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The line-driven instability in Sobolev approximation

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Abstract. Line-driven winds, e.g., of OB stars, are subject to a strong hydrodynamic instability. As a corollary to the comprehensive linear stability analysis performed by Owocki & Rybicki (1984), we present here a simplified derivation of the growth rates from applying a *second order* Sobolev approximation. This is applicable for perturbation wavelengths larger than the Sobolev length, and covers the physically most interesting regime of perturbations which can develop into strong reverse shocks, and heat the gas to X-ray temperatures. Since the usual WKB approximation is not applied, we furthermore find the existence of a limiting wavelength beyond which perturbances do not grow, but instead decay.

Key words: stars: early-type – hydrodynamics – instabilities – shock waves

1. Introduction

Lucy & Solomon (1970) noticed the existence of a new hydrodynamic instability for line-driven winds, which resembles the runaway mechanism described by Milne (1926) for static atmospheres.

MacGregor et al. (1979) and Carlberg (1980) derived the linear growth rates of this so-called line-driven instability by assuming flow perturbations are optically thin. Contrary to their results, Abbott (1980) found that growth rates of long-scale perturbations, for which the Sobolev approximation could be applied, are zero. Owocki & Rybicki (1984; OR in the following) 'bridged' these opposing results by showing that harmonic perturbations of wavelength λ shorter than one third to one half the Sobolev length, $L = v_{\rm th}/v'$ (primes indicate spatial differentiation), are highly unstable at the constant rate given by MacGregor et al. (1979) and Carlberg (1980); for $\lambda > L$, on the other hand, the growth rate drops as λ^{-2}, implying marginal stability for $\lambda \to \infty$.

Subsequent analytic work on the instability was concerned with: (1) the influence of scattering, e.g., flow stabilization due to line drag (Lucy 1984; Owocki & Rybicki 1985); (2) growth rates for non-radial velocity perturbations (Rybicki et al. 1990); and (3) growth rates for flows with optically thick continuum (Owocki & Rybicki 1991; Gayley & Owocki 1995).

Numerical wind simulations by Owocki et al. (1988) and Owocki (1991) showed the occurence of broad rarefaction regions over which the gas is highly accelerated, and is eventually decelerated again in a strong reverse shock, which heats the gas to X-ray temperatures. Subsequent radiative cooling compresses this gas into dense and narrow clouds. The idea that such clouds should correspond to the observed discrete absorption features in P Cygni profiles is meanwhile mostly ruled out (cf. Puls et al. 1993; Cranmer & Owocki 1996; and the volume edited by Moffat et al. 1994). On the other hand, after some initial difficulties (Cooper & Owocki 1992, 1994), it seems now plausible (Feldmeier et al. 1997) that the observed X-ray emission from O stars (cf. Hillier et al. 1993, and references therein) stems from instability-generated shocks.

The present paper returns to the issue of linear stability analysis, with a twofold aim: (1) to present an easy and straightforward derivation of growth rates, which, for the physically most interesting regime of moderate long-scale perturbations, complements the more elaborate analyses of OR and Lucy (1984); and (2) to show how the second order Sobolev approximation implies an *unstable growth* of inward propagating radiative-acoustic waves, where the latter were firstly described by Abbott (1980) and result already from a first order Sobolev treatment.

A related second order expansion was already performed earlier by Owocki (1991, priv. comm.) to show that the line force in (first order) Sobolev approximation is to one order in L more accurate for pure scattering lines than for pure absorption lines. The aim of this investigation was to explain certain difficulties which are encountered when one tries to reproduce the stationary wind solutions of Castor et al. (1975; CAK in the following) and Pauldrach et al. (1986) – which both apply first order Sobolev approximation – by using instead the exact line force for the case of pure line absorption (Owocki et al. 1988; Poe et al. 1990). In another context, the second order Sobolev approximation was also considered by Sellmaier et al. (1993).

Notice that the derivation given below is for pure absorption lines in a purely radial flow from a point source of radiation. According to Lucy (1984) and Rybicki et al. (1990), the growth rates are then *maximum* ones.

