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Properties of dense matter in a supernova core in the relativistic mean-field theory

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Abstract. We have used the Zimanyi & Moszkowski model in the relativistic mean-field theory to study the properties of asymmetric nuclear matter and the equation of state (EOS) for supernova matter. We find that even though muons are included in the chemical composition of neutron-star matter, this model still results in a small proton concentration so that only the modified Urca process occurs as a main neutrino reaction in neutron stars. This conclusion is opposite to that of the previous studies in the relativistic mean-field theory. The incompressibility of symmetric nuclear matter is consistent with both the experimental values extracted from the nuclear physics and the constraints placed on the EOS by observations of neutron star masses. If the EOS for supernova matter is described by this model, the prompt mechanism for Type II supervovae cannot work, but the EOS is likely to be favourable to the delayed explosion mechanism.

Key words: dense matter – relativity – stars: supernovae: general – stars: neutron

1. Introduction

According to the current models of Type II supernova explosions (Bethe 1990), the core collapse of a massive star between 8 and $30 M_{\odot}$ due to electron capture and photodisintegration proceeds until the nuclear matter density is reached, and a shock wave is produced as a consequence of the sudden stiffening of the equation of state (EOS). However, as the shock propagates through the iron core, it weakens and then perhaps stalls because of nuclear dissociation and neutrino losses. A complete analysis of the core collapse, bounce and shock movement requires not only detailed modeling of neutrino transport, general relativity and convection, but also careful investigation of EOS at densities higher than the nuclear matter density.

The EOS for dense matter in a supernova core plays important roles in the whole supernova-explosion process. First, a number of studies have attempted to delineate the exact role of the EOS in the collapse and the bounce phases (Baron,et al. 1985a,b; Baron et al. 1987; Myra & Bludman 1990; Bruenn 1989a,b; Cooperstein & Baron 1990; Swesty, et al. 1994), while the other studies (e.g., Takahara & Sato 1985; Gentile et al. 1993; Dai,et al. 1995) found that the evolution of the EOS, such as phase transitions, is likely to be in favor of explosions. Many of these investigations have been largely motivated by the observation that softening the EOS above the nuclear matter density in some cases leads to stronger bounces and shocks. Baron et al. (1985a,b, 1987) used the Cooperstein & Baron (1990) EOS, in which the bulk incompressibility and adiabatic index of supernuclear matter were varied, found that shock strength was correlated with EOS stiffness, and claimed that sufficiently soft EOSs could result in explosions before collapse to a black hole. Second, the structure of the post-bounce core depends upon the extent to which electron capture reactions deleptonize the core during the collapse epoch. A small electron-capture rate, which is determined by the bulk symmetry energy of nuclear matter (Bruenn 1989a; Swesty et al. 1994), results in a larger trapped lepton concentration at the core bounce. Third, in order to reenergize the shock if it stalls, one studied the so-called delayed mechanism, in which neutrino heating revives the shock (Bethe & Wilson 1985; Wilson 1985), and convection rapidly transports energy into the region behind it (e.g., Burrows & Fryxell 1992, 1993; Janka 1993; Janka & Müller 1993a,b; Wilson & Mayle 1993; Bruenn & Mezzacappa 1994; Herant et al. 1994; Burrows, et al. 1995). Moreover, the strength of the initial bounce and the structure of the post-bounce core are crucial to the operation of these two mechanisms.

Even after the supernova explosion, the EOS for supernuclear matter also plays important roles in determining the evolution of the matter at the birth stage (Burrows & Lattimer 1986). Such a birth event always companies with a neutrino burst of which the energy spectrum can be detected on the terrestrial experiments. In addition, theoretically, the properties of neutron stars such as the maximum mass, the maximum rotation frequency, the moment of inertia, and the proton concentration during the cooling, which affects the reaction rates of neutrino

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processes inside the stars, on one hand, are determined by the EOS together with the Tolman-Oppenheimer-Volkoff equation. On the other hand, many observations have put constraints on these properties (for a brief review see Cheng, Dai et al. 1996). Therefore, the determination of the EOS for dense matter is also crucial to the study of the birth, evolution and physics of neutron stars.

To determine an EOS for dense matter through the many-body theory of interacting hadrons, one has studied many approaches, among which the relativistic many-body approach on nuclear systems is of growing interest during recent years. The relativistic Brueckner-Hartree-Fock theory reproduces the saturation property of nuclear matter, which is not possible in the nonrelativistic approach unless one introduces three-body force by hand (Brockmann & Machleidt 1990). Moreover, rather promising results within the framework of this theory have been obtained by Müther, et al. (1990) and Li, et al. (1992), and the application of this theory to neutron stars has been investigated by Engvik et al. (1994) and Bao et al. (1994). On the other hand, the relativistic mean field (RMF) theory is successful both for elastic scattering and for nuclear ground-state property (Walecka 1974; Chin 1977; Serot & Walecka 1986). Hence, the RMF theory has been suggested to calculate the EOS for neutron-star matter (Walecka 1974). This approach contains both nucleonic and mesonic degrees of freedom and can be considered as phenomenological. The coupling constants and meson masses of the effective meson-nucleon Lagrangian are taken as free parameters which are adjusted to fit the properties of nuclear matter and finite nuclei.

