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ASTRONOMY AND ASTROPHYSICS

Alfvén wave transmission and particle acceleration in parallel shock waves

R. Vainio[1] **and R. Schlickeiser**[2]

[1] Space Research Laboratory, Department of Physics, FIN-20014 Turku University, Finland⋆
[2] Max-Planck-Institut für Radioastronomie, Postfach 2024, D-53010 Bonn, Germany

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Abstract. We study Alfvén wave transmission through parallel shocks. First, the paper corrects for mistakes made in a previous study by Campeanu & Schlickeiser on the same subject. It is shown, contrary to CS, that first-order Fermi acceleration by multiple shock crossings is generally more effective than stochastic acceleration in the downstream region. Moreover, we demonstrate that in a large region of the physical parameter space the scattering center compression ratio is much larger than the gas compression ratio. This leads to flat spectra for the cosmic rays accelerated by the first-order Fermi mechanism especially at low Alfvénic Mach number shocks.

Key words: acceleration of particles – shock waves – turbulence – cosmic rays

1. Introduction

Particle acceleration at shock waves is a universal mechanism for the origin of non-thermal particle populations since shock waves occur in many astrophysical sites (Drury 1983a, Blandford and Eichler 1987, Jones and Ellison 1991). In the conventional picture of diffusive shock wave acceleration energetic particles are confined by low-frequency plasma waves near the shock, gaining energy both by (1) multiple crossings of the shock (Axford et al. 1977, Krymsky 1977, Bell 1978, Blandford & Ostriker 1978), and by (2) stochastic second order Fermi acceleration in the downstream region of the shock (Campeanu & Schlickeiser 1992, Schlickeiser et al. 1993). The latter process is unavoidable due to the interaction of the upstream Alfvén waves with the shock, which produces a cross helicity state of downstream Alfvén waves different from $|H_c| = 1$ (see Campeanu & Schlickeiser 1992, hereafter referred to as CS).

The purpose of this paper is two-fold: in a first part of the paper we correct for a sign error in the original calculation of CS that qualitatively changes the respective importance of first-order (by multiple shock crossings) and (downstream) second-order Fermi acceleration. We also correct for an unphysical boundary condition in the wave transmission process used by CS, which changes the results quantitatively. In the second part of the paper we demonstrate the existence of an interesting range of parameter space that enables the generation of particle power laws by the first-order acceleration process with spectral indices Γ smaller than 2.

2. Alfvén wave transmission through a parallel shock

CS used the continuity of the transverse momentum,

$$\left[\rho u_{\rm n} \boldsymbol{u}_{\rm t} - B_{\rm n} \boldsymbol{B}_{\rm t}/4\pi\right] = 0$$

(notation as in CS), the continuity of the tangential electric field,

$$[u_{\rm n} \boldsymbol{B}_{\rm t} - B_{\rm n} \boldsymbol{u}_{\rm t}] = 0,$$

and the continuity of the mass flux,

$$[\rho u_{\rm n}] = 0,$$

to relate the upstream and downstream Alfvén wave parameters. For the gas flow velocity they used $\boldsymbol{u}_{{\rm n}_{1,2}} = (-u_{1,2}, 0, 0)$, but they adopted the ordered magnetic field as $\boldsymbol{B}^{\rm CS}_{{\rm n}_{1,2}} = \boldsymbol{B}_0 = (+B_0, 0, 0)$, which subsequently has led to a misidentification of forward moving (gas velocity and Alfvén speed parallel) and backward moving (gas velocity and Alfvén speed antiparallel) Alfvén waves.

Correcting for this sign error, i.e., adopting $\boldsymbol{B}_0 = (-B_0, 0, 0)$, we calculate anew the relations of the perturbations in the flow velocity, magnetic and electric field in the upstream and downstream region for an adiabatic shock. Because the precursor cosmic ray particle distribution amplifies the backward moving upstream Alfvén waves and damps the forward moving upstream Alfvén waves (Bell 1978) we again discuss the two cases of degenerated upstream cross helicity, $H_{{\rm c}_1} = +1$ and $H_{{\rm c}_1} = -1$. Where the corrections to the calculations by CS are trivial, we shall not repeat the detailed derivations but merely give the results in their correct form.

