

Gravitation spin effect on the magnetic inclination evolution of pulsars

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Abstract. If pulsar magnetic fields are associated with the intrinsic magnetic moment of neutrons inside the stellar crust, the gravitational spin effect can induce the magnetic axis to align with the rotation axis. Our theoretical results show that the decay time scale is inversely proportional to the magnetic field strength, and that longer initial period pulsars can delay the alignment process. Therefore the inclination angle of strong magnetic field pulsars should be distributed randomly but the weak field pulsars should concentrate towards lower values of inclination angle. Various statistical analyses comparing theoretical predictions and observational data have been done and satisfactory agreement between them obtained.

Key words: relativity – pulsars: general

1. Introduction

To test the observational effects predicted by Einstein's general relativity theory and other gravitational theories in pulsar systems by means of high-precision timing observations as well as radio polarization observation is an important trend and topic in modern relativistic astrophysics (see e.g. Damour, Gibbons & Taylor 1988). In this paper, we propose a gravitational spin effect as a possible mechanism to explain the evolution of the pulsar magnetic inclination, the angle between the pulsar's rotation axis and its magnetic axis, a quantity which is well determined either by the radio polarization observations developed by Lyne & Manchester (1988, hereafter LM88) or if the complete radio core beam is obtained, by the estimation of the standard pulsar radiation theory developed by Rankin (Rankin 1993, hereafter R93). Both methods are remarkably consistent with each other despite systematic differences in a number of cases (see comments in Bhattacharya & Van den Heuvel 1991). The magnetic inclination has been extensively investigated by astronomers

(LM88; R93; Candy & Blair 1986; Kuzmin & Wu 1992; Xu & Wu 1991), because it plays a very important role in determining the structure of pulsar magnetospheres and the detailed radiation processes which occur. On the other hand, the high precision pulsar observation data can provide evidence to support and/or test gravitational theories. The topics studied here connect gravitational theories and observational astronomy.

The magnetic inclination evolution has been studied theoretically by many other researchers who all conclude that the alignment of the magnetic axis with rotational axis is a consequence of the Maxwell radiation torque (Davies & Goldstein 1970; Michel & Goldwire 1970; Ruderman & Cheng 1988). However, it has been pointed out that the neutron star magnetic field may originate from the alignment of the neutron intrinsic magnetic moments in the stellar crust region (Boccaletti, De Sabbata & Gualdi 1965; Silverstein 1969; O'Connell & Roussel 1972), in which case the coupling between intrinsic spin and the gravitational field will influence the evolution of the direction of the magnetic field. Physically, the coupling effects include a special relativity effect (Mashoon 1988; Hehl & Ni 1990), a torsion effect predicted by gauge theories of gravity (Hayashi & Shirafuji 1979; Hehl et al 1976) and the magnetic self interaction. These combine to give the following evolutionary equations for the field (Zhang et al 1992; Zhang 1993),

$$\frac{d\boldsymbol{\mu}}{dt} = -\mathbf{E}(r, t) \times \boldsymbol{\mu} \quad (1)$$

$$\mathbf{E}(r, t) = 3\Sigma(r, t) - \Omega(t) + \boldsymbol{\omega}(t) \quad (2)$$

$$\mathbf{B}(t) = \int \boldsymbol{\mu} n(r, t) d^3r / \int d^3r \quad (3)$$

where $\Omega(t)$ is the angular velocity of the pulsar, $\Sigma(r, t)$ the rotation correlated axial torsion term, $\boldsymbol{\omega}(t) = g\mathbf{B}(t)/2m_n c$, the Larmor frequency, g the Lande g -factor, m_n the neutron mass and $\boldsymbol{\mu}$ the neutron magnetic moment. Eq. (1) shows that the magnetic moment is rotating with a combined angular frequency given in Eq. (2). The physical meaning of each term in Eq. (2) is following (Zhang 1993). The first term is the torsion-spin effect