In the standard model of Walecka (1974) the incompressibility of nuclear matter is overestimated. There are two ways to solve this question. First, Boguta & Bodmer (1977, hereafter BB) introduced cubic and quartic terms for the scalar field into the Lagrangian. This shifts the incompressibility to reasonable values in comparison with empirical data. Along this direction, many authors (Glendenning 1982, 1985, 1987a,b; Weber & Weigel 1989a,b; Kapusta & Olive 1990; Ellis, et al. 1991; Sumiyoshi, et al. 1992; Sumiyoshi & Toki 1994; Sumiyoshi, et al. 1995a; Cheng et al. 1996; Schaffner & Mishustin 1996) have studied the EOS for dense matter and the properties of neutron stars. Zimanyi & Moszkowski (1990, hereafter ZM) proposed an alternative nonlinear model, in which the non-linearity is contained in the connection between the effective nucleon mass and the scalar field. Thus the Lagrangian of this model has no extra terms, and consequently deals with fewer parameters as compared with the BB model. The ZM model also yields reasonable values of incompressibility and an effective nucleon mass for nuclear matter. This model was recently used to study the properties of neutron stars by Cheng et al. (1996), who suggested that observations on surface radiation of neutron stars may discriminate between these two models.

The application of the RMF theory to studies of supernova matter is also of much interest. Sumiyoshi & Toki (1994) and Chiapparini, Rodrigues & Duarte (1996) have studied the EOS for nonstrange dense matter in a supernova core by using the BB model. The scope of our work is to investigate the properties for asymmetric nuclear matter and to study the EOS of supernova matter, by using the ZM model. We arrange this paper as follows. In Sect. 2, we describe the framework based on the RMF theory. In Sect. 3, we calculate the properties of asymmetric nuclear matter and supernova matter. Astrophysical implications of our results are discussed in Sect. 4, and conclusions are given in the final section.

2. The ralativistic mean-field theory

In the RMF theory, the strong interaction is described by the exchange of mesons between nucleons through the Yukawa couplings. But in the model of Walecka (1974) only isoscalar-scalar and isoscalar-vector mesons are included. In order to describe actual nuclear systems, it is necessary to introduce proton-neutron asymmetry effects. This is done by adding the isovector-vector meson contribution. We follow the notation of Walecka (1974). The Lagrangian density of the system is given by

$$\begin{aligned}\mathscr{L} = & -\sum_{b=p,n}\{\bar{\psi}_b[\gamma_\mu(\partial^\mu - m^* - ig_v\gamma_\mu V^\mu - ig_\rho\gamma_\mu\tau_{3b}R^\mu]\psi_b\} \\ & -\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi + m_s^2\phi^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_v^2V_\mu V^\mu \\ & -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}m_\rho^2R_\mu R^\mu \end{aligned} \quad (1)$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu\,, \quad (2)$$

$$G_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu\,, \quad (3)$$

$$m^* = \frac{m}{1 + g_s\phi/m}\,. \quad (4)$$

Here ψ, ϕ, V^μ and R^μ denote the fields of baryons, attractive isoscalar-scalar mesons, repulsive isoscalar-vector mesons, and isovector-vector mesons with masses of m_s, m_v and m_ρ, respectively. The constants g_s, g_v and g_ρ are coupling constants for interactions between mesons and nucleons. The m^* is the effective nucleon mass. The Lagrangian density (1) with Eq. (4) is referred to as the ZM model.

In this work, we only investigate nuclear matter without strangeness, and we do not consider pion condensation, kaon condensation and fields of strange hadrons. Using the BB model, several authors (Glendenning 1985; Ellis et al. 1991) have studied neutron-star matter in the RMF theory with strangeness degrees of freedom and discussed contamination of strange hadrons in neutron stars. Using the ZM model, Prakash, et al. (1995) have studied quark-hadron phase transitions in protoneutron stars. The possibility of strange quark stars has been also discussed (e.g., Witten 1984; Alcock,et al. 1986; Haensel, et al. 1986). Pion condensation and kaon condensation in the RMF theory and their applications to neutron stars have been studied (Glendenning et al. 1983; Thorsson, et al. 1994; Dai & Cheng 1997).

Starting with Eq. (1), we derive a set of the Euler-Lagrange equations. The Dirac equation for the nucleon field is given by

$$[\gamma_\mu \partial^\mu - m^* - ig_v \gamma_\mu V^\mu - ig_\rho \gamma_\mu \tau_{3b} R^\mu]\psi_b = 0\,, \tag{5}$$

and the Klein-Gordon euqations for the meson fields are written as

$$\partial_\nu \partial^\nu \phi - m_s^2 \phi = \frac{\partial m^*}{\partial \phi} \sum_b \bar{\psi_b}\psi_b\,, \tag{6}$$

$$\partial_\nu F^{\mu\nu} - m_v^2 V^\mu = -ig_v \sum_b \bar{\psi_b}\gamma^\mu \psi_b\,, \tag{7}$$

$$\partial_\nu G^{\mu\nu} - m_\rho^2 R^\mu = -ig_\rho \sum_b \bar{\psi_b}\gamma^\mu \tau_{3b}\psi_b\,. \tag{8}$$

We consider static infinite matter so that we can obtain simplified equations, where the derivative terms vanish automatically, due to the translational invariance of infinite matter. Setting $\phi \to \phi_0$, $V_\mu \to iV_0\delta_{\mu 4}$, and $R_\mu \to iR_0\delta_{\mu 4}$, we have the meson fields expressed by the expectation values of the ground state,

$$\phi_0 = \frac{g_s/m_s^2}{(1+g_s\phi_0/m)^2} \sum_b < \bar{\psi_b}\psi_b >\,, \tag{9}$$

$$V_0 = \frac{g_v}{m_v^2} \sum_b < \psi_b^\dagger \psi_b >\,, \tag{10}$$

$$R_0 = \frac{g_\rho}{m_\rho^2} \sum_b < \psi_b^\dagger \tau_{3b}\psi_b > \,. \tag{11}$$

