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Cosmic ray acceleration by fast magnetosonic waves

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Abstract. Recently, Schlickeiser and Miller have calculated anew the acceleration rate of cosmic rays by fast magnetosonic plasma waves in a small-beta plasma, using a linear dispersion relation. They found that the transit-time damping of fast mode waves provides the dominant contribution to the stochastic acceleration rate of cosmic ray particles, both, in pure fast mode wave turbulence as well as in a mixture of isotropic fast mode turbulence and slab Alfven turbulence. Here it is shown that the use of the linear dispersion relation is fully justified for protons down to kinetic energies of a few tens of keV, whereas for electrons, the dispersive part of the dispersion relation should be included for energies below $10^3 V_A/c$ MeV. By retaining the largest scales of the turbulence only (for which the linear dispersion relation holds), an upper limit is computed, which shows that possible strong damping of the turbulence spectrum at high wavenumber would not reduce the efficiency of the acceleration process in a dramatic way, and that a strong modification induced by the waves dispersivity at high wavenumber could only be in the sense of an enhanced efficiency.

Key words: acceleration of particles – plasmas – turbulence – cosmic rays – Sun: flares

1. Introduction

Interaction of cosmic ray particles with MHD turbulence, in the interstellar medium as well as impulsive solar flares, is believed to produce strong acceleration of the particles. The mechanism of acceleration by fast magnetosonic plasma waves which, together with the Alfvén waves, are the less damped waves in a magnetized plasma (especially when the sound speed is lower than the Alfvén speed – low beta plasma), has been studied in the past decades by many authors (Hall & Sturrock 1967, Kulsrud & Ferrari 1971, Fisk 1976, Eilek 1979, Achterberg 1981 and Melrose 1994, for instance). However, for the sake of simplicity, the estimation of the acceleration rate was always made in the approximation of small Larmor radii. Dropping this approximation, Schlickeiser and Miller (1998 - hereafter referred to as SM) have recently reconsidered this problem, and found a high efficiency for the acceleration of low energy super-Alfvénic charged particles.

SM have calculated the quasilinear transport and acceleration parameters for cosmic ray particles resulting from the resonant interaction with oblique propagating low-frequency fast magnetosonic plasma waves in a background plasma with small plasma-beta $\beta = 2c_s^2/V_A^2 << 1$. In this limiting case the fast and slow magnetosonic waves in the fluid plasma (e.g. Thompson 1969) degenerate to the fast mode waves. The fast mode branch in a plasma extends from low-frequencies past the proton cyclotron frequency $\Omega_{0,p}$, and up to the electron cyclotron frequency (e.g. Thompson 1962; Swansson 1989, Ch. 2). In general, energetic charged particles of velocity v and Larmor frequency Ω resonantly interact with undamped waves of frequency ω if the gyroresonance interaction condition,

$$\omega - k_\parallel v_\parallel = n\Omega \tag{1}$$

with entire n running between $-\infty \leq n \leq \infty$, is fulfilled, where $v_\parallel = v\mu$ and $k_\parallel = k\eta$ are the parallel velocity component and the parallel wavenumber, respectively. Unlike for purely parallel propagating waves in slab turbulence, the presence of the compressive (along $\boldsymbol{B_0}$) magnetic field component of $\delta\boldsymbol{B}_w$ for fast mode waves allows the cosmic ray particles to resonantly interact with these waves through the $n = 0$ resonance. This effect is called transit-time damping (Fisk 1976; Achterberg 1981; Stix 1992, p.273) and is the magnetic analog of Landau damping (which involves the $n = 0$ resonance and a parallel electric field). SM demonstrated that the transit-time damping ($n = 0$) of fast mode waves provides the dominant contribution to the stochastic acceleration rate of cosmic ray particles, both, in pure fast mode wave turbulence as well as in plasma turbulence consisting of a mixture of isotropic fast mode turbulence and slab Alfven turbulence.