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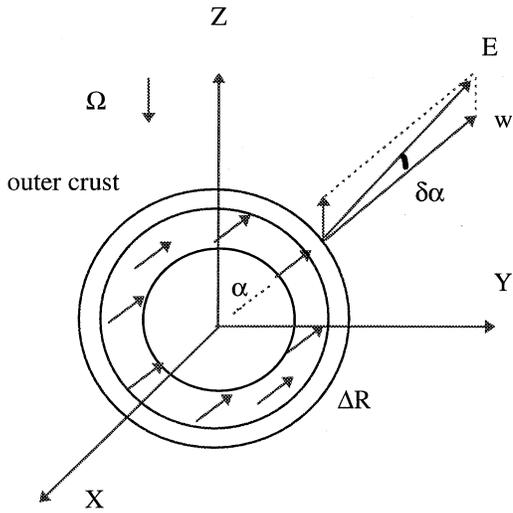


Fig. 1. A schematic representation of the model. The dipolar magnetic field, which is in the direction of vector ω , is formed in the crust region with crust depth ΔR . The inclination angle between rotational axis and the magnetic polar axis is α . The spin is precessing around the combined angular frequency vector \mathbf{E} . The angle between ω and \mathbf{E} is $\delta\alpha$.

which represents the non-Riemannian contribution of Metric-Affine Gravity. However, the Riemannian contribution of the metric part is the effect of a rotating gravitational field, which is equivalent to increase the rotation-spin effect by a factor of $(\frac{R_s}{R})\Omega$, where R_s is the Schwarzschild radius. The second term is the rotation-spin effect which represents the inertia centrifugal contribution of the rotation frame. The third term is the magnetic self interaction which represents the recovery effect when the spin deviates from the original main direction of the magnetic field. The post-Newtonian approximated torsion Σ inside a rotating star with constant density is given as follows (Nitsch 1980),

$$\Sigma(r, t) = A(r, t)\mathbf{e}_z + D(r, t)\cos\theta\mathbf{e}_r, \quad (4)$$

$$A(r, t) = \frac{G^2 M J(t)}{c^4 R^4} \left(4 - \frac{5r^2}{R^2} + \frac{5r^4}{7R^4}\right) \quad (5)$$

$$D(r, t) = \frac{G^2 M J(t)}{c^4 R^4} \left(\frac{5r^2}{R^2} - \frac{r^4}{7R^4}\right) \quad (6)$$

where G is the gravitational constant and c is the speed of light, and M and R are the mass and radius of the star, respectively. $J = I\Omega$ and $I = (2/5)MR^2$. Our model is shown schematically in Fig. 1. On the surface of the earth, the absolute value of Σ is about $10^{-24} \text{ rad s}^{-1}$, which is beyond the level of present-day experimental detectability. However, on the surface of a neutron star, $\Sigma \sim 10^{-2}\Omega \sim 1 \text{ rad s}^{-1}$, for $\Omega = 100 \text{ rad s}^{-1}$.

2. The evolution of the magnetic inclination

The evolutionary equations for the field are nonlinear differential and integral equations. It is difficult to find an exact solution. To solve these equations approximately, it is helpful to inspect

the order of magnitude of the quantities and the physical meaning of the equations. For a canonical pulsar, $\Sigma \sim 10^{-2}\Omega$ with $\Omega = 1 \sim 5 \times 10^3 \text{ rad s}^{-1}$ and $\omega = geB/2m_n c \sim 10^{16} \text{ rad s}^{-1}$. The approximation $\Sigma \ll \Omega \ll \omega$ is often used in the subsequent calculations. In spite of the small value of Σ and Ω compared with the Larmor frequency ω , the long time cumulative evolutionary effects, for $t \sim 10^{14}$ seconds for example, will be substantial. Next let us examine the physical meaning of the equations. We note that Eq. (1) is a precession equation, and each magnetic moment vector precesses around $\mathbf{E}(\mathbf{r}, t)$. However, such precession is incoherent because the coordinate dependent quantity $\Sigma(r, t)$ causes the phase difference in the different position. Secondly, the precession angle $\delta\alpha$ between the magnetic field \mathbf{B} and \mathbf{E} is very small, given approximately by