Next, according to the standard procedure of Walecka (1974), we derive the energy density of the system,

$$\begin{aligned}\epsilon &= \frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}\frac{g_v^2}{m_v^2}(\rho_p+\rho_n)^2 + \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}(\rho_p-\rho_n)^2 \\ &\quad + \frac{1}{8\pi^2}m^{*4}[f_1(k_{Fp}/m^*) + f_1(k_{Fn}/m^*)]\,,\end{aligned} \tag{12}$$

where

$$f_1(x) = 2x(x^2+1)^{3/2} - x(x^2+1)^{1/2} - \ln(x+\sqrt{x^2+1})\,, \tag{13}$$

and the pressure,

$$\begin{aligned}P &= -\frac{1}{2}m_s^2\phi_0^2 + \frac{1}{2}\frac{g_v^2}{m_v^2}(\rho_p+\rho_n)^2 + \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}(\rho_p-\rho_n)^2 \\ &\quad + \frac{1}{24\pi^2}m^{*4}[f_2(k_{Fp}/m^*) + f_2(k_{Fn}/m^*)]\,,\end{aligned} \tag{14}$$

where

$$f_2(x) = 2x^3(x^2+1)^{1/2} - 3x(x^2+1)^{1/2} + 3\ln(x+\sqrt{x^2+1})\,. \tag{15}$$

In Eqs. (12) and (14), k_{Fp} and k_{Fn} are the proton and neutron Fermi energies, and ρ_p and ρ_n are the proton and neutron number densities. The effective nucleon mass is calculated through the following equations:

Table 1. The parameters of the ZM model

g_s	g_v	g_ρ	m (MeV)	m_s (MeV)	m_v (MeV)	m_ρ (MeV)
5.824	6.417	1.373	938	420	783	763

$$m^* = \frac{m}{1+g_s\phi_0/m}\,, \tag{16}$$

and

$$\phi_0 = \frac{1}{2\pi^2}\frac{g_s m^{*3}}{m_s^2(1+g_s\phi_0/m)^2}[f_3(k_{Fp}/m^*) + f_3(k_{Fn}/m^*)]\,, \tag{17}$$

where

$$f_3(x) = x(x^2+1)^{1/2} - \ln(x+\sqrt{x^2+1})\,. \tag{18}$$

In order to investigate the properties of supernova matter, we must give some conditions. In a supernova core, neutrinos are trapped and form an ideal Fermi-Dirac gas. Thus, the weak process is $p + e^- \leftrightarrow n + \nu_e$. The chemical equilibrium requires

$$\mu_p + \mu_e = \mu_n + \mu_\nu \tag{19}$$

with the chemical potentials of proton and neutron being

$$\mu_p = \frac{g_v^2}{m_v^2}\rho + \frac{g_\rho^2}{m_\rho^2}(\rho_p - \rho_n) + \sqrt{{k_{Fp}}^2 + m^{*2}}\,, \tag{20}$$

and

$$\mu_n = \frac{g_v^2}{m_v^2}\rho - \frac{g_\rho^2}{m_\rho^2}(\rho_p - \rho_n) + \sqrt{{k_{Fn}}^2 + m^{*2}}\,, \tag{21}$$

where ρ is the total baryon number density. The second condition is charge neutrality,

$$\rho_e + \rho_\mu = \rho_p\,, \tag{22}$$

where ρ_e and ρ_μ are the electron and muon number densities respectively. The third condition is to fix leptonic concentration,

$$Y_l = \frac{\rho_e + \rho_\nu}{\rho} \equiv Y_e + Y_\nu \tag{23}$$

with ρ_ν being the neutrino number density.

From Centelles et al. (1992), we choose the parameter set in Table 1. Please note that there is a numerical factor of $1/2$ in front of the ρ-meson coupling constant in the Lagrangian density of Centelles et al. (1992) and Cheng et al. (1996). But, this factor has been absorbed into the ρ-meson coupling constant in Eq. (1), so in the present paper the value of this parameter has decreased by a factor of two as compared to the quoted value of Cheng et al. (1996). We have used this parameter set to calculate the binding energies, radii and diffusenesses of ^{40}Ca and ^{208}Pb. The results are well consistent with those of Centelles et al. (1992) (Cheng et al. 1996; for details see Yao 1996). As shown next section, this parameter set also yields very satisfactory saturation properties (incompressibility, saturation density and binding energy per nucleon) of symmetric nuclear matter.

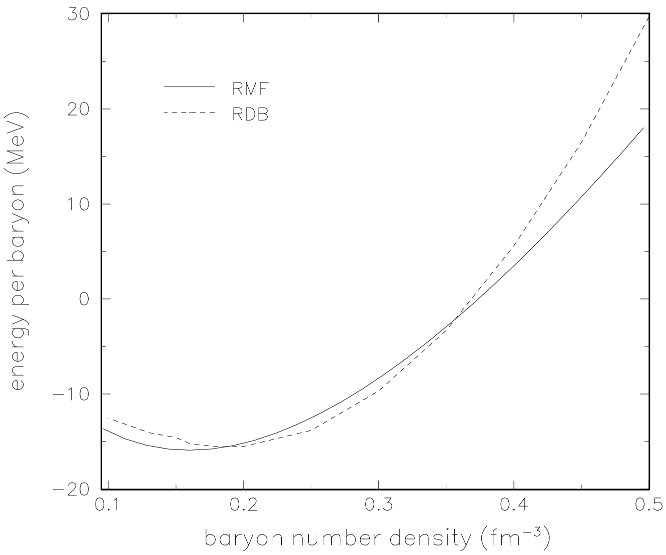

Fig. 1. The energy per nucleon versus baryon density for symmetric nuclear matter based on the ZM model, and the relativistic Dirac-Brueckner theory (RDB) of Li et al. (1992).

