

# Some notes on the relativistic apsidal motion of DI Herculis

A. Claret

Instituto de Astrofísica de Andalucía, C.S.I.C., Apartado 3004, E-18080 Granada, Spain

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**Abstract.** DI Her is a double-lined eclipsing binary presenting a very slow apsidal motion for which the relativistic contribution is of the order of the newtonian one. The observed apsidal motion is around 4 times that predicted by stellar evolutionary models and by General Relativity. This trend is known for non-relativistic systems although in a minor scale. The magnitude of such a disagreement led some authors to propose several mechanisms to explain it, including a revision of the gravitation theory. In this paper some aspects of these processes are analyzed using modern stellar models, recent absolute dimensions and apsidal motion rate. New alternative explanations for the problem as for example, the intrinsic observational difficulties to obtain very slow apsidal motion rates with confidence, are also introduced.

**Key words:** stars: binaries: close; eclipsing; evolution; DI Her – relativity

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## 1. Introduction

Over the last two decades the eclipsing binary DI Her has been object of intensive investigations due to the discrepancy between the observed apsidal motion rate and the theoretical predictions. Discrepancies between observed and theoretical rates were found in some eclipsing binary systems which exhibit apsidal motion indicating that real stars seem to be more mass concentrated than predicted by theoretical models. Such disagreements were investigated by Claret & Giménez (1993ab). For non-relativistic systems the improvements introduced in stellar evolution model such as new opacities, mass loss, core overshooting and rotation, were enough to reduce the differences between observation and theory down to acceptable levels. The new input physics contributed to decrease the theoretical values of  $k_2$ , i.e. the new models are more centrally condensed in mass than the previous ones. Of course, the high quality of the observational data were of prime importance to obtain such an agreement. For a detailed discussion see the papers by Claret &

Giménez (1993ab). Recent investigation by Claret (1997) indicates that also the relativistic systems, for which good absolute dimensions determination are available, compare very well with stellar models and the prediction by the General Relativity (GR).

However, such improvements were not sufficient to explain the case of DI Her. The observed apsidal motion rate is too slow when compared with the theoretical predictions. Several hypothesis were put forward in order to explain such a disagreement. For example, Guinan & Maloney 1985 have considered four possibilities:

1. A rapid circularization of the orbit
2. The presence of a circumstellar cloud between the components
3. The presence of a third star
4. Use of an alternative theory of gravitation.

On the other hand Shakura (1984) suggested that the misalignment of the orbital and spin angular momenta could explain the anomalous periastron shift and Company et al. (1987) explored in more detail this possibility.

These and other explanations for the problem of DI Her are revised in this paper making use of new observational data as well as modern stellar models. New possible alternatives also introduced.

## 2. The system DI Her and the observed apsidal motion rate

The components of DI Her are two stars with masses  $m_1=5.185\pm 0.108$  and  $m_2=4.534\pm 0.06 M_\odot$ . The radii are  $2.680\pm 0.046$  and  $2.477\pm 0.045 R_\odot$  respectively while the period is 10.55 days (Popper 1982). The observed apsidal motion rate is  $1.88\times 10^{-4}\pm 5.20\times 10^{-5}$  degrees per cycle (Guinan & Maloney 1985). Recent determination of times of minima gives  $\dot{\omega}_{obs} = 3.00\times 10^{-4}\pm 4.3\times 10^{-5}$  degrees per cycle (Guinan et al. 1994). The eccentricity is  $0.489\pm 0.002$ .

Concerning evolution, DI Her is a young system, as we can see in Fig. 1 where we represent the variation of the radius as a function of the time for both components. A common age of  $5\times 10^6$  years is found ( $\log \tau=6.7$ ). The present models were computed with  $X=0.7045$  and  $Z=0.015$  and the mixing-length parameter  $\alpha$  was 1.52 while for core overshooting we have adopted  $\alpha_{ov}=0.20$ . For a description of the stellar evolutionary model see Claret 1995. The age determination for very young systems is

not straightforward since in these cases it is difficult to decide what is the "good" isochrone. The error bars concerning DI Her in Fig. 1 show this fact, since we can find a range of acceptable ages within them. The uncertainty in age determination leads to an error around 10 % in the theoretical  $k_2$ . As we do not know what is the observed chemical composition of the system the above values are used only as an approximation. We have performed tests running models with other chemical compositions and we have found that the values of  $k_2$  are not too much dependent on composition (Claret 1995, Fig. 7). Given the magnitude of the discrepancy between theory and observation and the comments made above such uncertainties in the models do not affect seriously the present analysis. For the same models the theoretical values of  $k_2$  can be inferred. We obtain  $k_{21}=8.68 \times 10^{-3}$  and  $k_{22}=8.08 \times 10^{-3}$  for the primary and secondary respectively (Fig. 2).

