisms including Klein-Nishina cross-section, instead, we just repeat the calculations in the same way described at the beginning of this section till the mean energy of the scattered photons exceeds that of the electrons in the corona, and then let it simply be the mean energy of the electrons. Yet, this numerical approach is still too complicated to be used in constructing the disk-corona model. So, we propose a semi-analytical approach to approximate the numerical results.

Suppose the number of photons which suffer at least  $k$ -fold scatterings is  $N_k$ , the mean energy gained by the photons in the  $k$ -th scattering is  $N_k(1 + A)^{k-1}A\epsilon_i$ , where  $\epsilon_i$  is the mean energy of the input soft photons from the cold disk,  $A = 4\Theta + 16\Theta^2$ , is the amplified factor. Thus, the fraction  $D$  of the energy dissipated in the hot corona to the input soft photon energy flux from the cold disk is

$$D = \frac{1}{N_0} \sum_{k=1}^m N_k (1 + A)^{k-1} A, \quad (14)$$

where  $m$  is the scattering number of the photon before the Compton process saturates,  $N_0$  is the number of input soft photons from the cold disk. We know that the effective optical depth will be larger than the vertical optical depth  $\tau_0$ . The fraction of the soft photons suffering the first scattering is  $a(\tau_0)$  given by Eq. (13). The first scattered photon number  $N_1 = N_0 a(\tau_0)$ . We further assume  $N_{k+1} = N_k b(\tau_0)$  for  $k \geq 1$ , where the coefficient  $b(\tau_0)$  is assumed to be a constant for the multiple scatterings except the first one for the photons. Then,  $b(\tau_0)$  is only a function of the optical depth  $\tau_0$ . Therefore, Eq. (14) could be rewritten as

$$D = \frac{a(\tau_0)A[1 - b(\tau_0)^m(1 + A)^m]}{1 - b(\tau_0)(1 + A)}. \quad (15)$$

We take Eq. (15) as the 'seed' analytic form to fit the numerical solutions and find that

$$D = \frac{a(\tau_0)A[1 - b(\tau_0)^x(1 + A)^x]}{1 - b(\tau_0)(1 + A)}. \quad (16)$$

can well approximate the numerical results to an accuracy of  $< 2\%$  in the range of  $D = 0 - 5$  for the optically thin, hot corona. The two coefficients  $a(\tau_0)$  and  $b(\tau_0)$  in the formula are given by Eq. (13) and

$$b(\tau_0) = (0.2302\tau_0^{0.6956} + 0.5102\tau_0) \frac{a(\tau_0)}{\tau_0}, \quad (17)$$

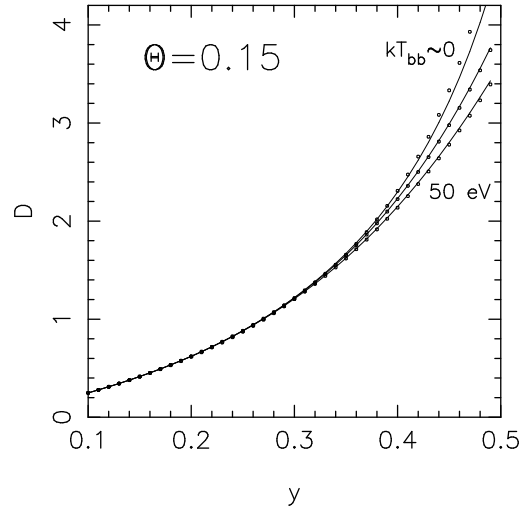
respectively. The mean effective scattering number  $x$  for the photon before it reaches saturation is

$$x = \log_{(1+A)} \left( \frac{0.511 \times 10^6 \Theta}{kT_{bb}} \right) + 0.8, \quad (18)$$

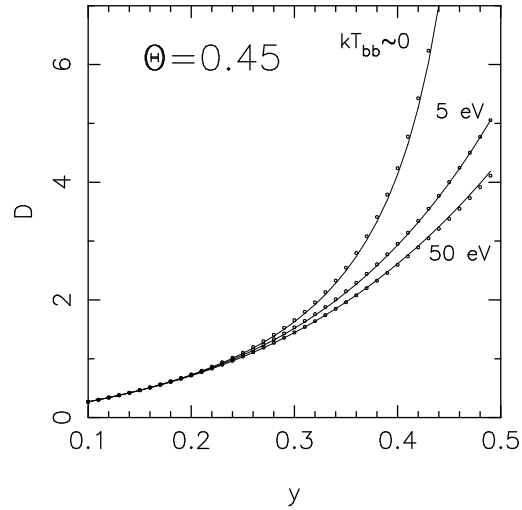
where 0.8 in the right side of the equation is simply induced to fit the numerical results better,  $kT_{bb}$  (in the unit of eV) is the temperature of the blackbody radiation from the cold disk. The comparisons between the semi-analytical approximations and the numerical results of the problem are plotted in Figs. 3 and 4.

#### 4. Discussion

We investigate the energy dissipations in the hot corona of the disk by iterating the radiative transfer equation. A somewhat simplified method is employed to study the saturations of Compton processes. In Fig. 3 both the numerical results and semi-analytical approximations are plotted. We find that the value of  $D$  decreases with the energy of the soft photons  $kT_{bb}$  for the same electron temperature  $\Theta$ . The reason is that in the limit of  $y \gg 1$ , all photons are in equilibrium with the electrons,  $D \sim 0.511 \times 10^6 \Theta / kT_{bb}$ . Therefore, a larger  $kT_{bb}$  for the input soft photons implies a smaller  $D$  for the fixed electron temperature  $\Theta$ . We note that the behaviours of  $D - y$  do not change significantly with the soft photons  $kT_{bb}$  for relatively lower electron temperatures (see Fig. 3 for  $\Theta = 0.15$ ). In this case, the photon suffers more scatterings before it reaches saturation, which indicates only a smaller fraction of input soft



**Fig. 3.** Comparisons between the results of the semi-analytical approximations and the numerical results (small circles) for the blackbody radiation temperatures of the soft photons  $kT_{bb} = 0, 5$  and  $50$  eV, with respect to the electron temperature  $\Theta = 0.15$ .



**Fig. 4.** Same as Fig. 3, but the electron temperature  $\Theta = 0.45$  instead.

photons is finally in energy equilibrium with the electrons in the corona. The  $D \sim 2$  is desired in two-phase model, which requires that the Compton parameter  $y$  to be  $0.3 - 0.4$  with respect to the different electron temperatures  $\Theta$  and the blackbody radiation temperatures  $kT_{bb}$  of the photons from the cold disk. For relatively higher electron temperature, there is a larger amplified factor  $A$ , and the Compton process will soon saturate within several scatterings except  $kT_{bb} \sim 0$ . Compared with the lower electron temperature case, more soft photons are driven to equilibrium with the electrons, and then no longer gain energy from hot electrons through further scatterings. We suggest that Eq. (16) combining Eqs. (13), (17) and (18) could be used as an approximate approach in the study of the structure and the spectrum of the disk-corona systems. Finally, we point that

the pair production and annihilation in the hot corona should be taken into account when the electron temperature in the corona exceeds  $10^9$  K. This will be the scope of the further work.

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