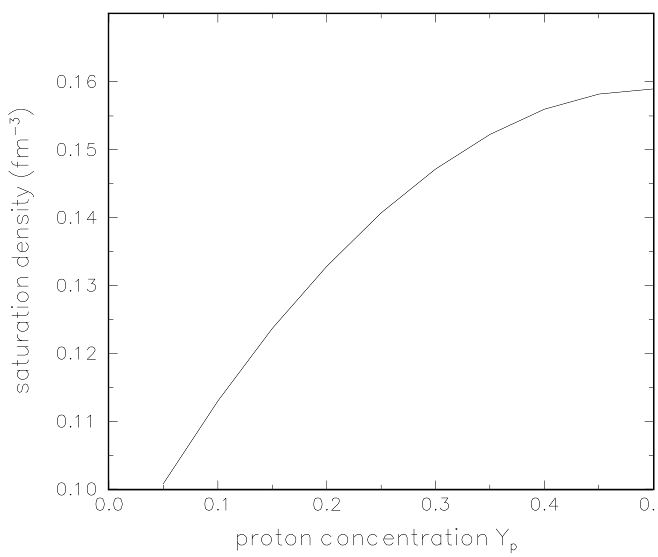

Fig. 2. The saturation density of asymmetric nuclear matter as a function of proton concentration.

3. Results

3.1. Properties of nuclear matter

In Fig. 1 we show the energy per nucleon of symmetric nuclear matter calculated in the RMF theory. As a comparison, we also display the result based on the relativistic Dirac-Brueckner (RDB) theory calculated by Li et al. (1992). It is obvious that the RMF result resembles the RDB one. In the RMF theory, the bulk incompressibility (K), saturation density (ρ_0) and binding energy per nucleon (E_b) of symmetric nuclear matter are 227.6 MeV, 0.1589 fm^{-3} and -15.93 MeV, respectively. These values are very close to those derived from the nuclear experiments. In addition, the bulk symmetry energy coefficient (E_v) at the saturation density is 29.35 MeV.

Next we apply the RMF theory to asymmetric nuclear matter. In Figs. 2 and 3 we show the dependence of the saturation density and the bulk incompressibility on the proton concentration Y_p, respectively. The saturation density decreases as the proton concentration decreases from Y_p = 0.5 to Y_p = 0.05 as shown in Fig. 2. The incompressibility at the saturation density rapidly decreases with decreasing the proton concentration from Y_p = 0.5. At Y_p = 0.05, the saturation density is 0.1008 fm^{-3} and the incompressibility is 114.2 MeV. These results are rather different from those of Sumiyoshi & Toki (1994), who used the BB model in the RMF theory and found that the saturation point of asymmetric matter disappears around Y_p = 0.05 where the incompressibility becomes almost zero.

3.2. Properties of neutron stars

We now apply the RMF theory to a study of neutron-star matter, which is composed of neutrons, protons, electrons and *muons* under the conditions of beta equilibrium and charge neutrality.

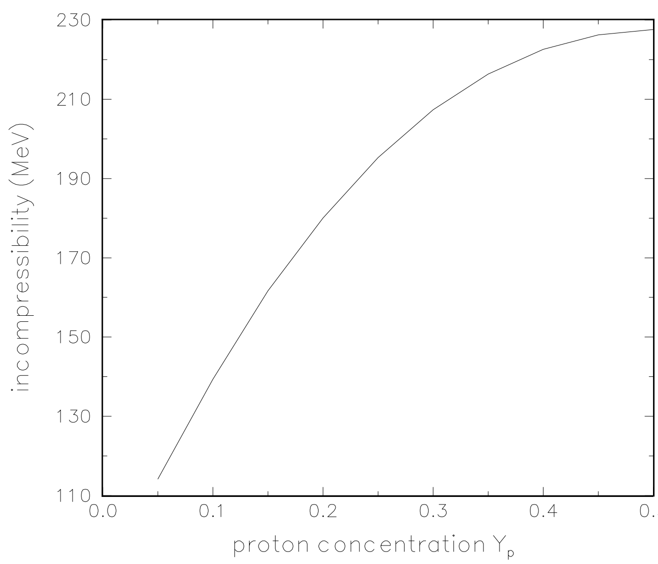

Fig. 3. The bulk incompressibility of asymmetric nuclear matter as a function of proton concentration.

Cheng et al. (1996) has already taken this application into account but did not consider the contribution of muons. Here we do not consider the contributions of hyperons or other exotic states such as meson condensations or quark matter.

In Fig. 4 we show the proton and electron concentrations as functions of baryon number density. As a comparison, we also give the results of Cheng et al. (1996). The proton concentration slightly increases with increasing ρ at high densities. At ρ = 1.0 fm^{-3}, the proton concentration is 0.085. This value is larger than 0.07 in the case without muons. In Sect. 4 we will discuss an astrophysical implication of this result.

Giving the EOS, we can calculate the hydrostatic structure of neutron stars by solving the Tolman-Oppenheimer-Volkoff

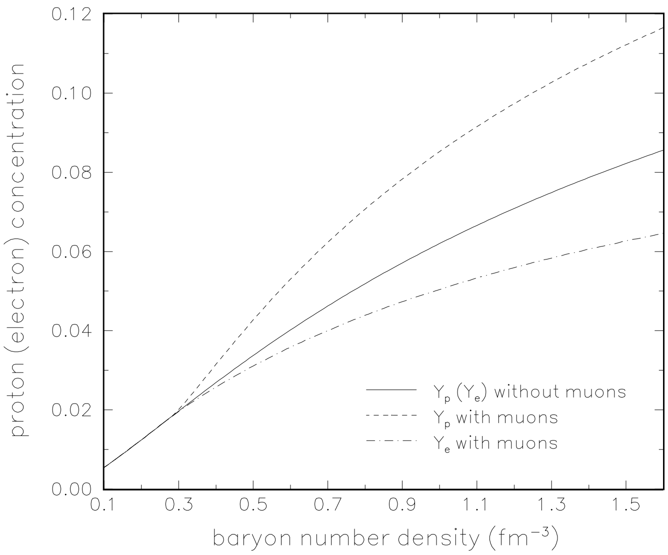

Fig. 4. The proton and electron concentrations as functions of baryon number density.