With these data we can compute the expected value for  $\dot{\omega}$  corrected by the relativistic contribution as

$$\dot{\omega}_{total\ GR} = \dot{\omega}_{dist} + \dot{\omega}_{GR} \quad (1)$$

where the symbol *dist* denotes tidal and rotational contribution and *GR* indicates the relativistic contribution to the periastron advance. The classical part due to distortions of the components can be written using the following equations:

$$\frac{P}{U} = c_{21}k_{21} + c_{22}k_{22} \quad (2)$$

$$\frac{360P}{U} = \dot{\omega} \quad (3)$$

where P is the orbital period and U is the apsidal motion period. The  $c_{2i}$  are given by

$$c_{2i} = \left[ \left( \frac{\Omega_i}{\Omega_K} \right)^2 \left( 1 + \frac{m_{3-i}}{m_i} \right) f(e) + \frac{15m_{3-i}}{m_i} g(e) \right] \left( \frac{R_i}{A} \right)^5 \quad (4)$$

The auxiliary functions  $f(e)$  and  $g(e)$  can be written as

$$f(e) = (1 - e^2)^{-2}$$

$$g(e) = \frac{(8 + 12e^2 + e^4)f(e)^{2.5}}{8}$$

and  $\Omega_i$  and  $\Omega_K$  denote the angular velocity of the component  $i$  and the keplerian one,  $A$  is the semi major axis of the orbit,  $R_i$  and  $m_i$  are the radius and mass of the component  $i$  respectively.

Following (Levi-Civita 1937, Kopal 1978) the relativistic contribution to the advance of the periastron is given by

$$\dot{\omega}_{GR} = 2.29 \times 10^{-3} \frac{(m_1 + m_2)}{A(1 - e^2)} \quad (5)$$

where  $m_i$  and  $A$  are given in solar units. Introducing numerical values in Eqs. 2, 3, 4 and 5 and using Eq. 1 we get  $\dot{\omega}_{total\ GR} = 1.25 \times 10^{-3}$  degrees per cycle. We have obtained this value using for  $\Omega_i/\Omega_K$  the values 3.50 and 3.78 respectively for the primary and secondary. The resulting theoretical apsidal motion rate is about 4 times larger than the observed one corrected by the relativistic correction.

### 3. Alternative explanations

It is known that several phenomena can change the rate of the apsidal motion. For example, if a third companion is present, the times of minima will vary periodically and this will affect the observed apsidal motion. In this section we discuss some of these physical mechanisms in order to try to explain the case of DI Her. Some of these processes have been discussed already by other authors. However, we shall comment them briefly for completeness. New alternatives are also investigated.

#### 3.1. Fast circularization of the orbit (Guinan & Maloney 1985)

The time of minima can change with time if a rapid decreasing of the eccentricity is present. If the discrepancy in the apsidal motion of DI Her is attributed to this mechanism a high derivative is needed ( $\dot{e} \approx -1.4 \times 10^{-4}/\text{year}$ ). We have studied recently the circularization in close binary systems (Claret et al. 1995, Claret & Cunha 1997) and for the specific case of DI Her we have found that the derivative, given by  $-e/\tau$  where  $\tau$  is the time scale for circularization, is around  $-10^{-14}/\text{year}$ . This result was obtained using the radiative damping mechanism. Taking into account both times we can deduce that the efficiency in circularizing the orbit is several order of magnitude smaller than that required to fit the observations. This conclusion holds for the case of the hydrodynamical or radiative damping processes, though both mechanisms predict different critical times for circularization.

#### 3.2. The third body (Guinan & Maloney 1985)

An extensive analysis of this possibility is given in the paper quoted above. The typical contribution to the apsidal motion is around  $1 \times 10^{-3}$  degrees/cycle, i.e., of the order of those due to other causes. From the dynamical point of view the hypothetical third body would change the orbital parameters and affect directly the depth of the eclipses. Such a behaviour has not been detected yet. However, it is difficult to discard or to confirm it definitively. Meanwhile the question is still open.

