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Period changes in both modes of RR_d stars in M 15

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Abstract. A new approach of the phase or $O - C$ diagram is presented. The Fourier phase diagram method combines the advantages of both the phase diagram and Fourier technique. The method is applied for 13 RR_d stars in the metal poor Oo II type globular cluster, M 15. Previously published periods are corrected up to 6 digits. The precise periods allowed us to get period changes with higher accuracy. Period changes were obtained not only for the first overtone but at the first time for the lower amplitude fundamental mode, too. The period change rate of the fundamental mode is significantly larger for each star than that of the first overtone. The P_1/P_0 ratio is increasing for most of the stars. Each period change rate is negative for the fundamental mode (except v51). For the first overtone both positive (5) and negative (6) period change rates were obtained. The most striking result is the period change behaviour of different sign for the fundamental and first overtone modes. The most remarkable case is v53 with β = - 0.31±0.06 day/Myr for the fundamental mode and β = + 0.14±0.01 day/Myr for the first overtone. This result suggests that caution is needed in connecting the sign of the period change rate and the direction of the stars's evolution in the HR diagram. Observational comparison is given for RR_{ab} and RR_c stars and the fundamental and first overtone modes of RR_d in M 15.

Key words: Galaxy: globular clusters: individual: M 15 – stars: oscillations – stars: variables: RR Lyr – methods: data analysis

1. Introduction

Since Martin's (1938) discovery that the periods of RR Lyrae stars in ω Centauri are predominantly increasing, many studies have been made on period changes of a great number of RR Lyrae stars in different clusters. It was hoped that the minute changes in the periods observed would give some information on the speed and direction of evolution of horizontal branch stars and provide sensitive test of the theory of stellar evolution, as it suggests that the periods of RR Lyrae stars are increasing if they evolve from blue to red in the HR diagram or decreasing if they evolve to the opposite direction.

The complex structure of some of the phase $O - C$ diagrams, however, suggests that some kind of "noise" could be superimposed on those secular variations of the period which we expect to exist as a result of evolutionary changes in the mean stellar radius (Balázs-Detre & Detre, 1966). This would mean that the phase diagrams are instead a convolution of stochastic processes and evolutionary changes. The source of "noise" is attributed to random fluctuations in the stellar structure (e.g. Sweigart & Renzini, 1979, Stothers, 1980). In spite of these disillusioning facts hopes have never been abandoned that a proper statistical evaluation of the parametrized values of the rate and direction of changes in the periods of a great number of RR Lyrae stars observed on a long baseline would eventually reveal the evolutionary effects.

In the determination of the average rate of period changes of RR Lyrae stars in globular clusters, in order to increase the statistical weight, both types of RR Lyrae stars, RR_{ab} and RR_c stars are usually combined and treated together, in spite of the fact that there are references to different period change behaviour between the groups of RR_{ab} and RR_c stars (Szeidl, 1975). In the case of RR_d stars only the changes in the dominant period (first overtone frequency) have been studied. As these investigations were carried out on the traditional way (the phase of maximum of the folded light curve was derived), the double mode phenomenon manifested itself as a large error on the phase $O - C$ diagram. The theoretical expectations suggest that the relative changes ($\alpha = (1/P) * dp/dt = (1/P) * \beta$) in the period (or frequency) of the fundamental and first overtone mode should be of the same sign and about of the same rate.

Although it is obvious that the period change behaviour of an RR_d star is not a simple mixture of that of RR_{ab} and RR_c stars, their different period change behaviour raised the idea of a simultaneous investigation of modes in RR_d stars. The globular cluster M 15 is one of the richest clusters in double mode RR Lyrae stars for which the observational baseline extends over nearly a century. In this paper we present a rigorous investigation of the changes in both the fundamental and first overtone frequencies of these stars.

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Fig. 1. Distribution of data in time

Table 1. Journal of sources of observation

Source	Observed RR_d stars
Bailey (1919)	v17,30,31,39,41,51,53, 54,58,61
Notni & Oleak (1957)	v41,54,58,61,67,96,101
Bronkolla (1959)	v17,30,31,39
Fritze (1962)	v51,53,54
Barlai (1989)	v17,30,31,39,51,53,54, 67,96,101
Mannino (1956)	v17,30,39
Grubissich (1956)	v31
Nobili (1957)	v51,53,54
Sandage et al (1981)	v17,30,31,39,41,51,53, 54,58,61,67
Smith & Wesselink (1977)	v31,39
Filippenko & Simon (1981)	v17,30,31,39,41,51,53, 54,58,67,96
Bingham et al. (1984)	v17,30,31,39,41,51,53, 54,67,96,101
Silbermann & Smith (1995)	v17,30,31,39,41,51,53, 54,67

2. Investigated data

The observations of RR Lyrae stars in M 15 were started as early as 1896 by Bailey (1919) and have been continued, more or less evenly distributed, up to our time. The data obtained by different observers at different epochs, however, differ from each other both in amount and quality.