$$\delta\alpha = \frac{\Omega}{\omega} \sim 10^{-14} \text{ rad} \quad (7)$$

if Ω and B are chosen to be 100 rad s^{-1} and 10^{12} Gauss , respectively. Thirdly, the incoherent precession of each magnetic moment will result in the integral magnetic field to be cancelled in the direction orthogonal to \mathbf{E} in a specific time interval δt required for the phase difference between the magnetic moment vectors located in the inner layer and the outer layer of magnetic region to reach π . When this occurs \mathbf{E} will shift towards the rotation axis by an angle $\delta\alpha$, and μ will precess around the new \mathbf{E} axis again for another time interval δt . Gradually, the magnetic axis, which is almost parallel to \mathbf{E} , will move towards the rotation axis. We can estimate δt as follows.

$$\langle \Delta E \rangle \delta t = \pi, \quad (8)$$

where $\langle \Delta E \rangle$ is the average phase velocity difference between the inner and outer magnetic region boundaries, is given by

$$\begin{aligned} \langle \Delta E \rangle &= \sqrt{2} \langle \Delta \Sigma \rangle = \sqrt{2} [\langle \Sigma(R) \rangle - \langle \Sigma(R - \Delta R) \rangle] \\ &= \frac{3}{10} \frac{\Delta R}{R} \left(\frac{R_s}{R}\right)^2 \Omega \end{aligned} \quad (9)$$

Here ΔR is the depth of the magnetic region and $R_s = \frac{2GM}{c^2}$ the Schwarzschild radius. In deriving Eq. (9), the contribution of the polar angle is taken to be $\frac{1}{\sqrt{2}}$ and $\frac{\Delta R}{R} \ll 1$ is used.

From Eqs. (7) to (9), we can obtain the decay rate of the magnetic angle as follows,

$$\frac{\delta\alpha}{\delta t} = -\frac{3}{10\pi\omega} \frac{\Delta R}{R} \left(\frac{R_s}{R}\right)^2 \Omega^2 \quad (10)$$

If magnetic dipole radiation is the only dissipation mechanism for a pulsar, its rotation energy loss (Sharpiro & Teukolsky 1983) is given by

$$\frac{d\Omega}{dt} = -\frac{B_s^2 \sin^2 \alpha R^6 \Omega^3}{6Ic^3} \quad (11)$$

Solving the differential Eqs. (10) and (11) simultaneously, we obtain the magnetic inclination evolutionary equation,

$$F[\alpha(t)] = F[\alpha(0)] - \Delta\alpha(t) \quad (12)$$

$$F[\alpha(t)] = \alpha(t) - \frac{1}{2} \sin 2\alpha(t) \quad (13)$$

$$\begin{aligned} \Delta\alpha(t) &= \frac{3}{10\pi\omega} \frac{\Delta R}{R} \left(\frac{R_s}{R}\right)^2 \frac{12Ic^3}{B_s^2 R^6} \ln \frac{\Omega_0}{\Omega(t)} \\ &= \frac{18}{5\pi\omega} \frac{\Delta R}{R} \left(\frac{R_s}{R}\right)^2 \frac{Ic^3}{B_s^2 R^6} \ln \frac{P(t)}{P_o}, \end{aligned} \quad (14)$$

where $\alpha(0)$ is the initial magnetic inclination angle, Ω_o (P_o) is the initial stellar angular velocity (period) and $\Omega(t)$ ($P(t)$) is the angular velocity (period) at any time t . Taking typical values of the pulsar parameters, e.g. $R \sim 10^6 \text{ cm}$, $B_s \sim 10^{12} \text{ gauss}$, $I \sim 10^{45} \text{ g cm}^2$, $M \sim 3 \times 10^{33} \text{ g}$, $\frac{\Delta R}{R} \sim 0.1$, we obtain

