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Third harmonic plasma emission in solar type II radio bursts

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Abstract. We discuss consequences of the recently reported experimental evidence for third harmonic plasma emission during shock–excited solar radio bursts (type II bursts). Spectrographic and partly imaging observations of three type II bursts displaying three drifting bands with frequencies related as 1 : 2 : 3 have been studied. The radio data of these events were simultaneously recorded by the digital radiospectrograph of the Observatory of Solar Radioastronomy in Potsdam–Tremsdorf and the multifrequency radioheliograph of the Paris–Meudon Observatory in Nançay. The data allow for determining the brightness temperature of radio emission in the three frequency bands. There are one to three orders of magnitude difference between the brightness temperature of the second and the third harmonic plasma emission in our burst sample.

Two non-linear processes – the coalescence of three plasma waves, and the coalescence of a plasma wave and an electromagnetic one at twice the plasma frequency – are considered to explain the occurrence of a third harmonic. The analysis shows that both processes can fit the observed brightness temperatures. The first process acts preferably at low phase velocities of plasma waves and sharp electron density gradients in the source, the second in the case of high plasma wave phase velocities. This means regarding both processes, the occurrence of the third harmonic in type II burst emission due to non-linear coronal plasma processes demands for some additional specific conditions in the shock or foreshock region. Finally, we propose a method to distinguish between the two invoked non-linear processes by a statistical investigation of a larger type II burst sample.

Key words: Sun: corona – Sun: radio radiation – shock waves

1. Introduction

Electromagnetic wave emission near the plasma frequency and its harmonic is a fundamental process in solar radio physics. The most clear-cut example are type II bursts, i.e. narrow emission bands ($\Delta f/f \le 0.3$) which slowly drift ($-\Delta f/\Delta t \le$ 1 MHz/s) from high to low frequencies in the decimetric-to-decametric frequency range. They are generally attributed to collisionless shock waves generated during a flare or a coronal mass ejection (reviews by Nelson & Melrose 1985, Mann 1995, Aurass 1997).

While type II bursts usually display two drifting bands whose frequency ratio is near 2:1, observations of the third harmonic were claimed only rarely (Bakunin et al. 1990; Chertok et al., 1990, Kliem et al. 1992, 1993, Aurass et al. 1994). Possible evidence for third harmonic emission was reported in type III bursts (Haddock & Takakura in Kundu 1965; Benz 1973; Takakura & Yousef 1974).

The understanding how higher harmonic plasma emission arises in the corona and under which conditions it can be observed gives a clue to nonthermal electrons and wave populations in the source (cf. Zheleznyakov & Zlotnik 1974; Zlotnik 1978; Kliem et al. 1992). One possibility is the coalescence of three Langmuir waves into an electromagnetic wave. Another one is the coalescence of an electromagnetic wave at twice the plasma (Langmuir) frequency ($2f_L$) with a plasma wave. Strong turbulence theories have also been qualitatively discussed (Kliem et al. 1992). Until now, the complexity of observed spectra and the ignorance of the brightness temperature has left different alternative mechanisms without observational constraint. Of course, there are two reasons to believe that clear–cut third harmonic emission is difficult to recognize, and a rare event, too: firstly, there is the mentioned spectral complexity of solar radio bursts coupled with insufficient spectral resolution of analogous broadband receivers. Secondly, the harmonic structure is usually associated with non-linear plasma processes. For their treatment, the weak turbulence approximation is valid in the coronal plasma (see, for instance Zheleznyakov 1977, 1996; Holman and Pesses 1983). Consequently, third harmonic emission (being a third order effect in terms of the plasma wave energy) will be detectable under specific conditions, only.

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In the present paper spectrographic observations of three type II bursts with triple harmonic bands are discussed. For two bursts imaging data are available, too. Sect. 2 gives an introduction to the instruments (Sect. 2.1), describes the spectra and the source locations of the different bands with respect to coronal plasma structures observed in X-rays (Sect. 2.2), and presents the measured brightness temperatures (Sect. 2.3). Possible mechanisms of the second and third harmonics in type II bursts are investigated in Sect. 3. The results are discussed in Sect. 4, used to estimate expected ratios of brightness temperatures at the third and second harmonic, and compared with the measurements given in Sect. 2. It is concluded (Sect. 5) that the non-linear interaction of plasma and electromagnetic waves in the coronal plasma can provide the occurrence of lanes with frequencies related as 3:2 on type II bursts.

2. Instruments and data analysis

2.1. Instruments

The observations were carried out with the spectrograph of the Tremsdorf Solar Radio Observatory (OSRA) and the Nançay Radioheliograph (NRH). The OSRA instrument consists of digitally recording swept-frequency spectrographs in the ranges 40–90, 100–170, 200–400 and 400–800 MHz, with a sweep rate of 10 s^{-1} (Mann et al. 1992). The NRH provides one-dimensional scans of the corona at five frequencies, with both its east-west and north-south branches, and time resolution of 0.1 s (The Radioheliograph Group 1993). The observed frequencies were 164, 236.6, 327, 410 and 435 MHz, the bandwidth 700 kHz.

2.2. Spectrum and location of different harmonics

2.2.1. Spectra

The dynamic spectra in Figs. 1 and 2 show type II bursts with more than two drifting bands. The bursts are associated with flares at or behind the limb, since X-ray brightenings (GOES, BATSE) are found, but no H_α flare is reported in *Solar Geophysical Data*. The 6 May 1996 burst occurs at the time of a CME observed by the SOHO/LASCO coronograph.

The instantaneous frequency ratios were measured at the high-frequency edges of the three bands of the type II bursts. Using the first two minutes of each event, we find the ratios 1 : 2.1 : 2.8 (27 Sep 1993) and 1 : 2.1 : 3.3 (28 Dec 1993). The ratio of the frequency drift rate is 1 : 2.1 : 3.3 in both events. Uncertainties of these evaluations come from fine structure masking the slowly drifting bands, such as herringbone bursts. During the 6 May 1996 herringbones cover the high-frequency edge of the harmonic entirely such that no reliable measurement is possible there. A gross evaluation in the body of the different bands leads to a similar ratio as in the other events. The frequency ratios are consistent with the interpretation of the bands as multiple harmonics. In the following we label the individual bands and their emitting sources by the terms f_L, $2f_L$ and $3f_L$, respectively.

2.2.2. Source positions

The spectra of the type II bursts on 27 Sep and 28 Dec 1993 are such that all bands are observed at the discrete frequencies of the NRH. The best imaging data were obtained during the 27 Sep 1993 event. During the 28 Dec 1993 burst the source dimensions are also measured and used in Sect. 2.3, but the source positions are difficult to determine because of ionospheric refraction. The NRH did not observe on 6 May 1996.

During the 27 Sep 1993 burst (Fig. 1) the NRH frequencies allow the simultaneous observation of the fundamental at 164 MHz and the second harmonic at 327 MHz. The third harmonic ($3f_L$) is observed 35 seconds later at 410 and 435 MHz. Localized sources are extracted from the one-dimensional NRH scans by fitting Gaussians (or the sum of a Gaussian and a constant). Fig. 1, bottom shows the positions of the f_L and $2f_L$ source superposed on a YOHKOH-SXT image taken 10 minutes before the type II burst. The source centroid of the $3f_L$ emission at 410 and 435 MHz is displaced by no more than 0.05 $R_\odot$ from the $2f_L$ source at 327 MHz. Given the intrinsic difficulty in defining the source centroid by two one-dimensional scans only, we conclude that the observations are consistent with the $2f_L$ and $3f_L$ emission coming from the same source.

The simultaneously observed f_L and $2f_L$ sources at 164 and 327 MHz are not found at the same position. At both frequencies the dominant type II source has a minor companion with less than 50% of the flux density of the main source. The main f_L source is cospatial with the minor $2f_L$ source and vice versa. The implications of this finding are discussed by Aurass et al. (1994). Since we focus on the comparison of $2f_L$ and $3f_L$ emissions, which do come from cospatial sources, we do not pursue this discussion here.

The diameters of the sources in Table 1 are given in units of the theoretical beamwidth (HPBW) and in arcminutes, with a correction for beam broadening when the image size is smaller than 1.5 times the beam. The diameters of the $2f_L$ and $3f_L$ sources at 327 MHz are indistinguishable, while the diameter of a given harmonic diminishes with increasing frequency. The same result has been reported from comparisons of the f_L and $2f_L$ sources at lower frequencies (Dulk 1982). The cospatiality of the $2f_L$ and $3f_L$ sources corroborates the interpretation of the type II spectrum in terms of multiple harmonic bands (1, 2, 3).