Throughout their analysis SM used the linear dispersion relation $\omega_j = jV_A k$, $j = \pm 1$, for fast mode waves, where V_A and k denote the Alfven speed and the magnitude of the wavevector $\boldsymbol{k}$, respectively, which holds as long as the wave frequency is well below the proton cyclotron frequency $\Omega_{0,p}$. As $|\omega_j|$ approaches Ω_p this simple dispersion relation is no longer valid, and above $\sim 10\Omega_{0,p}$ the branch enters the Whistler regime. It is the purpose of this work to determine the range of wavenumbers k with which the cosmic ray particles efficiently interact

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⋆ After March 1st, 1998 at: Institut für Theoretische Physik, Lehrstuhl 4: Weltraum & Astrophysik, Ruhr-Universität Bochum, D-44780 Bochum, Germany by transit-time damping, in the nondispersive approximation. We will show that protons of energies down to a few tens of keV (corresponding to $v \geq 10V_A$) only interact with fast magnetosonic mode waves of k much smaller than $k_c \equiv \Omega_{0,p}/V_A$, so that the use of the linear dispersion relation by SM is fully justified (unless dispersion produces catastrophic effects), and a sharp break of the turbulence spectrum would not modify the acceleration rate. However, low energy super-Alfvenic electrons can interact with larger k waves, since they have significantly smaller Larmor radii. We will precise the electron energy below which the dispersive part of the dispersion relation has to be taken into account to improve the evaluation of the transit-time damping acceleration time. We will also give an upper limit for the acceleration time by transit-time damping of these low-energy particles, which works as well when the small scales of the turbulence are strongly damped.

2. Transit-time damping acceleration

For an isotropic turbulence, and super-Alfvénic particles ($v \gg V_A$), the quasilinear Fokker-Planck coefficients can be written (see the paper by SM) as

$$D_{\mu\mu} \equiv \Re \int_0^\infty d\tau \, \langle \dot{\mu}(t)\dot{\mu}^*(t+\tau)\rangle = \frac{\pi\Omega^2(1-\mu^2)}{4B_0^2} \sum_{j=\pm 1}\sum_{n=-\infty}^{\infty}\int_{-1}^{1} d\eta \int_0^\infty dk \; g^j(k)(1+\eta^2) \delta(kv\mu\eta - \; \omega_j + \; n\Omega)[J_n^{'}(\frac{kv_\perp\sqrt{1-\eta^2}}{|\Omega|})]^2 \; , \quad (2)$$

and

$$D_{pp} \equiv \Re \int_0^\infty d\tau \, \langle \dot{p}(t)\dot{p}^*(t+\tau)\rangle = \frac{p^2V_A^2}{v^2} D_{\mu\mu} \; , \quad (3)$$

where <> denotes an ensemble average, $\Re$ the real part, and Ω the relativistic gyration frequency which, for a particle of mass m, is connected to $\Omega_{0,p}$ by

$$\Omega = \frac{m}{m_p}\frac{\Omega_{0,p}}{\gamma} \; .$$

The momentum diffusion coefficient $A_2(p)$, in the case of a vanishing cross helicity of the plasma waves, i.e. equal intensity of forward and backward waves, is determined by the pitch-angle average

$$A_2(p) = \int_0^1 d\mu \, D_{pp}(\mu) = \frac{\pi\Omega^2p^2V_A^2}{4v^2B_0^2}(q-1)\delta B^2 \frac{R_L}{v}(R_L k_{min})^{q-1} H_T \; , \quad (4)$$

with

$$H_T = \frac{v}{(q-1)\delta B^2 R_L (R_L k_{min})^{q-1}} \int_0^1 d\mu \, (1-\mu^2) \sum_{j=\pm 1}\sum_{n=-\infty}^{+\infty}\int d\eta \int dk \, g^j(k)(1+\eta^2) \delta(kv\mu\eta - \omega_j + n\Omega) J_1^2\left(\frac{kv_\perp\sqrt{1-\eta^2}}{|\Omega|}\right) \quad (5)$$

where

$$g^j(k) = (q-1)(\delta B)^2 k_{min}^{q-1} k^{-q} = g_0 k^{-q} \quad (6)$$

and R_L denotes the Larmor radius of the particle.

SM have shown that the transit-time damping term ($n = 0$) provides the main contribution to the momentum diffusion. The calculation of H_T in the approximation of the linear dispersion relation $\omega_j = jV_A k$ gave, for an infinitely wide power spectrum $g_0 k^{-q}$,

$$H_T = c_1(q)(1-\varepsilon^2)^{\frac{q+1}{2}} Q_{q/2}\left(\frac{1+\varepsilon^2}{1-\varepsilon^2}\right) \approx c_1(q)\ln\frac{1}{\varepsilon} \quad (7)$$

to lowest order in

$$\varepsilon \equiv \frac{V_A}{v} \quad (8)$$

and

$$c_1(q) = 2^{1-q}\frac{q}{4-q^2}\frac{\Gamma(q)\Gamma(2-q/2)}{\Gamma^3(1+q/2)} \; , \quad (9)$$

for $q < 2$. Q_α denotes the Legendre function of the second kind.