$$F[\alpha(t)] = F[\alpha(0)] - 0.023 \times B_{12}^{-1} B_{s12}^{-2} \ln \left[\frac{P(t)}{P_o} \right], \quad (15)$$

where B_{12} and B_{s12} are the internal and the dipolar magnetic fields of the neutron star in the units of 10^{12} Gauss, respectively. An alternative approach (Zhang 1992) to solve Eqs. (1)-(3) by perturbation expansion, in which α is assumed to be small, has given a solution similar to that of Eq. (15) but with a singularity occurring at $\alpha = \frac{\pi}{2}$, where the assumption is clearly no longer valid. So the solution obtained here is more physical. Furthermore, we want to remark that the internal magnetic field, in general, can be larger than the dipolar field.

If the inclination angle of a pulsar does not change drastically, a good approximate expression for the stellar angular velocity at an arbitrary time is given by (see e.g. Taylor & Manchester 1977; Shapiro & Teukolsky 1983),

$$\Omega^2(t) = \frac{\Omega_o^2}{1 + \frac{t}{\tau}}, \quad (16)$$

where τ is given by

$$\tau = \frac{3Ic^3}{B_s^2 \sin^2 \alpha_o R^6 \Omega_o^2}, \quad (17)$$

and the average value of $\sin \alpha_o$ can be taken as $\frac{1}{\sqrt{2}}$. Then the explicit time evolution formula for the magnetic inclination angle is given by

$$F[\alpha(t)] = F[\alpha(0)] - 0.023 \times B_{12}^{-1} B_{s12}^{-2} \ln [1 + 1.9 \times 10^{-4} \Omega_{o3}^2 B_{s12}^2 t(\text{yr})], \quad (18)$$

where $\Omega_{o3} = \frac{\Omega_o}{10^3 \text{ rad s}^{-1}}$.

3. Comparison with observations

Several methods of estimation of the magnetic inclination have been suggested (LM88; R93; Candy & Blair 1986). Although some differences exist among these methods, their common aspects are based on the geometric relations of the polar cap model, and the inclination angle values evaluated by the different methods are not far apart. Lyne & Manchester (LM88) were the first to

Table 1. The distribution of inclination α of pulsars from LM88 and R93

LM88	$N(B_{12} > 1)$	$N(B_{12} < 1)$
$\alpha \geq 45$	30	11
$\alpha < 45$	31	31
$\alpha \geq 43$	31	11
$\alpha < 43$	30	31
$\alpha \geq 26$	45	21
$\alpha < 26$	15	21
R93	$N(B_{12} > 1)$	$N(B_{12} < 1)$
$\alpha \geq 45$	36	21
$\alpha < 45$	54	32
$\alpha \geq 39$	45	24
$\alpha < 39$	45	39
$\alpha \geq 35$	51	27
$\alpha < 35$	39	26

give the values of magnetic inclination angle for a sample of 106 pulsars based on polarization data. Rankin (1993) suggested a new method to estimate the value of the inclination angle based on the properties of the core width of the radio beam, and gave results for a larger sample of 151 pulsars (R93). However, there is an uncertainty in Rankin's method which relies on the core component of the radio beam, since, it is not certain that all those 151 pulsars in R93 actually contain the core component. In Sect. 3.1, we will use both the samples of LM88 and R93. In Sect. 3.2, we only use LM88, which we believe it to be less ambiguous. Also we will assume that the internal field is $B \sim 10^{12}$ gauss for this section and that the dipolar field B_s can differ from the internal field because of the observational evidence (Chanmugam 1992; Phinney & Kulkarni 1994).