In the present analysis all the available data for the double mode RR Lyrae stars in M 15 are used as a coherent dataset. The journal of the sources of the observations used is given in Table 1.

The early observations of Bailey (1919) were neglected because they were rather sporadic and scarce (1896:4, 1897:5, 1904:2, 1908:1) and a proper grouping of the data was not possible. The observations of Makarova & Akimova (1965) were left out from consideration because they are given in a special photometric system.

All the known or suspected double mode RR Lyrae stars in M 15 are involved in our study but v26 (needed a special treatment because of a large abrupt period change) will be published separately. The number of data available for the stars investigated is summarized in Table 2. and their distribution in time is presented in Fig. 1.

According to our experience the photographic (blue) and the Johnson B light curves are fairly close to each other, so the m_{pg} and B data have been treated together. Since during the past 80 years the observations have been obtained by different techniques, different telescopes with different size and focal ratio, etc, which may have serious effects on photometry (e.g. crowding effect), the observational error of the coherent dataset could only be estimated. If we parametrize the mean error as $0.^{m}15$ which is generally accepted for photographic observations, we overestimate the error of the high quality data

Table 2. The number of data investigated for each stars

Star	B/m_{pg}	V
v17	785	460
v30	816	476
v31	855	484
v39	839	477
v41	229	316
v51	707	451
v53	819	472
v54	706	442
v58	163	258
v61	142	59
v67	568	459
v96	400	251
v101	460	53

segments (Sandage et al. (1981), Bingham et al. (1984)) and the CCD observations (Silbermann & Smith, 1995). The zero point shifts are treated independently by the final fit.

3. Fourier phase method

3.1. New period values

Since the observational data are seasonal or even several-year-long gaps could be found, the chance of miscounting the number of cycles increases; that is, the alias pattern of the frequency spectrum is more complicated. Although many careful analyses, using a part of the available data and different techniques (PDM, Fourier), have been made during the past years (Smith & Sandage, 1981; Cox et al., 1983; Nemec, 1985; Kovács et al., 1986; Clement & Walker, 1991; Jurcsik & Barlai, 1992; Silbermann & Smith, 1995; Purdue et al., 1995), there is still dispute paper by paper concerning the 1 or 2 cycles/year aliases of the lower amplitude fundamental mode, for example in v39, 30 and 31. In critical cases some extra criteria like symmetry arguments or aliases with physically realistic primary/secondary period ratios are used. At the same time, any kind of period changes could only be obtained if the precise value of the pulsation period is well-known, otherwise the period change is masked by the effect of incorrect value of the period.

The traditional phase diagrams or $O-C$ curves based on all of the available observational data can give a very accurate value and even the change of the dominant (first overtone) period. However, there is no possibility for the investigation of the lower amplitude fundamental mode. The combination of the traditional phase or $O-C$ diagram and the Fourier technique, which we call the Fourier-phase diagram method, can give not only very accurate values but the period changes of both excited modes in RR_d stars.

Since the effect of an incorrect frequency or a change in the frequency accumulates slowly, the data were devided into groups of different length in time according to the number of observations in a group, not more than two-three years were added together. Although RR_d stars in M 15 were more or less homogeneously observed the duration of data segments is not the same for each star, since the phase solution is sensitive to the data coverage (different for stars with different periods). In a critical case a longer data segment was accepted.

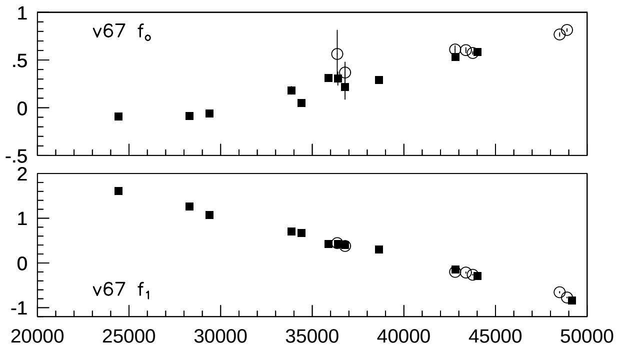

Fig. 2. Fourier-phase diagram of v67 for the fundamental mode and for the first overtone

In finding the new period values the program PERIOD (Breger, 1990) was used with an option of fixing the frequencies and amplitudes, only Φ_k phases (given in 3.2) were considered as free parameters. The reduced number of free parameters allowed us to use more sporadic observations, too. The starting values of the periods were accepted from the paper of Kovács et al. (1986), in some cases that of Nemec (1985) or Purdue et al. (1995). Amplitudes were fixed as it is described at the global fit in 3.2.

Although for checking the stability of the phase solution, 5 different sets of linear combinations have been studied, the $\{f\} = f_o, f_1, 2f_o, 2f_1$ linear combination has been applied in the final phase diagrams.