#### 3.3. Revision of the gravitation theory (NST84 and NST89 (Moffat 1984, 1989; this paper))

A different apsidal motion rate was proposed by Moffat 1984 using a new theory of gravitation based on a non-symmetrical tensor. The corresponding rate of variation of  $\omega$  is given by

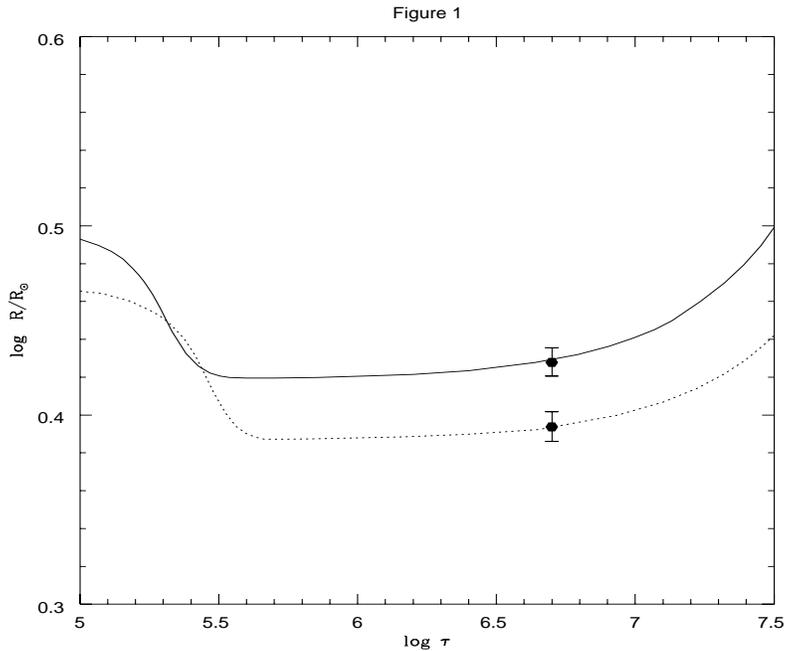
$$\dot{\omega}_{NST} = \frac{9.287 \times 10^{-3} (P/365.25) (m_1 + m_2)^{2/3} \lambda_{NGT}}{(P/2\pi)^{5/3} (1 - e^2)} \quad (6)$$

with

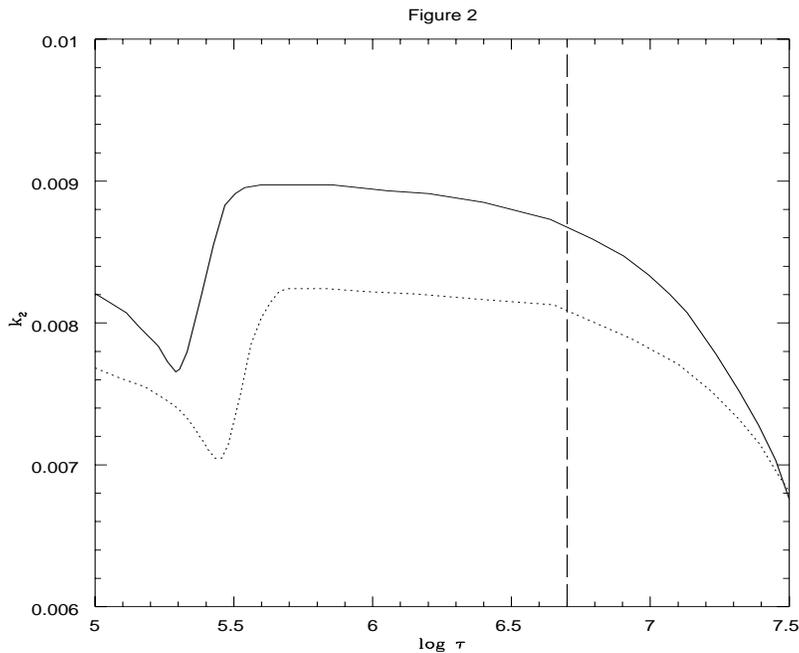
$$\lambda_{NST} = 1 - \frac{4.625 \times 10^{-35} l^4 (1 + e^2/4)}{(P/2\pi)^{4/3} (1 - e^2)^2 (m_1 + m_2)^{8/3}}$$

where P is given in days and the masses are in solar units.

Note that Eq. 6 reduces to the GR prediction when  $\lambda_{NST}$  tends to 1. The apsidal motion rate can even be reversed depending on the value of  $l$  which is inferred from a pre-calibration.