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2. Instability from second order Sobolev approximation

The reason that Abbott (1980) found no indication of wind instability is that he used the Sobolev approximation in lowest order, even for the flow perturbations. E.g., consider an optically thick line, with line force per unit mass $g_T \sim v'/\rho$ (Sobolev 1960; Castor 1974). By assuming harmonic perturbations, this implies a phase shift of 90° between velocity perturbations, δv, and the response of the line force, δg. Hence, the line force does no net work on the velocity perturbation over a full cycle, and the perturbation does not grow (OR).

To see how the instability arises, we consider at first the expression for the exact line force, before the Sobolev approximation is applied. Let $\tilde{x}$ be the frequency displacement from line center, ν_l, in Doppler units, $\Delta\nu_D = \nu_l v_{\rm th}/c$ ($v_{\rm th}$ the ionic thermal speed, c the speed of light), as measured in the observers frame. For radially directed photons, the force per unit mass due to photon absorption in a single line is,

$$g_l(r) = g_t(r) \int_{-\infty}^{\infty} d\tilde{x}\, \phi\left(\tilde{x} - \frac{v(r)}{v_{\rm th}}\right) e^{-\tau(\tilde{x},r)}. \tag{1}$$

g_t is the force due to an optically thin line, with κ the mass absorption coefficient, and F_ν the stellar flux at the line frequency,

$$g_t = \frac{\kappa\, \Delta\nu_D\, F_\nu}{c}, \tag{2}$$

and the radial optical depth is

$$\tau(\tilde{x}, r) = \int_{R_*}^{r} dr'\, \kappa(r')\rho(r')\phi\left(\tilde{x} - \frac{v(r')}{v_{\rm th}}\right), \tag{3}$$

with R_* the stellar radius. In first order Sobolev approximation, κ, ρ, and dv/dr are assumed to be constant over the narrow region, i.e., the Sobolev zone, over which photons of given frequency can be absorbed in the line transition. This implies

$$g_l = g_t\, \frac{1 - e^{-\tau_1}}{\tau_1}, \tag{4}$$

where

$$\tau_1(r) = \kappa(r)\, \rho(r)\, v_{\rm th}/v'(r). \tag{5}$$

Introducing then a velocity perturbation, δv, into (1) and (3) leads to a rather complex expression. To allow a further progress, MacGregor et al. (1979) and Carlberg (1980) assumed that the exponential term is not affected by the perturbation, i.e., the perturbation is optically thin.

As will be discussed in Sect. 4, the velocity jumps which are caused by the corresponding short-scale perturbations are rather small, namely of order the thermal speed, and cannot explain, e.g., the observed X-ray emission from OB stars.

We are therefore primarily interested in long-scale perturbations, which, despite of their reduced growth rates (cf. below), can still grow into saturation and give rise to large velocity, temperature, and also density jumps. Notice that $\delta\tau \approx 0$ can then no longer be assumed. However, as will be shown in the following, the growth rates are then easily derived from applying a second order Sobolev approximation.

The *bridging length* where this long-scale limit breaks down, and the λ^{-2} increase of the growth rate bends over to the constant, maximum rate given by Carlberg (1980), is set by the Sobolev length. This can be seen from the fact that the first order Sobolev approximation does *not* lead to an instability (Abbott 1980), while the second order approach leads to the correct growth rates for long-scale perturbations. The second order approximation differs from the first order one by terms in L.

2.1. Optical depth in second order Sobolev approximation

Let x be the frequency displacement in the comoving frame,

$$x = \tilde{x} - \frac{v(r)}{v_{\rm th}}, \tag{6}$$

and assume *small* perturbations, so that the wind velocity field remains monotonic. Then (3) can be transformed to a frequency integral,

$$\tau(x, r) = \kappa\, v_{\rm th} \int_x^\infty dy\, \frac{\rho}{v'}[s(y)]\, \phi(y), \tag{7}$$

where the integration variable is defined as $y = \tilde{x} - v(s)/v_{\rm th}$. As before, κ is assumed to be constant over the Sobolev zone, which is a reasonable assumption for resonance lines and transitions from metastable levels, both dominating the line force. Performing a Taylor series expansion of ρ/v' to first order (and abbreviating $\rho_r \equiv \rho(r)$, etc.),

$$\frac{\rho_s}{v'_s} = \frac{\rho_r}{v'_r}\left[1 + \left(\frac{\rho'_r}{\rho_r} - \frac{v''_r}{v'_r}\right)(s - r)\right]. \tag{8}$$

