Astron. Astrophys. 332, L21–L24 (1998)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

On the possibility of curvature radiation from radio pulsars

H. Lesch[1], **A. Jessner**[2], **M. Kramer**[2], **and T. Kunzl**[1]

[1] Institut für Astronomie und Astrophysik, Universität München, Scheinerstrasse 1, D-81679 München, Germany
[2] Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany

Received 2 September 1997 / Accepted 23 January 1998

Abstract. We consider the widespread hypothesis that coherent curvature radiation is responsible for the radio emission of pulsars. The comparison of energy conservation and the published data and luminosities explicitly proves that coherent curvature radiation **cannot** be the source for the radio emission of pulsars for frequencies below a few GHz. At higher frequencies coherent curvature radiation can be ruled out because neither the observationally deduced emission heights nor the observed radius to frequency mapping can be reproduced by this mechanism. Our argumentation is in accordance with the more general critics (e.g. Melrose 1992) that no adequate bunching mechanism has been identified for coherent curvature radiation. We present 5 examples (0329+29, 0355+54, 0540+23, 1133+16, 1916+10) of pulsars whose high frequency (larger than 1.4. GHz, up to 32 GHz) luminosities are well known, and as a low frequency example the faintest radio pulsar 0655+64 of the Taylor et al (1993) sample.

Key words: plasmas – radiation mechanism – pulsars

1. Introduction

Pulsar radio emission has a very high brightness temperature which implies that the emission mechanism must be coherent, that is the emission cannot be explained in terms of individual particles radiating independently (incoherently) of each other. There are several models for the origin of such a coherent mechanism (e.g. Asseo 1993 and references therein). Here we concentrate on one of the earliest and still most prominent models, namely coherent curvature radiation (Gunn and Ostriker 1971; Sturrock 1971; Ruderman and Sutherland 1975; Ginzburg and Zheleznyakov 1975; Buschauer and Benford 1976; Kirk 1980; Buschauer and Benford 1983). In the frame of a pulsar as a rotating dipole it is natural to consider the radiation of relativistic charged particles when they move along the curved field lines, thereby emitting curvature radiation. The necessary bunching mechanism was supposed to be provided by a two-stream instability, which excites plasma waves. They are supposed to bunch particles via their electrostatic fields (e.g. Ruderman and Sutherland 1975). This ansatz has been criticized for several reasons (Melrose 1992 and references therein) A fundamental difficulty is that the theory for bunching instabilities does not allow for any velocity dispersion of the emitting particles. No adequate bunching mechanism has been identified. An extreme from of bunching is required; specifically the relativistic particles of single sign need to form well separated bunches with a pancake shape with normal almost exactly along the field lines. It was concluded, that in view of these difficulties coherent curvature radiation should not be regarded as the favored mechanism for pulsar radio emission (Asseo et al 1980, 1983; Melrose 1992)).

Despite this irrefutable theoretical critics coherent curvature radiation is still one of the favorites of many phenomenological models (Rankin 1983a,b, 1986, 1988, 1989, 1992; Radhakrishan and Rankin 1990; Gil 1983, 1984, 1992; Gil and Snakowski 1990a,b). As a step towards a clarification we present here a simple argument against coherent curvature radiation as a source for radio emission from pulsar by means of energy conservation. We investigate whether coherent curvature radiation is able to produce the observed luminosities within the constraints of emission heights derived from the observations. Thus, we do not consider the criticism about the origin of bunches. We ask: is it possible to release enough power in order to explain the observed luminosities via coherent curvature radiation?

2. Coherent curvature radiation

Curvature radiation can be described in terms of emission by a relativistic particle moving around the arc of a circle chosen such that the actual acceleration corresponds to centripetal acceleration. A relativistic electron with Lorentz factor γ, constrained to follow a path with radius of curvature $R_{\rm c}$, radiates similarly to an electron in a circular orbit with frequency $c/2\pi R_{\rm c}$. The critical frequency where most of the radiation is emitted is approximately given by (e.g. Zheleznyakov 1996, p. 231)

$$\nu_{\rm c} \simeq \frac{3c}{4\pi R_{\rm c}}\gamma^3. \qquad (1a)$$

Send offprint requests to: H. Lesch

Or if we observe a particular frequency we find the corresponding Lorentz factor via

$$\gamma \simeq \left[\frac{\nu_{\rm c} 4\pi R_{\rm c}}{3c}\right]^{1/3}. \qquad (1b)$$

The curvature radius of the field lines in a magnetic dipole field can be expressed as (Smirnow 1973, p. 211)

$$R_c(\theta,\theta_s) = \frac{r_{ns}}{3}\cdot\frac{\sin\theta}{\sin^2\theta_s}\cdot\frac{(1+3\cos^2\theta)^{3/2}}{1+\cos^2\theta} \qquad (2)$$

where θ is the collateral angle of a point on the field line starting at θ_s at the pulsar surface. A reasonable approximation up to $50\cdot r_{ns}$ is given by

$$R_c(r) \simeq R_0\cdot\sqrt{\frac{r}{r_{ns}}}, \qquad (3)$$

where $R_0 = R_c(r_{ns}) = R_c(\theta_s,\theta_s)$ is the curvature radius at the pulsar surface.

