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ASTRONOMY AND ASTROPHYSICS

Minor planets ephemerides improvement*

From joint analysis of Hipparcos and ground-based observations

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Abstract. Two methods are considered for the improvement of the ephemerides of the asteroids observed by the Hipparcos satellite. They utilize different combinations of ground-based and space-based data. In particular, we develop a method based on an analysis of the variance matrix in the least-squares solution of Hipparcos observations taken alone. Application and comparison of the results for (2) Pallas and (324) Bamberga are given. It is shown that one of the methods is less sensitive to the weighting of the equations of condition. This procedure can be applied to the improvement of the ephemerides of asteroids when the observational data consist of subsets of very different accuracies.

Key words: astrometry – celestial mechanics, stellar dynamics – ephemerides – minor planets, asteroids – planets and satellites: individual: (324) Bamberga, (2) Pallas

1. Introduction

Observations at a single apparition of newly discovered asteroids, comets or Kuiper-belt objects, rarely yield an accurate ephemeris because the observation times are too short compared to the sidereal period of the celestial bodies. The main uncertainties arise in the determinations of the mean anomaly and semi-major axis (Muinonen et al. 1994; Muinonen 1996). Such poorly constrained orbits may result in the loss of a newly discovered object.

The Hipparcos satellite observed 48 bright minor planets, all located in the main belt (Hestroffer & Mignard 1997), and provided their positions in the frame of the ICRS (International Celestial Reference System) with a very high precision ($\approx$ 15 mas). However, the 3 year mission duration and the rather random distribution of the observations along the minor planet's orbit, prevent a unique determination of the 6 osculating elements at a given mean epoch (Hestroffer et al. 1995). Ground-based observations of these asteroids were carried out extensively for the preparation of the Hipparcos mission from 1983 to 1994 with the automatic meridian circle of Bordeaux observatory and from 1984 with the Carlsberg automatic meridian circle at La Palma observatory. A study of the observations of asteroids made with the two instruments from 1985 to 1990 (Viateau 1995) showed that their mean accuracy was about $0\farcs15$ in right ascension and $0\farcs25$ in declination for the observations made at Bordeaux, and $0\farcs25$ in both coordinates for observations made at La Palma. Such high precision astrometric measurements allow the determination of a perturbing asteroid's mass (e.g. Hilton 1996, Viateau & Rapaport 1997), based on an analysis of the mutual perturbations.

Improvement of the ephemerides of the Hipparcos asteroids can be achieved using a joint analysis of the whole observational data by two methods. The first and most commonly used method consists of a direct combination of all observations and corresponding equations of conditions. In this paper, we present a second method based on the particular 'Hipparcos solution' as obtained from the Hipparcos observations alone. Application to the asteroids (2) Pallas and (324) Bamberga, and comparison of the results obtained by both methods are given. We show that Hipparcos observations substantially constrain the orientation of the osculating trajectory. The method developed here can usefully be applied to the ephemerides improvement of the 48 asteroids observed by Hipparcos. Its application to other asteroids can be particularly powerful when one has to combine new and older data-sets of very different accuracies.

2. Ephemerides improvement

Hipparcos and ground-based observations are different in nature and we have developed a method that considers specific aspects of each. The most striking differences are that (1) Hipparcos constrains the orientation of the osculating plane much better than the ground-based observations and, (2) the latter better constrains the semi-major axis and the eccentricity which are very poorly determined from Hipparcos observations alone. The

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* Based on data from the Hipparcos astrometry satellite

method introduced here will constrain the best determined parameters (i.e., linear combination of the unknowns) from the Hipparcos observations, while the other free parameters must fulfill the equations of condition of the ground-based observations. This is achieved by solving the least squares problem of the ground-based observations, constrained by the Hipparcos solution set.

