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Magnetic activity in young stellar objects caused by tearing instabilities

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Abstract. Magnetic activity phenomena are characteristic for T-Tauri stars as well as protostellar class I objects. A model for the flare-like emissivity based on a macroscopic plasma instability, the resistive tearing mode, is proposed. The dispersion relation for the unstable modes for the appropriate partially ionized dusty plasma regime is presented and solved numerically. The results indicate that tearing instabilities operating in current sheets formed within magnetic flux tubes that basically constitute magnetospheres around young stellar objects are a possible explanation for the temporal variability and spatial extensions of bursty radiation emitting regions around T-Tauri stars.

Key words: stars: pre-main sequence; magnetic fields – MHD – instabilities – plasmas

1. Introduction

Low-mass pre-main-sequence stars are characterized by magnetic activity phenomena, i.e. temporal highly variable bursty radiative emissions (X-ray, radio, as well as optical and UV), during the stellar T-Tauri phase (Feigelson and DeCampli 1981; Montmerle 1991; White et al. 1992; André 1996). Recently, ASCA and ROSAT observations have proven that stellar objects also exhibit non-thermal flare-like radiation during an earlier phase of the stellar evolution, namely during the protostellar class I phase (Koyama et al. 1996; Grosso et al. 1997). It has often been argued that magnetic activity phenomena in the context of young stellar objects (YSO) should be understood in analogy to solar coronal flares. However, besides the fact that flares in YSO differ from solar flares in the total released energy amount, the harder spectrum as well as the size of the emitting region, the plasma parameter regime in the YSO-context is totally different from the one of the solar corona (cf. Montmerle 1991; Königl 1994; Koyama et al. 1996). In the YSO context we have to deal with a partially ionized dusty plasma filling the magnetospheres around T-Tauri stars and protostellar class I objects, since the relatively large size of the emitting regions indicates that the magnetic activity processes are not restricted to the more or less fully ionized narrow region immediately above the stellar surface.

A well known macroscopic plasma instability widely believed to be an important process which leads to the formation of coherent structure and eruptive energy conversion in laboratory (e.g. Yur et al. 1995) as well as astrophysical (e.g. Priest 1984) plasmas is the resistive tearing instability (Furth et al. 1963). In the context of solar flares it could be shown that the tearing instability indeed plays a dominant role (e.g. van Hoven 1981; Janicke 1982; Steinolfson and van Hoven 1984; Birk and Otto 1991). It is the aim of this contribution to discuss tearing instabilities in the context of YSO magnetic activity for the appropriate partially ionized dust plasma regime. In the next section the scenario of tearing unstable current sheets/ flux tubes in YSO magnetospheres is outlined. The governing equations are formulated in Sect. 3. In Sect. 4 the dispersion relation is presented (some technical details are given in an Appendix) and numerical solutions are shown. Eventually, Sect. 5 is devoted to a discussion of the results.

2. The role of tearing instabilities

The accretion disk of a YSO is somehow connected with the the dipole magnetic field of the central object (see Fig. 1). The interaction of the dipole field with the disk can be quite complicated (e.g. Shu et al. 1994; Lovelace et al. 1995) but the rotation of the disk should result in a significant twist of closed magnetic flux tubes anchored at the surface of the YSO thereby injecting continuously magnetic helicity into the flux tubes (cf. Hayashi et al. 1996; Li 1996). Consequently, a sheared magnetic field configuration with electric currents flowing develops.

This situation is favorable for magnetic reconnection and is very similar to the physics of rapid bursters (Aly and Kuijpers 1990; Kuijpers and Kuperus 1995), where neutron stars are interlinked magnetically with accretion disks, as well as to particle acceleration phenomena in the magnetospheres of active galactic nuclei (Lesch & Birk 1997). Moreover, it is well documented that solar flares are usually associated with sheared

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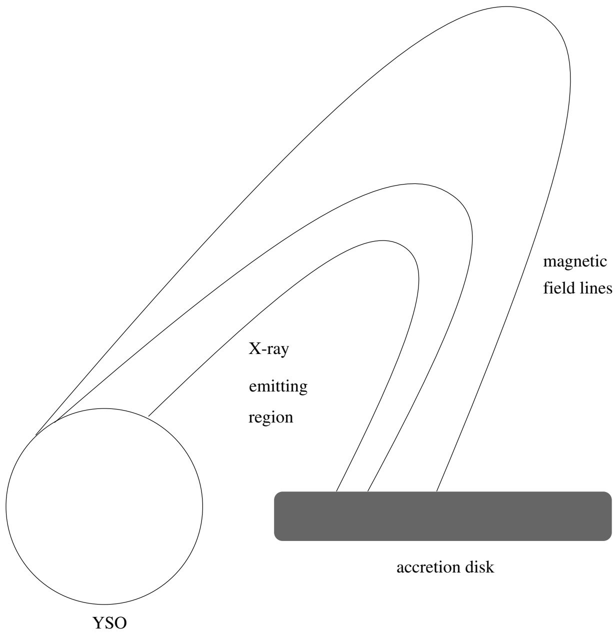

Fig. 1. Illustration of the magnetic interaction of the YSO with the accretion disk.

magnetic fields and thus, with current sheets (e.g. Sturrock 1972; Priest 1983, 1985; van Hoven 1979; Sturrock et al. 1984).