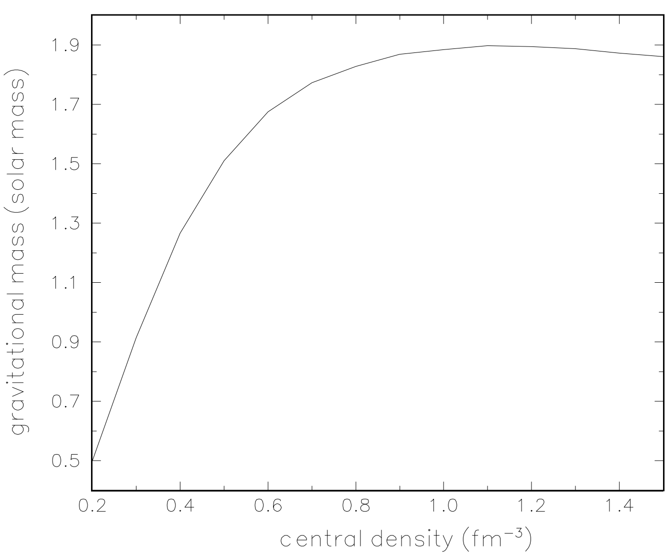

Fig. 5. Total mass versus central density of neutron stars.

equtaion. In our calculations, we use the EOS obtained by Haensel, et al. (1989) for the outer crusts of the stars, and the EOS derived by Baym, et al. (1971) for the inner crusts from the neutron-drip density to $0.14\,\mathrm{fm}^{-3}$. These EOSs have been rather accurately expressed as some polynomials by Bao et al. (1994). The structure of neutron stars is displayed in Fig. 5, which shows the mass as a function of the central density, and which shows that the maximum mass of neutron stars is $1.9M_\odot$ at a central density of $\rho_c \approx 1.1\,\mathrm{fm}^{-3}$ with a radius of $R \approx 11.0$km.

3.3. Properties of supernova matter

We use the RMF theory to study the properties of supernova matter, which consists of protons, neutrons, electrons, muons

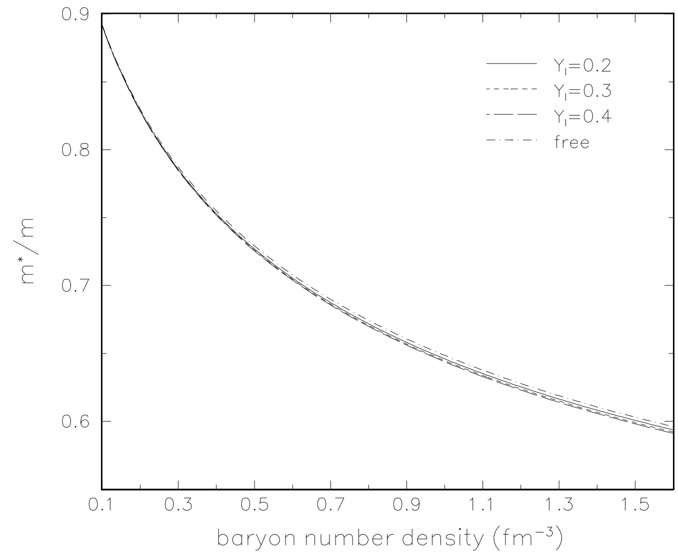

Fig. 6. The effective nucleon mass as a function of baryon number density for different lepton concentrations. "free" denotes the case of neutron-star matter.

and neutrinos. Here we want to point out that our calculations are done at zero temperature and zero entropy (i.e. neglecting thermal effects) and in the density range of $\rho \geq 0.1\,\mathrm{fm}^{-3}$. This is because below $0.1\,\mathrm{fm}^{-3}$ (i) the Fermi energies of particles may not be much larger than kT (where T is the matter temperature), (ii) matter is expected to contain nuclei, and (iii) the ZM model gives negative pressure.

For the ZM model, we find that the muon concentration is negligibly small. This conclusion is similar to that of Chiapparini et al. (1996) based on the BB model. Fig. 6 shows the effective nucleon mass as a function of baryon number density for different lepton concentrations. Clearly, the effective nucleon mass decreases as the baryon density increases, but is almost insensitive to the lepton concentration.

Now we define the sound velocity and the adiabatic index as

$$V_s \equiv \left(\frac{\partial P}{\partial \epsilon}\right)^{1/2}, \tag{24}$$

and

$$\Gamma \equiv \frac{\partial \ln P}{\partial \ln \epsilon}. \tag{25}$$

In supernova studies, in order to determine the sonic point of the hydrodynamical evolution of the matter, it is necessary to know the sound velocity as a function of density. In Fig. 7 we show the influence of the neutrino trapping on the sound velocity. This influence is not significant. Fig. 8 gives the dependence of the adiabatic index on baryon number density. It can be seen from this figure that the adiabatic index strongly increases at a density near $0.1\,\mathrm{fm}^{-3}$ in the case of neutron-star matter. The reason for this is that the pressure near this density in neutrino-free matter greatly decreases as compared with in neutrino-trapped matter