2.1. Hipparcos solution

The Hipparcos solution consists of a set of vectors $\mathbf{x}_{\rm H}$ that minimizes the L_2 norm of the residuals $|\mathbf{y}_{\rm H} - \boldsymbol{A}_{\rm H}\,\mathbf{x}|_2$. We will always refer to the L_2 norm in the rest of the paper, and remove the subscript for brevity. Direct inversion of the problem does not yield a realistic solution because the matrix of the partial derivatives $\boldsymbol{A}_{\rm H}$ is ill-conditioned (Hestroffer et al. 1995). Although the Hipparcos normal matrix $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}$ (the prime denotes the transpose) can be inverted, it is however almost singular. Thus, even replacing the small singular values of $\boldsymbol{A}_{\rm H}$, or the small eigenvalues of $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}$, with zeros, we still have to solve a least squares problem that is not significantly different from the initial one.

For sake of brevity, we also denote the matrix obtained after nulling singular values by $\boldsymbol{A}_{\rm H}$. Let $\mathscr{K}$ be the null space of the application associated to the singular matrix $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}$, and generated by the vectors $\mathbf{x}_{\rm K}$. We have the relation $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}\,\mathbf{x}_{\rm K} = \mathbf{0}$. Thus, the Hipparcos observations do not provide a unique solution, but a solution of the form:

$$\mathbf{x}_{\rm H} = \bar{\mathbf{x}}_{\rm H} + \alpha\,\mathbf{x}_{\rm K} \tag{1}$$

where $\bar{\mathbf{x}}_{\rm H}$ of dimension $n \times 1$ is the solution of minimal norm, and α is an arbitrary scalar. Further, let k be the dimension of the null space $\mathscr{K}$, so that the rank of $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}$ is $n-k$. Since the normal matrix is real and symmetric we also have $\bar{\mathbf{x}}'_{\rm H}\mathbf{x}_{\rm K} = 0$. Hence, when $\mathbf{y}_{\rm H}$ is the vector of the observed-minus-calculated positions (O-C), we have for not too large values of α:

$$\begin{aligned} \boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}\,\bar{\mathbf{x}}_{\rm H} &= \boldsymbol{A}'_{\rm H}\mathbf{y}_{\rm H} \\ \boldsymbol{A}_{\rm H}\,(\alpha\,\mathbf{x}_{\rm K}) &= \mathbf{0} \\ \boldsymbol{A}_{\rm H}\,\mathbf{x}_{\rm H} &\sim \mathbf{y}_{\rm H} \end{aligned} \tag{2}$$

2.2. Least squares with constraint

Next we determine the least-squares solution constrained by the previously determined Hipparcos solution. That is, among the solutions that satisfy the Hipparcos observations, we find the unique[1] one that minimizes the residuals of the ground-based observations. This corresponds to the statement:

Problem (a). *Given the normal matrix* $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}$ *of rank* $n-k$, *among all vectors that satisfy:*

$$\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H}\,\mathbf{x} = \boldsymbol{A}'_{\rm H}\,\mathbf{y}_{\rm H} \tag{3}$$

find the solution $\bar{\mathbf{x}}$, *such that:*

$$|\,\mathbf{y}_{\rm G} - \boldsymbol{A}_{\rm G}\,\bar{\mathbf{x}}\,| \text{ is minimal.} \tag{4}$$

[1] Eventually the solution of minimal norm. But in our case, for minor planets observed over many decades, there is always a unique solution.

In other words, since the least-squares problem reduced to the Hipparcos observations is (almost) rank-deficient, we complete the set by ground-based observations in order to determine the free unknown parameters. Next, we introduce the orthogonal decomposition of the normal matrix $\boldsymbol{A}'_{\rm H}\boldsymbol{A}_{\rm H} = \boldsymbol{V}\,\boldsymbol{W}^2\,\boldsymbol{V}'$, where $\boldsymbol{W}$ of dimension $n \times n$ is diagonal, and we write the partitioning of $\boldsymbol{V}$ into two blocks:

$$\boldsymbol{V} = [\boldsymbol{V}_1 \;:\; \boldsymbol{V}_2] \tag{5}$$

where $\boldsymbol{V}_2$, of dimension $n \times k$, generates the null space $\mathscr{K}$. Introducing also the particular Hipparcos solution of minimal norm $\bar{\mathbf{x}}_{\rm H}$ given by Eq. (3), the constrained least-squares estimate (LSE) is given by (Lawson & Hanson 1974):

$$\bar{\mathbf{x}}_{\rm LSE} = \bar{\mathbf{x}}_{\rm H} + \boldsymbol{V}_2\,(\boldsymbol{A}_{\rm G}\boldsymbol{V}_2)^{-}\,(\mathbf{y}_{\rm G} - \boldsymbol{A}_{\rm G}\,\bar{\mathbf{x}}_{\rm H}) \tag{6}$$