From (6), and again to first order,

$$s - r = -\frac{v_{\rm th}}{v'_r}\,(y - x). \tag{9}$$

We consider now only the Doppler core of the line, where de-shadowing effects are most pronounced, and therefore the growth rates are largest (OR). With $\Phi(x) \equiv \int_x^\infty dy\, \phi(y)$, and since $x\phi(x) = -\frac{1}{2}\phi'(x)$ for a Doppler profile, the optical depth in second order Sobolev approximation is,

$$\tau_2(x, r) \;=\; \tau_1(r)\left[\Phi(x) - \frac{v_{\rm th}}{2v'_r}\left(\frac{\rho'_r}{\rho_r} - \frac{v''_r}{v'_r}\right)\big(\phi(x) - 2x\Phi(x)\big)\right]. \tag{10}$$

Remember that $\tau_1(x, r) = \tau_1(r)\, \Phi(x)$ is the optical depth from a first order treatment.

The divergence of the expression $2x\Phi(x)$ in (10) for $x \to -\infty$ is an extrapolation artefact of the linear expansions performed in (8) and (9), and is compensated for by higher order terms in the Taylor series. In any case, this term enters the force response δg only via the combination $2x\Phi(x)\phi(x)$, which vanishes for $x \to -\infty$ (see below).

We separate v and ρ into their stationary components (subscript '0') plus a harmonic perturbation,

$$v(r,t) = v_0(r) + \delta v \exp\left[i\left(k_* \frac{v_*}{v_0(r)} r - \omega t\right)\right],$$

$$\rho(r,t) = \rho_0(r) + \delta \rho \exp\left[i\left(k_* \frac{v_*}{v_0(r)} r - \omega t\right)\right]. \quad (11)$$

Here, δv and k are assumed to be real, while $\delta\rho$ and ω are complex to allow for arbitrary phase shifts between velocity and density perturbations, and for unstable growth, respectively. The expression $k_* v_* r/v_0(r)$, where asterisks refer to an arbitrary, however *fixed* location in the stellar rest frame, accounts for the stretching of perturbations in the accelerating velocity field v_0 (i.e., λ/v_0 is independent of radius). For perturbations which originate in the photosphere, e.g., one could choose $v_* = v(R_*)$. Note that if the sound speed is small compared to *any* other speed (flow speed and wave speeds), pressure forces can be neglected, and the above expression for wave stretching is *exact*.

Inserting (11) into (10), and keeping only terms linear in δv and $\delta\rho$, gives

$$\frac{\delta\tau_2(x,r)}{\tau_1(r)} = \frac{\delta\rho}{\rho_0}\left[\Phi(x) - \frac{v_{\rm th}}{2v_0}\big(i\chi - q\big)\big(\phi(x) - 2x\Phi(x)\big)\right] - \\ -\frac{\delta v}{v_0}\left[\Phi(x) - \frac{v_{\rm th}}{2v_0}\big(i\chi - 4 - qt\big)\big(\phi(x) - 2x\Phi(x)\big)\right] i\chi. \quad (12)$$

Here, $\delta\tau_2$ is given modulo the exponential terms from (11), and the dimensionless wavenumber χ is defined as

$$\chi = k_* v_* \frac{v_0 - r v_0'}{v_0 v_0'}. \quad (13)$$

Since we will concentrate mostly on the case $r v_0' \ll v_0$, i.e., the outer wind, we can approximate

$$\chi \approx \frac{v_0}{v_0'}\, k = \frac{v_0}{v_{\rm th}}\, Lk. \quad (14)$$

Finally, we introduced in (12),

$$q = \frac{v_0 v_0''}{v_0'^2} = -(4\mathrm{r} - 3),$$

$$t = \frac{3v_0 - 2r v_0'}{v_0 - r v_0'} = \frac{3\mathrm{r} - 4}{\mathrm{r} - 3/2}, \quad (15)$$

where $\mathrm{r} = r/R_*$, and the second equalities hold for the CAK velocity law for a wind from a point source, $v_0(r) = v_\infty \sqrt{1 - \mathrm{r}^{-1}}$, with terminal wind speed v_∞.