Send offprint requests to: R. Vainio

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2.1. Forward upstream waves only

If backward moving upstream waves are absent ($\delta B_1^{\rm b,L} = \delta B_1^{\rm b,R} = 0$) we obtain for the transmission and reflection coefficients

$$T_{\rm f} \equiv \frac{\delta B_2^{\rm f}}{\delta B_1^{\rm f}} = \frac{r^{1/2}(r^{1/2}+1)}{2}\frac{M+1}{M+r^{1/2}} \quad (1)$$

$$R_{\rm f} \equiv \frac{\delta B_2^{\rm b}}{\delta B_1^{\rm f}} = \frac{r^{1/2}(r^{1/2}-1)}{2}\frac{M+1}{M-r^{1/2}} \quad (2)$$

instead of Eqs. (7a–7b) of CS, where $M = u_1/V_{\rm A_1}$ is the upstream Alfvénic Mach number, and

$$r = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1+2\beta/M^2} \quad (3)$$

the gas compression ratio in an adiabatic medium with the ratio of specific heats, γ, and the plasma beta

$$\beta = \frac{c_1^2}{V_{\rm A_1}^2}.$$

For the shock to exists we demand $M > \beta^{1/2}$, and for the flow to be super-Alfvénic we have to demand $M > r^{1/2}$. As was the case in CS, the magnetic helicity of the wave is conserved during the interaction with the shock. The total wave intensity changes by the factor

$$W_{\rm f} \equiv \frac{(\delta B_2^{\rm f})^2+(\delta B_2^{\rm b})^2}{(\delta B_1^{\rm f})^2} = T_{\rm f}^2 + R_{\rm f}^2 = \frac{r(M+1)^2}{4}\left[\left(\frac{r^{1/2}+1}{M+r^{1/2}}\right)^2+\left(\frac{r^{1/2}-1}{M-r^{1/2}}\right)^2\right] \quad (4)$$

and the cross helicity changes from $H_{\rm c_1} = +1$ through the shock to

$$H_{\rm c_2}^{\rm f} \equiv \frac{(\delta B_2^{\rm f})^2-(\delta B_2^{\rm b})^2}{(\delta B_2^{\rm f})^2+(\delta B_2^{\rm b})^2} = \frac{T_{\rm f}^2-R_{\rm f}^2}{T_{\rm f}^2+R_{\rm f}^2} = \frac{1-\psi_{\rm f}^2}{1+\psi_{\rm f}^2}, \quad (5)$$

where

$$\psi_{\rm f} \equiv \frac{R_{\rm f}}{T_{\rm f}} = \frac{r^{1/2}-1}{r^{1/2}+1}\frac{M+r^{1/2}}{M-r^{1/2}}. \quad (6)$$

2.2. Backward upstream waves only

In the opposite case of having no forward moving upstream waves ($\delta B_1^{\rm f,L} = \delta B_1^{\rm f,R} = 0$) we obtain

$$T_{\rm b} \equiv \frac{\delta B_2^{\rm b}}{\delta B_1^{\rm b}} = \frac{r^{1/2}(r^{1/2}+1)}{2}\frac{M-1}{M-r^{1/2}} \quad (7)$$

$$R_{\rm b} \equiv \frac{\delta B_2^{\rm f}}{\delta B_1^{\rm b}} = \frac{r^{1/2}(r^{1/2}-1)}{2}\frac{M-1}{M+r^{1/2}} \quad (8)$$

instead of Eqs. (16a–16b) of CS. The total wave intensity changes by the factor

$$W_{\rm b} \equiv \frac{(\delta B_2^{\rm f})^2+(\delta B_2^{\rm b})^2}{(\delta B_1^{\rm b})^2} = T_{\rm b}^2 + R_{\rm b}^2 = \frac{r(M-1)^2}{4}\left[\left(\frac{r^{1/2}+1}{M-r^{1/2}}\right)^2+\left(\frac{r^{1/2}-1}{M+r^{1/2}}\right)^2\right] \quad (9)$$

and the cross helicity changes from $H_{\rm c_1} = -1$ through the shock to

$$H_{\rm c_2}^{\rm b} = \frac{R_{\rm b}^2-T_{\rm b}^2}{T_{\rm b}^2+R_{\rm b}^2} = \frac{\psi_{\rm b}^2-1}{\psi_{\rm b}^2+1}, \quad (10)$$

where

$$\psi_{\rm b} \equiv \frac{R_{\rm b}}{T_{\rm b}} = \frac{r^{1/2}-1}{r^{1/2}+1}\frac{M-r^{1/2}}{M+r^{1/2}}. \quad (11)$$