3.1. Time evolution of pulsar inclination angle

The early statistical studies of observed data clearly support the decay of the inclination angle with time scales $10^6 \sim 10^7$ yrs (Proszyski 1979; Candy & Blair 1986). According to Eq. (18), the magnetic field strength strongly affects the evolution of this inclination angle. In order to inspect the theoretical prediction, we plot the inclination evolutionary curves in Figs. 2a and 2b, which show the distribution of inclination and apparent age, based on the data of LM88 and R93 respectively. The solid (empty) circles are pulsars with magnetic field stronger (weaker) than 10^{12} Gauss. It is clear that the solid circles are distributed rather randomly but the empty circles tend to concentrate at lower values of inclination angle. Table 1 shows that the mean inclination angle for stronger field pulsars ($B_{12} > 1$) is about 45° for LM88 and about 40° for R93, respectively, but the mean inclination of weaker field pulsars ($B_{12} < 1$) is about 26° for LM88 and about 35° for R93 respectively.

It is generally believed that the pulsars at birth possess a distribution over the entire 90° for the value of inclination angle $\alpha(0)$, rather than all being perpendicular to the spin axis. It has been suggested (Gil & Han 1996) that the magnetic axis is randomly oriented with respect to the spin axis according to

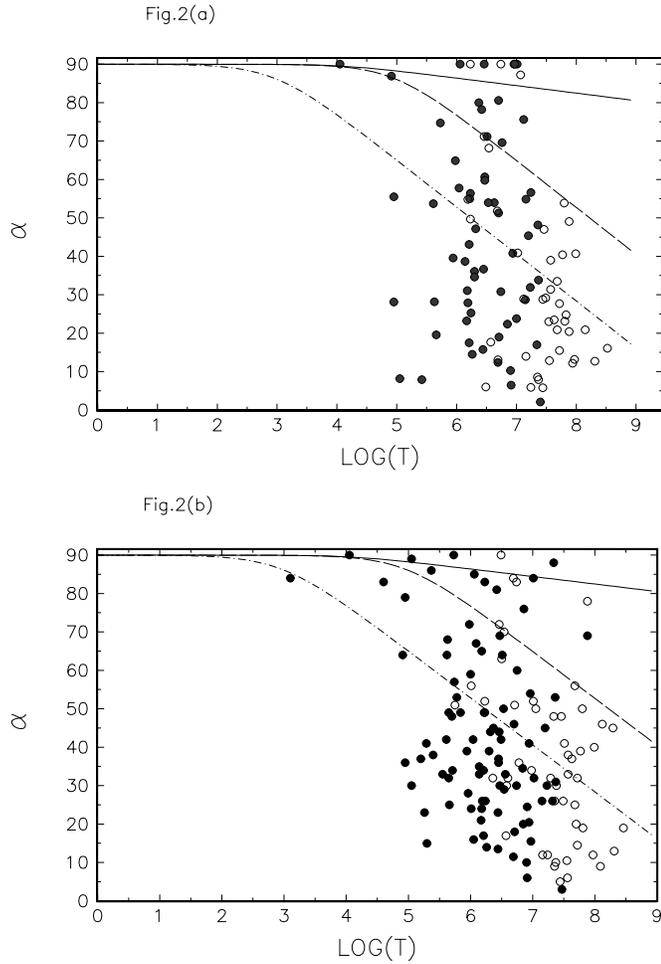


Fig. 2a and b. Magnetic inclination vs. age diagrams, **a** samples from LM88, **b** samples from R93. The solid circles are for $B_{12} > 1$ and the open circles $B_{12} < 1$, respectively. The solid curve, the dashed curve and the dot-dashed curve are the theoretical curves with $B_s = 10^{12}$ Gauss and $P_o=10\text{ms}$, $B_s = 0.4 \times 10^{12}$ Gauss and $P_o=10\text{ms}$, and $B_s = 0.4 \times 10^{12}$ Gauss and $P_o=1\text{ms}$, respectively.

a constant probability density function $f(\alpha(0))=2/\pi$. It means that the number of pulsars with inclination angle values larger and smaller than the average values of 45° should be equal, if the inclination angle does not decay. We compare our theoretical prediction (Eq. 18) with the observed data in Fig. 2a and Fig. 2b. The solid curve, the dashed curve and the dot-dashed curve are the theoretical curves corresponding to $B_s = 10^{12}$ Gauss with $P_o=10\text{ms}$, $B_s = 0.4 \times 10^{12}$ Gauss with $P_o=10\text{ms}$ and $B_s = 0.4 \times 10^{12}$ Gauss with $P_o=1\text{ms}$, respectively. The theoretical curves indicate that the decay of the inclination angle of weak field pulsars is much faster than that of strong field. Some weak field pulsars occur above the long dashed curve; these can result from causes of longer initial periods, which are also expected to be randomly distributed.