A generic spontaneous type of magnetic reconnection processes in current sheets is the resistive tearing instability which is caused by the Lorentz force between parallel electric currents. It has long been recognized as a potential candidate for initiating the release and conversion of magnetic energy stored in sheared astrophysical magnetic fields. This instability process can operate at locations where a magnetic field component reverses direction. It results in a reconnection of magnetic field lines across the plane of field reversal with a subsequent conversion of free magnetic energy to plasma heating and particle acceleration. The acceleration of charged particles in magnetic field-aligned electric fields is a generic feature of reconnection processes (Schindler et al. 1991). The formation of tearing unstable current sheets in magnetospheres of YSO is illustrated in Fig. 2. The main idea is that a differentially rotating disk/magnetosphere in dynamical equilibrium (cf. Li 1996) gives rise to a toroidal magnetic field component (cf. Fig. 2a). Since the star-disk magnetosphere is not self-similar (cf. Paatz and Camenzind 1996) one particular flux tube among others is shown in Fig. 2a. Different flux tubes are characterized by different strengths of the toroidal components. The associated current sheets form along the twisted magnetic flux tubes. Since the rotation continuously injects magnetic helicity into the closed magnetic fields magnetic non-equilibrium has to be expected. The unstable dynamics are characterized by reconnection processes. As illustrated in Fig. 2b the resulting configuration that should be examined with respect to tearing instabilities can be approximated nicely by a one-dimensional electric current sheet. The main component of the magnetic field is directed along the current sheet (i.e. in the z-direction), the shear component changes the sign at the y = 0-plane. Consequently, singular surfaces $\mathbf{k}\cdot\mathbf{B} = 0$ where unstable tearing modes may operate with the wave vector ($\mathbf{k}$) can be found in the plane perpendicular to the flux tube (Fig. 2b), i.e. the x-y-plane in the chosen geometry. The choice of a slab geometry for the analytical treatment can be justified by the fact that the tearing mode in plane current sheets is identical (besides the choice of coordinates) with the internal resistive kink mode (relevant for force-free equilibria in cylindrical geometry) in the long wave length limit, i.e. the m = 1 mode characterized by the fastest growing perturbations (cf. discussion in Priest 1987 and in Biskamp 1993). In course of the tearing dynamics charged particles can be accelerated and give rise to non-thermal radiation.

3. Governing equations

In the context of the magnetospheres of YSO as protostellar class I objects and T-Tauri stars one has to deal with partially ionized dusty magnetoplasmas characterized by a dynamical rather than static dust component that plays an outstanding role for the overall dynamics. We consider a frame comoving with the neutral component ($\mathbf{v}_n \neq 0$). Thus, the relevant fluid balance equations (cf. Birk et al. 1996, Shukla et al. 1997 for slightly different approximations) that describe the low-frequency (with respect to the dust gyro-frequency) dynamics of such multi-species quasineutral plasmas in ionization equilibrium are the continuity equations of the charged components (electrons, ions and dust):

$$\frac{\partial \rho_\alpha}{\partial t} = -\nabla \cdot (\rho_\alpha \mathbf{v}_\alpha) \quad ; \quad \alpha = e, i, d \tag{1}$$

where ρ and $\mathbf{v}$ are the mass density and the fluid velocity and the indices e, i and d denote the electron, ion and dust component, the total momentum balance equation of the charged fluids (electrons and ions are assumed to be inertialess as compared to the dust component):

$$\begin{aligned}\frac{\partial(\rho_d \mathbf{v_d})}{\partial t} = &- \nabla \cdot (\rho_d \mathbf{v_d} \mathbf{v_d}) - \nabla(p_e + p_i + p_d) \\ &+ \frac{c}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} \\ &- \nu_{dn}\rho_d \mathbf{v_d} - \nu_{in}\rho_i \mathbf{v_i} - \nu_{en}\rho_e \mathbf{v_e}\end{aligned} \tag{2}$$

where p, $\mathbf{B}$, $\nu_{\alpha\beta}$ and c denote the thermal pressure, the magnetic field, the collision frequencies of collisions between species α and β (elastic collisions as well as charge exchange) and the velocity of light, and the energy equations as, e.g., the dust energy equation (the other energy equations can be derived accordingly):

$$\begin{aligned}\frac{1}{\gamma_d - 1}\frac{\partial p_d}{\partial t} = &- \frac{1}{\gamma_d - 1}\nabla \cdot (p_d \mathbf{v_d}) - p_d \nabla \cdot \mathbf{v_d} \\ &+ \frac{m_i}{m_d + m_i}\zeta \mathbf{j}^2 + \frac{m_n}{m_n + m_d}\rho_d \nu_{dn} \mathbf{v_d}^2 \\ &- 2\frac{\rho_d \nu_{di}}{m_i + m_d}\left(\frac{k_B T_d}{\gamma_d - 1} - \frac{k_B T_i}{\gamma_i - 1}\right)\end{aligned}$$