2.3. Measurement of the brightness temperature

The flux densities measured independently by the two branches of the NRH were calibrated with the daily solar flux densities at 245, 410 and 610 MHz published in *Solar Geophysical Data*. The half-widths of the sources derived from the east-west scans of the NRH (which have a better spatial resolution than

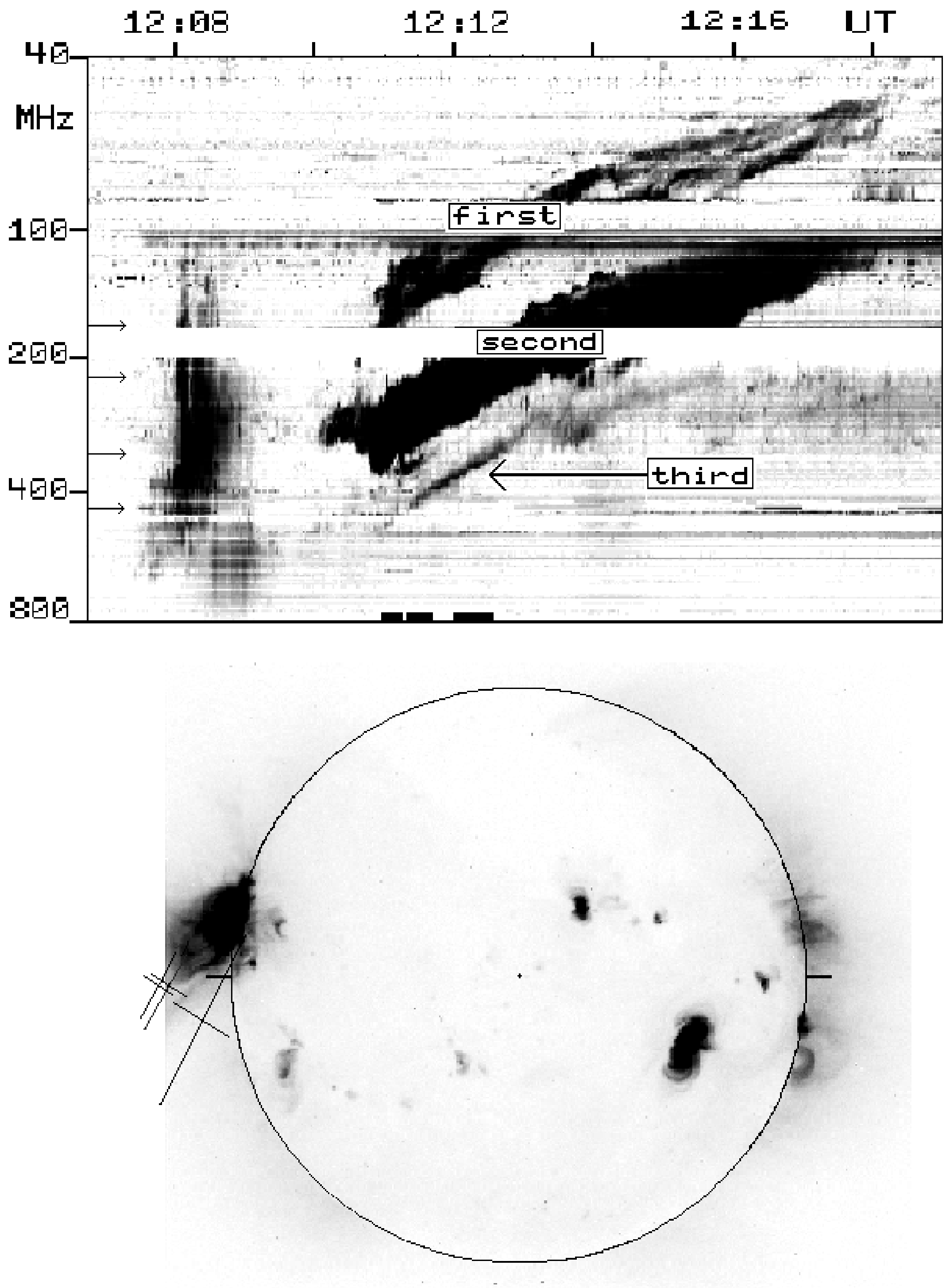

Fig. 1. Type II burst on 27 Sep 1993. Top: dynamic spectrum, after subtraction of pre-event fluxes. NRH observing frequencies and time intervals used to compute brightness temperatures (Table 1) are denoted by arrows and horizontal bars, respectively. Bottom: Source positions and half widths of the f_p source (164 MHz, big cross, 12:11 UT) and the $2f_p$ source (327 MHz, small cross, 12:11 UT) are superposed upon the YOHKOH-SXT image taken at 12:00:29 UT (AlMg filter, exposure 5.338 s). The $3f_p$ source at 435 MHz (second small cross, 12:11 UT) is about cospatial with the $2f_p$ source at 327 MHz.

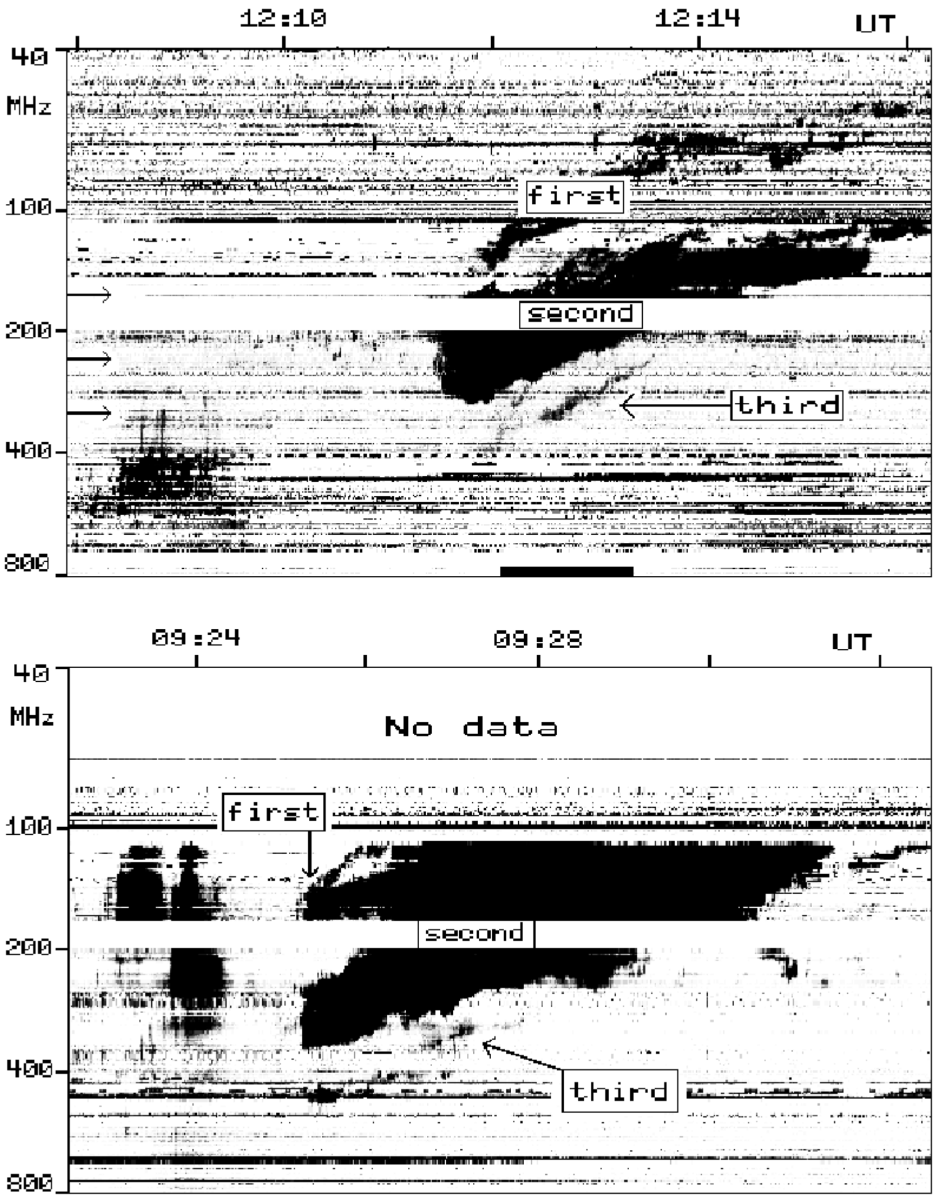

Fig. 2. Dynamic spectrograms of Type II bursts on 28 Dec (top) 1993 and 6 May 1996 (bottom).