**Fig. 1.** Models for DI Her. The radii are shown as a function of time. An acceptable common age of  $5 \times 10^6$  years is found. The adopted chemical composition was  $X=0.7045$  and  $Z=0.015$  with  $\alpha=1.52$  and  $\alpha_{ov}=0.20$ . The continuous line represents the primary, and the dotted one denotes the secondary.



**Fig. 2.** The theoretical apsidal motion constants  $k_2$  as a function of the time for the same models of Fig. 1. Note that inferred values of  $k_2$  are for the same age found in Fig. 1.

A linear fitting to the calibration performed by Moffat gives  $l = [0.2(m_1 + m_2) + 0.10] \times 10^9$  cm with the masses given in solar units. Using the observed values for DI Her and considering the above value for  $l$  one finds  $\dot{\omega}_{total\ NST} = \dot{\omega}_{dist} + \dot{\omega}_{NST} = 2.16 \times 10^{-4}$  degrees per cycle. This value seems to be in good agreement with the observational data within  $2\sigma$ . However, when one applies Eq. 6 to other systems the corresponding shift in the periastron positions are systematically too slow when compared with observed ones (Claret 1997).

Later Moffat (1989) modified his equations. The main difference concerning apsidal motion with respect to the 1984 formulation is that for equal masses and chemical compositions

they predict the same results as GR. However before to adopting it as definitive some remarks should be done:

1. The theory does not predict the apsidal motion a priori
2. DI Her (and other problematic systems) were used in the calibration
3. The contribution to the apsidal motion is strongly dependent on the number of cosmions which it is not well constrained
4. The theory present too many free parameters
5. There is a strong dependence of the calibration on the systems used (DI Her is always used)
6. The primaries and secondaries of systems used in the 1989 calibration follow different mass-number of cosmions re-

lationship. In fact the masses of the secondary of DI Her and the primary of AG Per differ only 0.2%. However, the respective number of cosmions differ in 65%.

The results based on the NST84(89) should be taken with care due to the problems indicated above. In the particular case of DI Her they do not formally explain this enigmatic system since it was used in the calibration. A more detailed discussion on this subject can be found in Claret (1997).

### 3.4. The effect of the circumstellar material (Guinan & Maloney 1985)

As already pointed out for non-relativistic systems (Claret & Giménez 1993a) the discrepancies between observed and theoretical apsidal motion rate can not be fully attributed to the change of the gravitational field of the stars due the presence of a circumstellar matter. The formula given by Hadjidemetriou (1967) gives

$$\frac{P}{U''} = -\frac{GP^2\sigma}{2\pi} \quad (7)$$

where  $\sigma$  is the density and  $U''$  is the associated period of apsidal motion. If the observed discrepancy is due to this effect the mean density of the circumstellar cloud would be as large as  $10^{-10} \text{ g cm}^{-3}$ . This density is high for typical interstellar medium between components of close binaries. Moreover, as reported by Guinan & Maloney (1985), there is no observational evidence of such a high density cloud around the components of DI Her.

### 3.5. Inclination of the axes of rotation (Sakura 1984, Guinan & Maloney 1985, Company et al. 1989)

Depending upon the orientation of the rotation axis of the stars with respect to the angular orbital momentum there will be a correction to the rotational term of  $\dot{\omega}$ . This correction may even be negative, i.e., it is possible a regression of the position of the periastron mainly if the system is a young one. This is the case of DI Her in spite of the uncertainties in the determination of its age. Thus, this possibility seems to be very attractive for explaining its anomalous apsidal motion.

Kopal (1978) has studied the problem. The correction depends on the quantity  $-1.5\sin^2(\theta_i+i)-0.5\sin 2(\theta_i+i)\tan(0.5i)$  where  $\theta_i$  is the angle of inclination of the rotational axis of the component  $i$  in the invariable plane of the system. Depending on the angles involved we can have a recession of the line of the periastron. This moved Sakura (1984) to invoke it as a possible explanation for the disagreement with observations. Some years later Company et al. 1989 returned to the problem of no alignment of the spin and angular orbital momentum. Their Eq. 1 summarized their results

$$\begin{aligned} \dot{\omega}_{total GR} &= \dot{\omega}_{tidal1} + \dot{\omega}_{tidal2} \\ &+ \dot{\omega}_{rot1}(1 - 3x^2 - 2x(x - x^2)^{1/2} \cot i \sin(\theta_1 + \omega)) \\ &+ \dot{\omega}_{rot2}(1 - 3y^2 - 2y(y - y^2)^{1/2} \cot i \sin(\theta_2 + \omega)) + \dot{\omega}_{GR} \quad (8) \end{aligned}$$