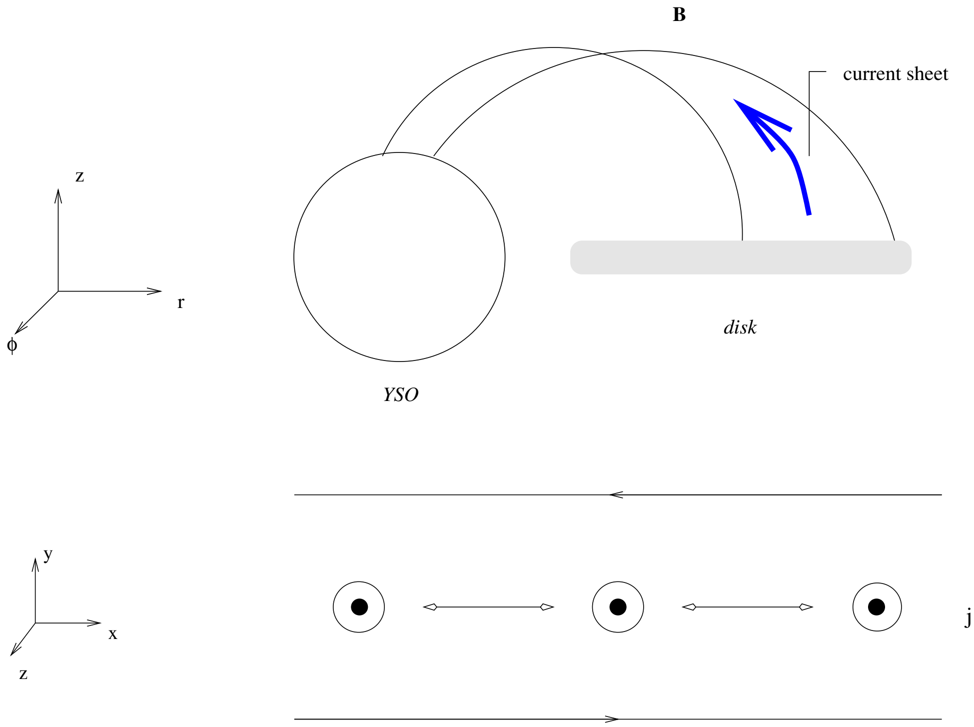

Fig. 2a and b. The magnetic interaction of the YSO with the accretion disk leads to the formation of twisted magnetic flux tubes (**a**) and thereby of rational surfaces (**b**) where unstable tearing modes may operate. In cylindrical geometry the differential rotation results in a finite B_Φ and thereby in a poloidal electric current.

$$- 2\frac{\rho_d \nu_{dn}}{m_n + m_d}\left(\frac{k_B T_d}{\gamma_d - 1} - \frac{k_B T_n}{\gamma_n - 1}\right)$$
$$- 2\frac{\rho_d \nu_{de}}{m_d}\left(\frac{k_B T_d}{\gamma_d - 1} - \frac{k_B T_e}{\gamma_e - 1}\right) \quad (3)$$

where γ, m, ζ, $\mathbf{j}$, and k_B denote the ratio of specific heats, the particle mass, the collisional resistivity, the current density and the Boltzmann constant, respectively. These equations have to be completed by Ohm's law or an induction equation that governs the dynamical evolution of the magnetic field. It can be derived (cf. Birk et al. 1996) from the inertialess ion momentum equation

$$0 = n_i e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}\right) - \nabla p_i - n_i m_i \nu_{id}(\mathbf{v}_i - \mathbf{v}_d)$$
$$-n_i m_i \nu_{in}(\mathbf{v}_i - \mathbf{v}_n) n_i m_i \nu_{ie}(\mathbf{v}_i - \mathbf{v}_e) \quad (4)$$

where $\mathbf{E}$ is the electric field and e the elementary charge. We neglect the Hall-like as well as the pressure term and obtain

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_d \times \mathbf{B}) + \frac{c^2}{4\pi}\eta \Delta \mathbf{B} \quad (5)$$

with the constant collisional resistivity $\eta = m_i^2(\nu_{id} + \nu_{in} + \nu_{ie})/e^2\rho_i$ which may alternatively be replaced by a turbulent resistivity caused by microinstabilities.

In deriving Eq. (5) we have used Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \quad (6)$$

and Amperère's law

$$\nabla \times \mathbf{B} = \frac{4\pi e}{c}(n_i \mathbf{v}_i - n_e \mathbf{v}_e - n_d z_d \mathbf{v}_d) \equiv \frac{4\pi}{c}\mathbf{j} \quad (7)$$

where z_d is the dust charge number. Additionally, we assumed that the dynamical friction is dominated by dust-neutral collisions, i.e. we are dealing with a plasma where the dust component is important for the overall dynamics rather than only an impurity effect.