A slightly incorrect (not precise enough) starting value of the period gives slightly different Φ_k values for the consecutive groups. If the starting periods are precise enough comparing to the ±1 cycle/year aliases, the differences accumulate to one cycle over the interval of some groups, lying along a segment of a straight line. After modulation by 2π the consecutive phases are situated on parallel segments of straight lines. The phase diagram in this stage looks like a saw-tooth function. During the demodulation the parallel segments of straight lines are shifted to a single straight line. If the uncertainty of the starting value of the period is comparable to the ±1 cycle/year aliases, it results in a 2π phase change over the length of a group, there is no way to get the correct period (miscounting of cycles). Another alias as a starting value is suggested.

The plot of phases ($\Phi_k\pm 2\pi$,4π...) versus the mean time of the observations in a group gives the Fourier phase diagram. The frequencies had to be increased or decreased comparing to the starting values. The value of frequency correction is given by the slope of the straight line which is very definite.

In Fig. 2. the Fourier-phase diagram of a remarkable RR_d star v67 is plotted for the first overtone and fundamental modes. The RR_d star v67 was chosen as an example where the amplitude of the fundamental mode is not essentially lower than that

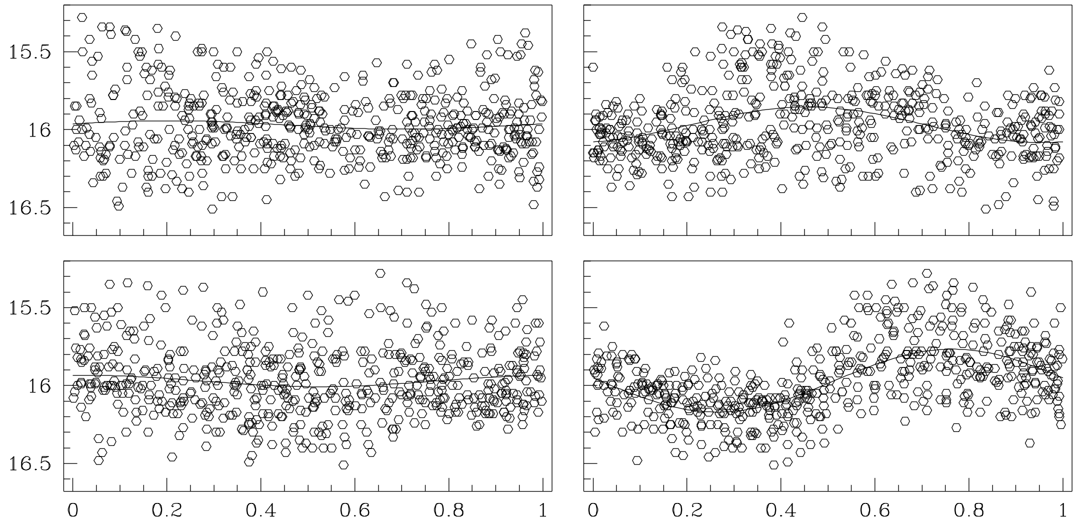

Fig. 3. Folded B light curves of v67 according to the phase of the fundamental mode (top) and the first overtone (bottom). Left side: starting values. Right side: new values

of the first overtone and previously there was no definite result for this star, nor for f_1.

In Fig. 3. the folded light curves of v67 according to the phase of the first overtone and the fundamental mode, respectively, could be seen. At the left side the starting values are used, while at the right side the new values have been used. The improvement of the folded curves with the new frequencies is remarkable.

The accepted frequency corrections for the RR_d stars in M 15 according to the Fourier-phase diagram method are given in Table 3. The finally accepted new values applied in the final fit, are presented in column (3) of Table 4.

3.2. The global fit of period changes

The changes of the Fourier phases were determined by a nonlinear least square fit to the light curve ($m_i = m(t_i)$) i.e. by minimizing the following function:

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left(m_i - A_{0,i} - \sum_{k=1}^{L} A_k \sin\left(2\pi f_k t_i + \Phi_k\right) \right)^2, \quad (1)$$

where the phase term is given by

$$\Phi_k = 2\pi(b_k^{(0)} + b_k^{(1)}(t_i - t_o) + b_k^{(2)}(t_i - t_o)^2). \quad (2)$$

The amplitudes (A_k) were held constant at values determined by a fit to the whole data set, while the zero shifts ($A_{0,i}$) were calculated for the data segments independently to minimize the errors due to the different observations. Only the $b_k^{(l)}$ parameters were taken as unknown values during the nonlinear iteration.