where the indices *tidal j* and *rot j* refer to the tidal and rotational contribution of the component *j*. The variable *x* and *y* are given by  $\mathbf{n} \cdot \mathbf{n}_1$  and  $\mathbf{n} \cdot \mathbf{n}_2$  respectively where  $\mathbf{n}$  is the unitary vector in the direction of the angular orbital momentum. The vectors  $\mathbf{n}_1$  and  $\mathbf{n}_2$  are the unitary vectors in the direction of the spin of the components. Thus  $\cos(\theta_1) = \mathbf{n}_1 \cdot \mathbf{ep} / (1 - x^2)^{1/2}$  and  $\cos(\theta_2) = \mathbf{n}_2 \cdot \mathbf{ep} / (1 - y^2)^{1/2}$  where  $\mathbf{ep}$  is the unitary vector in the direction of the periastron. A numerical analysis of the Eq. 8 carried out by the authors indicated that the discrepancy in the apsidal motion of DI Her could be attributed to the inclination of the rotation axis of the components if the corresponding angles are of the order of 70 degrees.

For DI Her the ratio of the orbital to rotational angular momentum is about  $10^3$ . This means that the time scales for synchronization and for the decay of the angle  $\theta$ , defined by the orbital and equatorial planes, are of the same order of magnitude. We have integrated the corresponding differential equations and have found that  $t_s \approx t_\theta \approx 10^8$  years. The corresponding critical time for circularization is about 25% larger than the mentioned ones. When compared with the derived age for DI Her  $t_\theta$  is around 20 times larger, that is, following the tidal evolution theory, it is possible that the rotation axes of DI Her are still inclined with respect to the plane of the orbit (of course, only if the initial angles were different from zero). We have re-analyzed this possibility through Eq. 8 using new models and apsidal motion rate and the result is invariant, that is, the required angles are around 70 degrees.

Observations of eclipsing binary stars during the eclipses can help us to elucidate the position of the rotational axes in a binary system. As in these phases the different surfaces velocities are eclipsed, there appears a net effect in form of a Doppler shift in the center of the line (Rossiter effect). Maloney & Guinan 1989 have observed the primary eclipse of DI Her. They reported that the preliminary results indicated that the orbital and equatorial planes are coplanar. These observations constrain this hypothesis severely but as in the third body hypothesis, only more systematic observations may elucidate the situation.

### 3.6. The theoretical $k_2$ derived from stellar models (this paper)

The discrepancies between observations and theoretical predictions for the apsidal motion for non-relativistic systems were reduced drastically due to the application of new stellar models by considering evolutive effects on  $k_2$  and new input physics. In fact, as the results by Claret & Giménez 1993ab indicate, modern stellar interior models are able to reproduce the observations of the apsidal motion rates, at least for those systems with well determined astrophysical properties. Therefore, the reasons for the disagreement present in DI Her should be attributed to other effects: even in the hypothetical case of no distortional contribution, the ratio between the observed and the theoretical rate (provided in this case only by the General Relativity) would be around 2.3.

### 3.7. Effects of the dynamic tides (this paper)

The classical apsidal motion rate is deduced in the framework of the equilibrium tides. Smeyers et al. 1991 and Ruymaekers 1993 extended it to the case of dynamic tides. Their main results are, that for short periods and sufficiently large eccentricities, apsidal motion rates derived in the framework of the dynamic tides are smaller than those predicted in the framework of equilibrium tides.

Recently Marshall et al. 1995 have examined the observations of DI Her outside the minimum realized during the years 1993-94. They reported "possible low amplitude light variations". However, these results and the associated periods are still uncertain due to a possible variability of the comparison star used.

However, concerning DI Her, one should keep in mind that even in the hypothetical case that such corrections led to  $k_2=0$  the discrepancy would still remain (see previous Subsection).

### 3.8. Effects of stellar rotation (this paper)

As the ratio of rotational to tidal distortion contribution is important in DI Her (around 0.56 and 0.58 for the primary and secondary respectively) it is convenient to investigate the changes in the predicted apsidal motion due to rotation. We have introduced rotation into our code of stellar evolution and we investigated its influence on the radius, luminosity and  $k_2$  (Claret & Giménez 1993a). Within the quasi spherical approximation the expected correction due to stellar rotation on the theoretical  $k_2$  depends on the parameter  $\lambda = 2v^2R/3GM$ , and for nearly homogeneous models we have found  $\Delta k_2 = -0.9\lambda$ . The theoretical correction is too small to explain the observed discrepancy in DI Her although for some systems it can help to remove the eventual disagreements.