2.2. The perturbed line force

Introducing the comoving frame frequency, x, in (1), and applying a small perturbation $\delta\tau(x,r)$, gives

$$\delta g_l(r) = -g_t(r) \int_{-\infty}^{\infty} dx\, \phi(x)\, e^{-\tau(x,r)}\, \delta\tau(x,r). \quad (16)$$

To avoid tedious expressions, we set $\tau = \tau_1$ in the exponential. Inserting also $\delta\tau = \delta\tau_2$ from (12) we find, for optically thick lines (subscript 'T'),

$$\frac{\delta g_T}{g_T} = -\big[1 - \epsilon(i\chi - q)\big]\frac{\delta\rho}{\rho_0} + i\chi\big[1 - \epsilon(i\chi - 4 - qt)\big]\frac{\delta v}{v_0}. \quad (17)$$

Note that actually we assumed $\tau \gg 1$ here; as above, δg_T is to be understood modulo the exponential terms. The (small) number ϵ is defined as

$$\epsilon = \frac{v_{\rm th}}{2v_0(r)} E(\tau), \quad (18)$$

where

$$E(\tau) = \tau^2 \int_{-\infty}^{\infty} dx\, \phi(x)\left[\phi(x) - 2x\Phi(x)\right] e^{-\tau\Phi(x)}. \quad (19)$$

The integral E, which is easily evaluated numerically, is of order unity for $\tau \gg 1$, and depends only weakly on τ, namely $E \sim \sqrt{\ln\tau}$ (Castor 1974).

The decisive fact in (17) is the occurence of a positive feedback between velocity and force perturbations, $\delta g_T/g_T = \epsilon\chi^2\delta v/v_0$. First, note the dependence of growth rates $\delta g/\delta v \sim \lambda^{-2}$, in accordance with the results by OR. Moreover, also the quantitative values agree well with those from the long-wavelength limit of the bridging law by OR, cf. their Eq. (28). E.g., for a line of optical depth $\tau = 10$ we find a growth rate which is 20% smaller than that given by OR; in view of the different approximations performed by them and in our derivation, this is completely admissible.

From (18), the growth rate is almost constant for all moderately optically thick lines, except for a dependence on $v_{\rm th}$ (atomic species), and the above weak dependence on τ. This allows us to estimate the response of the *total* line force on perturbations, which is needed below for the derivation of dispersion relations. Following CAK, we assume that the total line force is the simple sum of individual contributions from all lines, i.e., line overlap is neglected; and furthermore that the ratio of the force due to all thin lines to the force from all thick lines is $(1-\alpha)/\alpha$, where $0 < \alpha < 1$, and typically $\alpha \approx 2/3$ for O supergiants. Since, by Eq. (2), the force per unit mass due to an optically thin line is not affected by perturbations, one obtains $\delta g_L/g_L = \alpha\, \delta g_T/g_T$. For the CAK wind in the limit of vanishing sound speed, the total line force can be written $g_L = -g/(1-\alpha)$, with g the gravitational acceleration. Hence, the Euler equation reads $v_0 v_0' = \alpha\, g_L$, and the response of the total line force to a perturbation is

$$\delta g_L = v_0 v_0' \frac{\delta g_T}{g_T}, \quad (20)$$

with $\delta g_T/g_T$ from (17).

2.3. Dispersion relation

By inserting (17) and (20) into the linearized continuity and Euler equations for the harmonic perturbations $\delta\rho$ and δv *in* *the stellar rest frame*, and neglecting sphericity terms and gas pressure, we obtain

$$(-i\omega/v_0' + i\chi + 1)\,\frac{\delta\rho}{\rho_0} \;+\; (i\chi - 1)\,\frac{\delta v}{v_0} = 0,$$
$$(1 - i\epsilon\chi + aq)\,\frac{\delta\rho}{\rho_0} \;+ \tag{21}$$
$$+\;(-i\omega/v_0' + 1 + i\epsilon\chi[i\chi - 4 - qt])\,\frac{\delta v}{v_0} = 0.$$