the north-south scans) were used to infer the brightness temperatures, assuming a circular cross section of the radio source. When two sources were observed at a given frequency, one of them dominated the other by at least a factor 2. Therefore only the brightness temperature of the dominant source was evaluated. The measured quantities are listed in Table 1. Since the emission is fluctuating, the highest values (T_b^{max}) during the considered time intervals are given. In the two 1993 events the $2f_L$ source is brighter than the $3f_L$ emission, by factors of 140–1200 and 4–6, respectively. The values are more uncertain on 28 Dec 1993 than on 27 Sep 1993 because terrestrial interference and noise storm emission make the flux calibration more difficult. The relatively higher flux density of the $3f_L$ band in the 28 Dec burst, compared with that of 27 Sep 1993, is already borne out by the comparison of the $2f_L$ and $3f_L$ bands in the spectra. The brightness temperature ratio between the $2f_L$ and $3f_L$ sources is greater than the ratio of different split bands of the $2f_L$ emission, which was found to be less than a factor 2. No imaging observations are available for the 6 May 1996 event. The flux density ratio between $2f_L$ and $3f_L$ inferred from the OSRA spectral records at 327 MHz is in the range 150–250, which suggests a brightness temperature ratio similar to the 27 Sep 1993 burst.

3. Mechanisms of second and third harmonic plasma emission

The simultaneous radiation in two harmonically related frequency bands during type II (as well as in type III) bursts is attributed to a so-called plasma mechanism, which includes excitation of plasma (longitudinal or electrostatic) waves by some agent moving in the solar corona outwards (electron beams for type III and electrons running away from the shock wave front

Table 1. Brightness temperatures $\mathbf{T_b}$

Time UT	Freq. [MHz]	Spec. lane	T_b^{max} [10^9K]	Source size [arcmin]	Source size [HPBW]	Flux [sfu]
			27 Sep 1993			
12:10:57-	164	f_L	3.5	3.7	3.1	280
-12:11:14	327	$2f_L$	85	1.9	1.6	7100
	435	$3f_L$	0.7	1.6	1.8	73
12:11:20-	236	$2f_L$	380	2.3	2.7	22000
-12:11:40	327	$3f_L$	0.3	1.8	1.5	20
12:12:00-	236	$2f_L$	300	2.4	2.9	21000
-12:12:35	327	$3f_L$	0.9	2.0	1.7	82
			28 Dec 1993			
12:12:07	236	$2f_L$	0.9	2.2	2.7	47
12:12:37	327	$3f_L$	0.3	1.5	1.3	14
12:13:22	164	$2f_L$	1.6	2.3	1.9	46

1 sfu (solar flux unit) = 10^{-22}Wm^{-2}Hz^{-1};
HPBW - half-power beamwidth.

responsible for type II), and their subsequent transformation into electromagnetic (transverse) waves escaping from the corona, due to non-linear effects in the coronal plasma (Zheleznyakov 1977, 1996, Kundu 1965, Melrose 1980, Benz 1993). The origin of the second harmonic emission in both types is well understood as a result of coalescence (combinational scattering) of two plasma waves into transverse one at twice the plasma frequency:

$$l_1 + l_2 \rightarrow t_{II} \tag{1}$$

(here and below arabic numerals and the letter l refer to plasma or longitudinal waves, while Roman ones and the letter t refer to electromagnetic or transverse waves).

As for the third harmonic radiation, from the point of view of non-linear wave interaction in coronal plasma there are two ways to explain it which are allowed by conservation laws. The first is the two-step process

$$l_1 + l_2 \rightarrow t_{II};\;\; t_{II} + l_3 \rightarrow t_{III}, \tag{2}$$

which includes coalescence of two plasma waves l_1 and l_2 into an electromagnetic wave t_{II} at twice the plasma frequency and then coalescence of t_{II} and a plasma wave l_3 resulting in an electromagnetic wave t_{III} at the triple plasma frequency. The second way

$$l_1 + l_2 + l_3 \rightarrow t_{III} \tag{3}$$

is direct coalescence of three plasma waves l_1, l_2, l_3 into an electromagnetic wave t_{III}. As shown by Zheleznyakov and Zlotnik (1974) the third harmonic in type III bursts is most probably due to the process (2), because the effect (3) is less efficient in the source of these bursts. However, the second part of (2) can occur only for plasma waves, the phase velocities v_{ph} of which satisfy the inequality (Zheleznyakov and Zlotnik, 1974, Cairns, 1987, 1988):

$$0.22c = c/(2\sqrt{2} + \sqrt{3}) < v_{ph} < c/(2\sqrt{2} - \sqrt{3}) = 0.9c \tag{4}$$

(c is the velocity of light), otherwise conservation laws are not valid.

The condition (4) is not a strong restriction on plasma waves excited by the fast electrons and giving birth to type III bursts ($v_{ph} \sim c/3$). As for type II bursts, including so-called back-bone and herringbone structures, the situation is more complicated. The origin of the back-bone is unclear as yet, but herringbone structure is considered now (see, for instance, Mann 1995) to be due to electrons running away from the shock front and exciting plasma waves. According to measurements of frequency drift and modelling electron density distribution in the corona, the speed of the accelerated electrons is relatively low, $v_e \sim (0.03 \div 0.05)c$, and so is the phase velocity of the excited plasma waves (Mann, 1995). Assuming that type II bursts originate mainly from such electrons, we conclude that the process (2) cannot directly result in generation of the third harmonic. The same difficulty is met by interpretation of multiple plasma harmonic radiation in the Earth's bow shock. To avoid this constraint, Cairns (1987, 1988) suggested the following three-step process capable to explain observed harmonic structure:

$$l + s \rightarrow l';\;\; l'_1 + l'_2 \rightarrow t_{II};\;\; t_{II} + l' \rightarrow t_{III}, \tag{5}$$

where an incident plasma wave l interacts with a low-frequency ion sound wave s to generate a plasma wave l' with greater phase velocity; then these second or "fast" plasma waves l'_1 and l'_2 generate the second harmonic t_{II} and the third one t_{III}, like in the process (2), if plasma wave l' has appropriate phase velocity. Later Kliem et al. (1992) mentioned this idea in relation to type II bursts. At the same time the coalescence of three plasma waves (3) is permitted by conservation laws with no restrictions on the values of phase velocities.

The aim of the following investigation is to elucidate the relative part of mechanisms (3) and (5) in the source of type II bursts, and to explain observed intensities of the second and third harmonic lanes in the events described in Sect. 2. Subsect. 3.1 gives a set of known formulas for the second harmonic generation. Coalescence of three plasma waves into electromagnetic emission is investigated in 3.2. The three-step process (5) is considered in 3.3.