The frequency correction and variation is determined by the $b_k^{(l)}$ parameters:

$$f_k^{new}(t) = f_k + b_k^{(1)} + 2b_k^{(2)}(t - t_0) \quad (3)$$

Table 3. Frequency corrections applied to the previously published values

Star	Δf_1 10^{-3} c/d	Δf_o 10^{-3} c/d
v17	-0.1663	+0.0441
v30	+0.0279	-0.0250
v31	+0.0943	-0.0201
v39	+0.0442	-0.0583
v41	-0.0173	-0.3641
v51	+0.2780	-0.0298
v53	+0.1989	-0.1325
v54	-0.0125	-0.0614 part 1
	-0.0563	-0.1312 part 2
v58	+0.0724	+0.0128
v61	-0.0376	+0.0900
v67	-0.0946	+0.0340
v96	-0.4074	+0.1669
v101	-0.0809	-0.1730

The errors of the periods and the rates of period variations ($\sigma(P_k)$ and $\sigma(\beta_k)$) were calculated by a Monte-Carlo simulation. Different realizations of Gaussian noise were added to the observed magnitudes and then the fit was recalculated. Then $\sigma(P_k)$ and $\sigma(\beta_k)$ was obtained from the standard deviation of the resultant $b_k^{(1)}$ and $b_k^{(2)}$ factors. The standard deviation of the added noise was selected according to the observational noise ($\sigma = 0.075 - 0.1$).

We tested the method on an artificial data set given by our fit to the v53 light curve. The times of the observations were used in the test signal. With different amount of added noise (σ_n) in the test signal the dependence of the errors $\sigma(P_k)$ and $\sigma(\beta_k)$ on the observational noise was determined. As expected, we found linear relations: $\sigma(P_0) \approx 8.0\ 10^{-6}\sigma_n$, $\sigma(P_1) \approx 2.0\ 10^{-6}\sigma_n$, $\sigma(\beta_0) \approx 0.2\sigma_n$ and $\sigma(\beta_1) \approx 0.04\sigma_n$.

The amplitudes (A_k) are held time independent in our calculations, so it is important to test how the results depend on amplitude variations. For this test, each segment of the light curve was multiplied by different random factors $(1+z)$, where z is a Gaussian noise with standard deviation σ_z. We again found linear dependences: $\sigma(P_0) \approx 1.8\,10^{-6}\sigma_z, \sigma(P_1) \approx 0.3\,10^{-6}\sigma_z$, $\sigma(\beta_0) \approx 0.04\sigma_z$ and $\sigma(\beta_1) \approx 0.007\sigma_z$.

Even for a high value of the amplitude fluctuation the errors of the period and period change rates remain small, *i.e.* the V and the B light curves can be mixed, without altering the results significantly. Although the displayed values are valid only for the data distribution of the V53 light curve, they show the general tendencies of error propagation.

4. Period changes

The final results of the global fit of period changes are summarized in Table 4. In column 3 the new period values ($1/f_k^{new}(t_o)$ in Eq. 3.) are listed for the first moment of the observation, that is t_0 = 20724.652 for v17 - v61, t_0 = 24381.546 for v67 - v101, along with their errors. The rate of period change, β and α calculated in days/Myr and cycles/Myr are given in columns 5 and 6, respectively, along with their mean errors. The β and α values derived by Silbermann and Smith (1995) are also listed in Table 4. (columns 7 and 8). The present values are in good agreement with the rates (for v17,31,39,51 and 53) obtained by them but the present values are given with higher accuracy.

The newly derived periods were accepted for the final Fourier-phase diagrams. The phase changes of both fundamental and first overtone modes for 13 RR_d stars in M 15 are presented in Fig. 4. We should emphashize that the correlation between the curvature of the phase changes and period change is of the opposite sign from that seen in $O-C$ diagrams.

4.1. Comments on individual variables

Detailed inspection of the phase change diagrams in Fig. 4., taking into account the observational errors, suggests the following interpretations of individual stars. According to us, those stars could be seen as well-established double mode RR Lyrae stars where both phase change diagrams show simple, regular structure. Generally good agreement of our periods for the first overtone pulsation with those derived in previous studies of long term period changes has been obtained.

v17.) Although the double mode nature is well-established, remarkable amount of correction in the period of both the fundamental (2% of the cycle/year alias) and the first overtone (6%) modes had to be applied. With the new value of periods the phase change diagrams are given with extremely low errors and the long-term behaviour is well-described by a negative parabola for the first overtone and positive parabola for the fundamental mode. This means period increase in the first overtone and period decrease in the fundamental mode. The period increase rate of the first overtone agrees with the value obtained by Silbermann and Smith (1995). The period decrease rate of the fundamental mode is one of the largest value in our sample.

v30.) Only minor corrections (1% of the cycle/year alias value) in both frequencies were applied. The phase change diagrams show unique structure in our sample contradicting Silbermann and Smith (1995) result where "some scatter about a straight line but no significant evidence for period change" has been found. No doubt about the double mode nature but a sinusoidal fit is given for both modes with low error bars. The sinusoidal fit of the $O-C$ curves or phase diagrams (in single mode RR Lyrae stars) used to be a sign of the binary nature of the star. In our case, however, the two sinusoidal fits (practically with the same amplitude, $2.4 * 10^{-6}$ and $2.7 * 10^{-6}$ days for the first overtone and fundamental mode, respectively) are the same but are shifted with respect to each other, which rules out the binary nature. The cycle length of the phase change is 72 years. At this moment there is no definite explanation for v30.