We introduce a dimensionless phase speed, ω_1, and growth rate, ω_2,

$$\omega_1 = \frac{\mathrm{Re}(\omega/v_0')}{\chi}, \qquad \omega_2 = \frac{\mathrm{Im}(\omega/v_0')}{\epsilon\chi^2}, \tag{22}$$

and set the determinant of (21) to zero, which gives

$$\omega_1^2 - \omega_1(1-4\epsilon) - 2\epsilon(1+\omega_2) + \epsilon^2\chi^2\omega_2(1-\omega_2) - 2\chi^{-2} \;+$$
$$+\,\epsilon q(\omega_1 t - t - \chi^{-2}) = 0,$$
$$\omega_1(2 + 2\epsilon\chi^2\omega_2 - \epsilon\chi^2) + \epsilon\chi^2(1 - \omega_2 + 4\epsilon\omega_2) + 5\epsilon \;+$$
$$+\,\epsilon q\big(\omega_2\epsilon t\chi^2 + 1 + t\big) = 0. \tag{23}$$

The essential result from this equation system can be found analytically, by assuming that

$$\chi \gg 1, \qquad \epsilon \approx 0, \qquad \epsilon\chi \approx 0, \tag{24}$$

and keeping terms in $\epsilon\chi^2$ (see below for a justification). This reduces (23) to

$$\omega_1^2 - \omega_1 = 0,$$
$$\omega_1\left(2\omega_2 - 1 + \frac{2}{\epsilon\chi^2}\right) + 1 - \omega_2 = 0. \tag{25}$$

The two solution branches correspond to fast growing waves which propagate inward, and slowly decaying waves which propagate outward,

$$\omega_1 = 0,\;\; \omega_2 = 1 \qquad \text{(inward)},$$
$$\omega_1 = 1,\;\; \omega_2 = -2/\epsilon\chi^2 \qquad \text{(outward)}. \tag{26}$$

By inserting these results into (21), and by using again (24), the ratio of relative perturbation amplitudes is found to be

$$\frac{\delta\rho/\rho_0}{\delta v/v_0} = -1 \qquad \text{(inward)},$$
$$= i\chi \qquad \text{(outward)}. \tag{27}$$

For an interpretation of this result, note that the continuity equation reads *in the comoving frame* and after applying the WKB approximation (i.e., mean flow gradients neglected), $d\delta\rho/dt + \rho_0\, d\delta v/dz = 0$, or, for harmonic perturbations, $\delta\rho/\rho_0 = \delta v/v_\phi$, with phase speed $v_\phi = \omega/k$.

From (26), the *unstable* waves propagate inward at a phase speed $v_\phi = 0$ in the observer's frame, or $v_\phi = -v_0$ in the comoving frame, i.e., they stand with respect to the star. Eventually, these waves should steepen into reverse shocks, as is indeed found from numerical simulations. The phase shift between density and velocity fluctuations is 180°, similar to the case of ordinary, inward propagating sound waves.

The *damped* waves, on the other hand, propagate outward at a phase speed $v_\phi = v_0$ in the stellar rest frame, or $v_\phi = 0$ in the comoving frame, i.e., they stand with respect to the wind. (In order to derive this from the amplitude relations (27), one has to set $\chi^{-1} = 0$.) By including pressure terms, Abbott (1980) found more exactly $v_\phi = a^2/v_0$ in the comoving frame. Also, the phase shift of these waves is 90°, instead of being 0° for ordinary, outward propagating sound waves.

Abbott (1980) termed both these long-scale modes *radiative-acoustic* waves.

Finally, (24) remains to be justified. By assuming $\epsilon \lesssim v_{\rm th}/v_0 \approx 0$, we are restricted to the highly supersonic, outer wind. Then, $\epsilon\chi \approx 0$, since rv_0' can be neglected in χ, so that $\chi \approx v_0 k/v_0'$, and $\epsilon\chi \approx Lk$. The latter quantity is small compared to unity in the considered long wavelength limit. Finally, with $\chi \gg 1$ we are restricted to perturbations for which the WKB approximation is valid, i.e., to wavelengths shorter than the wind scale height, v_0/v_0'.

2.4. Limiting wavelength for unstable growth

Yet, since we did not apply the WKB approximation to derive (21), this system contains more information regarding the long wavelength regime than does the analysis of OR. Especially, one can derive an upper limiting wavelength, λ_c, above which even inward propagating waves are no longer unstable, but decay instead.