2.3. Alternative notation

Evidently, the transmission and reflection coefficients (1–2) and (7–8) can be conveniently written in form

$$T \equiv \frac{\delta B_2'}{\delta B_1} = \frac{r^{1/2}(r^{1/2}+1)}{2}\frac{M+H_{\rm c_1}}{M+r^{1/2}H_{\rm c_1}} = \frac{r^{1/2}+1}{2r^{1/2}}\frac{V_1}{V_2'} \quad (12)$$

$$R \equiv \frac{\delta B_2''}{\delta B_1} = \frac{r^{1/2}(r^{1/2}-1)}{2}\frac{M+H_{\rm c_1}}{M-r^{1/2}H_{\rm c_1}} = \frac{r^{1/2}-1}{2r^{1/2}}\frac{V_1}{V_2''}, \quad (13)$$

where V_i denote the shock frame phase velocities of the relevant upstream ($i = 1$) and downstream ($i = 2$) wave modes, and primes and double primes are used to denote the transmitted and reflected downstream waves, respectively; i.e., $V_1 = u_1 + H_{\rm c_1}V_{\rm A_1}$, $V_2' = u_2 + H_{\rm c_1}V_{\rm A_2}$, and $V_2'' = u_2 - H_{\rm c_1}V_{\rm A_2}$. With this representation the wave amplification factor and the downstream cross helicity may be written in compact form

$$W = T^2\left(1+\psi^2\right) \quad (14)$$

$$H_{\rm c_2} = H_{\rm c_1}\frac{1-\psi^2}{1+\psi^2}, \quad (15)$$

where

$$\psi \equiv \frac{R}{T} = \frac{r^{1/2}-1}{r^{1/2}+1}\frac{M+r^{1/2}H_{\rm c_1}}{M-r^{1/2}H_{\rm c_1}} = \frac{r^{1/2}-1}{r^{1/2}+1}\frac{V_2'}{V_2''}. \quad (16)$$

In the rest of the paper we shall use this notation.

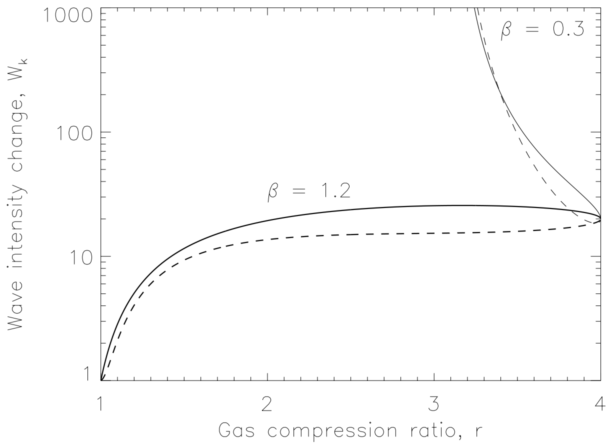

Fig. 1. Total wave amplification factor at constant wave number for a shock with a constant upstream plasma beta (c.f. Eq. (3)). Dashed and solid lines give the results for $H_{c_1} = +1$ and -1, respectively.

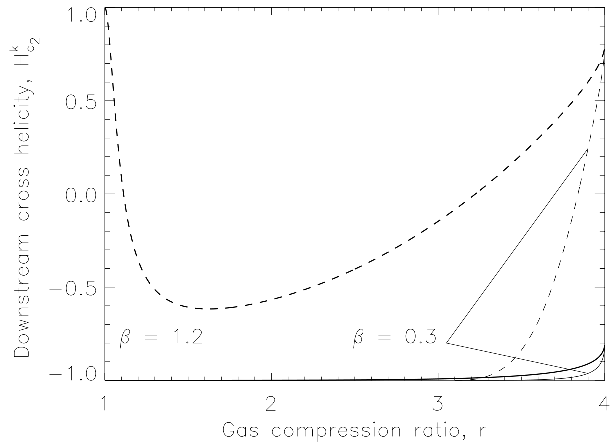

Fig. 2. Downstream cross helicity at constant wave number for a shock with a constant upstream plasma beta (c.f. Eq. (3)). Dashed and solid lines give the results for $H_{c_1} = +1$ and -1, respectively.

2.4. Shock's effect on wave numbers

In CS, the authors assumed the conservation of wave number during the interaction of the Alfvén waves with the shock. In this work, we demand the conservation of wave frequency, $\omega_1 = \omega_2$. For circularly polarized waves this is a direct consequence of the coplanarity theorem, which states that the downstream magnetic field has to be in the plane defined by the upstream magnetic field and the shock normal. For now, we will ignore magnetic helicity since it had no effect on the transmission of the waves, and the wave spectra are taken to be of form