v31.) Minor corrections (3 and 1% for the first overtone and fundamental mode, respectively) were enough to get the accepted new periods. The double mode nature seems to be well-established but the overtone is dominant comparing to the fundamental mode. The amplitude ratio is rather high. The low amplitude of the fundamental mode is manifested through the higher error bars of the phase change curve of the fundamental mode. The first overtone's phase curve is fitted by a negative parabola with similar low value of period change rate as given by Silbermann and Smith (1995).

v39.) Minor corrections (2% in both modes) were applied. We can confirm the previously published (Silbermann & Smith (1995), Purdue at al.(1995)) results concerning the double mode nature and the first overtone. The double mode nature is well-established and the evidence for a period decrease is weak. The phase change diagram is fitted by a positive parabola. The period decrease of the fundamental mode, in spite of the larger scatter around the curve, seems to be significant.

v41.) This star is usually left out from the investigation of double mode stars. Silbermann & Smith (1995) defined only three epochs and the star was not included in their results. In the present investigation more numerous epochs are defined and the period decrease of the first overtone (correction was 1%) with rather large period change rate seems to be definitely significant. However, the Fourier phases are obtained with lower accuracy for the fundamental mode. No fit is given for the phase change of the fundamental mode (13% correction was applied). Any attempt failed to find a period value which gives Fourier phases with higher accuracy. The ill-defined solution could be attributed to the low amplitude of the fundamental mode.

v51.) In the latest paper concerning the RR_d stars in M 15, Purdue at al. (1995) could not confirm the double mode nature during 1991-1992. The problem could be caused by the incorrect period determinations. Many trials were carried out to obtain the new values of the periods. Finally the first overtone period of Nemec (1985) and the +1 cycle/day alias of Jurcsik & Barlai's (1992) fundamental periods were accepted as proper starting values. A rather large correction (10%) of the first overtone frequency was applied. For the fundamental mode a smaller (1%) correction proved to be satisfactory. The phase change diagrams show regular structure, the double mode nature of v51 does not