However one can give a similar formulation that involves matrixes and vectors of lower size. It can be shown that if $\hat{\mathbf{x}}_2$ is a solution of Problem (a), then it also minimizes the norm :

$$|\,(\boldsymbol{A}_{\rm G}\boldsymbol{V}_2)'\,\mathbf{y}_{\rm G} - (\boldsymbol{A}_{\rm G}\boldsymbol{V}_2)'\boldsymbol{A}_{\rm G}\,\mathbf{x}_2\,| \tag{7}$$

and hence, introducing the unique solution $\bar{\mathbf{x}}_{\rm G}$ that minimizes $|\mathbf{y}_{\rm G} - \boldsymbol{A}_{\rm G}\,\mathbf{x}|$, we have:

$$\boldsymbol{A}'_{\rm G}\boldsymbol{A}_{\rm G}\,\bar{\mathbf{x}}_{\rm G} = \boldsymbol{A}'_{\rm G}\,\mathbf{y}_{\rm G}$$

and putting:

$$\boldsymbol{\sigma}^2 = (\boldsymbol{V}'_2\,\boldsymbol{A}'_{\rm G}\boldsymbol{A}_{\rm G}\,\boldsymbol{V}_2)^{-1}$$

we find:

$$\bar{\mathbf{x}}_{\rm LSE} = \bar{\mathbf{x}}_{\rm H} + \boldsymbol{V}_2\,\boldsymbol{\sigma}^2\,\boldsymbol{V}'_2\,\boldsymbol{A}'_{\rm G}\boldsymbol{A}_{\rm G}\,(\bar{\mathbf{x}}_{\rm G} - \bar{\mathbf{x}}_{\rm H}) \tag{8}$$

2.3. Variance matrix

From Eq. (6) the variance of the solution vector $\bar{\mathbf{x}}_{\rm LSE}$ is given by:

$$\boldsymbol{\sigma}^2_{\bar{\mathbf{x}}_{\rm LSE}} \sim \boldsymbol{\sigma}^2_{\bar{\mathbf{x}}_{\rm H}} + \boldsymbol{V}_2\,\boldsymbol{\sigma}^2\,\boldsymbol{V}'_2\,\boldsymbol{\sigma}^2_{\mathbf{o}} \tag{9}$$

where $\boldsymbol{\sigma}^2_{\mathbf{o}} = \sigma_o^2\,\boldsymbol{Id}_n$, σ_o being the error per unit weight, and we have assumed that $\boldsymbol{\sigma}^2_{(\mathbf{y}_{\rm G} - A_G\,\bar{\mathbf{x}}_{\rm H})} \sim \boldsymbol{\sigma}^2_{\mathbf{y}_{\rm G}} = \boldsymbol{\sigma}^2_{\mathbf{o}}$. If this assumption fails, then putting:

$$\boldsymbol{B} = \boldsymbol{A}'_{\rm G}\boldsymbol{A}_{\rm G}\boldsymbol{V}_2$$

the variance is given by:

$$\begin{aligned} \boldsymbol{\sigma}^2_{\bar{\mathbf{x}}_{\rm LSE}} = {} & \boldsymbol{\sigma}^2_{\bar{\mathbf{x}}_{\rm H}} + \sigma_{\mathbf{o}}^2\,\boldsymbol{V}_2\,\boldsymbol{\sigma}^2\,\boldsymbol{V}'_2 + \\ & \boldsymbol{V}_2\,\boldsymbol{\sigma}^2\,\boldsymbol{B}'\,\boldsymbol{\sigma}^2_{\bar{\mathbf{x}}_{\rm H}}\,\boldsymbol{B}\,\boldsymbol{\sigma}^2\,\boldsymbol{V}'_2 \end{aligned} \tag{10}$$