$$I_1(k) = \mathscr{U}(k - k_1)\mathscr{I}_1 k^{-q} \quad (17)$$

$$I'_2(k) = \mathscr{U}(k - k'_2)\mathscr{I}'_2 k^{-q} \quad (18)$$

$$I''_2(k) = \mathscr{U}(k - k''_2)\mathscr{I}''_2 k^{-q}, \quad (19)$$

where k is the wave number, $\mathscr{U}(k)$ is the step function, k_1, k'_2, and k''_2 are the minimum wave numbers of the incident, transmitted and reflected waves, respectively. We consider values $1 < q < 2$ for the spectral index. The normalization coefficients are given by $\mathscr{I}_1 = (q-1)(\delta B_1)^2 k_1^{q-1}$, $\mathscr{I}'_2 = (q-1)(\delta B'_2)^2 {k'_2}^{q-1}$, and $\mathscr{I}''_2 = (q-1)(\delta B''_2)^2 {k''_2}^{q-1}$. From the frequency conservation condition, we have

$$k'_2 = k_1 \frac{V_1}{V'_2} = k_1 r \frac{M + H_{c_1}}{M + r^{1/2} H_{c_1}}$$

$$k''_2 = k_1 \frac{V_1}{V''_2} = k_1 r \frac{M + H_{c_1}}{M - r^{1/2} H_{c_1}}.$$

Thus, we may write the relations

$$\frac{\mathscr{I}'_2}{\mathscr{I}_1} = \left(\frac{\delta B'_2}{\delta B_1}\right)^2 \left(\frac{k'_2}{k_1}\right)^{q-1} = \left(\frac{\delta B'_2}{\delta B_1}\right)^2 \left(\frac{V_1}{V'_2}\right)^{q-1}$$

$$\frac{\mathscr{I}''_2}{\mathscr{I}_1} = \left(\frac{\delta B''_2}{\delta B_1}\right)^2 \left(\frac{k''_2}{k_1}\right)^{q-1} = \left(\frac{\delta B''_2}{\delta B_1}\right)^2 \left(\frac{V_1}{V''_2}\right)^{q-1}.$$

These equations give the wave transmission coefficients at *constant wave number*. Using Eqs. (12–13) we obtain

$$T_k \equiv \left(\frac{I'_2(k)}{I_1(k)}\right)^{1/2} = \frac{r^{1/2}+1}{2r^{1/2}} \left(\frac{V_1}{V'_2}\right)^{(q+1)/2} = \frac{r^{1/2}+1}{2r^{1/2}} \left(r \frac{M + H_{c_1}}{M + r^{1/2} H_{c_1}}\right)^{(q+1)/2} \quad (20)$$

$$R_k \equiv \left(\frac{I''_2(k)}{I_1(k)}\right)^{1/2} = \frac{r^{1/2}-1}{2r^{1/2}} \left(\frac{V_1}{V''_2}\right)^{(q+1)/2} = \frac{r^{1/2}-1}{2r^{1/2}} \left(r \frac{M + H_{c_1}}{M - r^{1/2} H_{c_1}}\right)^{(q+1)/2}, \quad (21)$$

where k has to be larger than the largest one of the cut-off wave numbers, $k > \max(k_i)$. We can also define the total wave amplification factor and downstream cross helicity at constant wave number $k > \max(k_i)$,

$$W_k \equiv \frac{I'_2(k) + I''_2(k)}{I_1(k)} = T_k^2 (1 + \psi_k^2) \quad (22)$$

$$H^k_{c_2} \equiv H_{c_1} \frac{I'_2(k) - I''_2(k)}{I'_2(k) + I''_2(k)} = H_{c_1} \frac{1 - \psi_k^2}{1 + \psi_k^2}, \quad (23)$$

where

$$\psi_k \equiv \frac{R_k}{T_k} = \frac{r^{1/2}-1}{r^{1/2}+1} \left(\frac{M + r^{1/2} H_{c_1}}{M - r^{1/2} H_{c_1}}\right)^{(q+1)/2} = \frac{r^{1/2}-1}{r^{1/2}+1} \left(\frac{V'_2}{V''_2}\right)^{(q+1)/2}. \quad (24)$$

Note that these representations will be changed if we consider wave numbers smaller than the largest one of the cut-off values. It can be seen that the transmission coefficients at constant wave number agree with the ones calculated for the integrated spectra only if $q \to 1$. We have plotted W_k and $H^k_{c_2}$ for different shock parameter values in Figs. 1 and 2, respectively.

The total wave amplification plots are qualitatively similar to those of CS. However, because of their misidentification of the waves' propagation direction (their forward wave in fact should be the backward wave and vice versa), the drastic changes in the downstream cross helicity state for low Mach number shocks occur for forward, and not for backward upstream waves. When the critical limit, $M \to r^{1/2}$ is approached, the downstream helicity state always approaches $H^k_{c_2} \to -1$, not +1 as was concluded by CS. As we shall see below, this has significant impact on the relative efficiency of the first-order and second-order Fermi acceleration mechanisms at the shock.