Assume for the moment that curvature terms of the velocity field can be neglected, i.e., $q = 0$. For not too small χ, the numerical solution of (23) shows that $-\omega_1 \ll 1$ for the inward mode, and ω_1^2 can be neglected in (23). With $\epsilon \approx 0$, and setting the growth rate $\omega_2 = 0$, we find from (23),

$$\chi_c = \sqrt{2}\,\epsilon^{-1/4}. \tag{28}$$

Assuming again that $\chi \approx v_0 k/v_0'$, one finds that (as was to be expected) the critical wavelength λ_c is set by the scaleheight of the wind, v_0/v_0'. E.g., for the CAK velocity law,

$$\lambda_c = 2\sqrt{2}\,\pi\,\epsilon^{1/4}\,(r - R_*)\,\frac{r}{R_*}. \tag{29}$$

Except close to the star, λ_c is rather large. An easy calculation shows that, for $\lambda \lesssim \lambda_c$, even the growth rates from (17) are so small that such perturbances would not grow significantly over a wind flow time. Hence, λ_c from (29) is of not much practical interest. (Note also that for such large values of λ_c, one would have to include sphericity terms in the above derivation.)

Finally, the numerical solution of (23) shows that *curvature* terms, v_0'', are rather unimportant in the outer wind, and lead to a small downward revision of λ_c only. Interestingly, however, curvature terms imply that also outward propagating waves can become unstable within a certain wavelength regime. Yet, since the corresponding growth rates are very small, this issue is of academic interest only.

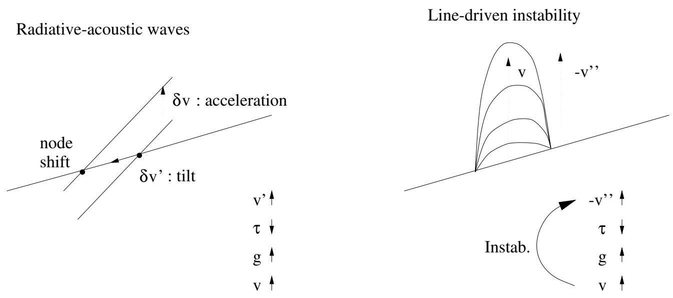

Fig. 1. *Left:* the occurence of long-scale, inward propagating waves in *first* order Sobolev approximation. *Right:* the line-driven instability in *second* order Sobolev approximation. Upward (downward) pointing arrows indicate that the corresponding quantity rises (drops) as consequence of a flow perturbation.

We close this section with a perspective. Present numerical simulations of winds subject to the line-driven instability are cpu-time expensive because the line force is integrated directly, without applying any Sobolev approximation. Our above results imply, however, that the quantitatively correct, linear growth rates can be alternatively obtained from a second order Sobolev treatment. It is furthermore known (Owocki et al. 1988) that these linear rates hold over almost the full growth regime, until they quickly drop to zero when saturation is reached, i.e., when the thermal band becomes optically thin. It seems therefore plausible to perform instability simulations using the cheap Sobolev line force in second order instead of a more elaborate line transfer.

One has to keep in mind, however, that such a method, based on higher-order extrapolation of local conditions, does not incorporate the inherently nonlocal physics that occurs within the nonlinear growth of the instability. Furthermore, in order that a meaningful comparison with non-local integral methods, especially the Smooth Source Function method of Owocki (1991), can be done, it may be necessary to first develop the second order Sobolev forms for the diffuse force terms.

3. Why are radiative-acoustic waves unstable?

We give here some simple, heuristic arguments for the occurence of unstable, radiative-acoustic waves in line-driven winds.

Assume that at some location, r, in the wind, the velocity field experiences an accidental perturbation, and becomes slightly steeper, as is the case, e.g., at the zero crossing of a harmonic perturbation. Since the first order Sobolev force per unit mass due to a thick line is $\sim v'/\rho$, the perturbed gas experiences a larger force, and is accelerated to higher velocities. From the left panel of Fig. 1, one sees that then the node around which the band is tilted is shifted inward. Correspondingly, if the velocity field becomes shallower at some node, the line force drops, the gas is decelerated, and the node is also shifted inward. Thus, long-scale harmonic perturbations of the velocity field, for which the Sobolev approximation can be applied, induce inward propagating waves.