4. Derivation and solution of the dispersion relation

In deriving the dispersion relation for the resistive tearing instability we concentrate on two-dimensional perturbations and assume incompressibility. We apply a linear mode perturbation analysis starting from a one-dimensional equilibrium configuration (cf. preceding section) characterized by $\mathbf{B} = B_{eq} tanh(y)\mathbf{e}_x + \hat{B}\mathbf{e}_z$ and $n_d = \hat{n}_d + cosh^{-2}(y)$ where $\hat{B}$ and $\hat{n}_d$ are constant. The magnetic field as well as the dust velocity can be expressed in terms of the magnetic vector potential A and the velocity stream function U:

$$\mathbf{B} = \nabla A \times \mathbf{e}_z + \hat{B}\mathbf{e}_z \;\; ; \;\; \mathbf{v}_d = \nabla U \times \mathbf{e}_z + w\mathbf{e}_z \, . \quad (8)$$

In appropriate normalized units (the velocity is normalized to the dust Alfvén velocity $v_A = B_{eq}/\sqrt{4\pi\rho_{d0}}$, the growth rate to the inverse dust Alfvén transit time $\tau_A = l/v_A$, where l is the half-thickness of the equilibrium current layer; S denotes the normalized inverse diffusivity, i.e. the magnetic Reynolds number, $S = 4\pi l v_A/c^2\eta$) the governing linearized equations (cf. Otto 1991) are the z-component of the curl of Eq. (2):

$$\nabla \cdot \left[q\rho_{d0} \left(1 + \frac{\nu_{dn}}{q + \nu_{nd}} \right) \nabla U_1 \right] = (\nabla j_0 \times \mathbf{e}_z) \cdot \nabla A_1 + \mathbf{B}_0 \cdot \nabla \nabla^2 A_1 \quad (9)$$

and the z-component of the curl of Eq. (5):

$$S(qA_1 - \mathbf{B}_0 \cdot \nabla U_1) = \nabla^2 A_1 \quad (10)$$

which decouple from all the other equations (the indices 0 and 1 denote equilibrium and perturbed quantities). In deriving Eq. (9) we have assumed $\nu_{dn} \gg \nu_{in}, \nu_{en}$. Eqs. (9) and (10) determine an eigenvalue problem or, to be more specific, a boundary layer problem. They have to be expanded in an appropriate scaling. The asymptotic matching of the solutions yields the dispersion relation.

We assume the perturbed quantities to vary as

$$\phi_1(x, y) = \tilde{\phi}(y) \exp(qt + ikx) \quad (11)$$

where q denotes the complex growth rate and k the wave number of the mode.

Following the procedure given by Otto (1991) we obtain the dispersion relation (see Appendix for some technical details) similar to the solution given by Otto and Birk (1992) for partially ionized plasmas without any dust component

$$\frac{q^{1/2} k^{1/2} \rho_{d0}^{1/4} \pi}{\eta^{1/2}} \Lambda^{1/2} = (1 - \Lambda^2)(1 - k^2) \frac{\Gamma\left(\frac{\Lambda+1}{4}\right)}{\Gamma\left(\frac{\Lambda+3}{4}\right)} \quad (12)$$

with

$$\Lambda = \frac{q^{3/2}}{k\eta^{1/2}} \left(1 + \frac{\nu_{dn}}{q + \nu_{nd}} \right)^{1/2} \quad (13)$$

which is valid for $k \leq \mathscr{O}(1)$, $\eta \ll 1$ and $\epsilon \ll 1$ in normalized units where ϵ is the width of the inner layer of the boundary problem, i.e. the region where the resistive term in Eq. (10) cannot be neglected. The growth rates of the unstable modes should be reduced with comparison to the tearing instability in totally ionized plasmas (Furth et al. 1963) due to dust-neutral friction. If the ion-neutral collision frequency is comparable to the dust-neutral one, the growth rates are expected to be even more reduced. The z-component of the magnetic field $\hat{B}$, i.e. the component parallel to the current sheet, does not influence the dispersion relation.

We have to solve numerically (by means of an iterative damped Newton method) the dispersion relation (12) for typical parameter sets relevant in the YSO context. One should note that these physical parameters can be quite variable and depend on the distance from the stellar object as well as wind and accretion properties. In order to get a feeling for the quantitative results of tearing instabilities operating in YSO magnetospheres we choose the following set of parameters which seems to be reasonable in the T-Tauri context (cf. Königl 1994; Paatz and Camenzind 1996): The neutral gas particle density as well as the dust particle density are chosen as $n_n = 10^9 \mathrm{cm}^{-3}$ and the ion particle density as $n_i = 10^{11} \mathrm{cm}^{-3}$. For the magnitude of the magnetic field we choose a relatively moderate value $B_{eq} = 10$G. Note that B_{eq} is the shear component of the magnetic field and not the main component of the stellar dipole field. Thus, the dust Alfvén velocity reads $v_A = B_{eq}/\sqrt{4\pi\rho_d} \approx 2 \cdot 10^6 \mathrm{cms}^{-1}$, if we assume, for example, that the dust grains are heavier than the ions by a factor of ~ 1000. Additionally, we assume the dust charge number as $z_d \approx 100$ (the actual value is not of importance for the macroscopic dynamics under consideration).