On the other hand, making use of the Lagrange multipliers with the constrains $\boldsymbol{V}'_1\,\bar{\mathbf{x}} = \boldsymbol{V}'_1\,\bar{\mathbf{x}}_{\rm H}$, where $\boldsymbol{V}_1$ of dimension $n \times (n-k)$ is given in Eq. (5), and putting:

$$\boldsymbol{\tau}^2 = \left[\boldsymbol{V}'_1\,(\boldsymbol{A}'_{\rm G}\boldsymbol{A}_{\rm G})^{-1}\,\boldsymbol{V}_1\right]^{-1}$$

Table 1. Number of observations used for calculation of either $\bar{\mathbf{x}}_{G}$, $\bar{\mathbf{x}}_{GLS}$ or $\bar{\mathbf{x}}_{LSE}$

	Hipparcos	Ground	Total
(2) Pallas	67	985	1052
(324) Bamberga	75	1211	1286

Table 2. Perturbing minor planets used for the construction of the ephemerides

Minor planet	Mass $[10^{-10}\, M_{\odot}]$
(1) Ceres	5.0
(2) Pallas	1.2
(4) Vesta	1.35
(10) Hygiea	0.47
(11) Parthenope	0.026
(52) Eunomia	0.14
(511) Davida	0.18
(704) Interamnia	0.35

we find the solution:

$$\bar{\mathbf{x}} = \bar{\mathbf{x}}_{G} - (\boldsymbol{A}'_{G}\boldsymbol{A}_{G})^{-1}\, \boldsymbol{V}_1\, \boldsymbol{\tau}^2\, \boldsymbol{V}'_1\, (\bar{\mathbf{x}}_{G} - \bar{\mathbf{x}}_{H}) \quad (11)$$

It is known that the use of Lagrange multipliers suffer the drawback of providing distorted statistics in the variance matrix. Following Eq. (9), the variance of the LSE solution will be larger or equal to the one of the Hipparcos solution of minimal norm $\bar{\mathbf{x}}_{H}$.

3. Observations and ephemerides

Two minor planets (2) Pallas and (324) Bamberga have been selected for the application of the method. No particular selection criteria was applied. These two minor planets are however representative of Hipparcos observations and have a long history of ground-based astrometric observations. The improvement of the ephemerides of the other Hipparcos asteroids based on the two methods is already underway (Viateau et al., in prep.).

The observational data used for Pallas consists of the 101 normal places published in Landgraf (1987), covering the time interval 1802-1978, and the meridian observations made at Bordeaux and La Palma. This dataset has already been used for a joint determination of the orbital elements of Pallas and the mass of Ceres (Viateau & Rapaport 1995). The complete observational data was used for Bamberga. It was provided by the Minor Planet Center through its Extended Computer Service. All the observations were given weights corresponding to their mean estimated accuracy. For the normal places of Pallas, we used the weights given by Landgraf (1987). The meridian observations of Pallas and all the observations of Bamberga were selected and given weights in the same way as detailed in Viateau & Rapaport (1997). A single mean position was constructed for each transit from the FAST and NDAC Hipparcos observations (ESA 1997). The number of resulting 'observations' is given in Table 1.