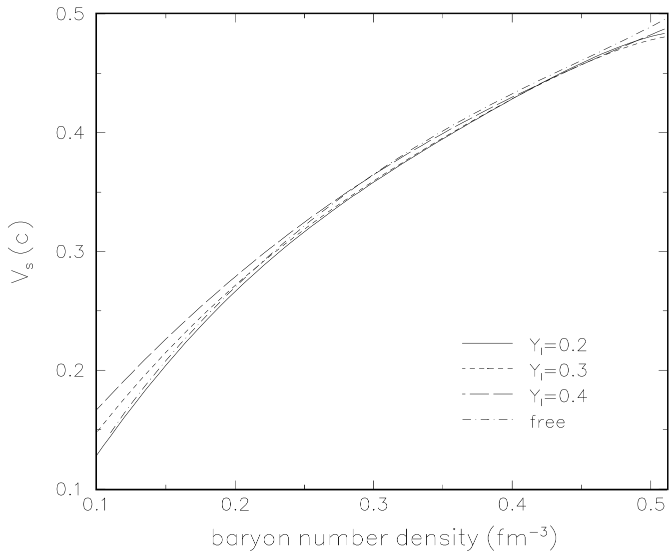

Fig. 7. The sound velocity as a function of baryon number density for different lepton concentrations. "free" denotes the case of neutron-star matter.

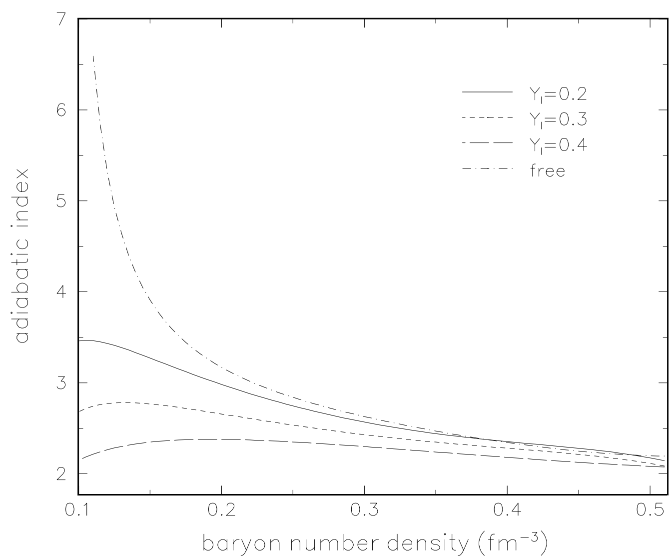

Fig. 8. The adiabatic index as a function of baryon number density for different lepton concentrations. "free" denotes the case of neutron-star matter.

(cf. Fig. 9) and then becomes negative at lower densities. We will simply discuss an implication of this behavior of the adiabatic index next section.

In Fig. 9 we show the effect of neutrino trapping on the equation of state. When neutrinos are taken into account their contribution to the pressure stiffens the EOS at low densities more significantly than that in Chiapparini et al. (1996). The numerical values for the pressure and energy density as functions of baryon number density are listed in Tables 2-5.

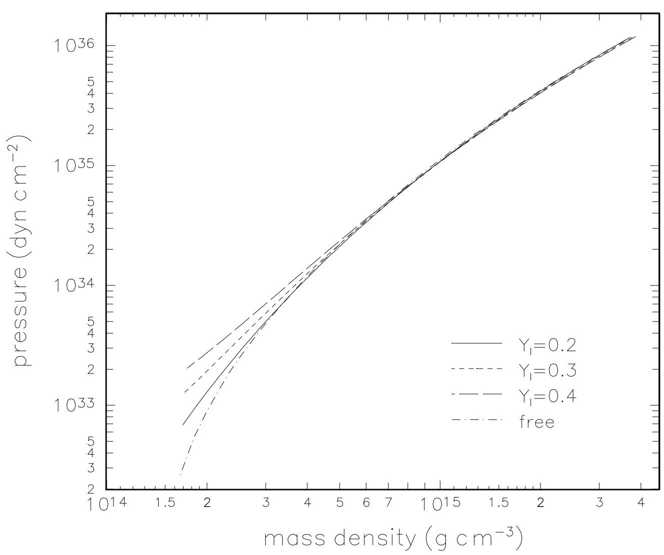

Fig. 9. The pressure as a function of mass density for different lepton concentrations. "free" denotes the case of neutron-star matter.

4. Discussion

We have used the ZM model in the relativistic mean-field theory to study the properties of asymmetric nuclear matter and the equation of state of supernova matter. We have also investigated the structure of neutron stars.

We now discuss astrophysical implications of our EOS for neutron star matter. First, we make an estimate of the maximum rotation frequency ($\Omega_{\rm max}$) of neutron stars, which is determined by the mass shedding condition. Haensel & Zdunik (1989) first noticed that for realistic EOSs of dense matter, the numerically calculated values of $\Omega_{\rm max}$ can be fitted by an empirical formula

$$\Omega_{\rm max} = C\left(\frac{GM_{\rm max}}{R_{\rm max}^3}\right)^{1/2}, \tag{26}$$

where $M_{\rm max}$ is the maximum mass of the nonrotating neutron stars with the same EOS, $R_{\rm max}$ is the radius corresponding to $M_{\rm max}$, and C is a dimensionless phenomenological constant. Haensel and collaborators (Haensel & Zdunik 1989; Haensel, et al. 1995; Lasota, et al. 1996) further found that the best fit is for $C = 0.468 + 0.378x_s$, where $x_s \equiv 2GM_{\rm max}/R_{\rm max}c^2$. Therefore, for our EOS, $\Omega_{\rm max} = 9.13 \times 10^3\ {\rm s}^{-1}$, which leads to the minimum rotation period of 0.69 ms. Second, for a neutron star with the gravitational mass of $1.4M_{\odot}$, our EOS reproduces the ratio of the inner crust to total moments of inertia to be about 8.6%, which is consistent with the result from the analysis of observational data of the glitches of four pulsars in Link, et al. (1992). Third, we found that the proton concentration at the density of $\sim 1\,{\rm fm}^{-3}$ in neutron stars is about 0.085. This value is smaller than 15%, which is required for the operation of direct Urca process (Lattimer et al. 1991). Thus, the modified Urca process is a main neutrino reaction in neutron stars. This conclusion is opposite to that of many authors (e.g., Boguta 1981; Glendenning 1985; Sumiyoshi & Toki 1984; Sumiyoshi