With these parameters collisions with dust grains are more frequent than electron-ion Coulomb collisions and electron/ion-neutral collisions. The dust collision frequencies can be calculated from the appropriate Landau collision integrals (e.g. Benkadda et al. 1996). The dependence of the ion-dust collision frequency, which determines the collisional resistivity in our case, on dust charge number (the neutral gas temperature was chosen as $T_n = 100K$) and dust particle density is illustrated in Fig. 3, whereas Fig. 4 shows the functional dependence of the dust-neutral collision frequency on neutral gas particle density and temperature.

The ion-dust collisions dominate the electrical resistivity which is of the order of $\eta = 7 \cdot 10^{-8}$s for the above parameters. This implies a magnetic Reynolds number of $S \approx 400$ for a half-thickness of the current sheet of the order of $l \approx 10^9$cm. The Alfvénic transit time in this case reads $\tau_a = l/v_A \approx 500$s.

The dispersion relation for the tearing mode in the considered parameter regime is solved for a variety of magnetic Reynolds numbers of $S = 100$, $S = 1000$, and $S = 10000$ (Fig. 5). A normalized growth rate of $q \approx 0.035$ (for $S = 100$) implies a growth time of the unstable tearing mode of approximately 4 hours. Higher growth rates (due to lower magnetic Reynolds numbers as a result of ion-dust collisions) lead to an even faster development of the mode. Moreover, it should be noted that for the above study a relatively small magnetic field B_{eq} has been chosen. A magnetic field of $B_{eq} = 100$G would instead result in a time scale of the order of 1000s which is comparable to very fast flares (e.g. Feigelson and DeCampli 1981). In principle, varying plasma parameters, e.g. due to some variation in the accretion rate, may explain temporal variety of magnetic activity phenomena from hours up to intraday variability on the grounds of tearing theory. The normalized wave numbers of the most unstable modes (cf. Fig. 5) correspond to a wave length of $2 \cdot 10^{10}$cm. This implies that the spatial extent of the emitting flux tube is of the order of a few 10^{10}cm for the parameters chosen which fits very well to VLBI radiosizes of the emitting regions around (weak) T-Tauri stars (cf. André 1996).

5. Discussion

Magnetic activity in the context of YSO should be understandable on the grounds of plasma physics. It was the aim of this contribution to show that resistive instabilities can be excited in YSO magnetospheres thereby playing an important role in the

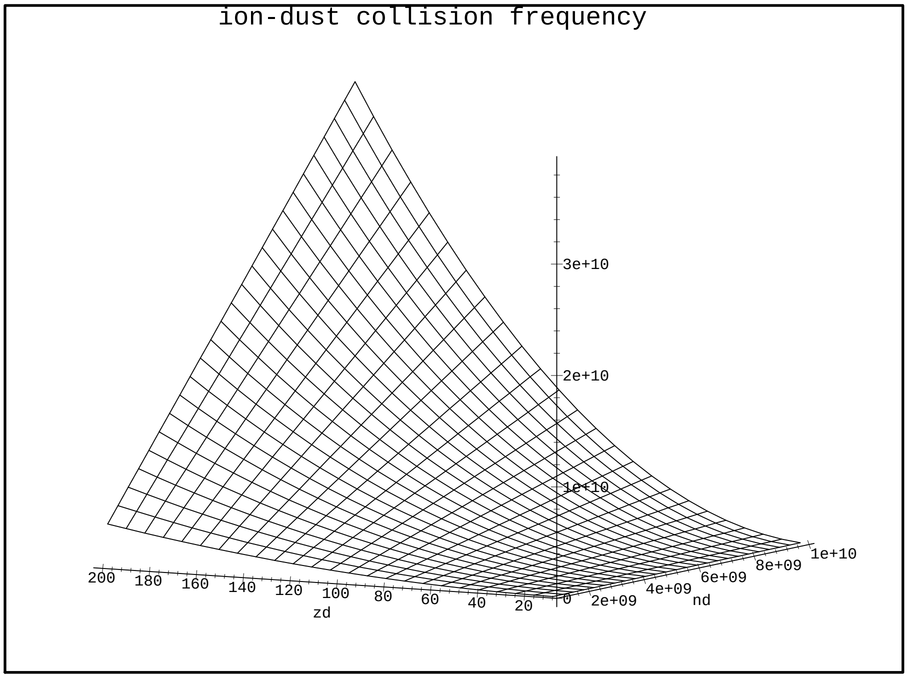

Fig. 3. The functional dependence of the ion-dust collision frequency on dust charge number and dust particle density.