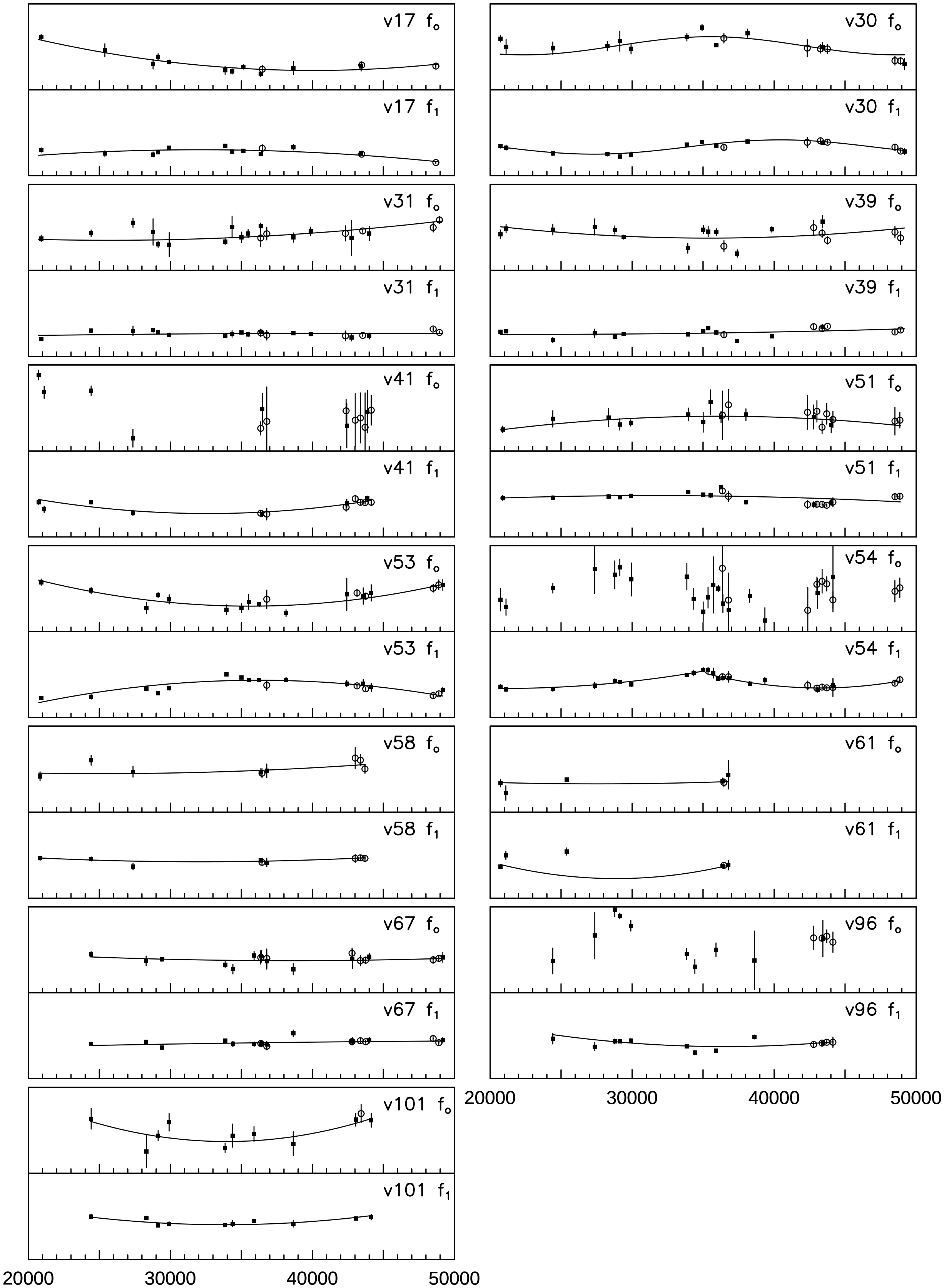

Fig. 4. Phase changes of both fundamental and first overtone modes for 13 RR_d stars in M 15. Time is JD - 2400000. The range for the phases is 2π.

seem to be doubtful. The period change rate of the first overtone perfectly agrees with the value obtained by Silbermann and Smith (1995). The larger error bars of the fundamental mode phase change diagram could be the consequence of the relatively lower amplitude and/or the improper phase coverage in the groups. In spite of the larger error bars and scatter of points, a definite period increase with significant period change rate was derived.

v53.) Although in most cases periods of the first overtone and fundamental mode obtained by the same authors were used, in this case the starting values of the first overtone given by Kovács et al. (1986) and that of the fundamental mode given by Purdue et al. (1995) were accepted. Minor correction (5%) for f_o, but a larger one (7%) for f_1 were applied to get the new values. The Fourier phase diagram for both modes display clean, regular structure, that is, a positive parabola for the fundamental mode and a negative parabola for the first overtone with small scatters of the points along the parabola and small error bars of the derived phase in each group. The period changes of different sign for the first overtone and fundamental modes are even more pronounced than in v17. The period change rate of the first overtone perfectly agrees with the value obtained by Silbermann and Smith (1995). The period change rate of the fundamental mode is the largest significant value in our sample.

v54.) Not too much improvement has been achieved for this star. Only minor corrections (0.5% and 2% for the first and second part of the phase change curve, respectively) for f_1 have been applied. Different trials, regarding some cycle/year aliases of the fundamental mode as starting values, were carried out. Finally the mean value of the +1 cycle/year alias of Purdue et al.'s (1995) period and the -1 cycle/year alias of Nemec's (1985) period was accepted as a starting value, but only a slightly better solution was found. (Corrections were 2% and 5% for the first and second part.) Concerning the large error bars of the Fourier phases, no fit is given for the phase change curve of the fundamental mode. The phase change of the first overtone is best fitted by two parabolas for the first and second parts of the data which are interpreted as different rates of period decrease. In between a $6 * 10^{-6}$ days abrupt period increase occurred. The sharp break in the Fourier phase diagram (the abrupt period increase) rules out the possibility of a sinusoidal fit.

v58.) and v61.) As a consequence of less observation, only a few epoch could be derived for these stars. Although the Fourier phases are accurate enough, the period change rates do not seem to be significant. More observations are needed to get significant solutions.

v67.) Minor corrections (4% for f_1 and 1% for f_o) were applied for Kovács at al. (1986)'s values. More numerous epochs, than derived by Silbermann and Smith (1995) give a definite Fourier phase diagram with high accuracy for both modes. The period change rates have different sign as in the cases of v17 and v53 but the values are rather low and do not seem to be significant. In any case, the regular phase change diagrams suggest that the new periods are well-defined values.

v96.) Starting values were accepted as given by Nemec (1985). One of the largest correction (15%) has been applied for the first overtone. Although a minor correction (6%) was employed for the fundamental mode, too, the new value did not result in regular structure in the phase change diagram. Further observations are needed to find the precise period of the fundamental mode which has lower amplitude than the fundamental mode in the other RR_d stars. The period change of the first overtone is derived for the first time, since Silbermann and Smith (1995) did not include this star in their investigation. The first overtone period is decreasing with a significant period change rate.

v101.) The starting values are also taken from Nemec (1985). This was one of the cases, where larger correction (6%) had to be applied for the fundamental mode than for the first overtone (3%). Although the fit is given to the phase diagram of the fundamental mode, the period change rate is at the limit of the significance. The first overtone shows a period decrease with significant period change rate.