The instability arises then from second order (curvature) terms. The optical depth τ_2 from (10) drops for a concave deformation ($v_0'' < 0$) of the thermal band, i.e., an elevation. (We have assumed here that the 'mean' v' over the perturbation remains constant). In consequence, the line force rises, and the perturbed gas is accelerated to higher speeds. This enhances the elevation, and the curvature increases *further*. The right panel of Fig. 1 displays this feedback between perturbations in v and v''. Similarly, for a convex deformation, i.e., a trough, the line force drops, and the trough becomes deeper.

In later stages, when fast gas starts to overtake slow gas, the original sine-wave perturbation is transformed into a sawtooth. Here, the gas is highly accelerated over a broad velocity elevation, and subsequently decelerated in a narrow, reverse shock front. The corresponding kind of behavior is found for the solutions of the inviscid Burgers equation, $\partial u/\partial t + u\,\partial u/\partial z = 0$.

4. Short-scale vs. long-scale perturbations

Carlberg (1980) pointed out that the growth of optically thin, short-scale perturbations (OR) should saturate at small velocity amplitudes of a few thermal speeds. By then, the perturbed gas is essentially shifted into the unshadowed continuum, and any further velocity shift does not provide more flux or force.

In contrast, the long-scale perturbations analysed in the previous section can grow to much larger amplitudes before they become optically thin.

This suggests that the fast-growing, short-scale perturbations can be considered as *noise* superimposed on the process of a *slow* and *coherent* tilt of the thermal band over large distances, as induced by perturbations with $\lambda \gg L$. This view is supported by numerical simulations of O supergiant winds where a spectrum of base frequencies is injected into the flow (Feldmeier et al. 1997). Here, the longest waves grow into saturation, giving rise to shocks with velocity jumps of up to v_∞.

Obviously, one has to ensure that the growth time of such long-scale perturbations is still short as compared to the flow time. For O star winds, the longest base perturbations which can still grow into saturation give typical distances between subsequent, strong reverse shocks of $\approx 1\,R_*$ at distances from the star where the wind has essentially reached its terminal speed (note: wavelength stretching). On the other hand, optically thin perturbations which grow at maximum rate give structural length scales in the outer wind of shorter than $10^{-2}\,R_*$.

Furthermore, it is presently not clear whether velocity amplitudes are the most relevant measure of importance of the wind structure, e.g., because only a very small amount of wind material is actually involved in such large velocity amplitudes (Owocki et al. 1988). Thus other measures, for example dissipation of wind kinetic energy, might be more strongly influenced by the structure at smaller scale.

5. Summary

While unstable growth rates for both the limiting regimes of short-scale perturbations, $\lambda < L$, and very long-scale perturbations, $\lambda \gg L$, were formerly discussed using simplifying assumptions (i.e.: optically thin perturbations, resp. lowest order Sobolev approximation), we have found that the physically most interesting regime of moderate long-scale perturbations, $\lambda > L$, which was hitherto only treated within the elaborate analyses of OR and Lucy (1984), is also accessible to a simplified approach using second order Sobolev approximation.

While the standard, first order approach demonstrates the existence of so-called radiative-acoustic waves, the second order treatment shows the inward branch of these waves to be unstable: accidental, positive perturbations in the velocity field of the wind imply larger absolute curvatures; this reduces the optical depth, raises the line force, and consequently leads to a *further* acceleration.

Somewhat astonishingly therefore, the radiative-acoustic waves according to Abbott (1980) and the line-driven instability seem to have a similar basis, stemming respectively from first and second-order Sobolev approximation. On the other hand, Owocki & Rybicki (1986) showed by using a Green's function analysis that the true signal propagation speed in an unstable wind which is driven by pure absorption lines is the ordinary *sound* speed. Future work has to clarify this dichotomy.

Our analysis allows one to abandon the usual WKB approximation, and to account for wave stretching. We hope that the simplicity of this approach also leads to an applicability in cases of, e.g., more complex flow geometries, as may occur in line-driven winds from accretion disks in quasars or cataclysmic variables.

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