3. Influence of a "high" wavenumber cutoff

We want now to introduce a cutoff k_{max} in the spectrum, and calculate $H_T(k_{max})$ in order to study the convergence to the infinite spectrum value, H_T. A convergence, reached while the linear dispersion relation $\omega_j = jV_A k$ still holds, will validate the derivation by SM, whereas a too slow convergence will indicate that a further account of dispersive effects is required.

So, we have to estimate

$$H_T(k_{max}) = \int_0^1 d\mu \frac{1-\mu^2}{|\mu|} H(|\mu|-\varepsilon)(1+\frac{\varepsilon^2}{\mu^2}) \left[(1-\mu^2)\left(1-\frac{\varepsilon^2}{\mu^2}\right)\right]^{q/2} \int_0^{k_{max}R_L\sqrt{(1-\mu^2)(1-\varepsilon^2/\mu^2)}} ds \, s^{-(1+q)} J_1^2(s) \quad (10)$$

with $R_L \equiv v/|\Omega|$ and $k_c \equiv \Omega_{p,0}/V_A$. This is just the $n = 0$ term of (5) in the nondispersive approximation and for a vanishing cross helicity, after the change of variable $s = kv_\perp\sqrt{1-\eta^2}/|\Omega|$. As SM, we neglect the cutoff at the large scales of the turbulence, and replaced the lower boundary of the integral in (10) by 0. The fact that

$$\int_0^{s_M} ds \, s^{-(1+q)} J_1^2(s) = \frac{s_M^{2-q}}{4(2-q)} \times {}_PF_Q(\{3/2,(2-q)/2\};\{2,3,2-q/2\};-s_M^2) \quad (11)$$

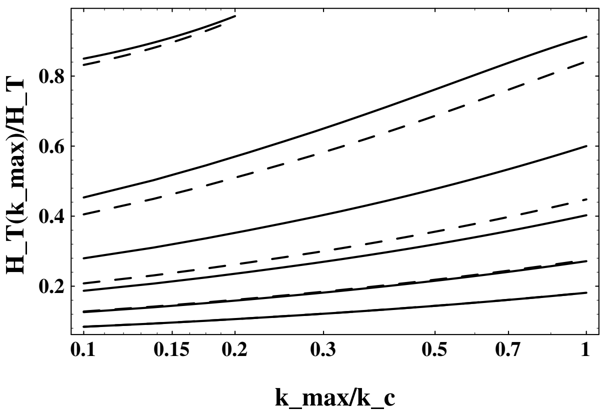

Fig. 1. Continuous line: the ratio $H_T(k_{max})/H_T$ for electrons and $\varepsilon_A = 10^{-3}$, as a function of k_{max}/k_c, for kinetic energies starting on the top left with 10 MeV, and decreasing downwards by decades. Only the first curve with the kinetic energy of 10 MeV gives a sufficient convergence to 1 at small k_{max} to justify the use of a linear dispersion relation. Dashed line: the same with $\varepsilon_A = 10^{-2}$, and a maximal kinetic energy of 100 MeV on the top. Again, only the first curve indicates a sufficient convergence. At smaller energies, dispersive effects should be taken into account.

is a fast converging function – its asymptotical value is reached up to a few percent for $s_M \geq 2$ – will make the nondispersive approximation reasonable in many cases, but not all. The upper boundary in (10) can be written as a function of the particle kinetic energy E_c like

$$s_M = \frac{k_{max}}{k_c}\frac{m}{m_p \varepsilon_A}\left[\left(1+\frac{E_c}{mc^2}\right)^2 - 1\right]$$

$$\sqrt{(1-\mu^2)(1-\varepsilon^2/\mu^2)} \qquad (12)$$

with $\varepsilon_A \equiv V_A/c$, because

$$R_L k_c = \frac{\Omega_{p,0}}{|\Omega|}\frac{v}{V_A} = \frac{m}{m_p}\frac{\gamma}{\varepsilon} \; . \qquad (13)$$

So, the convergence will be much faster for protons than for electrons, since s_M is proportional to m/m_p. Furthermore, for low-energy electrons, the convergence of $H_T(k_{max})$ will be too slow for ε/γ less than a few m/m_p.