### 3.9. Viscosity of the stellar interior (this paper)

The usual formalism for the apsidal motion rate was derived under the assumption that stellar viscosity is low (see Kopal 1978). Hosokawa (1985) using two extreme cases for the stellar viscosity - inviscid and rigid body - derived an approximation for  $\dot{\omega}$ . The main difference with respect to Eq. 4 is that a correction is introduced in the tidal term. The correction can be written as  $(1-\nu)$  where  $\nu$  is the "viscosity". Its extreme values are zero for a perfect fluid and 1.4 for a rigid body.

In the hypothetical and improbable case that stars behave like rigid bodies, the correction would lead to  $\dot{\omega}_{totalGR} = 7.4 \times 10^{-4}$  degrees per cycle which is still a large value when compared with the observed rate.

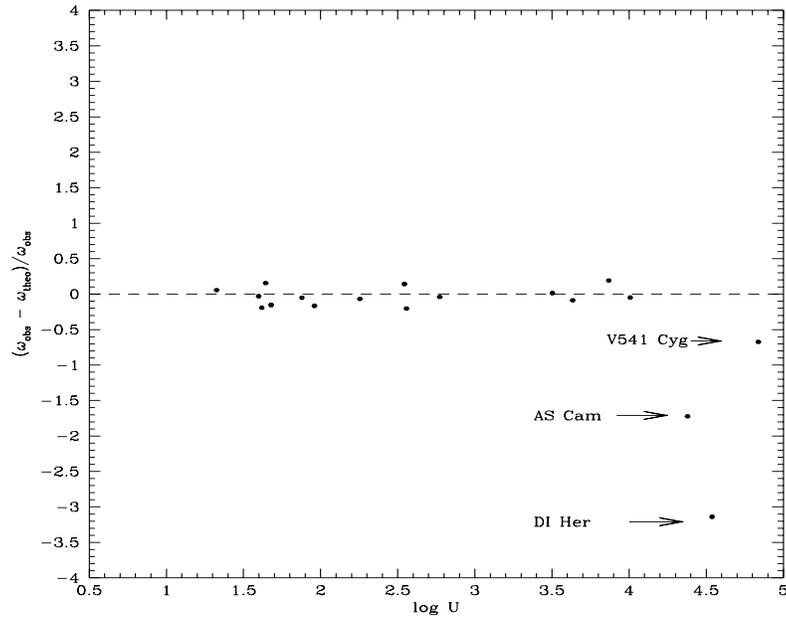
### 3.10. Observational aspects of the apsidal motion rates (this paper)

The range of periods of apsidal motion is very large: some decades up to almost  $10^5$  years. The time spent in observing eccentric systems which exhibit apsidal motion is comparable in some case with U itself but in other ones the ratio is very small.

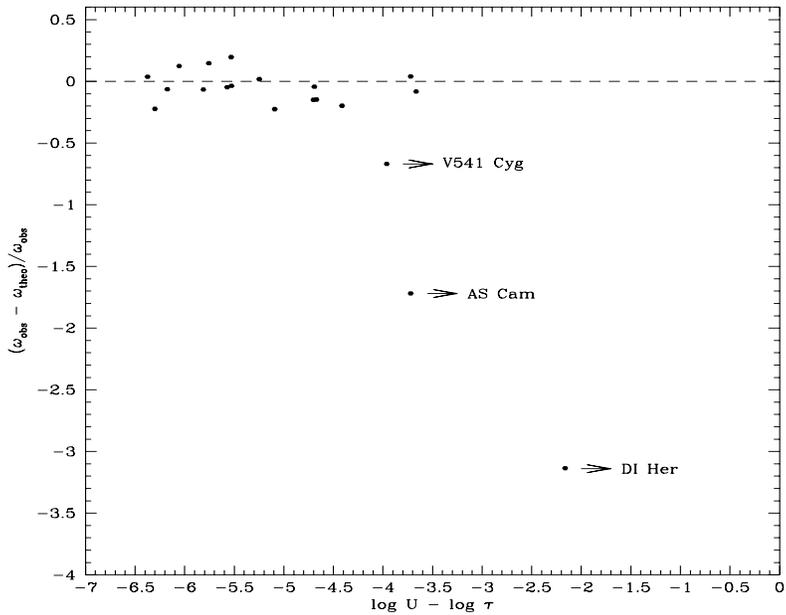
Let us examine some observational conditions under which the apsidal motion rates are obtained and that may affect its interpretation.