3.1. Coalescence of two plasma waves

It is assumed that in the type II burst source the plasma waves are excited by electrons moving relative to ions in the vicinity of the shock front. Then the electromagnetic wave at the second harmonic occurs according to the process (1). The conservation laws for this case have the form:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_{II};\;\; \omega_1 + \omega_2 = \omega_{II} \simeq 2\omega_L, \tag{6}$$

where $\mathbf{k}$ and ω with appropriate subscripts are wave vectors and frequencies of plasma waves 1,2 and the electromagnetic wave II, $\omega_L = (4\pi e^2 N/m_e)^{1/2}$ is the Langmuir (plasma) frequency, N is the electron density, e and m_e are charge and mass of the electron. The process described by (1), (6) has been extensively studied in the literature. In particular, it follows from Zheleznyakov (1977, 1996) that if spatial spectrum of plasma waves is of isotropic character and combinational scattering (1) is optically thin, the brightness temperature of radiation at twice the plasma frequency is determined by:

$$T_{II} \simeq \frac{8\pi^2}{15\sqrt{3}} \frac{\kappa e^2 f_L^2 T_1^2 L_{II}}{m_e^2 c^3 v_T^2 v_{ph}}, \tag{7}$$

where $f_L = \omega_L/2\pi$, κ is Boltzmann's constant, $v_T = (\kappa T/m_e)^{1/2}$ is the electron thermal velocity, T_1 is plasma wave brightness temperature, L_{II} is a characteristic size of the region (along the line-of-sight) in which plasma waves at a given frequency ω_1 can exist (see more details in Zheleznyakov and Zlotnik, 1974; Zaitsev, 1975). From the dispersion relation for plasma waves

$$\omega_1^2 = \omega_L^2 + 3k_1^2 v_T^2 \tag{8}$$

it is easy to estimate the size L_{II} in the inhomogeneous corona with characteristic scale $L_N = N(dN/dl)^{-1}$ of the electron density change:

$$L_{II} \simeq \frac{6k_1^2 v_T^2}{\omega_L^2} L_N \tag{9}$$

Substituting $k_1 \simeq \omega_L/v_{ph}$ and L_{II} in (7), we obtain:

$$T_{II} \simeq \frac{4(2\pi)^2}{5\sqrt{3}} \frac{e^2\kappa}{m_e^2 c^3} \frac{f_L^2}{v_{ph}^3} T_1^2 L_N. \tag{10}$$

This relation is obtained for an isotropic spectrum of plasma waves. If they are excited in some preferable direction, then the expression for T_{II}, unlike (10), contains a multiplier $\Psi(\theta) = \sin^2\theta\cos^2\theta$, where θ is the angle between the direction of the plasma wave vector $\mathbf{k}_1$ and the line-of-sight $\mathbf{k}_{II}$. In this case the value T_1 necessary to provide the observed brightness temperature at the second harmonic is dependent on the angle θ, i.e. must be greater in some directions than prescribed by (10).

Note that the values T_{II} and f_L are known from observations. The value v_{ph} is usually estimated by measuring the frequency drift velocity and modelling the electron density distribution in the corona. The scale L_N is also determined by this modelling. This means that using (10) we can find (under some reasonable assumptions) the brightness temperature of initial plasma waves.

It should be noted that momentum conservation also imposes an upper limit on the phase velocity of plasma waves capable to participate in the process $l_1 + l_2 \rightarrow t_{II}$:

$$v_{ph} < \frac{2c}{\sqrt{3}}. \tag{11}$$

3.2. Coalescence of three plasma waves

The conservation laws for the process (3) have the form:

$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{k}_{III}; \quad \omega_1 + \omega_2 + \omega_3 = \omega_{III} \simeq 3\omega_L \tag{12}$$

Unlike process (2), in which the plasma wave l_3 (coalescing with electromagnetic wave t_{II} and giving birth to the wave t_{III}) must fit some requirements, the process (3) and conservation laws (12) don't put any kinematic restrictions on wave number values of the interacting plasma waves. The only condition is quasi-isotropy of the plasma wave spectrum: a sum of three great-magnitude vectors $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ can give small-magnitude vector $\mathbf{k}_{III}$ only if they are not parallel or anti-parallel to each other (as it would happen in the case of a quasi-one-dimensional spectrum).

Following the methods worked out for this kind of problems (Tsytovich, 1967, Zheleznyakov, 1977, Melrose, 1980), we find the emissivity into the wave t_{III}:

$$a_{III}^{\omega} = \frac{k_{III}^2}{v_{III}^{gr}} \frac{dW_{\mathbf{k}_{III}}}{dt}, \tag{13}$$

where the wave number k_{III} and group velocity v_{III}^{gr} at the frequency $\omega_{III} \simeq 3\omega_L$ are

$$k_{III} = 2\sqrt{2}\frac{\omega_L}{c}; \quad v_{III}^{gr} = \frac{2\sqrt{2}}{3}c, \tag{14}$$

and the value $dW_{\mathbf{k}_{III}}/dt$ is described by:

$$dW_{\mathbf{k}_{III}}/dt = \int \Pi(\mathbf{k}_{III}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) W_{\mathbf{k}_1} W_{\mathbf{k}_2} W_{\mathbf{k}_3} d\Lambda; \tag{15}$$

$$d\Lambda = \delta(\omega_{III} - \omega_1 - \omega_2 - \omega_3) \times \\ \delta(\mathbf{k}_{III} - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3. \tag{16}$$

Here plasma wave energy densities in wave vector space $W_{\mathbf{k}_i}$ are connected with corresponding brightness temperatures of plasma waves T_i:

$$W_{\mathbf{k}_i} = \frac{\kappa T_i}{(2\pi)^3}, \tag{17}$$

and probability $\Pi(\mathbf{k}_{III}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is equal to:

$$\Pi(\mathbf{k}_{III}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{8\pi^2 \omega_{III}}{\partial(\omega_{III}^2 \epsilon_\perp)/\partial\omega_{III}} \times \\ \frac{(4\pi)^3 \omega_1\omega_2\omega_3 |\mathbf{S}|^2}{|\partial\epsilon_\parallel/\partial\omega_1||\partial\epsilon_\parallel/\partial\omega_2||\partial\epsilon_\parallel/\partial\omega_3|}. \tag{18}$$

The non-linear conductivity vector $\mathbf{S}$ is to be found by solving the kinetic equation and a system of electrodynamic equations, developing the total current into a series of electric field powers and finding the cubic component in the electric fields

$\mathbf{E}_{\mathbf{k}_1}, \mathbf{E}_{\mathbf{k}_2}, \mathbf{E}_{\mathbf{k}_3}$, which generate the electric field $\mathbf{E}_{\mathbf{k}_{III}}$. Omitting the working calculations, we write the final expression for $\mathbf{S}$:

$$\mathbf{S}(\mathbf{k}_{III}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -(\frac{e}{im_e})^4 \frac{2\pi i e^2 N^2}{\omega_{III}\omega_1\omega_2\omega_3 k_1 k_2 k_3} \times$$
$$\{\{\frac{\mathbf{k}_{1\perp}[(\mathbf{k}_2, \mathbf{k}_2+\mathbf{k}_3)k_3^2(\frac{\omega_2}{\omega_3}) + (\mathbf{k}_3, \mathbf{k}_2+\mathbf{k}_3)k_2^2(\frac{\omega_3}{\omega_2})]}{(\omega_2+\omega_3)^2(\mathbf{k}_2+\mathbf{k}_3)^2\epsilon_{\|}(\omega_2+\omega_3)} +$$
$$\frac{\mathbf{k}_{1\perp}2(\mathbf{k}_2, \mathbf{k}_2+\mathbf{k}_3)(\mathbf{k}_3, \mathbf{k}_2+\mathbf{k}_3)}{(\omega_2+\omega_3)^2(\mathbf{k}_2+\mathbf{k}_3)^2\epsilon_{\|}(\omega_2+\omega_3)}\}\times \tag{19}$$
$$(k_{III}^2/\omega_{III} - k_1^2/\omega_1 - 2k_{III}k_{1\|}/\omega_{III}) +$$
$$\text{the same}(1 \leftrightarrow 2) + \text{the same}(1 \leftrightarrow 3)\}.$$

Here $k_{1\perp}$ and $k_{1\|}$ denote the components of the plasma wave vector perpendicular and parallel to $\mathbf{k}_{III}$. The term $(\mathbf{k}_2, \mathbf{k}_2+\mathbf{k}_3)$ denotes the scalar product of vectors $\mathbf{k}_2$ and $\mathbf{k}_2+\mathbf{k}_3$. Integration in (15) is performed over the total wave vector volume occupied by plasma waves, taking into account conservation laws by means of the δ-functions.