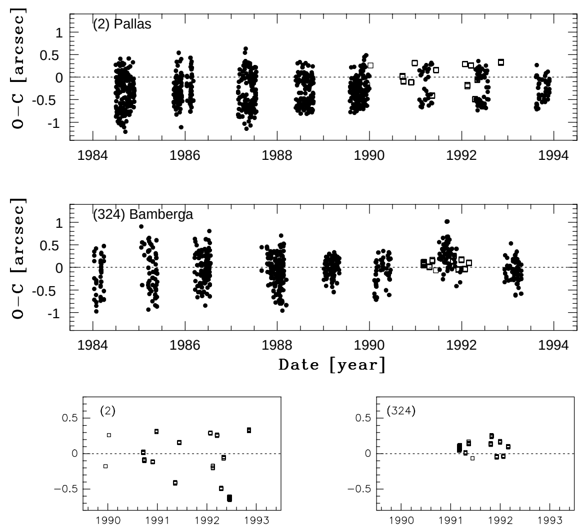

Fig. 1. Observed minus calculated positions O-C for (2) Pallas and (324) Bamberga over the period 1984–1994, using the orbital elements given in the "Ephemerides of Minor Planets for 1995". The O-C in right ascension and in declination obtained from meridian observations are shown together in the same plot. The one-dimensional O-C for the Hipparcos observations (over the period 1989.9–1993.5) are shown by open squares, details are given on the lower panels

Table 3. Global rotation between the ICRS(Hipparcos) and FK5 Catalogues in equatorial coordinates. The orientation components are given at the epoch T_o = J1991.25 (TT)

Orientation [mas]	Spin [mas/year]
$\varepsilon_{o_x} = -18.8 \pm 2.3$	$\omega_x = -0.10 \pm 0.10$
$\varepsilon_{o_y} = -12.3 \pm 2.3$	$\omega_y = +0.43 \pm 0.10$
$\varepsilon_{o_z} = +16.8 \pm 2.3$	$\omega_z = +0.88 \pm 0.10$

The ephemerides were constructed by taking into account the perturbations by the major planets as given by the DE200 planetary ephemeris and some other minor planets. The selection and evaluation of the masses of the perturbing asteroids (see Table 2) are given in Viateau & Rapaport (1997). The initial conditions were taken from the "Ephemerides of Minor Planets for 1995" (Batrakov 1994). A transformation between the FK5 and the ICRS reference frames was applied to the coordinates of the ground based-observations $\mathbf{u}_{FK5}$ using a time-dependent rigid rotation $\boldsymbol{R}$:

$$\mathbf{u}_{ICRS} = \boldsymbol{R}\big(\varepsilon(t)\big)\, \mathbf{u}_{FK5}$$

where $\varepsilon(t) = \varepsilon_o + \omega\,(t - T_o)$, and T_o = J1991.25 (TT). The components of the orientation and spin vectors (Mignard et al. 1997) are listed in Table 3. The differences O-C are shown in Fig. 1 for the most accurate observations, i.e. the meridian circles observations after 1984 and the Hipparcos observations. Zonal corrections were not applied.

Table 4. Corrections to the osculating elements at epoch J2 450 000.5 (TT). Solutions $\bar{\mathbf{x}}_{GLS}$, $\bar{\mathbf{x}}_{LSE}$ derived from the GLS and LSE methods. The symbol (^{o}) refers to the solution obtained without weighting of the equations of conditions. The particular Hipparcos solution of minimal norm $\bar{\mathbf{x}}_H$ is given in the last row of each group

	Δa	Δe	Δi	$\Delta\Omega$	$\Delta\omega$	ΔM
	[AU]	—	[deg]	[deg]	[deg]	[deg]
	$\times 10^8$	$\times 10^7$	$\times 10^5$	$\times 10^5$	$\times 10^4$	$\times 10^5$
(2) Pallas						
GLS^o	-8.91	-1.36	7.83	-8.42	-.834	6.29
GLS	-8.39	-1.63	.552	-7.81	-1.12	8.38
LSE^o	-9.95	-1.71	.566	-7.39	-1.13	8.24
LSE	-8.38	-1.55	.480	-7.45	-1.16	8.47
Hipp	-30.6	-2.20	.514	-7.40	-1.30	14.0
(324) Bamberga						
GLS^o	-2.17	-1.28	1.31	-3.80	.042	4.35
GLS	-2.01	-.965	2.27	8.10	-1.51	6.13
LSE^o	-2.17	-.526	2.29	8.60	-1.35	5.16
LSE	-2.05	-.763	2.28	8.53	-1.53	6.06
Hipp	13.2	.385	2.20	7.91	-.674	-.629