The isotropisation of the particles will be effective, and thus the use of the pitch-angle averaged Fokker-Planck coefficients justified, if $v \gg V_A$, i.e. at least $v > 10V_A$ – This is also a condition of validity of the derivation by SM, since (4) results from a first order expansion in ε. In terms of energy, this condition reads

$$E_c > 100\varepsilon_A^2\frac{mc^2}{2} \; . \qquad (14)$$

Making use of the relation (10), (11) and Mathematica, we can calculate $H_T(k_{max})/H_T$ as a function of k_{max}, for different Alfvén velocities and particle energies ranging from $E_c > 100\varepsilon_A^2 mc^2/2$ up to a few MeV.

3.1. Protons

For protons, we find that for an energy of $5.10^4\varepsilon_A^2$ MeV (corresponding to $v \approx 10V_A$), wavenumbers below $0.1k_c$ contribute to 79% of the value of H_T, below $0.2k_c$ to 93%, and below $0.5k_c$ to more than 99% of H_T. For higher energies, the contribution of small k is even stronger – already 90% at $0.1k_c$ and 98% at $0.2k_c$ for $10^5\varepsilon_A^2$ MeV, and over 99.9% at $0.2k_c$ for $10^6\varepsilon_A^2$ MeV. The use of the linear dispersion relation by SM is thus fully justified for protons, and higher m/Z particles, unless the inclusion of dispersive effects produces catastrophic modifications – A modification by several, at least 3 orders of magnitude would be necessary to yield a noticeable effect on H_T, hence on the acceleration time at an energy of $10^5\varepsilon_A^2$ MeV.

3.2. Electrons

For electrons, as predicted from the form of the integral boundary, the convergence of $H_T(k_{max})$ is not as fast. The curves in Fig. 1 represent $H_T(k_{max})/H_T$ as a function of the normalised upper wavenumber k_{max}/k_c for the two values $\varepsilon_A = 10^{-3}$ and $\varepsilon_A = 10^{-2}$ and different electron energies under 10 MeV and 100 MeV, respectively. $H_T(k_{max})$ approaches H_T by less than 10% for $k_{max} < 0.2k_c$ only at energies of 10 MeV (resp. 100 MeV) and higher. At lower energies, small wavenumbers do not even give the main contribution to H_T and it would be necessary to include dispersive effects in the calculation of the Fokker-Planck coefficients. However, since the integrand in (10) is positive, the intersection of the curves in Fig. 1 with the ordinate axis, corresponding to $k_{max} = 0.1k_c$, or even with the line $k_{max} = 0.2k_c$, provides a lower limit for H_T, hence an upper limit for the acceleration time.

The acceleration time is given by

$$T_{acc} = \frac{p^2}{A_2(p)} \; , \qquad (15)$$

hence, for the isotropic, zero cross helicity turbulence that we consider,

$$T_{acc} = \left(\frac{m}{m_p}\right)^{2-q}\frac{\gamma^{2-q}}{\varepsilon_A^2}\left(\frac{v}{c}\right)^{3-q}\frac{T_0}{H_T(k_{max})} \qquad (16)$$

with

$$T_0 = \frac{4}{\pi(q-1)}\left(\frac{B_0}{\delta B}\right)^2\left(\frac{ck_{min}}{|\Omega_{p,0}|}\right)^{2-q}\frac{1}{ck_{min}} \; . \qquad (17)$$

We plotted on Fig. 2 and 3 the acceleration time and its upper limit for $\varepsilon_A = 10^{-3}$ and $\varepsilon_A = 10^{-2}$. Whenever the result by SM is validated, it is plotted in continuous line. When the result by SM would have to be improved by the inclusion of the dispersive effects in the dispersion relation, it is represented by a dashed line. In mixed line, we show the upper limit that can be deduced from Fig. 1 taking k_{max} at $0.2k_c$. This upper limit in no case means that we expect the real acceleration time, in a broad power law spectrum, to be higher than the acceleration time obtained by SM. On the opposite, given the "small" discrepancy between the prediction by SM and this upper limit, it