Table 2. The pulsars sample with large values of $\alpha(0)$ from LM88 & XW91

PSR	Logt	LogB	$\alpha(t)$	P
0149-16	7.01	12.02	90.0	0.833
0523+11	7.88	11.21	49.1	0.354
0833-45	4.05	12.53	90.0	0.089
1508+55	6.37	12.29	80.0	0.740
1727-47	4.91	13.07	86.9	0.830
1747-46	6.96	12.00	90.0	0.742
1811+40	6.76	12.19	69.6	0.931
1900+01	6.46	12.24	90.0	0.729
2003-08	8.31	11.60	12.7	0.581
2016+28	7.77	11.47	40.4	0.588
2306+55	7.57	11.50	39.0	0.475
2310+42	7.68	11.31	33.5	0.349
2044+15	7.99	11.67	40.7	1.138
0834+06	6.47	12.47	60.7	1.274
1237+25	7.36	12.07	48.2	1.382
1700-32	7.46	11.97	47.0	1.212
1839+56	7.19	12.23	56.6	1.653
1845-19	6.47	13.01	59.8	4.308
1905+39	7.57	11.91	31.4	1.236
1913+167	7.80	11.91	53.9	1.616
1916+14	4.95	13.20	55.5	1.181
1917+00	6.42	12.50	78.2	1.272
1919+21	7.20	12.13	45.4	1.337
1942-00	7.49	11.88	29.2	1.046
2321-61	7.16	12.40	54.9	2.347

3.2. Effect of magnetic field strength on the observational distribution of the inclination angle

It is important to know the initial value $\alpha(0)$ of the magnetic inclination in order to check our theoretical relation between inclination and field strength. A recent analysis of the evolution of the magnetic inclination angle based on the model of Candy & Blair(1986) and the polarization data of LM88 was done by Xu & Wu (1991) and the initial values $\alpha(0)$ of pulsar's inclination were obtained. From this work, we choose those pulsars with $\alpha(0) > 64^\circ$ as our statistical sample in order to make the evolution analysis more clear. We have also ignored one pulsar with a low field, $\text{LOG}(B)=10.52$, which is suspected to be recycled pulsar. The parameters of these pulsars are listed in Table 2, where the values of magnetic inclination are taken from LM88.

Fig. 3 shows the dependence of the observed magnetic inclination angle on the magnetic field for the sample listed in Table 2. The open circles represent pulsars with period shorter than 1 second, and the solid circles represent the pulsars with the period longer than 1 second. The solid curve and the dashed curve are theoretical curves taking $\alpha(0) = 90^\circ$ with $\frac{P}{P_o} = 3$ and $\alpha(0) = 60^\circ$ with $\frac{P}{P_o} = 300$, respectively. The observational distribution shows two important features which support our theoretical model. The first one is the inverse correlation between magnetic inclination angle and the field. The second one is that the period is a determining factor influencing evolution of α for pulsars with very low magnetic field. These two results are con-