3. Cosmic ray acceleration parameters

Like CS, we consider the consequences of the Alfvén wave transmission through the shock to the acceleration of cosmic rays. If the ordered magnetic field, $\boldsymbol{B}_\mathrm{n}$, is homogeneous the steady-state equation governing the phase space density $f(x,p)$ may be written written in form (CS)

$$\frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} - \left(u_\mathrm{n} + \frac{1}{4p^2}\frac{\partial}{\partial p}(p^2 vA)\right)\frac{\partial f}{\partial x} + \left(\frac{p}{3}\frac{\partial u_\mathrm{n}}{\partial x} + \frac{v}{4}\frac{\partial A}{\partial x}\right)\frac{\partial f}{\partial p} + p^{-2}\frac{\partial}{\partial p}p^2\theta\frac{\partial f}{\partial p} + Q(x,p) = 0, \tag{25}$$

where x is distance from the shock along the shock normal, p is particle momentum, v is particle velocity, $Q(x,p)$ represesents sources and sinks, and the general forms of the acceleration and transport parameters, spatial diffusion coefficient $\kappa(x,p)$, rate of adiabatic deceleration $A(x,p)$, and momentum diffusion coefficient $\theta(x,p)$, may be found, e.g., in CS (see also Schlickeiser 1989). In the upstream region, we may write (CS)

$$\kappa_1(x,p) = \frac{vR_\mathrm{L}S_1}{2\pi(q-1)}\left(\frac{B_0}{\delta B_1}\right)^2(k_1R_\mathrm{L})^{q-1} \tag{26}$$

$$A_1(x,p) = -H_{c_1}\frac{4}{3}\frac{V_{A_1}}{v}p \tag{27}$$

$$\theta_1(x,p) = 0, \tag{28}$$

where R_L is the Larmor radius of the particle,

$$S_1 = \frac{4}{(2-q)(4-q)}\frac{1}{1-\sigma^2}, \tag{29}$$

and σ is the magnetic helicity state of the waves. Hence, the upstream ($x>0$) transport equation reads

$$\frac{\partial}{\partial x}\kappa_1\frac{\partial f}{\partial x} - \left(u_1 + H_{c_1}V_{A_1}\right)\frac{\partial f}{\partial x} + \frac{p}{3}\frac{\partial}{\partial x}\left(u_1 + H_{c_1}V_{A_1}\right)\frac{\partial f}{\partial p} = -Q_1(x,p). \tag{30}$$

In the downstream region we obtain

$$\kappa_2(x,p) = \kappa_1(0,p)/W_k \tag{31}$$

$$A_2(x,p) = r^{-1/2}\frac{1-\psi_k^2}{1+\psi_k^2}A_1(0,p) \tag{32}$$

$$\theta_2(x,p) = \vartheta\frac{W_kV_{A_1}^2p^2}{r\kappa_1(0,p)}, \tag{33}$$

where

$$\vartheta \equiv \frac{2\psi_k^2S_1}{q(q+2)(1+\psi_k^2)^2}. \tag{34}$$

The parameters have been obtained from the ones calculated by CS by replacing their wave transmission parameters with the ones calculated at constant wave number. Note that this is valid only for particles with $r_\mathrm{L} < \max(k_i)^{-1}$, which we shall assume. Thus, the downstream ($x<0$) transport equation reads

$$\frac{\partial}{\partial x}\frac{\kappa_1(0,p)}{W_k}\frac{\partial f}{\partial x} - \frac{1}{r}\left(u_1 + r^{1/2}H_{c_1}\frac{1-\psi_k^2}{1+\psi_k^2}V_{A_1}\right)\frac{\partial f}{\partial x} + \frac{p}{3r}\frac{\partial}{\partial x}\left(u_1 + r^{1/2}H_{c_1}\frac{1-\psi_k^2}{1+\psi_k^2}V_{A_1}\right)\frac{\partial f}{\partial p} + \frac{\vartheta}{p^2}\frac{\partial}{\partial p}\frac{W_kV_{A_1}^2p^4}{r\kappa_1(0,p)}\frac{\partial f}{\partial p} = -Q_2(x,p), \tag{35}$$

which, as already noted by CS and confirmed by our analysis, inevitably contains the momentum diffusion term. However, the strength of the momentum diffusion term is smaller than calculated by CS in case of upstream forward moving waves only.