Bearing in mind that in an isotropic plasma $\epsilon_{\|} = 1 - \omega_L^2/\omega^2, \epsilon_\perp = k^2c^2/\omega^2$, we can easily calculate the derivatives in the denominator of (18). Further, assuming the plasma wave spectrum to be isotropic in space and their energy density $W_{\mathbf{k}_1}$ to be independent of k_1 inside some interval Δk_1, we can perform integration in (15), using δ-functions and then taking constants out of the integral. As a result, (15) reduces to the following approximate expression:

$$\frac{dW_{\mathbf{k}_{III}}}{dt} \simeq \frac{2^9\pi^7}{3}\frac{\omega_L k_1^3\Delta k_1}{v_T^2}(W_{\mathbf{k}_1})^3|\mathbf{S}|^2. \tag{20}$$

When estimating input of $|\mathbf{S}|^2$ using(18), we have to take into account that since $|\mathbf{k}_{III}| \ll |\mathbf{k}_1|, |\mathbf{k}_2|, |\mathbf{k}_3|$, the conservation laws (12) demand wave vectors $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ of interacting plasma waves to be in the form of about equal-side triangle. In this case $|\mathbf{k}_2+\mathbf{k}_3| = k_1, (\mathbf{k}_2+\mathbf{k}_3, \mathbf{k}_2) = k_1^2/2,\ (\mathbf{k}_2+\mathbf{k}_3, \mathbf{k}_2) = k_1^2/2, (\mathbf{k}_3,\ \mathbf{k}_2+\mathbf{k}_3) = k_1^2/2$. Substituting these values (and the same for $1 \leftrightarrow 2, 1 \leftrightarrow 3$) into (18), we obtain a simplified formula, suitable for estimations:

$$|\mathbf{S}| \simeq \frac{1}{16\pi}\frac{e^2}{m_e^2}\frac{k_1^2}{\omega_L^3} \tag{21}$$

Accordingly, (19) transforms to the form

$$\frac{dW_{\mathbf{k}_{III}}}{dt} \simeq \frac{2\pi^5}{3}(\frac{e}{m})^4\frac{k_1^7\Delta k_1}{v_T^2c^3\omega_L^3}W_{\mathbf{k}_1}^3 \tag{22}$$

Using (13)-(14) and (21) it is possible now to write the emissivity a_ω^{III} in terms of the brightness temperature of plasma waves:

$$a_{III}^\omega \simeq \frac{\sqrt{2}}{8(2\pi)^4}(\frac{e}{m_e})^4\frac{k_1^7\Delta k_1\kappa^3}{v_T^2c^3\omega_L^3}T_1^3 \tag{23}$$

This formula coincides with (18) in Zheleznyakov and Zlotnik (1974) up to a numerical multiplier of the order of unity.

In (15) it is assumed that the process of coalescence is "optically thin", i.e. that the decay of the electromagnetic wave into plasma waves is insignificant (the estimations afterwards confirm this assumption under the conditions of type II burst sources). Then the brightness temperature of the third harmonic radiation is equal to:

$$T_{III} = \frac{\pi c^2}{f_{III}^2\kappa}a_{III}^\omega L_{III}, \tag{24}$$

where L_{III} is the size of the region where plasma waves can exist and coalesce. It is reasonable to put $L_{III} \simeq L_{II}$ (9). Then, assuming $\Delta k_1 \sim k_1$ and $k_1 \sim \omega_L/v_{ph}$, we obtain for T_{III}:

$$T_{III} \simeq \frac{\sqrt{2}\pi^2}{6}\frac{\kappa^2}{c}(\frac{e}{m_e})^4\frac{f_L^3}{v_{ph}^{10}}T_1^3L_N \tag{25}$$

Apparentely, as it should be expected, T_{III} is the third power of plasma wave brightness T_1. Note that T_{III} is strongly dependent on the phase velocity of plasma waves ($\propto v_{ph}^{-10}$), the less v_{ph} the more efficient the process of coalescence. It should be pointed out also, that relation (24) is obtained for isotropic spectrum of plasma waves. If the primary plasma waves are excited preferably in one direction, then the process considered above can happen in the following way: these primary waves creat an isotropic background of second plasma waves with brightness temperature $\tilde{T}_1 < T_1$ as a result of spontaneous scattering; then the primary plasma wave $\mathbf{k}_1$ chooses from this background the waves at such angles relative to $\mathbf{k}_1$ which can participate in the process $l_1 + l_2 + l_3 \to t_{III}$; in this case $T_{III} \propto T_1(\tilde{T}_1)^2$.

3.3. Coalescence of the second harmonic electromagnetic radiation and plasma wave

Let us turn to the process (5), suggested by Cairns (1987, 1988) for explaining multiple plasma harmonic radiation in the Earth's bow shock, and mentioned by Kliem et al. (1992) in connection with plasma parameters typical for type II solar bursts.

It should be noted, first of all, that this process is worth considering only in the case where the non-linear transfer of plasma waves into those with great phase velocities is optically thick, that is energy density or brightness temperature of second or "fast" plasma waves is not less than that of primary or "slow" plasma waves. Otherwise the process of the third harmonic radiation contains the nonlinearity of the forth order of smallness (unlike cubic nonlinearity in case of type III plasma waves) and cannot be significant.

Note also, that ion-sound waves can exist only in nonisothermal plasma where electron and ion thermal temperatures are not equal to each other: $T_e \neq T_i$. Under the conditions of the solar corona, including the sources of intense radio emission, such an inequality is hardly expected. In isothermal plasma the equivalent effect of induced scattering on ions

$$l_1 \to l' + i \tag{26}$$

can serve as the first step in the process (5) in order to transform plasma waves with relatively low phase velocities into those with small wave numbers capable to satisfy inequality (4) and participate in the generation of the third harmonic.

Let us estimate (following methods by Zaitsev, 1975, 1977) the efficiency of the process (25) assuming that the initial width of k_1–spectrum of plasma waves is rather great (compared with the change of k_1 in one act of scattering) and non-linear transfer along the spectrum towards smaller k' is of differential character. In this case we can describe the change of averaged spectral density $W_{k_1} = k_1^2 W_{\mathbf{k}_1}$ by (Kaplan and Tsytovich, 1973):

$$\frac{\partial W_{k_1}}{\partial t} = \alpha^l W_{k_1} \frac{\partial W_{k_1}}{\partial k_1}, \tag{27}$$

where the probability α^l is:

$$\alpha^l = \frac{4\pi^2}{27} \frac{e^2 \omega_L}{m_e m_i v_T^4 (1 + T_e/T_i)^2}. \tag{28}$$

It follows from (26) that the typical time scale of W_{k_1} change is:

$$\gamma_1 = \frac{1}{W_{k_1}} \frac{\partial W_{k1}}{\partial t} \sim \alpha^l \frac{\partial W_{k_1}}{\partial k_1}. \tag{29}$$

This describes the well-known effect of amplification of plasma waves in the part of the spectrum where $\partial W_{k_1}/\partial k_1 > 0$ and their decay in the part of the spectrum where $\partial W_{k_1}/\partial k_1 < 0$. It means that if the primary plasma waves are excited in some interval Δk_1 with maximum at some $\tilde{k_1}$ in the middle of this interval, then the spectrum evolves in such a way that its maximum shifts to the smaller wave numbers or greater phase velocities. Our goal is to estimate the optical depth of the plasma layer, where plasma waves at a given frequency ω_1 can exist, relative to the process of shifting $\tilde{k_1} \simeq \omega_L/v_{ph}$ ($v_{ph} \sim 10^9$ cm/s) to smaller $\tilde{k}'$ capable to satisfy the condition (4).

The corresponding coefficient of spatial decay or growth is:

$$\mu_1 = \gamma_1/v_1^{gr} = \alpha^l (\partial W_{k_1}/\partial k_1)/v_1^{gr}, \tag{30}$$

where the group velocity of plasma waves is $v_1^{gr} = 3v_T^2/v_{ph}$, and the optical depth:

$$\tau_1 = \int \mu_1 dl = \int \frac{\alpha^l}{v_1^{gr}} \frac{\partial W_{k_1}}{\partial k_1} dl. \tag{31}$$

Integration over the ray path in (30) may be replaced by integration over wave numbers k_1 using the relation $dl = 6L_N k_1 v_T^2 dk_1/\omega_L^2$ implied by the condition $\omega_1^2 = \omega_L^2(l) + 3k_1^2(l)v_T^2 = const$. This procedure is equivalent to taking L_{II} (9) as a size of the region in the inhomogeneous plasma where a plasma wave at the frequency ω_1 can propagate. Substituting α^l (27) and $v_1^{gr} = 3k_1 v_T^2/\omega_L$ into (30), we can reduce it to the form:

$$\tau_1 = \frac{8\pi^2}{27} \frac{e^2 L_N}{m_e m_i v_T^4 (1 + T_e/T_i)^2} \int \frac{\partial W_{k_1}}{\partial k_1} dk_1. \tag{32}$$