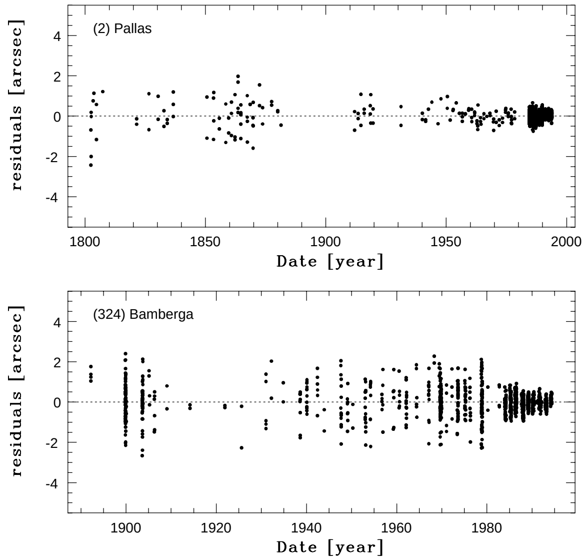

Fig. 2. Residuals for Pallas and Bamberga and the whole data set with the new osculating elements

4. Results

Depending on the retained rank for the matrix $\boldsymbol{A}'_H \boldsymbol{A}_H$, the solution $\bar{\mathbf{x}}_{LSE}$ will range between $\bar{\mathbf{x}}_H$ and $\bar{\mathbf{x}}_G$. On the other hand, a direct combination of the observations would yield the equations of condition:

$$\begin{pmatrix} \boldsymbol{A}_G \\ \boldsymbol{A}_H \end{pmatrix} \mathbf{x} = \begin{pmatrix} \mathbf{y}_G \\ \mathbf{y}_H \end{pmatrix}$$

and the global least-squares solution (GLS):

$$\bar{\mathbf{x}}_{GLS} = (\boldsymbol{A}'_G \boldsymbol{A}_G + \boldsymbol{A}'_H \boldsymbol{A}_H)^{-1} \, (\boldsymbol{A}'_G \mathbf{y}_G + \boldsymbol{A}'_H \mathbf{y}_H)$$

which also depends on the adopted relative weights between the different observations and instruments.

The solutions for the corrections to the osculating elements of the minor planets (2) Pallas and (324) Bamberga are given in Table 4. Comparison of the results obtained by weighting or not weighting the equations (in particular the inclination and the perihelion) shows that the LSE method is more stable than the GLS one, and thus less sensitive to the weighting. The two solutions obtained after weighting of the equations of conditions are in good agreement; the largest difference appears on the correction for the eccentricity but remains acceptable. The standard errors are also found to be of similar size. This agreement is not only due to the fact that both solutions are derived from the same observational data. As a consequence, (1) the applied weights for the GLS solutions appear to be appropriate, and (2) the good agreement yields a higher confidence in the solution accuracy.

Hipparcos observations are of significant value for the determination of the orientation of the osculating trajectory, but less for the determination of the semi-major axis. The corrections Δi and $\Delta\Omega$ in Table 4 obtained by considering only the Hipparcos data are close to the solutions obtained with the whole data set. Also the improvement in the determination of the inclination, and to a lesser extent of the argument of the ascending node, is noticeable in Table 5, in contrast to the improvement of the semi-major. The new osculating elements in the ICRS(Hipparcos) frame, as derived from the LSE solution, and their corresponding standard errors, are given in the ecliptic coordinates for the epoch J2 450 000.5 (TT) in Table 6.

5. Discussion

The solutions for the improvement of the ephemerides obtained by both methods (GLS or LSE) are equivalent in terms of the expectancy and the corresponding variance. The residuals (given in Fig. 2), as obtained from both methods, are also found to be very similar. Their norms are equal at the milliarcsecond (mas) level, and they do not differ by more than a few mas in absolute value for the Hipparcos data. The method of least-squares with equality constraints is similar to the one consisting of applying a highly weighted a priori covariance matrix to the constraint equations. With the weights adopted here, we have roughly a minimal value on the ratio of 200 between the weights of the Hipparcos and the ground-based observations. Due to the particular constraints applied, the solution of the LSE method is in closer agreement with the Hipparcos observations, independent of the adopted weighting of the equations of conditions.