Table 2. The energy density and pressure of neutron-star matter based on the ZM model

ρ (fm^{-3})	mass density ($g\,cm^{-3}$)	pressure ($dyn\,cm^{-2}$)
0.10	0.166E+15	0.257E+33
0.15	0.250E+15	0.248E+34
0.20	0.336E+15	0.694E+34
0.25	0.422E+15	0.139E+35
0.30	0.511E+15	0.233E+35
0.35	0.602E+15	0.354E+35
0.40	0.695E+15	0.500E+35
0.45	0.791E+15	0.672E+35
0.50	0.889E+15	0.871E+35
0.55	0.990E+15	0.110E+36
0.60	0.109E+16	0.135E+36
0.65	0.120E+16	0.162E+36
0.70	0.131E+16	0.193E+36
0.75	0.142E+16	0.225E+36
0.80	0.153E+16	0.261E+36
0.85	0.165E+16	0.299E+36
0.90	0.177E+16	0.339E+36
0.95	0.189E+16	0.382E+36
1.00	0.201E+16	0.428E+36
1.05	0.214E+16	0.476E+36
1.10	0.227E+16	0.526E+36
1.15	0.240E+16	0.579E+36
1.20	0.254E+16	0.634E+36
1.25	0.267E+16	0.692E+36
1.30	0.281E+16	0.753E+36
1.35	0.296E+16	0.815E+36
1.40	0.310E+16	0.881E+36
1.45	0.325E+16	0.948E+36
1.50	0.340E+16	0.102E+37
1.55	0.356E+16	0.109E+37
1.60	0.372E+16	0.117E+37

et al. 1995a; Cheng et al. 1996), who found that the proton concentration in neutron stars based on BB model in the RMF theory is so large that the direct Urca process occurs. This may provide an observational signature for the ZM model.

We next discuss the effects of our EOS for supernova matter on Type II supernovae. First, we explore the role of the bulk incompressibility on the "prompt" phase of Type II supernovae. Baron et al. (1985b) studied the supernova explosions systematically by using the following parameterized EOS:

$$P = \frac{K(Y_p)\rho_0(Y_p)}{9\gamma}\left\{\left[\frac{\rho}{\rho_0(Y_p)}\right]^\gamma - 1\right\}, \qquad (27)$$

where the saturation density $\rho_0(Y_p)$ and the incompressibility $K(Y_p)$ determine the behavior of the saturation properties of asymmetric nuclear matter at the proton concentration, Y_p, and the adiabatic index γ expresses the stiffness of the EOS. They proposed that the value of the incompressibility at $Y_p \sim 0.33$ is a key quantity of prompt explosions, and further found that $K(0.5)$ should be less than 180 MeV for the successful prompt explosions of massive stars with $15M_\odot$. This value is too small as compared to the experimental value extracted from the nuclear physics, and the softness of the EOS expressed by Eq. (27) with K = 180 MeV and γ = 2.5 violates the constraint from the $1.44M_\odot$ mass of PSR 1913+16 (Swesty et al. 1994). In our work, the value of K at Y_p = 0.33 is 210 MeV and the value at Y_p = 0.5 is 227.6 MeV. Thus, we can conclude that if the dense matter in a supernova core is described by the ZM model in the RMF theory, the prompt explosion mechanism cannot work. Furthermore, Swesty et al. (1994) used the Lattimer & Swesty's (1991) EOS and numerically showed that there is no

Table 3. The energy density and pressure of supernova matter based on the ZM model (Y_l = 0.2)

ρ (fm^{-3})	mass density ($g\,cm^{-3}$)	pressure ($dyn\,cm^{-2}$)
0.10	0.169E+15	0.686E+33
0.15	0.255E+15	0.296E+34
0.20	0.341E+15	0.735E+34
0.25	0.429E+15	0.141E+35
0.30	0.519E+15	0.233E+35
0.35	0.611E+15	0.351E+35
0.40	0.705E+15	0.495E+35
0.45	0.801E+15	0.665E+35
0.50	0.899E+15	0.861E+35
0.55	0.100E+16	0.108E+36
0.60	0.110E+16	0.133E+36
0.65	0.121E+16	0.161E+36
0.70	0.132E+16	0.191E+36
0.75	0.143E+16	0.223E+36
0.80	0.154E+16	0.259E+36
0.85	0.166E+16	0.296E+36
0.90	0.177E+16	0.337E+36
0.95	0.189E+16	0.380E+36
1.00	0.202E+16	0.425E+36

Table 4. The energy density and pressure of supernova matter based on the ZM model (Y_l = 0.3)

ρ (fm^{-3})	mass density ($g\,cm^{-3}$)	pressure ($dyn\,cm^{-2}$)
0.10	0.172E+15	0.128E+34
0.15	0.259E+15	0.391E+34
0.20	0.347E+15	0.864E+34
0.25	0.437E+15	0.157E+35
0.30	0.529E+15	0.253E+35
0.35	0.623E+15	0.374E+35
0.40	0.719E+15	0.521E+35
0.45	0.817E+15	0.694E+35
0.50	0.917E+15	0.893E+35
0.55	0.102E+16	0.112E+36
0.60	0.113E+16	0.137E+36
0.65	0.123E+16	0.165E+36
0.70	0.134E+16	0.195E+36
0.75	0.146E+16	0.228E+36
0.80	0.157E+16	0.263E+36
0.85	0.169E+16	0.301E+36
0.90	0.181E+16	0.342E+36
0.95	0.193E+16	0.385E+36
1.00	0.206E+16	0.431E+36