The total power radiated by a particle of energy $E = \gamma m_e c^2$ along a curved field line is (q_e denotes the charge)

$$P_{\rm c} = \frac{2}{3}\gamma^4\frac{q_e^2 c}{4\pi\epsilon_0 R_c^2}. \qquad (4)$$

The classical Goldreich-Julian charge density in the pulsar magnetosphere (Goldreich and Julian 1969)

$$n_{GJ} = \frac{2\epsilon_0\Omega B_0}{q_e}\cdot\frac{r_{ns}^3}{r^3} \qquad (5)$$

provides us with a typical particle density of $n_{GJ}(r_{ns}) = 3.6\cdot 10^{11} cm^{-3}$ if P=0.5 s and $B = 10^{12}$ G. It enables us to estimate the typical size of the emission region for *incoherent curvature emission*. For the lowest luminosity $L = 10^{25} erg/s$ (0655+64 (Taylor et al. 1993) we find $(\frac{L}{P_c\cdot n_{GJ}})^{1/3} = 4000 km$. Using Eqs. (1b), (4) and (5) together with the simplifying assumption that the emission region extends across the line of sight as well as along it, we find that the predicted profile widths for incoherent curvature emission turn out to be within

$$w_{inc}(\nu) = 2\cdot\tan^{-1}\Big(\big(\frac{81cR_c^2}{4\pi\nu^4}\big)^{1/9}\big(\frac{3L}{4q_e\Omega B_0}\big)^{1/3}\Big) \qquad (6)$$

For frequencies below 10^{16} Hz we predict profile widths of roughly 180 degrees and only when we approach 10^{19} Hz ($h\nu = 4keV$) can we expect widths of around 25 degrees. Typical pulsar profile widths at radio frequencies are however of the order of 5-10 degrees (e.g. Izvekova et. al. 1994, Seiradakis et. al. 1995). Because of the narrow profiles of the pulsed emission we must rule out any consideration of *incoherent curvature emission* as the source of the observed pulsar radio luminosities.

As incoherent emission is obviously insufficient it might be worthwhile to turn or attention to *coherent* curvature emission which has been suggested as an emission mechanism (see references above). Coherence exists in volumes of the order of $V_c = c^3\gamma^2/\pi\nu^3$ (Melrose 1992). The power output of such a coherence volume is then given by:

$$P_N = \frac{2}{3}\gamma^4\frac{(N_c\cdot q_e)^2 c}{4\pi\epsilon_0 R_c^2} = P_c\cdot N_c^2 \qquad (7)$$

with $N_c = V_c\cdot n_{gj}$ being the number of particles within a coherence volume. To account for the observed luminosity L one requires $N_v = \frac{L}{P_c\cdot N_c^2} = 7\cdot 10^{11}$ of these volumes, which will give us an estimate of the total number of particles involved

$$N = N_c\cdot N_v = \frac{L}{P_c\cdot N_c} \qquad (8)$$

and the minimal size of the emission region

$$\Lambda_c = (N_v\cdot V_c)^{1/3} \qquad (9)$$

Energy conservation alone provides us with an important constraint that will enable us to restrict the free parameter γ and to check the consistency of the model.

The energy loss of a charge within the emission region is now

$$P_\Lambda = \frac{\Lambda_c}{c}P_c N_c \qquad (10)$$

which must never exceed the energy of the particle $\gamma m_e c^2$ itself.