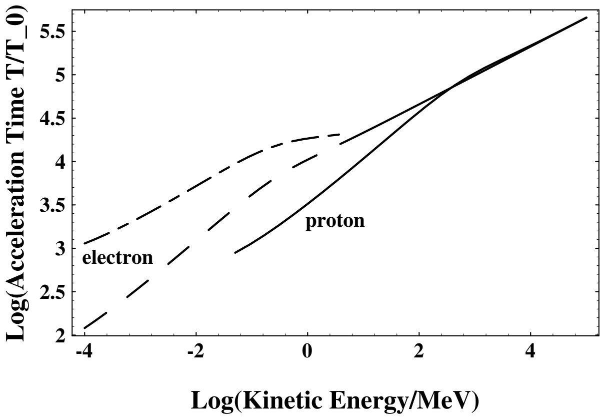

Fig. 2. The acceleration time scale resulting from transit-time damping of fast mode waves as a function of kinetic energy for protons (lowest continuous curve) and electrons, for a power spectral index $q = 5/3$, and $\varepsilon_A = 10^{-3}$. Continuous line is used when the result by SM is confirmed, whereas a dashed line means that dispersive effects should be taken into account. This is the case for electrons below a few MeV. The mixed line represents the upper limit of the acceleration time, as it can be deduced from a derivation using a linear dispersion relation, i.e. assuming a spectrum which cuts off at $k_{max} = 0.2k_c$. There is no such upper limit for protons, since wavenumbers above $0.2k_c$ do not contribute to the acceleration process.

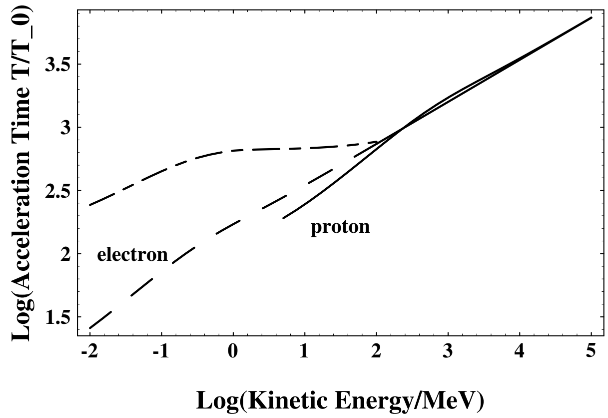

Fig. 3. Same as Fig. 2, but for $\varepsilon_A = 10^{-2}$.

means that if a full dispersion relation induces strong modifications, it can only be downwards, i.e. by improving the process of acceleration. But if the small scales of the turbulence are strongly damped, resulting in a sharp break of the spectrum above $k < k_c$, the real acceleration time might be close to this upper limit. Further interpretation of the curves on Fig. 1 to discuss the precise contribution of higher k is not allowed. Such a discussion would require to plot the corresponding curves for the exact dispersion relation, valid at high k, which is beyond the scope of this paper.

Finally, for smaller ε_A, the use of the linear dispersion relation would be justified down to lower energies (≈ one order of magnitude per decade), but the Alfvén speed has to remain large compared to the sound speed for the linear dispersion relation to apply at least at small k. Taking typical magnetic and temperature estimates in the interplanetary medium, the solar corona and the interstellar medium (e.g. Benz 1993), 10^{-3} seems the lowest reasonable value of ε_A to keep β small.

4. Results and conclusions

To learn about the contribution of the different wavenumbers in the process of CR acceleration by obliquely propagating fast magnetosonic modes, we reduced the spectrum to a finite width in k, and computed in the approximation of a linear dispersion relation the ratio $H_T(k_{max})/H_T(\infty)$ – which contains all the effects of the turbulence spectrum and is proportional to the inverse of the acceleration time – as a function of the upper boundary k_{max} of the reduced spectrum, for different particle energies and Alfvén velocities. As this ratio approaches 1, the part of the spectrum above k_{max} becomes non efficient in the process of acceleration. For super-Alfvénic protons and higher m/Z particles, we found that the use of the linear dispersion relation in the derivation by SM is well justified, and that a sharp break of the spectrum at small scales would not modify their result. For electrons, however, dispersive effects should be included in the dispersion relation below 10 MeV for $V_A = 10^{-3}c$ and 100 MeV for $V_A = 10^{-2}c$, in order to improve the estimate of the acceleration time. Finally, the evaluation of the ratio $H_T(k_{max})/H_T(\infty)$ allows us, both, to validate the derivation by SM whenever the overwhelming contribution comes from the large scales of the turbulence, and also to determine the upper limit of the acceleration time when the small scales of the turbulence are strongly damped, or when small k do not dominate the acceleration process. Hence, we can predict that a break in the turbulence spectrum would not change the acceleration rate in a catastrophic way, and that significant effects of the dispersion of the waves could only be towards an enhanced efficiency of the process of acceleration by oblique propagating fast magnetosonic modes, so that the conclusions of SM are essentially confirmed.

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