The full system of Eqs. (26–35) together with the appropriate boundary conditions has been solved analytically by Schlickeiser et al. (1993). The solution indicated that the most important parameter controlling the first-order Fermi acceleration by multiple shock crossings is the scattering center compression ratio

$$r_k \equiv \frac{V_1}{V_2} = \frac{u_1 + H_{c_1}V_{A_1}}{u_2 + H^k_{c_2}V_{A_2}} = r\frac{M + H_{c_1}}{M + r^{1/2}H^k_{c_2}}, \tag{36}$$

which in general is different from the gas compression ratio r. Neglecting momentum diffusion in the downstream region, the canonical steady-state shock spectrum, $f(0,p) \propto p^{-s}$, where $s = 3r_k/(r_k-1)$ is controlled exclusively by r_k. This gives for the differential intensity, $dJ/dE = p^2 f \propto p^{-\Gamma}$, where $\Gamma = s - 2 = (r_k+2)/(r_k-1)$.

If the scattering center compression ratio equals the gas compression ratio, like at the infinite Mach number limit in Eq. (36), the spectral index is bounded to values $\Gamma_\mathrm{gas} = (r+2)/(r-1) \geq 2$, since $1 < r \leq 4$ for an adiabatic shock with $\gamma = 5/3$. We have plotted the scattering center compression ratio for different shock parameter values in Fig. 3 and the corresponding spectral indices in Fig. 4. Especially in a low beta plasma, the scattering center compression ratio may attain large values, and consequently the resulting particle spectral indices Γ approach the infinite compression ratio limit, $\Gamma \to 1$. Quite interestingly, if the plasma beta is close to unity we obtain a spectral index between 1.5 and 3 almost independent of the gas compression ratio ($\Gamma_\mathrm{gas} = 8$ corresponds to $r = 10/7 \approx 1.43$).

The principal difference between the gas compression ratio and the scattering center's compression ratio, being equivalent to the difference between the effective wave velocity and the gas velocity, and the possible consequences for the spectral index of the differential energy spectrum of accelerated particles, has been already noted by Bell (1978), see his Eqs. (11) and

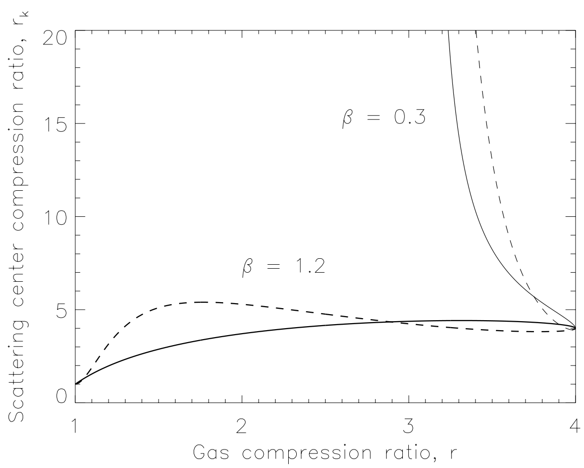

Fig. 3. Scattering center compression ratio for a shock with a constant upstream plasma beta (c.f. Eq. (3)). Dashed and solid lines give the results for H_{c_1} = +1 and −1, respectively.

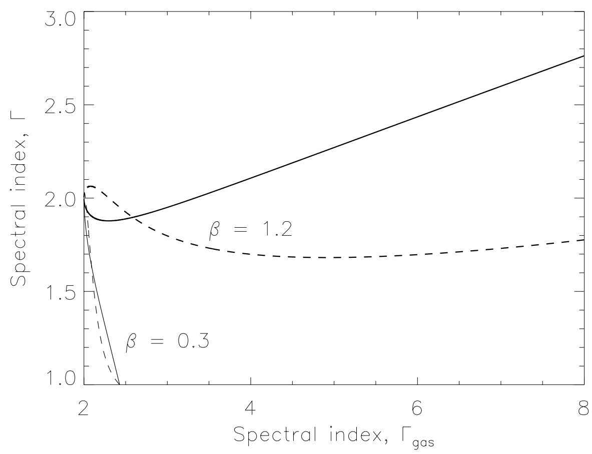

Fig. 4. Cosmic ray spectral index produced by a shock with a constant upstream plasma beta (c.f. Eq. (3)), neglecting stochastic acceleration in the downstream region. Dashed and solid lines give the results for H_{c_1} = +1 and −1, respectively.

(12); although he did no quantitative calculations of this effect. By calculating the correct transmission coefficients of Alfvén waves through the shock we have shown here that precisely this effect can account for the generation of particle spectral indices flatter than $\Gamma = 2$.