Fig.3

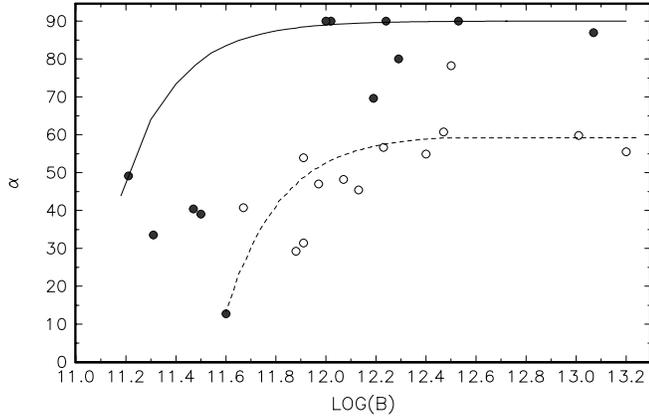


Fig. 3. The magnetic inclination angle vs. field strength diagram for pulsars with $\alpha(0) > 64^\circ$. The solid circles are pulsars with period less than 1 second and the open circles pulsars with period longer than 1 second. The solid curve and the dashed curve are theoretical curves with $\alpha(0) = 90^\circ$ and $\frac{P}{P_0} = 3$ and $\alpha(0) = 60^\circ$ and $\frac{P}{P_0} = 300$, respectively.

Table 3. The average values of α and $\text{Log}(B)$ from LM88

LOG(B)	11.0-11.5	11.5-12.0	12.0-12.5	12.5-13.2
$\langle \alpha \rangle$	23.18	39.64	46.51	41.54
σ	11.70	25.62	24.68	28.17
$\langle B \rangle$	11.38	11.85	12.22	12.75
σ_B	0.154	0.121	0.132	0.238
N	17	25	47	13

sistent with our theoretical prediction given by Eq. (15). In this figure, it is more clearly seen that pulsars with higher magnetic field strength ($B_{12} > 1$) have larger values of inclination angle, and those with the low values of magnetic field ($B_{12} < 1$) have smaller values of inclination angles.

Fig. 4a shows the average of the inclination angle versus the product of the magnetic field and $\sin\alpha$, which is the directly observable quantity, for the samples of LM88 when they are divided into four groups according to the field strength. Table 3 shows the number of pulsars, the average values of the magnetic field, the angle α , and their corresponding standard deviations for each group. For the lowest field group, the average value of α is significantly different from 45° when the error bar is included. This fact indicates that the α values of pulsars with low magnetic field ($\text{Log } B = 11.0-11.5$) are clearly decreased. The solid curve, the dashed curve and the dotted curve are theoretical curves with $\frac{P}{P_0} = 10, 50$ and 150 , respectively. Fig. 4b is similar to Fig. 4a except $B\sin\alpha$ is replaced by B corresponding to an average $\alpha = 45^\circ$. The solid curve, the dashed curve and the dot-dashed curve represent $\frac{P}{P_0} = 2, 5$ and 10 respectively. Again, the decay of the inclination of weak field pulsars seems clear and is consistent with the theoretical prediction.

Fig.4(a)

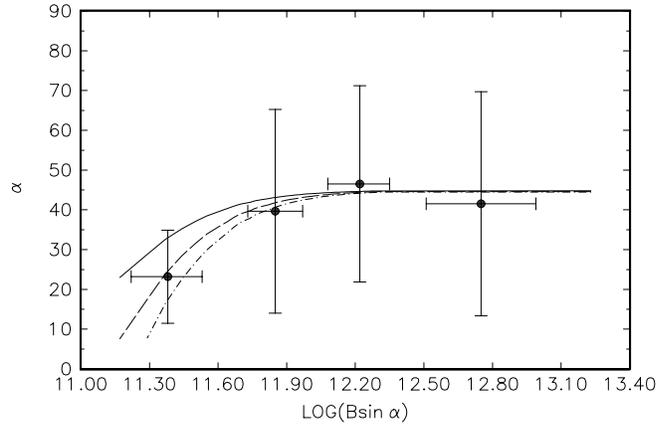


Fig.4(b)