The bunching of particles requires an external electric force which confines the particles in a coherence volume. Although electrostatic instabilities have been criticized for several reasons, the principal argument, that electrostatic plasma wave can be responsible for the bunching of particles has not been questioned, only processes that excite the waves are under discussion. The Doppler shifted plasma waves oscillate with the local plasma frequency $\omega_{\rm pe} = \gamma\sqrt{\frac{nq_e^2}{\epsilon_0\gamma m_e}}$ of the particles and therefore their electrostatic fields present the ideal bunching force. When the particles are in resonance with the wave they experience the wave electric field most intensively and they are confined (bunched) in the coherence volume $\propto \lambda^3 \propto (c/\omega_{\rm pe})^3$. The same resonance condition holds for the propagation of electromagnetic waves with frequency ν (where $\nu \geq \omega_{\rm pe}/2\pi$), which defines a certain radius in the pulsar magnetosphere, at which the coherent radiation can escape from the plasma. Particle bunching and wave propagation is determined by the same condition $\nu \geq \omega_{\rm pe}/2\pi$ (e.g. Melrose 1992). Since the bunching leads to a very fast and efficient emission the place at which the condition is fulfilled is also the place of emission, i.e. the emission height. For larger radii only lower frequencies can be emitted (see Eqs. (1a) and (3)). An increase in density corresponds to an increase in frequency at a given radius (Eq. (5)). With the determination of the minimum emission height from $\nu \geq \omega_{\rm pe}/2\pi$ using Eq. (1b) to eliminate γ

$$\nu = \frac{3^{2/5}}{4\pi c^{1/5}}\Big(\frac{8q_e\Omega B_0 r_{ns}^3}{3m_e}\Big)^{3/5} r^{-9/5}R_c^{1/5} \qquad (11)$$

we can express the ratio of emitted energy over the total energy as

$$\eta_1 = \frac{P_\Lambda}{\gamma m_e c^2} \qquad (12)$$

which cannot exceed unity. From the condition $\eta_1 \leq 1$ we obtain a condition for the luminosity by inserting Eqs. (1–9) into Eq. (10), leaving

$$L_{\rm crit} = \frac{\gamma^3 m_e^3 c^9}{V_{\rm c}P_{\rm c}^2 N_{\rm c}} = \frac{243}{32}\frac{\pi\epsilon_0 m_e^3 c^2}{q_e^3\Omega B_0}\nu^3\frac{r^3}{r_{ns}^3}R_c \qquad (13)$$

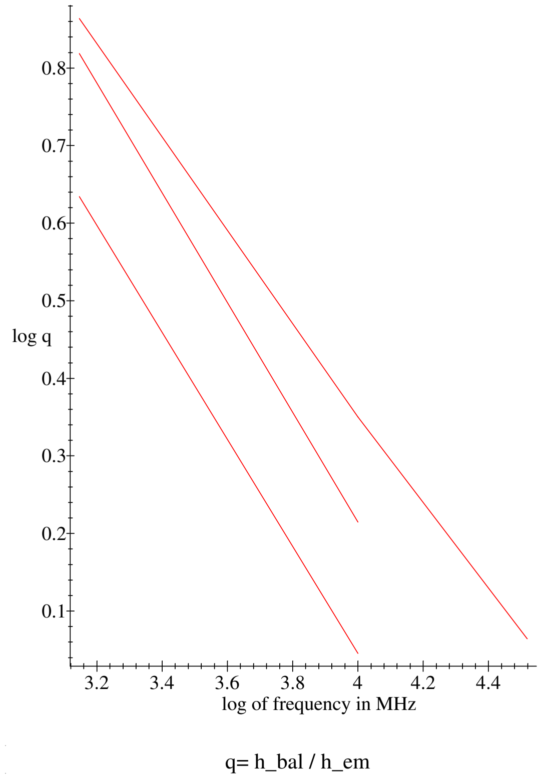

Fig. 1. h_{bal}/h_{em} versus frequency for three pulsars (from left to right: 1929+10; 0540+32; 0329+54). $q > 1$ means that curvature radiation can **not provide enough energy at the corresponding frequency**