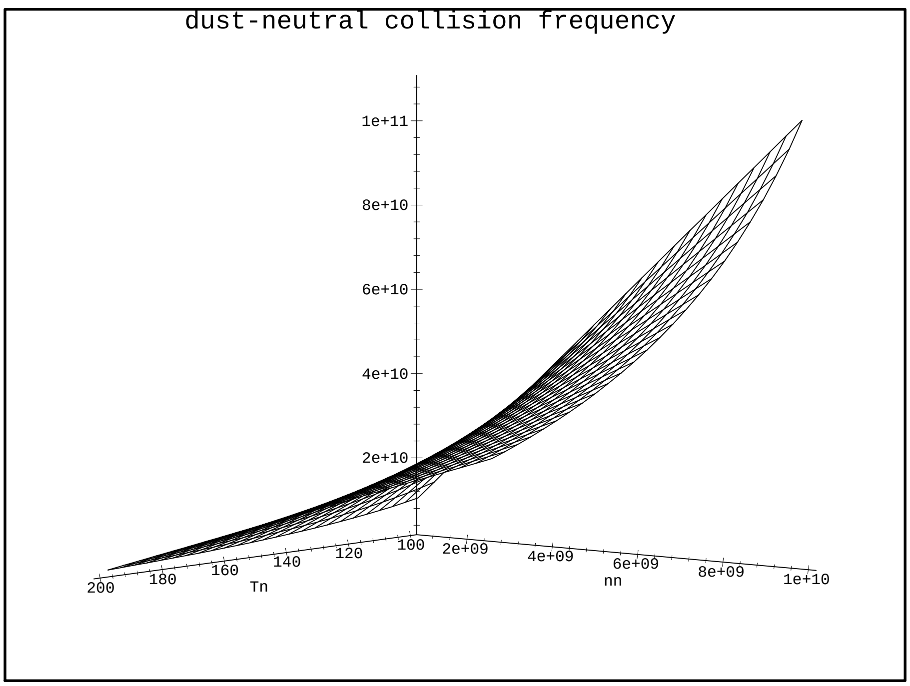

Fig. 4. The functional dependence of the dust-neutral collision frequency on neutral gas particle density and temperature.

generation of spontaneous temporal variable non-thermal radiative emission. It was shown that the instability for quite 'typical' plasma parameters can evolve on temporal scales comparable to the observed 'flare'-like activity (Montmerle 1991; André 1996). This means that the rate of energy conversion is fast enough to explain the observed time scale of magnetic activity, in principle. Moreover, the wave length of the most unstable mode is comparable to the expected extent of the emitting region for a reasonable thickness of the current sheet formed within a magnetic flux tube. Contrary to the solar corona applications the tearing instability has not to be triggered by some microturbulence giving rise to anomaleous resistivity in order to operate on temporal scales comparable to the observations (e.g. Kuperus 1976, Birk and Otto 1991). However, it should be noted that the physical parameters may vary significantly resulting in different temporal and spatial scales. On the other hand, observations indeed shows variability of magnetic activity processes within the same YSO which in the context of the introduced model can be explained as the consequence of different local plasma parameters (e.g. ionization rate, dust density).

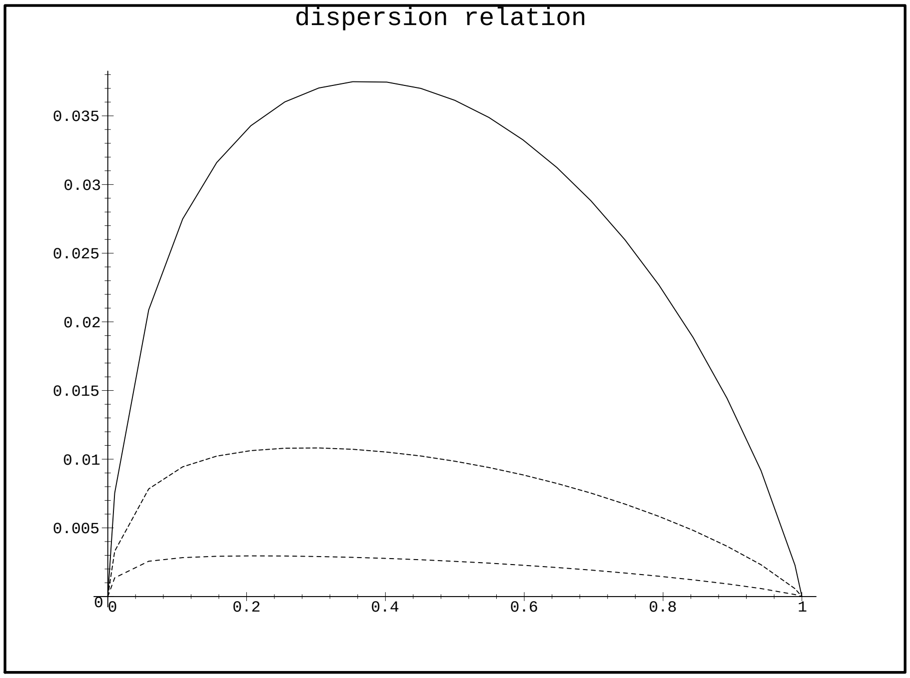

Fig. 5. Dispersion relation of the tearing mode for magnetic Reynolds numbers of $S = 100$ (solid line), $S = 1000$ (dashed-dotted line) and $S = 10000$ (dashed line). The growth rate $\Re q$ (normalized with respect to the inverse Alfvénic transit time) is plotted against the wave number k (normalized to the inverse half-thickness of the current layer)