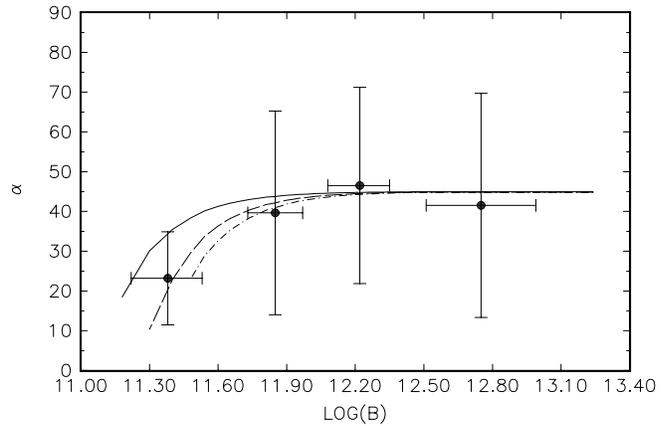


Fig. 4. a The average inclination vs. the observed variable $B\sin\alpha$. The solid curve, dashed curve and dotted curve are theoretical curves with $\frac{P}{P_0} = 10, 50$ and 150 , respectively. **b** Similar to Fig. 4a except $B\sin\alpha$ replaced by B and taken $\sin\alpha = \frac{1}{\sqrt{2}}$. The solid curve, dashed curve and dotted curve are theoretical curves with $\frac{P}{P_0} = 2, 5$ and 10 , respectively.

4. Discussion and conclusions

So far we have focused on the evolution of the magnetic inclination of canonical pulsars. It appears that our model cannot explain millisecond pulsars which are generally believed to be spun up by accretion, (so they are called recycled pulsars,) because these millisecond pulsars have typical magnetic field around 10^8 to 10^9 gauss, characteristic ages of at least 10^8 years, and some of these pulsars possess large inclination angle extending even to 90° . First, we want to point out that it is very misleading to use the spin-down age ($\frac{P}{2\dot{P}}$) to estimate the age of weak field millisecond pulsars, the actual age of those pulsars should be younger than the spin-down age. Second, it has been suggested that the accretion can reduce the surface magnetic field of pulsars (Shibazaki et al 1989; Zhang et al 1994; Zhang 1998; Van den Heuvel & Bitzaraki 1995; Taam & Van den Heuvel 1986). In this case, most of the stellar magnetic field may be just buried inside the crust. In other words, the crustal

Table 4. Recycled pulsar parameters

Pulsar	P(ms)	LOG($\frac{P}{2P}$)	LOG(B)	α
1953+29	6.1	9.52	8.63	90 ^a
1937+214	1.56	8.37	8.61	90 ^b
1913+16	59	8.04	10.36	46 ^c
0820+02	865	8.12	11.48	46 ^c
0655+64	196	9.65	10.07	61 ^d

a=Chen & Ruderman (1993)

b=Lyne & Manchester (LM88)

c=Rankin (1993)

d=Wu et al. (1997)

field B could be larger than the original field by one or two order of magnitudes because it has been squeezed into a smaller volume resulting in a surface field B_s of only 10^8 gauss. Thirdly, the mass of the star is very likely larger than 1.4 solar masses due to accretion which reduces $\frac{\Delta R}{R} \sim 0.03$ (Cheng & Dai 1997). Therefore, we can expand the logarithmic term in Eq. (18) and obtain

$$F[\alpha(t)] \approx F[\alpha(0)] - 1.2 \times 10^{-6} B_{12}^{-1} \Omega_{03}^2 t(\text{yr}), \quad (19)$$

We can see that the initial angular velocity plays a very important role in determining if the inclination will decay. Table 4 lists those millisecond pulsars with known inclination angle. It is interesting to point out that even if the crustal magnetic field is actually 10^{14} gauss, our model cannot explain the inclination angles of PSR1953+29 and PSR1937+214 unless their ages are less than 10^8 years. Perhaps the magnetic field of these two pulsars originates from quantized flux tubes in the superconducting core and the interaction between the flux and the superfluid vortex lines can force the magnetic fluxoids to always be perpendicular to the rotation axis (Ruderman 1991).

Finally, we conclude that our model predictions can explain the evolution of the magnetic inclination of most pulsars including those accretion spun up pulsars, but can only explain those pulsars with periods less than a few millisecond if their ages are actually less than 10^8 years old.

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