1. A common problem of the systems showing apsidal motion is that, during the recollection of time of minima to obtain  $\dot{\omega}_{obs}$ , **different techniques** (mainly data reduction) have been used with **different instruments**. This means that different detectors with different levels of confidence were used. Indeed as quoted by Guinan & Milone 1985 the differences in  $\dot{\omega}_{obs}$  for DI Her obtained using photographic, visual and photo-electric measurements and only photometric data can reach 270% ( $1.75 \times 10^{-2}$  to  $6.5 \times 10^{-3}$  degrees per year).
2. It should be mentioned that these observations were carried out during about 27 years only, that is, they only cover 0.1% of the apsidal motion period (in case of photometric observations). This is another important point that, in our opinion, can limit definitive conclusions on DI Her and on other systems with slow apsidal motion.
3. The new observations of  $\dot{\omega}$  obtained by Guinan et al. in 1994 reduce the difference observed-predicted rates. They found that the ratio  $\dot{\omega}_{theo}/\dot{\omega}_{obs}$  - which was 6.7 - is reduced to 4.2. This is essentially due to the new observations and data analysis of  $\dot{\omega}$  since the theoretical values for  $k_2$  and observed values of radii, eccentricity and masses are essentially the same.
4. In addition, there is some controversy about the best method to analyze the data of times of minimum in order to obtain the apsidal motion rate (Maloney & Guinan 1991). These authors found a observational apsidal motion rate for AS Cam different than that by Krzesinski et al. 1990. The later authors found that the discrepancy for AS Cam was reduced if a different value of eccentricity was used. Maloney & Guinan interpreted this result as an erroneous use of the least square method. This system also presents discrepancies with respect to the GR prediction.
5. Another alternative to understand the behavior of DI Her could be based on the ratio between the time scales involved: the apsidal motion period and the time interval since apsidal motion have been measured. As mentioned before, for DI Her this ratio is very small. This means that the time spent in observing the system is very short when compared with the apsidal motion period. Of course this could lead to poor results. In order to illustrate this situation in the light of the present discussion we plot in Fig. 3 the discrepancies between observed and theoretical apsidal motion rates as a function of  $\log U$ . All systems with good absolute dimensions determination - relativistic and non-relativistic ones - are represented. In spite of the uncertainties in their absolute parameters V541 Cyg and AS Cam are also presented since these systems present high deviations.

This figure shows very interesting trends. Up to  $U \approx 10000$  years there is no evidence of any systematic dependence of the discrepancies on the period U. However, for DI Her, V541 Cyg and AS Cam there is a dependence of the deviations with the period of apsidal motion. These systems,



**Fig. 3.** Relative differences between the observed and the predicted internal structure constant as a function of the apsidal motion period  $U$  for relativistic and non-relativistic systems. The positions of DI Her, V541 Cyg and AS Cam are indicated.



**Fig. 4.** Relative differences between the observed and predicted apsidal motion rates as a function of the relative orbital and evolutionary time scales.

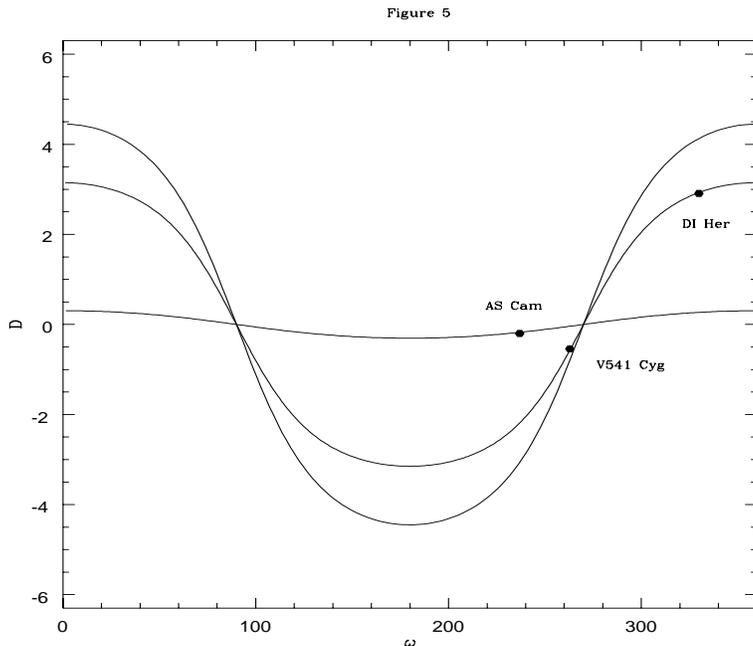
which present the largest deviations, also present the largest  $U$ 's.