Table 5. The energy density and pressure of supernova matter based on the ZM model (Y_l = 0.4)

ρ (fm^{-3})	mass density ($g\,cm^{-3}$)	pressure ($dyn\,cm^{-2}$)
0.10	0.174E+15	0.204E+34
0.15	0.263E+15	0.515E+34
0.20	0.354E+15	0.104E+35
0.25	0.446E+15	0.180E+35
0.30 0	0.540E+15	0.281E+35
0.35	0.637E+15	0.408E+35
0.40	0.735E+15	0.561E+35
0.45	0.836E+15	0.740E+35
0.50	0.939E+15	0.945E+35
0.55	0.105E+16	0.118E+36
0.60	0.115E+16	0.143E+36
0.65	0.126E+16	0.172E+36
0.70	0.138E+16	0.203E+36
0.75	0.149E+16	0.236E+36
0.80	0.161E+16	0.272E+36
0.85	0.173E+16	0.311E+36
0.90	0.186E+16	0.352E+36
0.95	0.198E+16	0.396E+36
1.00	0.211E+16	0.442E+36

discernible difference in the shock stall radius for the different incompressibility.

Second, at one time it was thought that the strength of supervova shocks also strongly depends on the bulk symmetry energy of nuclear matter. For example, Bruenn (1989a) investigated the role of the symmetry energy using the Cooperstein & Baron EOS, and found that decreasing the symmetry energy leads to weaker shocks. However, the work of Swesty et al. (1994) indicates that the shock stall radius is practically independent of the symmetry energy. In addition, Swesty et al. (1994) also found that the rate of electron capture which deleptonizes the core during the collapse epoch increases with increasing the symmetry energy. Our EOS in the RMF theory gives a smaller value of the symmetry energy (29.35 MeV), which may result in a smaller electron-capture rate during the collapse and a larger trapped lepton concentration (Y_l) at the core bounce. This further leads to a stiffer EOS of the post-bounce core than for a smaller Y_l (see Fig. 9), and a larger radius of the protoneutron star for a given mass.

Third, the delayed explosion mechanism has been of particular interest in recent years. In this mechanism the stall shock is reheated through neutrino energy deposition behind it and is revived over hundreds of milliseconds. Clearly, a high neutrino luminosity is important for a successful shock. Using Sumiyoshi, Suzuki & Toki's (1995b) results based on the BB model, we can make an estimate of shock energy. Since our value of the symmetry energy is close to that for their parameter set TMS, Y_l at the core bounce in our work is $\sim 10\%$ larger than for their parameter set TM1. According to Burrows & Goshy's (1993) analytic theory of one-dimensional neutrino-heated supernovae, the total energy (E_s) which the shock obtains during the reheating stage is nearly proportional to $Y_l^{3.5}$. Thus, E_s in our work may be $\sim$ 30% larger than that for the parameter set TM1 of Sumiyoshi et al. (1995b). Therefore, our EOS in the RMF theory is likely to be favourable to the operation of the delayed explosion mechanism. Of course, convection in the region between the shock front and the neutrinosphere can rapidly transport neutrino energy into the region behind the shock and thus the shock will obtain more energy.

Fourth, Fig. 8 implies that during the evolution of a protoneutron star into a neutron star the adiabatic index near the baryon number density of 0.1 fm^{-3} strongly increase. This behavior might result in the Rayleigh-Taylor instability in a layer of the inner crust. This is because during the evolution of a protoneutron star neutrinos in the region close to 0.1 fm^{-3} diffuse more rapidly than neutrinos at higher densities do so that under the effect of gravity and pressure the mass density at a baryon number density near 0.1 fm^{-3} can be larger than that at a higher baryon number density at some evolution stage. This Rayleigh-Taylor instability might have an important impact on the evolution of the protoneutron star and the origin of a magnetic field of the neutron star (Thompson & Duncan 1993).

Finally, we note that Goussard, et al.(1997) recently studied the maximum rotation frequency of uniformly rotating protoneutron stars in general relativity and further gave an empirical formula for this frequency. This formula actually coincides with used in cold neutron stars (e.g., Eq. [26]). Thus, our EOS for dense matter in supernova cores can also be applied to a study of the maximum rotation frequency of uniformly rotating protoneutron stars.

5. Conclusions

We have compared two RMF models for dense matter: one is the BB model in which cubic and quartic terms for the scalar field are included in the Lagrangian as nonlinear terms, and another is the ZM model in which the nonlinearity is contained in the connection between the effective nucleon mass and the scalar field. In this paper we have used the latter model to study the properties of asymmetric nuclear matter and the equation of state for supernova matter. We find that even though muons are included in the chemical composition of neutron-star matter, the ZM model still results in a small proton concentration so that only the modified Urca process occurs as a main neutrino reaction in neutron stars. This conclusion is opposite to that based on the BB model. This may provide an observational signature for the ZM model.

The incompressibility of dense matter based on the ZM model is consistent with both the experimental values extracted from the nuclear physics and the constraints placed on the EOS by observations of neutron star masses. If the EOS for supernova matter is described by the ZM model, the prompt mechanism for Type II supervovae cannot work. But this EOS is likely to be favourable to the delayed explosion mechanism because this model can lead to a large trapped lepton concentration at the core bounce and a large radius of the protoneutron star.

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