Astron. Astrophys. 331, L37–L40 (1998)

ASTRONOMY
AND
ASTROPHYSICS

Letter to the Editor

On some features of free precession of a triaxial body: the case of Her X-1

N.I. Shakura[1], **K.A. Postnov**[2], **and M.E. Prokhorov**[1]

[1] Sternberg Astronomical Institute, Moscow University, 119899 Moscow, Russia
[2] Faculty of Physics, Moscow State University, 119899 Moscow, Russia

Received 6 November 1997 / Accepted 23 December 1997

Abstract. We show that the free precession of a triaxial body can naturally explain the anomalously rapid change of the X-ray pulse profile of Her X-1 observed by the HEAO-1 in September 1978 without requiring a large change in the moment of inertia.

Key words: stars: neutron – stars: individual: Her X-1

Hercules X-1 is the most famous and well studied X-ray pulsar containing an accreting neutron star with a spin period of $P_{ns} = 1.24$ s in a circular orbit around a $2M_{\odot}$ main-sequence star. The orbital period of the binary system is 1.7 days. Discovered in 1972 (Tananbaum et al. 1972), it nevertheless has not been completely understood until now. This mostly concerns the origin of its long-term 35-day X-ray periodicity, which has broadly been discussed in the literature. The possible reason for this long-term period was suggested to be either (1) the neutron star free precession (Brecher 1972) or (2) the precession of a tilted accretion disk controlled by the outer parts (i.e. by the precession of HZ Her (Roberts 1974) or by some intrinsic reasons (Boynton et al. 1980)). Notice that both the neutron star free precession and a complex precessing motion of the accretion disc may in fact simultaneously operate in Her X-1/HZ Her binary system (Shakura et al. 1997, in preparation).

A strong evidence favouring the free precession model was found by Trümper et al. (1986) in the EXOSAT observations of the X-ray pulse profile phase and shape changing over the 35-day period. In contrast, Soong et al. (1987) claimed that their observations of Her X-1 by HEAO-1 X-ray satellite in 1978 do not support this model. In September 1978, they observed an unusually short high-on state of Her X-1 (the X-ray emission faded down very rapidly during 7 days instead of 10, and the pulse profile shape changed over 20 hrs, as contrasted to about 17 days for the EXOSAT observations), which, if interpreted in terms of the free precession model, would correspond, as the authors claim, to a very large change in the moment of inertia of the neutron star body corresponding to an oblateness of $\sim 8 \times 10^{-6}$. Such a large moment of inertia changing would lead in turn to the pulse period change by an order of magnitude higher than was actually observed during this period (i.e. in August-September 1978) 8×10^{-7} s.

The purpose of this Letter is to show that in fact the free precession model cannot be so easily rejected if one considers the possible *triaxiality* of the neutron star body. Then the observed episode of an unusually rapid X-ray pulse shape change in Her X-1 can naturally be explained by a sudden small deviation of the moment of inertia along one axis without changing the characteristic period of the free precession (i.e. conserving the gross oblateness of the body). The magnetic pole simply starts moving non-uniformly along a non-planar trajectory which apparently manifests itself as the rapid change in the X-ray pulse shape because the X-ray beam goes rapidly down to the rotational equator of the neutron star (then the observer sees two poles producing two equal X-ray pulses over one spin period, as was observed by the EXOSAT during the low-on state of Her X-1 in March 1984) and travels the way it usually takes a 17-day interval in a much shorter time of ~ 1 day. Of course, the total moment of inertia of the neutron star remains practically unchanged and, subject to the angular momentum conservation, no appreciable X-ray pulse period change should be observed.

Consider first the more familiar case of an axially symmetric body with $I'_3 > I'_2 = I'_1$ rotating around the angular momentum $\mathbf{M}$ (see Fig. 1). Then in the rotating frame the trajectories the magnetic pole of the neutron star moves along represent plane circles on the neutron star surface with the center at the largest moment of inertia (I'_3). This situation corresponds to a "normal" free precession in Her X-1 and the magnetic pole P uniformly goes along such a circle passing in one day a path marked by the short thick arrow. The precession period is simply $P_{pr} \approx P_{ns} I_{||}/(\cos b (I_{\perp} - I_{||})) >> P_{ns}$, where $I_{||}$ and $I_{\perp}$ are the components of the moment of inertia parallel and normal to the total angular momentum and b is the angle between the largest moment of inertia (I'_3 in our case) and the angular momentum.