Estimating integral in (31) as some averaged value W_{k_1} and substituting $W_{k_1} = k_1^2 \kappa T_1/(2\pi)^3$, we obtain:

$$\tau_1 = \frac{4\pi}{27} \frac{e^2 \kappa}{m_e m_i v_T^4 (1 + T_e/T_i)^2} \frac{f_L^2 T_1^2 L_N}{\alpha^2}, \tag{33}$$

where

$$\alpha = v_{ph}/v_T. \tag{34}$$

The necessary condition for induced scattering to be efficient is $\tau_1 > 1$. Estimating τ_1 at frequency $f_L \simeq 160$ MHz, thermal velocity $v_T = 3 \cdot 10^8$ cm/s and inhomogeneity density scale $L_N \sim (10^9 - 10^{10})$ cm gives:

$$\tau_1 \sim 9 \cdot (10^{-11} \div 10^{-10}) T_1/\alpha^2, \tag{35}$$

Consequently, at $\alpha \sim 3$ ($v_{ph} \sim 10^9$ cm/s) the inequality $\tau_1 > 1$ is valid if the brightness temperature of primary waves is:

$$T_1 > (10^{10} \div 10^{11}) K. \tag{36}$$

The same (by the order of magnitude) estimation is obtained for the process $l_1 \to l' + s$, where a plasma wave l_1 decays into another plasma wave l' and an ion-sound wave s, if one uses the relations given by Tsytovich (1967) for the typical time of re-distribution of plasma waves to the interval of great phase velocities.

The condition (35) is probably valid in the sources of type II bursts (see estimations in Sect. 4). Thus, indeed, slow plasma waves excited by some agent can quickly transfer to plasma waves with great phase velocities, which can participate in the process $t_{II} + l' \to t_{III}$.

It is important that what we have obtained is an estimation of non-linear transfer of plasma waves to those with small wave numbers only from above. We didn't take into account mechanisms which can impede the considered process, for instance, decay of plasma waves due to electron-ion collisions or the effect of compensation, which results from the fact that the sections of the plasma wave spectrum with $\partial W_{k_1}/\partial k_1 > 0$ give a positive input into the value τ_1, while those with $\partial W_{k_1}/\partial k_1 < 0$ give a negative one (this can be easily seen from the integral in (31), which is equal to zero if integration is performed over the whole interval of wave numbers covered by plasma waves). Nevertheless, we assume that the energy density of plasma waves with small k' doesn't differ remarkably from that of primary plasma waves.

It should be noted that a similar evaluation for the coalescence of two primary plasma waves into the second harmonic electromagnetic wave shows that the process $l_1 + l_2 \to t_{II}$ is optically thin, and therefore $T_{II} \ll T_1$.

Thus, the following scheme of the third harmonic origin in type II bursts seems to be reasonable: plasma waves with relatively low phase velocity are excited by some slowly moving agent; these waves quickly transfer to regions of the spectrum with greater phase velocity, but are permanently restored in the resonance part of the spectrum due to instability, the second harmonic arising from coalescence of primary "slow" waves and the third one being a result of coalescence of the second harmonic electromagnetic wave and "fast" plasma wave:

$$l_1 \to l'; \ \ l_1 + l_2 \to t_{II}; \ \ t_{II} + l' \to t_{III} \tag{37}$$

Note that although the scheme of third harmonic formation outlined above is rough, it is sufficient to estimate the expected level of different lanes in type II bursts. Actually, the stationary spectrum of plasma waves in the source associated with shock waves must be found, considering the specific mechanism of instability responsible for plasma wave excitation as well as every possible mechanism of linear and non-linear decay of plasma waves and their spectral transfer. We also leave aside the origin of the fundamental in type II bursts, resulting probably from induced scattering of primary plasma waves on ions. Both problems will be considered elsewhere.

The emissivity of electromagnetic waves arising from the last step of the process (37) can be evaluated using the known probability of coalescence of plasma and electromagnetic waves into an electromagnetic wave (Tsytovich, 1967):

$$w^t_{t,l}(\mathbf{k}_{III}, \mathbf{k}_{II}, \mathbf{k}') = \frac{\hbar e^2 (2\pi)^6 \omega_L (k')^2}{16\pi m_e^2 \omega_{III}(\mathbf{k}_{III})\omega_{II}(\mathbf{k}_{II})} \times$$
$$(1 + \frac{(\mathbf{k}_{II}\mathbf{k}_{III})^2}{k^2_{II}k^2_{III}})\delta(\mathbf{k}_{\mathrm{III}} - \mathbf{k}_{\mathrm{II}} - \mathbf{k}')\delta(\omega_{III} - \omega_{II} - \omega'). \tag{38}$$

With regard to (16), the well-known transfer equation for energy density (Zheleznyakov 1977, 1996)

$$\frac{dW_{\mathbf{k}_{III}}}{dt} = \int w^t_{t,l'} \frac{\omega_{III}}{\omega_{II}\omega'} W_{\mathbf{k}_{II}} W_{\mathbf{k}'} \frac{d\mathbf{k}_{II} d\mathbf{k}'}{\hbar (2\pi)^3} \tag{39}$$

can be written in terms of brightness temperatures:

$$\frac{dT_{III}}{dt} \simeq \frac{3\kappa}{2\hbar\omega_L} \int w^t_{t,l}(\mathbf{k}_{III}, \mathbf{k}_{II}, k') T' T_{II} \frac{d\mathbf{k}' d\mathbf{k}_{II}}{(2\pi)^6}. \tag{40}$$

Performing integration in (40) over $d\mathbf{k}_{II}$ with $\delta(\mathbf{k}_{III} - \mathbf{k}_{II} - \mathbf{k}')$ and assuming $(\mathbf{k}_{II}, \mathbf{k}_{III})^2/k^2_{II}k^2_{III} \sim 1$ (for estimation from above) reduces equation (39) to the form:

$$\frac{dT_{III}}{dt} \simeq \frac{\kappa e^2}{32\pi m_e^2 \omega_L^2} \int T' T_{II} \delta(\omega_{III} - \omega_{II} - \omega')(k')^2 d\mathbf{k}', \tag{41}$$

where every function in the integrand is taken at $\mathbf{k}_{II} = \mathbf{k}_{III} - \mathbf{k}'$. Integration over $d\mathbf{k}'$ is performed taking into account $d\mathbf{k}' = (k')^2 \sin\theta d\varphi d\theta dk'$, where φ and θ are the angles in a cylindrical system of coordinates with axis along $\mathbf{k}_{III}$. If plasma wave spectrum is isotropic, integration over φ gives multiplier 2π. But $k^2_{II} = (\mathbf{k}_{III} + \mathbf{k}')^2 = k^2_{III} - (k')^2 - 2k_{III}k'\cos\theta$ depends on k' and θ. Integration over $d\theta$ is performed with the function $\delta(\omega_{III} - \omega_{II} - \omega')$. This results in the multiplier $\partial\omega_{II}/\partial\theta = (\partial\omega_{II}/\partial k^2_{II})(2k_{III}k'\sin\theta) = \sqrt{2}k'c\sin\theta$ in the denominator of the integrand in (40) and the following emissivity:

$$\frac{dT_{III}}{dt} = \frac{1}{16\sqrt{2}} \frac{\kappa e^2}{m_e^2 c \omega_L^2} \int (k')^3 T' T_{II} dk'. \tag{42}$$

The limits of integration in (39)-(41) are determined by the boundaries of the wave number intervals where plasma waves satisfying conservation laws are concentrated. According to (4), k' is confined by the values $k'_{min} \simeq 1.1(\omega_L/c)$ and $k'_{max} \simeq 4.5(\omega_L/c)$. Since we don't know dependence on k' from our qualitative consideration, evaluation of the integral in (42) is made as follows: $\int (k')^3 T' T_{II} dk' \sim (k')^3 T' T_{II} \Delta k'$, where the intergrand is taken at some point in the middle of the interval $\Delta k' = k'_{max} - k'_{min}$. We will put $k' = a(\omega_L/c)$, where the number a can change from 1.1 up to 4.5 and assume $\Delta k' \sim k'$. Then (42) reduces to the form:

$$\frac{dT_{III}}{dt} \sim \frac{\pi a^2}{4\sqrt{2}} \frac{\kappa e^2}{m_e^2 c^5} f_l^2 T' T_{II}. \tag{43}$$