It remains to answer the question if during the tearing dynamics the observed luminosity can be produced. It is very probable that the observed radio and X-ray flares with luminosities of $L^{rad} \approx 10^{29-32}$ergs^{-1} in magnetospheres of (weak) T-Tauri stars (as well as protostellar class I objects) are of nonthermal origin (Montmerle 1991; André 1996; Koyama 1996). Consequently, the GHz-radio emission, for example, must be emitted by MeV-electrons which in the proposed scenario can be accelerated during the reconnection process (cf. Schindler et al. 1991; Lesch and Birk 1997) along the magnetic flux tubes. In fact, for the above cited parameters a magnetic field-aligned potential structure with

$$U = \int E_{||} ds = \int \eta j_{||} ds \approx 3 \cdot 10^4 \text{ statvoltcm}^{-1} \quad (14)$$

evolves (the integral has to be calculated along magnetic field lines penetrating the reconnection region), if we assume the length of acceleration region λ_{acc} to be of the order of the wave length of the most unstable mode which should be comparable to the width of the current sheet, in which electrons can gain energies up to 10MeV, in principle. However, for any effective particle acceleration the acceleration length λ_{acc} must not exceed the loss length due to synchrotron radiation $\lambda_{syn} = 5 \cdot 10^8 c/B^2\gamma$ (γ is the Lorentz factor of the accelerated particles) or inverse Compton scattering $\lambda_{IC} = 3 \cdot 10^7 4\pi R^2 c^2/L^{rad}\gamma$ (R is the length scale of the emitting region), in the first place. For a magnetic field of $B = 100$G we obtain $\lambda_{syn} \approx 10^{14}$cm. The inverse Compton scattering loss length is $\lambda_{IC} \approx 3 \cdot 10^{28} R^2/L^{rad} \approx 10^{18}$cm for $R \sim \lambda_{acc}$ and $L^{rad} \approx 10^{30}$ergs^{-1}. Thus, we can conclude that electrons can be accelerated within the reconnection current sheets/magnetic flux tubes up to the observed energies.

Eventually, it should be noted that whereas the quantitative result were obtained for (weak) T-Tauri parameters the tearing scenario is also applicable for magnetic activity in the context of protostellar class I objects. In fact, in this case comparable spatial and temporal scales should be involved but the relevant plasma parameters seem to be not as well fixed down as in the case of T-Tauri stars.

Appendix A

Eqs. (9) and (10) determine a boundary layer problem (cf. Nayfeh 1981). It can be solved by expanding the linearized equations for the outer region and a thin (resistive) region inside the current layer and by asymptotically matching the solutions of the expanded equations (cf. Janicke 1982; Otto 1991). In order to solve Eqs. (9) and (10) in the outer region these equations are expanded in a scaling $y = (1/\epsilon)\Theta$ with $\epsilon \ll 1$. In the following the tilde is omitted for the perturbed quantities denoted by the index 1. Assuming $\eta \leq q$ and $q^2\nu_{dn}/\nu_{nd} \ll k^2$ Eq. (9) for the outer region reduces to

$$\Delta A_1 + \frac{dj_0}{dA_1} = 0 \quad \text{(A1)}$$

with the solution

$$A_1^o = c^o e^{-ky}(tanhy + k) \quad \text{(A2)}$$

In order to solve Eq. (9) and (10) in the inner region one applies a scaling $y = \epsilon\Theta$ with $\epsilon \ll 1$ and $\epsilon^2 k^2 \ll 1$ and Taylor expansions for symmetric (with respect to $y = 0$) quantities (like ρ_d and η) of the form $h(y) = h^{(0)} + \frac{1}{2}y^2 h^{(2)}$ and for B_x of the form $B_x = B_x^{(1)}y$. With these expansions Eqs. (9) and (10) can be combined to give (for more details see Otto 1991):

$$\begin{aligned} & yq^2\rho_d^{(0)}\eta^{(0)}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)\frac{d^3A_1}{dy^3} \\ - & \ q^2\rho_d^{(0)}\eta^{(0)}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)\frac{d^2A_1}{dy^2} \\ - & \ y\left[q^3\rho_d^{(0)}\eta^{(0)}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)+k^2y^2\right]\frac{dA_1}{dy} \\ + & \left[q^3\rho_d^{(0)}\eta^{(0)}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)+k^2y^2\right]A_1 \\ - & \ ky^2\hat{c}=0 \end{aligned} \quad \text{(A3)}$$

where $\hat{c}$ is an integration constant. Eq. (A3) can be related to the differential equation for the hypergeometric functions which provides us with the solution (cf. again Otto 1991):

$$A_1^i = \hat{c} + c_0 y + \mathscr{R} \quad \text{(A4)}$$

where $\mathscr{R}$ is a complicated combination of Kummer functions and the constant c_0 is given by

$$c_0 = \frac{\hat{c}\pi(q\eta)^{5/4}}{k^{1/2}}\left[\rho_d^{(0)}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)\right]^{1/4}\frac{1}{1-\lambda^2}\frac{\Gamma\left(\frac{\lambda+3}{4}\right)}{\Gamma\left(\frac{\lambda+1}{4}\right)} \quad \text{(A5)}$$

with

$$\lambda = \frac{q^{3/2}}{k\eta^{1/2}}\left(1+\frac{\nu_{dn}}{q+\nu_{nd}}\right)^{1/2} . \quad \text{(A6)}$$