Above, we neglected the stochastic acceleration in the downstream region. The respective importance of first and second-order mechanisms may be studied by comparing the acceleration time scales, i.e., $\tau = p/\langle \mathrm{d}p/\mathrm{d}t\rangle$ of the two acceleration processes. For the first-order process we have (e.g., Forman & Webb 1985)

$$\tau_\mathrm{s} \equiv \frac{3}{\Delta V}\left(\frac{\kappa_1}{V_1}+\frac{\kappa_2}{V_2}\right) = \\ = \frac{\kappa_1}{(u_1+H_{\mathrm{c}_1}V_{\mathrm{A}_1})^2}\frac{3r_k}{r_k-1}\left(1+\frac{r_k}{W_k}\right).$$

(This is different from the one used in CS.) Systematic momentum gain in stochastic acceleration has the rate $\langle \mathrm{d}p/\mathrm{d}t\rangle = p^{-2}\partial(p^2\theta_2)/\partial p$ and, therefore, the second-order process has the time scale $\tau_\mathrm{F} = p^3/\left[\partial(p^2\theta_2)/\partial p\right]$ (again, slightly different from the simple diffusive time scale used by CS). Hence, the ratio of the two time scales is given by

$$\frac{\alpha(q,v)\,\tau_\mathrm{F}/\tau_\mathrm{s}}{(1-\sigma^2)B(q)} = \\ (M+H_{\mathrm{c}_1})^2\frac{r_k-1}{1+r_k/(T_k^2+R_k^2)}\,\frac{r}{15r_k}\left(\frac{1}{T_k^2}+\frac{1}{R_k^2}\right), \qquad (37)$$

where $B(q) = 5q(q+2)(2-q)(4-q)/24$ is a slightly varying function of q, $B(1.5)\approx 1.4$, $B(5/3)\approx 1.0$, and $B(1.8)\approx 0.63$, and $\alpha(q,v) = \left[\partial\log(p^4\kappa^{-1})/\partial\log p\right]/3$, which equals $(1+q)/3$ at non-relativistic particle velocities and $(2+q)/3$ at relativistic velocities, i.e., $2/3 < \alpha < 4/3$.

The ratio of acceleration time scales (37) for different shock parameter values is plotted in Fig. 5. For high Mach number shocks, the acceleration is dominated by the first order mechanism, if the magnetic helicity of the waves is not large, $\sigma^2 \ll 1$. If the upstream waves are dominantly backward waves and the magnetic helicity is not large, the acceleration is always dominated by the first-order Fermi process regardless of the Mach number. Note, however, that if the magnetic helicity of the waves is close to $\sigma = \pm 1$, the dominant acceleration process is the downstream stochastic acceleration. This occurs because the degenerate magnetic helicity states lead to large regions in the velocity space, where particles do not see any waves at all (Schlickeiser 1989). This, in turn, may make it impossible for the particles to get reflected back to the shock. However, in that part of the velocity space that the particles do see the waves, they see waves travelling in both directions in the downstream region. Thus, second-order mechanism still accelerates particles. Stochastic acceleration also dominates in case of forward upstream waves, if the Mach number of the shock and the plasma beta are not large. However, since one expects the upstream waves to be predominanty backward travelling and not having degenerate magnetic helicity values ($\sigma \neq \pm 1$), we expect the first-order process to provide the dominant part to the acceleration in realistic physical environments, and that the contribution of the second-order Fermi process is much smaller than estimated by CS.

4. Discussion and conclusions

The present calculations indicate that the transmissions of the Alfvén waves through the shock is qualitatively different from the one discussed in CS. It was concluded by CS that a forward wave field experiences little changes in its cross helicity state as it is convected through the shock, and that the downstream wave field with large intensity would be the forward travelling one. In case of backward upstream waves, the wave field was calculated to be effectively reversed by the shock, if the

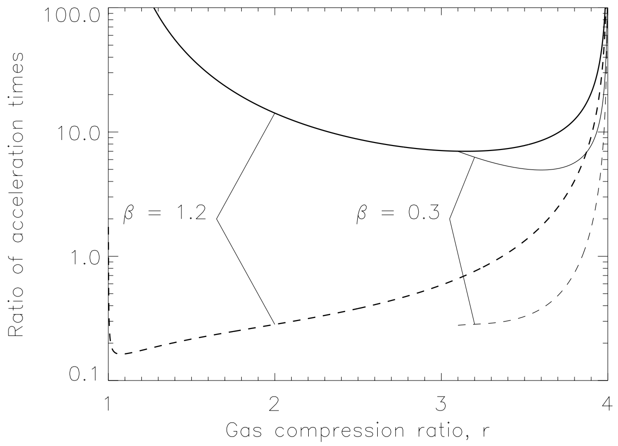

Fig. 5. The ratio of acceleration times for a shock with a constant upstream plasma beta (c.f. Eq. (3)). Dashed and solid lines give the results for H_{c_1} = +1 and −1, respectively.