Using (13)-(14) and (23) we obtain the emissivity and the brightness temperature of the third harmonic:

$$T_{III} \simeq \frac{\sqrt{2}}{3c} \frac{dT_{III}}{dt} L_{III} \tag{44}$$

but unlike the process $l_1 + l_2 + l_3 \to t_{III}$, here one may put $L_{III} \sim L_N$, since the spectrum of the second harmonic consists of waves coming from all elements of the nonuniform source (see Zheleznyakov,Zlotnik 1974 for more details). Hence, we obtain:

$$T_{III} \sim \frac{\pi^2 a^4}{12} \frac{\kappa e^2}{m_e^2 c^6} f_L^2 T' T_{II} L_N. \tag{45}$$

The numerical multiplier here is determined only to within an order of magnitude, because of rather approximate integration over the plasma wave spectrum. This multiplier depends on the form and width of the "fast" plasma wave spectrum. Besides, the value T' cannot be found without a sequential self-consistent analysis of plasma wave transfer along the spectrum. For approximate evaluation we put T' of the order of brightness temperature of primary plasma waves T_1, and excluding T_{II} with help of (10), we get the expression for T_{III} similar to (24)

$$T_{III} \sim \frac{4\pi^4 a^4}{15\sqrt{3}} \frac{\kappa^2 e^4}{m_e^4 c^9} \frac{f_L^4 T_1^3 L_N^2}{v^3_{ph}} \tag{46}$$

The phase velocity v_{ph} here bears no relation to plasma waves coalescing with electromagnetic waves at the second harmonic; it enters in (45) from relation (10) for T_{II} and refers to plasma waves creating the second harmonic.

4. Discussion

4.1. The two mechanisms for third harmonic plasma emission presented here

The presented considerations allow numerical estimations of the expected brightness temperatures of second and third harmonic radiation assuming probable type II burst source parameters. The estimates can be compared with the observed temperatures.

First, we find the brightness temperature of primary plasma waves using the observed value of T_{II}. From (10) we have:

$$T_1 \sim 10^{18}\sqrt{\alpha^3 T_{II}/f_L^2 L_N} \tag{47}$$

For estimations we take two scales of electron density inhomogenities in the corona: there is the usually accepted value of $L_N \sim 10^{10}$ cm, and – with regard to increased density gradients in the vicinity of the shock front – a value one order of magnitude less, say $L_N \sim 10^9$ cm. Taking the fundamental frequency $f_L = 160$ MHz and $T_{II} = 3 \cdot 10^{11}$ K (for the time period 12:12:00 –12:12:35 UT; see Table 1) and assuming $\alpha \sim 3$ ($v_{ph} \sim 10^9$ cm/s) we obtain $T_1 \sim (0.3 \div 1) \cdot 10^{12}$ K for $L_N \sim (10^{10} \div 10^9)$ cm, correspondingly. If the second harmonic results from the coalescence of plasma waves with greater phase velocities ($\alpha > 3$), then the temperature T_1 required to provide the observed value T_{II} increases. The brightness temperature of primary plasma waves T_1 can be higher if their $\mathbf{k}_1$ spectrum is quasi-one-dimensional, and if the line of sight does not coincide with the directivity maximum of the t_{II} wave emissivity. Thus, the value obtained from (47) is most probably a lower limit of the brightness temperature T_1 of primary plasma waves capable to provide the observed T_{II}. So, we can assume $T_{II} \ll T_1$, and neglect the back–scattering of the t_{II} wave into l_1 and l_2.

The ratio $\delta = T_{III}/T_{II}$ can be evaluated for both considered non-linear processes. It is easy to obtain it using (10), (24) and (45):

$$\tilde{\delta} = \delta(t_{II} + l' \to t_{III}) \simeq 10^{-25} a^4 f_L \sqrt{\alpha^3 T_{II} L_N} \tag{48}$$

$$\tilde{\tilde{\delta}} = \delta(l_1 + l_2 + l_3 \to t_{III}) \simeq 0.03\sqrt{T_{II}/\alpha^{11} L_N}. \tag{49}$$

Evidently, the ratios depend quite differently on the source parameters except for the common term $\sqrt{T_{II}}$. This means that the detection probability of the third harmonic rises with the brightness temperature of the second harmonic. For the 27 Sept. 1993 burst we take according to Table 1 for the fundamental frequency $f_L = 160$ MHz and for the second and third harmonic brightness temperatures $T_{II} \simeq 3 \cdot 10^{11}$K and $T_{III} \simeq 0.9 \cdot 10^9$ K. Hence we get $\delta \simeq 3 \cdot 10^{-3}$. For the 28 Dec 1993 event this ratio reaches the value $\delta \sim 0.2$.

If we take for the plasma parameters (which seem to be probably realized in the source plasma of type II bursts): $\alpha \sim 3$ ($v_{ph} \sim 10^9$cm/s) and $L_N \sim (10^9 \div 10^{10})$ cm and choose the middle point $a = 2.5$, then we get:

$$\tilde{\delta} \sim (0.5 \div 1) \cdot 10^{-4}; \quad \tilde{\tilde{\delta}} \sim (2 \div 10) \cdot 10^{-4}. \tag{50}$$

Comparing these values with the observed $\delta \simeq 3 \cdot 10^{-3}$ we conclude that with reasonable v_{ph} and L_N the coalescence of three plasma waves is most probably responsible for the third harmonic radiation. However, the calculated ratios strongly depend on the magnitude of the plasma wave phase velocity. If the latter decreases (remember that it must be greater than the electron thermal velocity, otherwise plasma waves cannot exist because of Landau damping in the background plasma), then the ratio $\tilde{\delta}$ decreases, but $\tilde{\tilde{\delta}}$ increases sharply. For example, at $\alpha = 2$ the value $\tilde{\tilde{\delta}}$ increases tenfold. Assuming less α and greater electron density gradients (i.e. less L_N, for instance, $L_N \sim (10^7 \div 10^8)$ cm, which is possible near the shock wave front), $\tilde{\tilde{\delta}}$ is increased by one to two orders of magnitude thus fitting the value $\delta \sim 10^{-1}$ which was observed during the 28 Dec 1993 burst.

In the opposite case of great phase velocities the efficiency of the process $l_1 + l_2 + l_3 \to t_{III}$ drops, and the third harmonic can be due only to the process $t_{II} + l' \to t_{III}$. This is true for type III burst sources (Zheleznyakov and Zlotnik, 1974). Note, that the value α and the phase velocity in $\tilde{\delta}$ are determined by the plasma waves responsible for the second harmonic generation. It is not excluded that the wave t_{II} is generated by "faster" plasma waves than those excited by the slowly moving agent. In this case the parameter α in $\tilde{\delta}$ and $\tilde{\tilde{\delta}}$ is not the same and the value $\tilde{\delta}$ may be remarkably increased.

For example, if $\alpha \sim 30$ (as in type III bursts) and $L_N \sim 10^{10}$ cm, then $\tilde{\delta} \sim 10^{-3}$, what is closer to the observed value. Besides, the value $\tilde{\delta}$ can be increased if we take – in evaluating the integral in (41) – the integrand not in the middle point $a = 2.5$ of the interval $\Delta k'$ allowed by (4), but closer to its upper boundary. For instance, under the most favourable conditions, when $a = 4$, $\alpha = 30 (v_{ph} \simeq c/3)$ and $L_N \sim 10^{10}$ cm, it follows $\tilde{\delta} \sim 10^{-2}$. This fits nicely with the data. In case of a quasi-one-dimensional plasma wave spectrum the ratio $\tilde{\delta}$ can still be greater due to the higher level of primary plasma waves in some distinguished directions which is necessary to provide the observed values T_{II} (see our remark on relation (10)).

Thus, the relative part played by the two mechanisms of third harmonic generation depends on the circumstances. For example the ratio

$$\tilde{\tilde{\delta}}/\tilde{\delta} \sim 3 \cdot 10^{23}/\alpha^7 f_L L_N a^4 \tag{51}$$

is equal to $25 \div 2.5$ at $\alpha = 3$, $a = 2.5$, $L_N = (10^9 \div 10^{10})$ cm, correspondingly. For $v_{ph} > 10^9$ cm/s this ratio decreases remarkably.