Comparison between the solutions and their respective standard deviations obtained by both methods provides a test of the confidence we can have on their accuracy. As noted in Sect. 4 the agreement on the solutions is good for all of the parameters. In general this is also the case for the standard deviations of the unknowns, although one can note that the GLS method yields the smaller values. Nevertheless, for both minor planets

Table 5. Standard errors for the determination of the osculating elements. (ground-based) ground-based data; (GLS, LSE) combination of ground-based and Hipparcos data. The rank retained for calculation of $\bar{\mathbf{x}}_{\rm H}$ is given in the last column

	σ_a [AU]	σ_e –	σ_i [deg]	σ_Ω [deg]	σ_ω [deg]	σ_M [deg]	Rank (Hipparcos)
(2) Pallas							
ground-based	$2.6\,10^{-9}$	$1.8\,10^{-8}$	$2.8\,10^{-6}$	$4.0\,10^{-6}$	$6.2\,10^{-6}$	$6.1\,10^{-6}$	–
GLS	$2.4\,10^{-9}$	$1.0\,10^{-8}$	$6.0\,10^{-7}$	$1.9\,10^{-6}$	$2.0\,10^{-6}$	$1.6\,10^{-6}$	–
ratio	1.1	1.8	4.7	2.1	3.1	3.8	
LSE	$6.1\,10^{-9}$	$1.5\,10^{-8}$	$6.3\,10^{-7}$	$2.1\,10^{-6}$	$2.1\,10^{-6}$	$1.3\,10^{-6}$	1
ratio	0.4	1.2	4.5	1.9	3.0	4.7	
(324) Bamberga							
ground-based	$2.7\,10^{-9}$	$3.3\,10^{-8}$	$4.0\,10^{-6}$	$1.7\,10^{-5}$	$1.8\,10^{-5}$	$5.2\,10^{-6}$	–
GLS	$2.5\,10^{-9}$	$1.3\,10^{-8}$	$6.3\,10^{-7}$	$3.6\,10^{-6}$	$5.9\,10^{-6}$	$2.2\,10^{-6}$	–
ratio	1.1	2.5	6.4	4.7	3.1	2.4	
LSE	$6.1\,10^{-9}$	$1.8\,10^{-8}$	$6.5\,10^{-7}$	$3.7\,10^{-6}$	$7.7\,10^{-6}$	$2.8\,10^{-6}$	2
ratio	0.4	1.8	6.2	4.6	2.3	1.9	

Table 6. New osculating elements for (2) Pallas and (324) Bamberga at epoch J2 450 000.5 (TT) in the ICRS(Hipparcos) system

	a [AU]	e –	i [deg]	Ω [deg]	ω [deg]	M [deg]
(2) Pallas	2.771076516	0.23379775	34.8079348	173.2982355	309.6971839	23.9878047
$\pm$	$3\,10^{-9}$	$1\,10^{-8}$	$6\,10^{-7}$	$2\,10^{-6}$	$2\,10^{-6}$	$2\,10^{-6}$
(324) Bamberga	2.685140280	0.33706002	11.0995428	328.1326653	44.1006171	322.3858406
$\pm$	$3\,10^{-9}$	$1\,10^{-8}$	$6\,10^{-7}$	$4\,10^{-6}$	$6\,10^{-6}$	$2\,10^{-6}$

the formal precision σ_a on the determination of the change in the semi-major axis, is degraded in the LSE method when compared to GLS or to the solutions obtained by considering the ground-based observations or the Hipparcos observations only. A similar situation arises when one aims to determine a correction to the mass of the perturbing asteroid (1) Ceres (which is beyond the scope of this study); a more optimal solution is obtained if this unknown parameter is not added in the Hipparcos equations of conditions. On the other hand, the improvement on σ_a with GLS obtained by including the Hipparcos observations is rather small. This is mostly a consequence of the mission duration and the particular scanning law of the satellite since (1) Hipparcos observations provide data irregularly spread along the trajectory and englobing less than one sidereal period, and (2) they provide a one-dimensonal abscissa over a great circle oscillating with an angle of 43° around a plane perpendicular to the ecliptic. Also the larger value of σ_a obtained with LSE is a consequence of distorted statistics in the variance matrix, reflecting the fact that Hipparcos provides only little information on the mean motion or semi-major axis of the minor planet.