4.2. General remarks

The phase changes are described by smooth curves over 80 years. A second order polynomial fit is satisfactory for most of the stars. The most striking result is the different period change behaviour of the fundamental and first overtone modes in rate and sign.

- The present values of β and α are in good agreement with the values (for v17, 31, 39, 51 and 53) obtained by Silbermann and Smith (1995) but their accuracy is improved.
- Only four stars show the same direction of period change for both modes (v39, 101, 58: decreasing and v51: increasing), however, the rates are not the same for the first overtone and fundamental modes.
- The absolute value of the period change rate of the fundamental mode is significantly larger for each star than that of the first overtone mode disregarding the sign of the period change.
- The P_1/P_0 period ratio is increasing for the stars where both periods are significantly determined.
- Each period change rate is negative for the fundamental mode except V51.
- For the period change rates of the first overtone both positive (5) and negative (6) values are derived.
- Four stars (v17, 31, 53 and 67) have period change rates of different sign for the fundamental and first overtone modes. In these cases the P_1/P_0 period ratio increase is the largest.
- The most remarkable case is v53 with the largest values of period change rates. β = - 0.31 $\pm$ 0.06 d/Myr for the fundamental mode and β = + 0.14 $\pm$ 0.01 d/Myr for the first overtone.

One can ask, whether it is possible to use the fit of the first overtone, to prewhiten the data and then to use the traditional $O-C$ method to determine the period changes in the fundamental mode. There are multiple reasons which make it impossible. First of all in real (noisy) data the maxima (or minima) are not well defined - and we use information only the neighbouring points around the extrema (contrary the Fourier phase method

Table 4. Period changes of RR_d stars in M 15

star	mode	period day	amplitude B-mag.	β d/Myr	α c/Myr	β^* d/Myr	α^* c/Myr
17	F	0.575587±.000002	0.10	-0.23±0.05	-0.40±0.09		
17	1o	0.4288928±.0000006	0.24	0.07±0.01	0.16±0.03	0.06±0.03	0.14±0.08
30	F	0.5446574±.0000007	0.07				
30	1o	0.4059805±.0000001	0.21				
31	F	0.547206±.000002	0.07	-0.09±0.05	-0.16±0.10		
31	1o	0.4081780±.0000004	0.26	0.01±0.01	0.02±0.02	0.02±0.02	0.05±0.05
39	F	0.522982±.0000003	0.09	-0.12±0.06	-0.24±0.11		
39	1o	0.3895713±.0000005	0.23	-0.01±0.01	-0.03±0.03	-0.02±0.02	-0.05±0.05
41	F	0.5241±.0003	0.04				
41	1o	0.3917547±.0000008	0.23	-0.13±0.02	-0.33±0.06		
51	F	0.530774±.000004	0.05	0.14±0.09	0.26±0.17		
51	1o	0.3969552±.0000005	0.23	0.03±0.01	0.07±0.03	0.02±0.02	0.05±0.05
53	F	0.555411±.000003	0.08	-0.31±0.06	-0.56±0.11		
53	1o	0.4141215±.0000006	0.20	0.14±0.01	0.33±0.03	0.10±0.03	0.25±0.06
54	F 1	0.5350±.0004	0.02				
54	F 2	0.53502±.00004	0.19				
54	1o2	0.399568±.000001	0.05	-0.12±0.06	-0.30±0.14		
54	1o2	0.399575±.000001	0.21	-0.30±0.07	-0.75±0.17 **		
58	F	0.546686±.000005	0.09	-0.07±0.14	-0.12±0.25		
58	1o	0.407250±.000001	0.22	-0.04±0.03	-0.10±0.08		
61	F	0.537624±.000007	0.15	-0.07±0.29	-0.12±0.54		
61	1o	0.399635±.000002	0.25	-0.29±0.11	-0.72±0.27		
67	F	0.542374±.000002	0.12	-0.05±0.06	-0.09±0.12		
67	1o	0.4046055±.0000007	0.21	0.01±0.02	0.01±0.05		
96	F	0.530853±.000008	0.06				
96	1o	0.396364±.000001	0.26	-0.11±0.03	-0.27±0.08		
101	F	0.53765±.00001	0.05	-0.54±0.41	-1.01±0.77		
101	1o	0.4007420±.0000008	0.25	-0.12±0.03	-0.29±0.07		

* Silbermann and Smith (1995)
** abrupt period jump ΔP_1 = 6 10^{-6} occurs at t=35200
t_0 = 20724.652 for v17 - v61
t_0 = 24381.546 for v67 - v101

uses all the information from the light curve). The times of the maxima are also affected by the nonlinear coupling terms (f_0+f_1 etc.), which are unknown. Those terms have only minor effects on the Fourier phase method, because the sine functions with well separated frequencies are almost orthogonal to each other even on the base of uneven sampling.