Let now the body of the neutron star experience some quake resulting in a practically instantaneous change in all moments of inertia with $I_3 > I_2 > I_1$ (Fig. 1). As is well known (see Landau and Lifshits 1965), in this case the motion of the an-

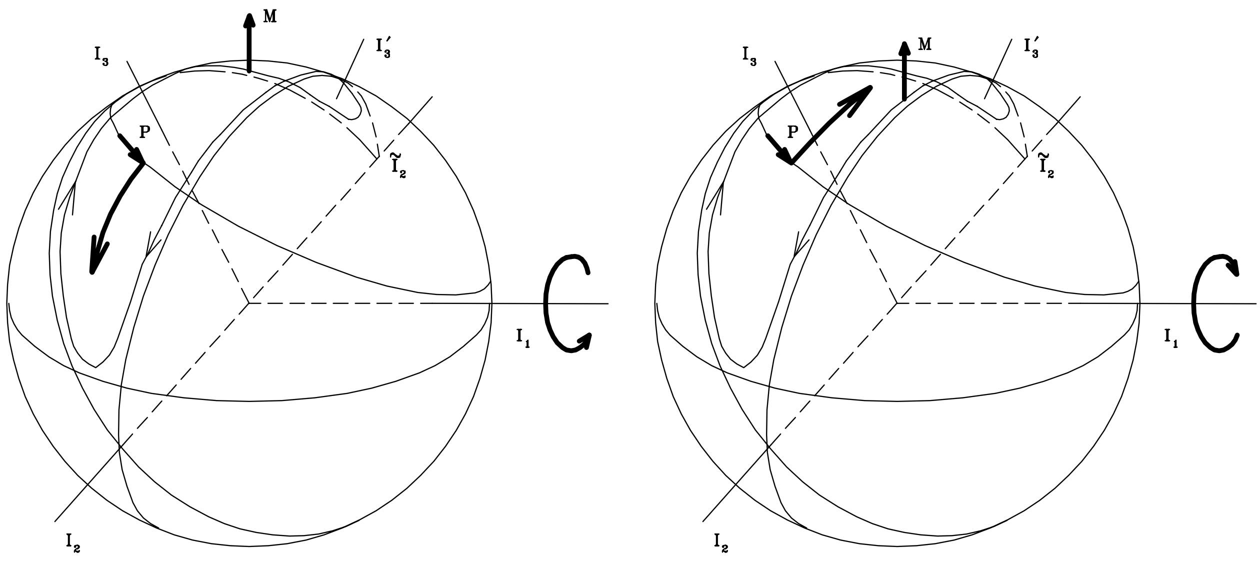

Fig. 1. A schematic view of the neutron star body. $\mathbf{M}$ is the angular momentum vector. The case of axisymmetric free precession: $I_3' > I_2' = I_1'$, the magnetic pole moves along a plane trajectory; the small thick arrow shows the way the pole passes in 1-day time interval. The case of the triaxial free precession: $I_3 \gtrsim I_2 > I_1$, two separatrices appear crossing at I_2 and $\tilde{I}_2$; a non-planar trajectory of $\mathbf{M}$ relative to the new axes of inertia is shown with the thin arrows indicating the direction of the angular momentum motion. In the left panel, the case when $\mathbf{M}$ goes toward $\tilde{I}_2$ is shown, i.e. the neutron star body turns anti-clockwise around an axis close to I_1. In the right panel, $\mathbf{M}$ moves toward I_2 and the star turns clockwise around I_1. The long thick arrow indicates the rapid motion of the magnetic pole P toward the rotational equator.

gular momentum vector relative to the axes of inertia becomes more complicated: two families of non-planar trajectories appear isolated by two separatrices passing through I_2, one around I_3, another around I_1. The motion along a trajectory around the maximal moment of inertia (I_3) becomes very nonuniform (see Fig. 2): the closer the trajectory to the separatrix, the more nonuniform the motion along it is. In Fig. 2 we show how the angle between the angular momentum and the magnetic pole θ changes with time over one precession period (see the Appendix for more detail). The angular momentum rapidly passes most part of the trajectory and slows down its motion near the turning point close to the points I_2 and $\tilde{I}_2$ (in the limiting case when the pole goes exactly along the separatrix, it would stay infinitely long at the separatrix crossing points I_2 and $\tilde{I}_2$, being in the state of indifferent equilibrium). In the middle panel of Fig. 2 we also reproduce the phase change of $\cos\theta$ in the axisymmetrical case with the angles taken from Trümper et al. (1986)(the thick sinusoid). Clearly, the quake must have taken place somewhere between $\Psi_{35} = 0$ and $\Psi_{35} = 0.1$, which indeed corresponds to the observations of Soong et al. (1987)[1].