Denoting by $\boldsymbol{v}$ the residuals for the whole data set, by $\boldsymbol{p}$ the matrix of weights, and by $(m-n)$ the redundancy; the estimator of the error per unit weight:

$$\tilde{\sigma}_0^2 = \boldsymbol{v}'\boldsymbol{p}\,\boldsymbol{v}/(m-n) \qquad (12)$$

was, for both minor planets, close to the initial value $\sigma_0 = 0\farcs5$, which again reflects a reasonable choice of the relative weights. We have thus adopted the values for the precision as obtained by the GLS method in Table 6. Providing the Hipparcos observations with higher weights in the GLS method would correspond to stronger constraints. On the other hand, it would suffer the drawback of distorting the variance matrix and increasing the condition number. Extrapolating this result, it is interesting to note that the GLS method could not been applied as such (i.e. weights inversely proportional to the standard deviation), if the space-based astrometry would have been more precise by a factor 100, as expected for GAIA type measurements (Lindegren & Perryman 1996).

Systematic effects such as a photocentre offset due to the oblique illumination by the Sun, error in the mass of Ceres for the calculation of its perturbation, or relativistic precession of the perihelion, were not taken into account in this study. The Hipparcos residuals still show systematic effects or errors that were not modeled (see Fig. 3). Contrarily to the ground-based observations, the Hipparcos observations were obtained at large solar phase angles, i.e. when the phase effect is maximal. For Pallas a further improvement is obtained when this phase effect is taken into account (Hestroffer, in prep.).

6. Conclusion

A new approach for the improvement of the ephemerides of the Hipparcos asteroids has been developed. This method, which consists of a least-squares solution constrained by the Hippar-

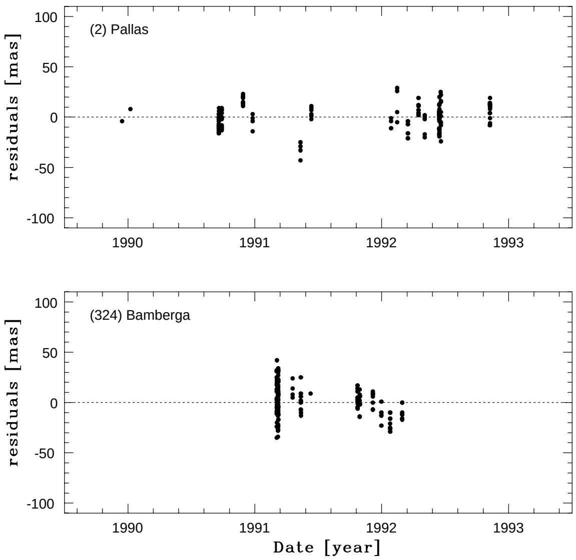

Fig. 3. Same as Fig. 2 for the Hipparcos data set only

cos solution, has been compared to the more usual solution obtained by a direct combination of the entire observational data set. The solutions for the osculating elements of (2) Pallas and (324) Bamberga obtained by both methods are found to be in good agreement, enhancing our confidence on their accuracy. It is stressed that the angles are given here with a precision of a few 10^{-6} degrees. On the other hand, Hipparcos observations do not significantly contribute to the improvement of the semi-major axes.

A similar approach can be employed when one aims to combine observational data of different nature, and in particular of very different precision. Modern ground-based or space-based observations of asteroids allow, or will soon allow, astrometric observations to the 10 mas precision level or better. As in the case of the Hipparcos observations, they will be limited in time until an observation campaign over a few decades has been carried out. Since insufficient accuracy in the osculating elements is the major source of error, such observations still have to be completed by older and less precise data for many purposes. One would then separate the data-sets in different groups, where the highest constraint is given by the more recent and accurate observational techniques.

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