compression ratio was moderately strong ($r = 2 - 3$) and the plasma beta was small. This implied that the scattering center compression ratio was small, even smaller than unity for certain conditions, and that the first-order Fermi acceleration by the shock crossings was inefficient in such shocks or could even turn to deceleration. Noting that the physically more plausible case is the one of backward upstream waves, CS concluded that stochastic second-order Fermi acceleration in the downstream region would generally be more efficient than first-order Fermi acceleration by multiple shock crossings. We have shown here that these results by CS are due to their misidentification of the wave's propagation direction, and we have corrected their calculations for this mistake.

Based on the corrected Alfvén wave transmission coefficients we have shown that it is always the backward downstream wave field that seems to have an infinite amplitude in a $\beta < 1$ plasma at the critical limit, $M \to r^{1/2}$. This is also qualitatively reasonable, since in this limit the downstream backward waves are group standing in the shock frame; as the upstream waves are continuously flowing in to the shock, the part of their energy that is converted to the downstream backward waves is ineffectively transported away from the shock. This results in large intensities of the downstream backward waves regardless of the cross helicity state of the upstream medium. Our study showed that the downstream stochastic acceleration works slower than the acceleration by multiple shock crossings under physically plausible conditions. However, we confirmed the conclusion of CS that in no case can stochastic acceleration be totally absent from the downstream region, and that this second-order mechanism would work even in the extreme case of degenerate magnetic helicity state $\sigma = \pm 1$, where the first-order mechanism ceases to operate. Second-order acceleration would also be important in case of forward upstream waves if the Alfvénic Mach number of the shock was small.

We also demonstrated that the spectra resulting from the first-order mechanism were generally harder than predicted by conventional theory (c.f. Fig. 4). If the plasma beta is close to unity the spectral index seems to vary slowly with the gas compression ratio and be between 1.8 and 2.8 for forward upstream waves in a wide range of $1.4 < r \leq 4$. The most interesting result of our study, perhaps, is the possibility of generating extremely hard power-law spectra in $\beta < 1$ plasmas by the first-order mechanism, if the critical Mach number limit, $M \to r^{1/2}$, is approached. These kind of conditions can be met, for example, in the solar corona, where the Alfvén speeds are of the same order as the shock speeds and much larger than the sound speed, and also near supernova shocks propagating into an interstellar medium with low plasma beta. Thus, the model, being able to generate particle energy power-law spectra harder than the originally limiting value $\Gamma = 2$, avoids the discrepancy noted by, e.g., Lerche (1980), Drury (1983b), and Dröge et al. (1987) that the original shock wave acceleration theory in its simplest test-particle form is not in accord with the observed flat particle spectra in shell-type supernova remnants and bright spiral galaxies, and with the independence of spectral indices from evolutionary effects in these sources. Non-linear shock acceleration model of Ellison et al. (1996), where the back-reaction of the accelerated particles is taken into account in determining the hydrodynamic flow at the shock, can produce gas compression ratios larger than $r = 4$ and, hence, also hard particle spectra despite the fact that it assumes scattering centers frozen-in to the plasma.

For a given value of the plasma beta, Eq. (3) gives a unique relationship between the gas compression ratio and the Alfvénic Mach number of the shock. For $\beta < 1$, there is a singularity in the backward wave intensity at the critical Mach number, which can be also given in form $M \to (4 - 3\beta)^{1/2}$, when Eq. (3) holds. However, this singularity occurs only because the wave pressure has not been taken into account in Eq. (3). In a situation where the critical limit is approached, e.g., if the shock is slowing down in a uniform medium or the shock is propagating towards a decreasing plasma beta, the gas compression ratio will adjust to a lower value due to the additional downstream pressure produced by the waves. This way the shock will avoid the singularity and continue to propagate as a super-Alfvénic shock. An interesting subject for the future is to find out the upper limits for the wave amplification factor and the scattering center compression ratio including the wave pressure effects in Eq. (3), i.e., to see how close the Mach number can get to $r^{1/2}$. In future we will extend the analysis to oblique shock geometries and also study other MHD wave modes. In particular, fast MHD waves are interesting, since their phase speeds exceed the fluid speed in the downstream region and, thus, there can be only forward fast MHD waves in the downstream region. Including compressional waves as a minor component to the upstream field probably somewhat reduces the effective scattering center compression ratio but at the same time stochastic acceleration in the downstream region will be enhanced. If the compressional wave modes dominate in the upstream region, the situation may be completely different.

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