The regularities outlined here should give a new line of the investigation of pulsation and evolution connection. Comparisons with RR_d stars in other globular clusters should give dependence on the metallicity.

5. Discussion

The smooth, simple structure of the period change curves for both modes seems to rule out the presence and importance of any kind of stochastic processes suggested by Balázs-Detre & Detre (1966) as an explanation of complex structure of $O-C$ curves. There are no large period change rates that would need the mixing events associated with the semiconvective zone of the stellar core as an explanation (Sweigart & Renzini, 1979). However, the abrupt period increase of the first overtone of v54 needs the mixing events theory. According to Lee (1991) the observations of period changes are consistent with his evolutionary models only if the observational errors are of order ±0.07 days/Myr. For his model $0 < \beta < 0.1$, while after the addition of a random error the dominant range of period change is $-0.2 < \beta <$ +0.3 for M 15. Now we can determine the period changes with higher precision, and then the distribution of period changes disagrees with the evolutionary prediction.

However, Lee's evolutionary model is derived independently from the type of pulsation mode for single mode RR Lyrae stars. At the present view the sign and less definitely the amount of the period change rate, independently from the type of pulsation (RR_{ab} - fundamental, RR_c - first overtone), are closely connected to the direction of evolution on the HRD. The period change behaviour of the different modes of multimode pulsators have been theoretically investigated for AC And models (Kovács & Buchler, 1994). Both theoretical and observational period changes were found to be the same in size and sign for AC And. Different sizes of period changes for two modes in 4 CVn (Breger, 1990) were reported. The sign of period changes were the same for both modes, although it contradicted to the theoretically predicted direction.

The most striking result of the present investigation, the very definite period change rates of different sign for the fundamental and first overtone modes in the double mode RR Lyrae stars, v53 and v17, in M 15 suggests that caution is needed in connecting the sign of period change rate and the direction of the star's evolution in the HR diagram.

Whether the period changes of the different types of pulsation modes have special regularities during the evolution and the different modes of the double mode pulsators have the same regularities as the different types of single mode pulsators or not is open for theoretical investigations.

However, on the observational basis we can compare our knowledge of the different period change behaviour of RR_{ab} and RR_c stars, which was neglected in the last years, with that of the fundamental and first overtone modes of RR_d stars in M 15.

The difference in the period change behaviour that is associated with the pulsation mode was first reported by Szeidl (1975). Barlai (1977) did note that RR_{ab} type variables (fundamental mode) tended to show larger period changes (both positive and negative) than did the RR_c type stars (first overtone). Smith & Sandage (1981) also found that the RR_{ab} type variables exhibit a greater range of period changes than do the RR_c types but, as he remarks, the difference in the mean rate of period change for RR_{ab} and RR_c types variables is too small to be significant. However, Silbermann and Smith (1995) reported that the standard deviation about the mean for the α and β values are more than twice as large for the RR_{ab} variables as for the RR_{cd} stars. The wider range for the RR_{ab} variables is also evident in their histogram of the rate of period change (see their Fig. 3.).

Comparing the result of the present investigation for the period change rate of the fundamental mode in RR_d stars to RR_{ab} stars, we emphasize the significantly larger period change rate for the fundamental mode than for the first overtone mode (Table 4. and Fig. 4.). There is, however, a definite difference in the sign of the period change rates. According to Silbermann and Smith (1995) the mean value of β for RR_{ab} stars equals $+0.08\pm0.10$ while the fundamental modes of RR_d in M 15, with except one star, have negative sign of period change rates.

According to Silbermann and Smith (1995) the rates of period changes observed for the eight RR_d variables are similar to those of RR_c variables (RR_c: β=+0.04$\pm$0.02 and RR_d: β=+0.05$\pm$0.02). If we take into account that RR_c stars are pulsating in first overtone and that in RR_d stars the dominant mode is the first overtone, the agreement would suggest some kind of regularity for the first overtone in both RR_c and RR_d stars.

Although our results of β for the first overtone of RR_d stars, except one example (v54 and V26 is not involved in this paper), agree well with Silbermann & Smith's (1995) values, our larger sample reveals an equal number of stars with positive (5) and negative (6) period changes. Barlai (1984) argued for equal numbers of positive and negative β values for single mode RR Lyrae stars.

It seems that first overtones behave also in a different way in RR_c and RR_d stars, at least according to these samples.

Further observations at least for some of the RR_d stars in M 15 and similar investigations in other clusters are needed to clear up how general the conclusions of the present investigation are.

Theoretical investigations (effect of changes in the composition gradient (Cox, 1997) and variation in temperature and luminosity of both linear and nonlinear models (Kolláth, 1997)) for the explanation of the unambiguous case of v53 with the largest value of period change rates of different sign have been started.

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