Our analysis has shown that both considered mechanisms can be responsible for the occurrence of the third harmonic in type II radio bursts. The coalescence of three plasma waves is preferable at low phase velocities and sharp electron density gradients in the source. The coalescence of plasma and t_{II} electromagnetic waves is more efficient for high phase velocities. In principle, we do not know the phase velocities of coalescing plasma waves. We can estimate the phase velocity of primary plasma waves (and the velocity of runaway electrons generating these waves; see Mann, 1995) from the frequency drift of herringbones in the dynamic spectrum, only. However, using the results (48) and (49) some conclusions are possible.

– Concerning the bandwidth of the second and the third harmonic lane: For a source with a vertical extent of $L \sim L_N$ the band width reaches the value $\Delta\omega_{III} \sim \omega_L$. Since $L_{III} \sim L_N \gg L_{II}$ and $\Delta\omega_{III} \gg \Delta\omega_{II}$, the third harmonic lane resulting from the process $t_{II} + l' \to t_{III}$ must be more diffuse on the dynamic spectrum than the second harmonic lane. But in all observed type II bursts with three lanes this is not true: the third harmonic is as rich in narrowband features as the second one. This is an argument against the process $t_{II} + l' \rightarrow t_{III}$. Of course, this should not be overestimated taking into account both the variety of reasons that may cause the occurrence of fine structures and the rather approximate character of our theoretical considerations.

– Concerning an enhanced occurence frequency of type II bursts with three harmonic lanes at the solar limb: The three events discussed here and one of the two events mentioned in the literature before (Chertok et al. 1990) are situated above the limb. This fact can indicate some kind of directivity of the third harmonic radiation. Zlotnik (1978) has shown that in type III bursts quasi-one-dimensional plasma waves (excited mainly along the direction of electron beam movement) lead to a specific directivity of the third harmonic [1]. The third harmonic is visible inside the conic angle $\theta_0 \sim 37^0$ relative to the plasma wave vector direction, and in some interval $0 < \theta_0 < 37^0$ the intensity of the t_{III} wave may be comparable to or even higher than that of the t_{II} wave [2]. This effect explains type III bursts with a second and third harmonic flux in the same order (reported by Takakura & Yousef, 1974). The problem of directivity concerning the process $l_1+l_2+l_3 \rightarrow t_{III}$ for one-dimensional plasma waves has not yet been solved.

  If we accept for the moment for type II bursts the same concept as for type III bursts, we can get important information about the character of the shock wave: since the third harmonic can be observed with some preference in the direction of electron beam movement the electrons seem to move mainly along the line of sight (towards the Earth). This means that in a source above the limb the magnetic field (along which accelerated electrons run away from a shock front) is parallel to the solar surface. If we have additional information about the direction of the shock front motion, we are able to learn whether the shock wave was running parallel or perpendicular to the magnetic field. However, since the spatial spectrum of plasma waves responsible for type II bursts is unknown this remains a preliminary qualitative hypothesis, only.

– Concerning the observing frequency of type II bursts with three harmonically related lanes: Note that the ratio $\tilde{\delta}$ (48) is proportional to the fundamental frequency f_L. That implies a more probable occurrence of the third harmonic at high frequencies. The ratio $\tilde{\tilde{\delta}}$ (49) does not depend on f_L. Besides, the strong dependence of $\tilde{\tilde{\delta}}$ on α (which is connected with the plasma wave phase velocity) reveals that the process $l_1 + l_2 + l_3 \rightarrow t_{III}$ is the more probable explaining the third harmonic the less there is the herringbone drift rate in the dynamic spectrum. Both facts can be used to distinguish between the two invoked non-linear processes by a statistical investigation of sensitive digital type II burst observations.

  At the present state of the theory of type II bursts and the knowledge of source parameters we are not yet able to make a definite choice.

[1] due to the process $l_3 + t_{II} \rightarrow t_{III}$

[2] Of course, in the third harmonic there is less energy than in the second one.

4.2. Comments on other mechanisms

Let us briefly discuss the other mechanisms considered in the literature for explaining type II bursts with three lanes. Kliem et al. (1992) suggested collapsing Langmuir solitons as a candidate for harmonic radiation. However, usually the conditions of strong turbulence are hardly realized in the coronal plasma (see, for instance, estimations by Holman and Pesses, 1983). Besides, Kliem et al.'s (1992) consideration of the increase of the plasma wave frequency $f^2 = f_L^2(1 + 3k_l^2\lambda_D^2)$ up to $f \sim 3f_L$ [3] due to the motion of the wave along the dispersive curve towards larger k_l is not quite correct: k_l increases indeed, but the collapsing soliton displaces the plasma and forms a cavity (Zakharov 1984). Therefore f_L in the above relation decreases, and as a result the plasma wave frequency f decreases in the course of the collapse. Finally, if such a collapse is finished and f is grown up to $3f_L$ it will be impossible to see separated lanes on the dynamic spectrum – there should appear a wide band continuum.

A further idea on the origin of the third harmonic in type II bursts was suggested by Bakunin et al. (1990). They qualitatively describe the occurrence of three lanes in type II bursts as signature for the generation of the first harmonic of the electron cyclotron frequency $\omega_B = eB/m_e c$ in regions satisfying the condition of double plasma resonance $s\omega_B = \omega_L$. But an extraordinary wave cannot escape from the level $\omega = \omega_B$, because its point of reflection $\omega_L^2/\omega^2 = 1 - \omega_B/\omega$ is located higher in the corona (closer to an observer) than the level $\omega = \omega_B$. This means that the harmonic – if observed – should be fully polarized in contrast with the observations. Further, the waves escaping from gyroresonance layers $s = 1, 2$ are absorbed when propagating through the corona outwards by the layers $s = 2, 3, 4$. It is difficult to believe that the whole source is in the so-called "window of transparency" along the magnetic field where the gyroresonance absorption is supressed. In any case since the radiation at the fundamental mode can not be detected according to this model it is in serious contradiction with the observations.

5. Conclusions

Improved digital radio spectral observations (OSRA Potsdam–Tremsdorf) combined with high time resolution multi–frequency imaging observations (NRH Paris–Meudon) have doubtless shown the existence of lanes of type II bursts with a frequency ratio of 1 : 2 : 3. In the imaging data we have seen the second and the third harmonic emission escaping from the same volume. This justifies their interpretation in terms of electromagnetic wave emission near the fundamental, the second and third harmonic of the local plasma frequency. The brightness temperature of the third harmonic source is smaller than

[3] λ_D is the Debye wavelength

that of the second harmonic source by up to three orders of magnitude, but in one of three events the difference is less than one order of magnitude (Sect. 2).

Two non-linear processes – the coalescence of three plasma waves, and the coalescence of a modified plasma wave and an electromagnetic one at twice the plasma frequency – are considered to explain the occurrence of the third harmonic emission. The analysis has shown that both of them can fit the observed brightness temperatures of the second and third harmonic. In case of low phase velocities of plasma waves and sharp electron density gradients in the source the process $l_1 + l_2 + l_3 \rightarrow t_{III}$ is more preferable. Plasma waves with greater phase velocity (possibly arising from the non-linear transfer of primary plasma waves towards smaller wave numbers) yield a third harmonic due to the process $t_{II} + l' \rightarrow t_{III}$. Using relations given in Sects. 3 and 4 one can estimate the expected brightness temperatures. We predict properties of both mechanisms which can be used for identifying the more probable mechanism by a statistical analysis of type II burst spectra.

Therefore, the occurrence of the third harmonic due to non-linear effects in the coronal plasma demands for some specific conditions in the shock front (e.g. small velocities of the agent exciting plasma waves, or sharp gradients of electron density). Possibly this is one reason why the third harmonic occurrence in type II bursts is not so frequently reported. Another reason can be a rather narrow radiation pattern of the t_{III} wave excited by quasi-one-dimensional plasma waves. But a simple third reason is that only carefully analyzed sensitive digital radio spectral records reveal such comparativly faint effects, and that high time resolution multifrequency meter wave imaging data are only available for a comparatively short time.

In order to make firm conclusions on the reason of harmonic structure appearing in type II bursts and to retrieve the physical conditions in the shock waves exciting such bursts one must observe more events with multiple harmonics, prepare the proposed statistical analysis, and – concerning the theory – solve the self-consistent problem of a stationary frequency and spatial spectrum of plasma waves responsible for type II bursts.

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