Expansion for $y/\epsilon \gg 1$ of the inner solution gives

$$A_1^i(y) = \hat{c} + c_0 y + \mathscr{O}[(\epsilon/y)^2] \quad \text{(A7)}$$

and expansion of the outer solution for small arguments of y leads to

$$A_1^o(y) = c^o(k + (1-k^2)y + \mathscr{O}[y^2]) . \quad \text{(A8)}$$

The dispersion relation is obtained by a matching of the expanded solutions in significant order

$$\frac{c_0}{\hat{c}} = \frac{1-k^2}{k} \quad \text{(A9)}$$

which gives Eq. (12).

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References

Aly, J.J., Kuijpers, J.: 1990, A& A, 227, 473

André, P.: 1996, in *Radio Emission from the Stars and the Sun*, ASP Conf. Ser. 93, eds. A.R. Taylor and J.M. Paredes, 273

Benkadda, S., Gabbai, P., Tsytovich, V.N., Verga, A.: 1996, Phys. Rev. E 53, 2717

Birk, G.T., Otto, A.: 1991, Phys. Fluids B, 3, 1746

Birk, G.T., Kopp, A., Shukla, P.K.: 1996, Phys. Plasmas 3, 3564

Biskamp, D.: 1993, *Nonlinear Magnethydrodynamics*, Cambridge University Press, Cambridge, chap.4

Feigelson, E.D., DeCampli, W.M.: 1981, ApJ 243, L89

Furth, H.P., Killeen, J., Rosenbluth, M.N.: 1963, Phys. Fluids 6, 459

Grosso, N., Montmerle, Y., Feigelson, E.D., André, P., Casanova, S., Gregorio-Hetem, J.: 1997, Nature 387, 56

Hayashi, M.R., Shibata K., Matsumoto R.: 1996 ApJ 468, L37

Janicke, L.: 1982, Sol. Phys. 76, 29

Königl, A.: 1994, in *Theory of Accretion Disks*, ed. W.J. Duschl, J. Frank, F. Meyer, E. Meyer-Hofmeister and W. M. Tscharnuter, Kluwer, Dordrecht, 53

Koyama, K., Hamaguchi, K., Ueno, S., Kobayashi, N., Feigelson, E.D.: 1996, PASJ 48, L87

Kuijpers, J., Kuperus, M.: 1995, A& A 286, 491

Kuperus, M: 1976, Sol. Phys. 47, 79

Lesch, H., Birk, G.T.: 1997, A& A, in press

Li, J.: 1996, ApJ 456, 696

Lovelace, R.V.E., Romanova, M.M., Bisnovatyi-Kogan, G.S.: 1995, MNRAS 275, 244

Montmerle, T.: 1991, in *The Physics of Star Formation and Early Stellar Evolution*, eds. C.J. Lada and N.D. Kylatis, Kluwer, Dordrecht, 675

Nayfeh, A.H.: 1981, *Introduction to Perturbation Theory*, Wiley, Canada, chap. 12

Otto, A.: 1991, Phys. Fluids B 3, 1739

Otto, A., Birk, G.T.: 1992, J. Geophys. Res. 97, 8391

Priest, E.R.: 1983, Sol. Phys. 86, 33

Priest, E.R.: 1984, Rep. Prog. Phys. 48, 955

Priest, E.R.: 1985, in *Unstable Current Systems and Plasma Instabilities in Astrophysics*, IAU 107, eds. M.R. Kundu, G.D. Holman, Reidel, Dordrecht, 233

Priest, E.R.: 1987, *Solar Magnetohydrodynamics*, Reidel, Dordrecht, chaps. 7.5, 10

Paatz, G., Camenzind, M.: 1996, A & A 308, 77

Schindler, K., Hesse, M., Birn, J.: 1991, ApJ 380, 293

Shu, F., Najita, J., Ostriker, E., Wilkin, F., Ruden, S., Lizano, S.: 1994, ApJ 427, 781

Shukla, P.K., Birk, G.T., Kopp, A.: 1997, Phys. Scripta, in press

Steinolfson, R.S., van Hoven, G.: 1984, ApJ 276, 391

Sturrock, P.A.: 1972, Sol. Phys. 23, 438

Sturrock, P.A., Kaufman, P., Moore, R.L., Smith, D.F.: 1984, Sol. Phys. 84, 341

van Hoven, G.: 1979, ApJ 232, 572

van Hoven, G.: 1981, in *Solar Flare Magnetohydrodynamics*, ed. E.R. Priest, Gordon and Breach, London, chap.4

White, S.M., Pallavicini, Kunud, M.R.: 1992, A & A 259, 149

Yur, G., Rahman, H.U., Birn, J., Wessel, F.J. Minami, S.: 1995 J. Geophys. Res. 100, 23727

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