Perhaps a way to see such effects more clearly is to examine Fig. 4, where we plot the relative discrepancies as a function of  $\log(U/\tau)$  where  $\tau$  is the age of the system. For DI Her, for example, the orbital time scale begins to be comparable with the stellar evolution time scale.

6. An additional observational difficulty may come from the position of the longitude of periastron. The observations of apsidal motion are expected to be unfavorable for  $\omega$  near 0,  $\pi$  and  $2\pi$ . In Fig. 5 we represent the quantity  $D$ , given by

$$D = \frac{P}{\pi} \left[ \tan^{-1} \left( \frac{e \cos \omega}{(1 - e^2)^{1/2}} \right) + \frac{e \cos \omega (1 - e^2)^{1/2}}{(1 - e^2 \sin^2 \omega)} \right] \quad (9)$$

as a function of  $\omega$ . Recall that  $\dot{\omega}_{obs}$  is dependent on the time derivative of  $D$ . In that figure we show the position of V541 Cyg, AS Cam and DI Her in the plane  $\omega \times D$ . Although AS Cam does not lie in an unfavorable position the corresponding values of  $D$  are too small. On the other hand V541 Cyg - which present the smallest deviation - is in a more favorable position and the values of  $D$  are larger than in the previous case. DI Her is not in a very favorable position to obtain apsidal motion information and the values of  $D$  are of the same order of that for V541 Cyg.



**Fig. 5.**  $D$  (Eq. 9) as a function of the longitude of periastron for DI Her, AS Cam and V541 Cyg.

#### 4. Conclusions

We have re-analyzed the case of the puzzling apsidal motion of DI Her using new observational and theoretical data. Some possible alternatives already explored to explain the discrepancy were reviewed briefly. We have also introduced new possible explanations for the problem. When compared with observational data some hypothesis present some inconsistencies or are directly in conflict with the data. There are some hypothesis that can not stand comparison even with theoretical data as for the case of high viscosity for the stellar interior, or the high density required if we assume that a circumstellar material is the responsible for the slow apsidal motion of DI Her.

Some more plausible explanations are debilitated by recent observations. As for example, that based on the possibility of no alignment of the orbital and spin momenta. In fact, the observation of the Rossiter effect in DI Her performed by Maloney & Guinan 1989 restrict seriously such a hypothesis although systematic observations are needed to enable us more conclusive analysis.

We can also quote the use of an alternative equation for the apsidal motion rate based on the work by Moffat (1984) which, when applied to others systems, gives an over-correction that leads to poorer results than when we use General Relativity. On the other hand the formulation by Moffat (1989) is also dependent on a pre-calibration, using DI Her itself among other systems. This theory does not predict the apsidal motion a priori and there is a strong dependence on the baryon to cosmions ratio which is not a well constrained physical parameter (Claret 1997).

The presence of a third body although is an acceptable explanation, depends obviously on observational evidence.

The magnitude of the period of apsidal motion  $U$ , the position of the periastron  $\omega$  and the values of  $D$  may affect the

apsidal motion measurements. However due to the small number of points in Fig. 3, 4 and 5 it is not possible to conclude definitively that some of these effects are responsible for the high deviations of  $\dot{\omega}_{obs}$ . But the tendency shown (mainly in Fig. 3) deserves certainly a more careful investigation under the observational and theoretical point of view. Still concerning these questions, some natural questions can be made as for example: how to evaluate the errors in  $\dot{\omega}_{obs}$  taking into account the relatively short observation time (compared with  $U$ )? What is the degree of confidence when the observations of apsidal motion are obtained near unfavorable positions of the periastron?

The main goal of this paper is not to solve the case of DI Her. The explanation for the case of DI Her is certainly more complex than commented here and it is possible that more than one perturbing process is acting simultaneously on the system. With this paper we mainly try to stimulate and renew the discussion on the observational and theoretical methods often used in the analysis of apsidal motion in close binary systems.

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