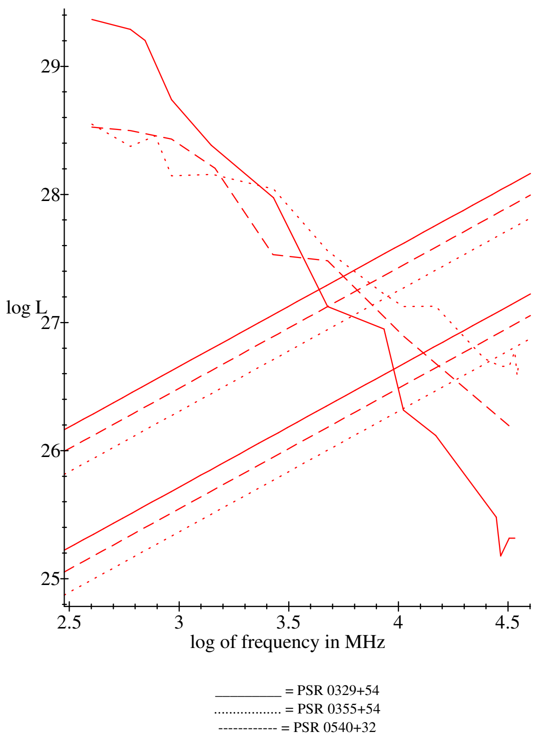

Fig. 2a. Luminosities of the three pulsars mentioned, compared with the maximum possible luminosity that can be obtained by coherent curvature radiation. The upper straight lines are calculated with $n = 10^4 n_{GJ}$, whereas for the lower ones $n = n_{GJ}$ is assumed.

3. Application to six pulsars

Clearly, the observed luminosities must not exceed $L_{\rm crit}$ if coherent curvature radiation is to be a dominant mechanism. We can now compare the observed luminosities at different frequencies with the corresponding critical luminosity provided by coherent curvature radiation. For frequencies up to 32 GHz we use the sample of Kramer et al. (1996): 0329+29;0355+54; 0540+23; 1133+16; 1916+10. As a limiting case we use also the weakest pulsar in the catalogue of Taylor et al. (1993): 0655+64 which has a maximal luminosity at 408 MHz of $3 \cdot 10^{25}\ ergs^{-1}$.

Energetically, curvature radiation is not ruled out wherever $L_{crit} > L_{observed}$. We will call the minimal surface distance for which we would have $L_{crit}/L_{observed} > 1$ the energy balance distance h_{bal} to contrast it from the minimal emission distance h_{em},– found by inverting Eq. (11), where the curvature radiation frequency matches the plasma frequency. We present the ratio h_{bal}/h_{em} versus frequency for five pulsars in Fig. 1. Apart from the highest frequencies we find that it is impossible to fulfill the condition $h_{em}/h_{bal} < 1$ for frequencies below several GHz.

4. Conclusions

We note that nearly everywhere the theoretical emission heights are too large if compared with the corresponding observational estimates, (Kramer et al. 1996; Kijak and Gil 1997). This is particularly striking when one considers the frequency regime below 1.4 GHz. Here the observations often yield luminosities that cannot be achieved anywhere within the bounds of pulsar magnetosphere. As it is unlikely that all of the beam energy is going to be emitted as curvature radiation we have to realize that the actual emission heights would even be greater than from these simple estimates. Furthermore, from Eq. (3) and (11) we obtain a radius to frequency mapping exponent of $\chi = -10/17 = -0.59$, whereas the observed exponent is -0.15 ± 0.1 (Gil and Kijak 1992; Kramer et al. 1996). The radius to frequency method determines the frequency dependent radius $R(\nu)$, at which the emission should originate: $R(\nu) \simeq R_0 + C \cdot \nu^{\chi}$ (C is a constant with dimension mHz^{χ} (Phillips 1992).

Although at high frequencies (above 10 GHz) L_{crit} exceeds $L_{observed}$ by many orders of magnitude, (i.e. there is no energetic argument to exclude curvature radiation), we rule out coherent curvature radiation since it neither reproduces the observed radius to frequency mapping nor the estimated emission heights.

In Fig. 2a and 2b we show the frequency dependent $L_{\rm crit}$ given by Eq. (13) versus frequency for the 6 pulsars with different magnetospheric densities. The solid lines are for $n = n_{\rm GJ}$ and the dashed lines for $n = 10^4 n_{\rm GJ}$. The latter case is used since several models for γ-radiation involve massive pair production above the polar cap region, i.e. much more secondary particles are produced than primary particles (Daugherty and Harding 1994, 1996; Usov and Melrose 1996; Zhang et al. 1997; Miyazaki and Takahara 1997). Obviously even for the high den-

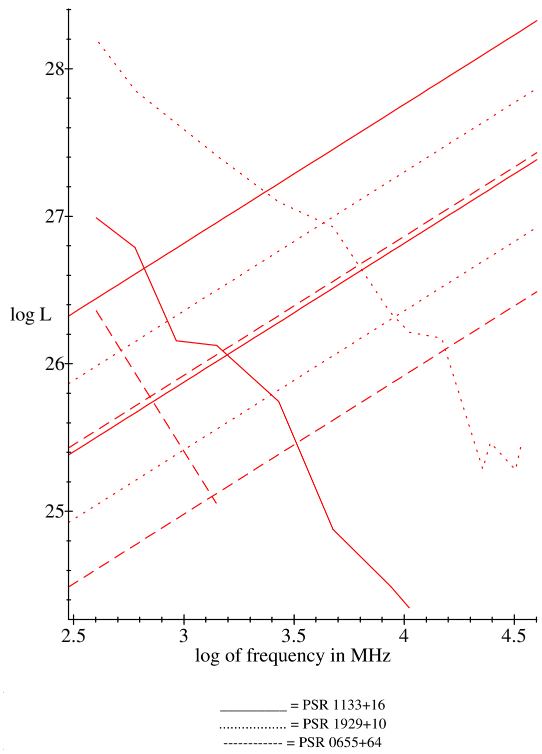

Fig. 2b. The same as Fig. 2a, but for three other pulsars.