In the triaxial case, the angular momentum vector can move in two opposite directions depending on at which part of the trajectory the quake happened (left and right schemes in Fig. 1). Accordingly, in the rotating frame with the z-axis along $\mathbf{M}$, the magnetic pole will rapidly move downward (left part of Fig. 1) or upward (right part of Fig. 1) since the neutron star body turns around some axis (close to I_1 in Fig. 1). Requiring that the magnetic pole lies near the rotational equator shortly after the quake (in order to make it possible to observe an X-ray pulse with two equal peaks), it should be located near the circle passing through I_3 and I_1 axes of inertia (as the angular momentum vector "freezes" near the axis I_2).

The long thick arrow in Fig. 1 illustrates the way the magnetic pole now passes over one day. In Her X-1, the transition from one trajectory to another occurs between September 22 and 23, 1978, which explains the apparent 10-fold increase in the free precession rate. After that the moments of inertia relaxes to their "usual" values and the magnetic pole returns to another planar "axially-symmetric" trajectory lying not far from the old one (because the angular momentum spent most time near the separatrix "crotch").

During the triaxial motion described above the precession period should not change appreciably since the gross difference in the parallel and perpendicular moments of inertia remains practically the same. After the body of the neutron star has returned into its axisymmetric form, it should be recognized that the X-ray pulse should generally be phase-shifted. This effect can in principle be detected by accurate timing of X-ray pulses in different 35-day cycles.

In the free precession model for Her X-1 the vector of the neutron star angular momentum should be inclined to the line of sight by an angle of ~ -40 degrees tilted away from the observer (see Trümper et al. 1986 for a detailed discussion of all relevant angles in the case of the axially symmetric free precession). That the angular momentum of the neutron star proves to be tilted with respect to the orbital angular momentum

[1] We remind that conventionally the precession phase $\Psi_{35} = 0$ corresponds to the maximum X-ray flux so that the main X-ray turn-on starts at $\Psi_{35} = -0.15$

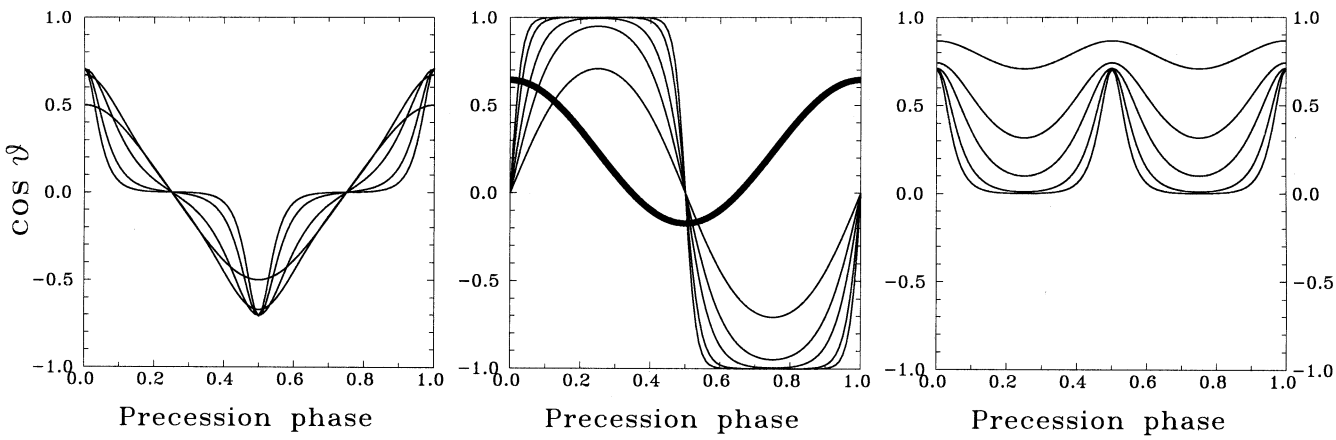

Fig. 2. The dependence of the angle θ between the magnetic pole P and the vector of angular momentum $\mathbf{M}$ on the precession phase Ψ_{35} in the case of the triaxial precession. The relative differences in moments of inertia are both 10^{-6}. The magnetic pole position is close to I_1 (left panel), I_2 (middle panel), and I_3 (right panel). The five curves in each figure are shown for the trajectories around I_3 (see Fig. 1) characterized by different maximal angles χ_{max} between $\mathbf{M}$ and the axis I_3: $\cot\chi_{max} = 1, 1/3, 1/10, 1/100, 1/1000$. The closer the trajectory to the separatrices, the more nonuniform the motion of $\mathbf{M}$ along it is. The thick sinusoid in the middle panel depicts the phase behaviour of $\cos\theta$ for axisymmetric precessional motion with the angles taken from Trümper et al. (1996): $\cos\theta = \cos(25^o)\cos(75^o) + \sin(25^o)\sin(75^o)\cos\Psi_{35}$. The quake must have taken place close to $\Psi_{35} = 0.05$

is naturally explained in the framework of the free precession model because the torques applied to a strongly magnetized rotating neutron star by the accretion disk change the sign for some critical inclination (~ 55 degrees) of the magnetic dipole axis to the neutron star spin axis (Lipunov 1992).

Note to conclude that free precession of neutron stars makes them an interesting potential source of gravitational radiation (Jones 1998), and the confirmation of the free precession model for Her X-1 should stimulate such studies.

Acknowledgements. The work was partially supported by Russian Fund for Basic Research through Grant No 95-02-06053-a, by the INTAS Grant No 93-3364 and by the Russian Ministry of Science NTP "Astronomija", project 1.4.4.1. The authors thank Dr. A.N.Rakhmanov for help in drawing the figures and the anonymous referee for valuable notes.

Appendix

For $I_3 > I_2 > I_1$, given the total energy

$$2E = I_1\Omega_1^2 + I_2\Omega_2^2 + I_3\Omega_3^2\,,$$

and angular momentum

$$M^2 = I_1^2\Omega_1^2 + I_2^2\Omega_2^2 + I_3^2\Omega_3^2\,,$$

the motion of the angular momentum vector in the coordinate system related to axes of inertia in the rotating frame is given by the following system of equations (Landau and Lifshits 1965):

$$\Omega_1 = \sqrt{\frac{2EI_3 - M^2}{I_1(I_3 - I_1)}}\,\mathrm{cn}\,\tau$$

$$\Omega_2 = \sqrt{\frac{2EI_3 - M^2}{I_2(I_3 - I_2)}}\,\mathrm{sn}\,\tau$$

$$\Omega_3 = \sqrt{\frac{M^2 - 2EI_1}{I_3(I_3 - I_1)}}\,\mathrm{dn}\,\tau\,,$$

where cn τ, sn τ, and dn τ are elliptic Jacobi functions and the dimensionless time τ is

$$\tau = t\sqrt{\frac{(I_3 - I_2)(M^2 - 2EI_1)}{I_1 I_2 I_3}}\,.$$

Specifying the relative differences $\Delta I_{12}/I_3$, $\Delta I_{23}/I_3$ and expressing the precession phase in units of the dimensionless precession period

$$\Pi = 4\int_0^{\pi/2} \frac{du}{\sqrt{1 - k^2\sin^2 u}}$$

with the parameter k defined as

$$k^2 = \frac{(I_2 - I_1)(2EI_3 - M^2)}{(I_3 - I_2)(M^2 - 2EI_1)}\,,$$

we calculate the curves shown in Fig. 2 for the position of the magnetic pole on the neutron star surface and the angular momentum vector as explained in the figure caption.

References

Boynton P.E., Crosa L.M., Deeter J.A., 1980, ApJ, 237, 169
Brecher K., 1972, Nature, 239, 325
Jones I., 1998, PhD Thesis, Cardiff University of Wales
Landau L.D., Lifshits E.M., 1965, Mechanics (Moscow: Nauka)
Lipunov V.M., 1992, Astrophysics of Neutron Stars (Berlin: Springer Verlag)
Roberts J.W., 1974, ApJ, 187, 575
Soong Y., Gruber D.E., Rothschild R.E., 1987, ApJ, 319, L77
Tananbaum H., Gursky H., Kellog E.M., Levinson R., Schreier E., Giacconi R., 1972, ApJ, 174, L143
Trümper J., Kahabka P., Ögelman H., Pietsch W., Voges W., 1986, ApJ, 300, L63

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.