sity case the observed luminosities at frequencies lower than about 6 GHz cannot be explained by coherent curvature radiation and since it cannot explain emission heights and radius to frequency mapping, we exclude this mechanism as origin for pulsar radio emission.

Other mechanisms like for example nonlinear electrostatic instabilities which drive strong Langmuir turbulence, as proposed by Asseo (1993) should be considered as promising alternatives to coherent curvature radiation at low radio frequencies where the bulk of the radio emission is received. A comparison with this mechanism and the observations will be shown in a following paper.

References

Asseo, E., 1993, MNRAS 264, 940
Asseo, E., Pellat, R., Rosado, M., 1980, ApJ 239, 661
Asseo, E., Pellat, R., Sol, H., 1983, ApJ 266, 201
Benford, G., Buschauer, R., 1983, A&A 118, 358
Buschauer, R., Benford, G., 1976, MNRAS 177, 109
Daugherty, J.K., Harding, A.K., 1994, ApJ 429, 325
Daugherty, J.K., Harding, A.K., 1996, ApJ 458, 278
Gil, J.A., 1983, A& A 123, 7
Gil, J.A., 1984, 1984, A& A 131, 67
Gil, J.A., 1992, in The Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, eds. Hankins, T.H., Rankin, J.M. and Gil, J.A., IAU Colloq. **128**, p. 395
Gil, J.A., Snakowski, J.K., 1990a, A& A 234, 237
Gil, J.A., Snakowski, J.K., 1990b, A& A 234, 269
Gil, J.A., Kijak, J., 1992, A& A 256, 477
Ginzburg, V.L., Zheleznyakov, V.V., 1975, ARAA 13, 511
Goldreich, P., Julian, W.H., 1969, ApJ 157, 869
Gunn, J.E., Ostriker, J.P., 1971, ApJ 165, 523
Izvekova, V.A., Jessner, A., Kuzmin, A.D., et al., 1994, A&A Suppl. 105,235
Kijak, J., Gil, J., 1997, MNRAS (in press)
Kirk, J.G., 1980, A&A 82, 262
Kramer, M. Xilouris, K.M., Jessner, A., et al., 1996, A&A 306, 867
Melrose, D.B., 1992, in The Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, eds. Hankins, T.H., Rankin, J.M. and Gil, J.A., IAU Colloq. **128**, p. 306
Michel, F.C., 1978, ApJ 220, 1101
Miyazaki, J.F., Takahara, F., 1997, MNRAS 290, 49
Phillips, J.A., 1992, ApJ 385, 282
Radhakrishan, V., Rankin, J.M., 1990, ApJ 352, 258
Rankin, J.M., 1983a, ApJ 274, 333
Rankin, J.M., 1983b, ApJ 274, 359
Rankin, J.M., 1986, ApJ 301, 901
Rankin, J.M., 1988, ApJ 325, 314
Rankin, J.M., 1990, ApJ 352, 247
Rankin, J.M., 1992, in The Magnetospheric Structure and Emission Mechanisms of Radio Pulsars, eds. Hankins, T.H., Rankin, J.M. and Gil, J.A., IAU Colloq. **128**, p. 133
Ruderman, M.A., Sutherland, P.G., 1975, ApJ 196, 51
J.H. Seiradakis, J.A. Gil, D.A. Graham, et al., 1995, A&A Suppl., 111, 205-227
Smirnow, W.I., 1973 Lehrgang der höheren Mathematik, Akademie-Verlag (Berlin)
Taylor, J.H., Manchester, R.N., Lyne, A.G., 1993, ApJS, 88, 529
Usov, V.V., Melrose, D.B., 1996, ApJ 464, 306
Zhang, B., Qiao, G.J., Lin, W.P., Han, J.L., 1997, ApJ 478, 313
Zheleznyakov, V.V. 1996 Radiation Processes in Astrophysical Plasmas, Kluwer, Dordrecht, p. 231