

On the determination of carbon isotopic ratios in cool carbon stars

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Abstract. We discuss the determination of carbon isotopic ratios $^{12}\text{C} / ^{13}\text{C}$ in N-type Galactic carbon stars. Ohnaka & Tsuji (1996) reported $^{12}\text{C} / ^{13}\text{C}$ ratios smaller by a factor 2 or 3 than the determinations of Lambert et al. (1986). Using synthetic spectra of carbon stars, we analyse systematic errors in their *iso-intensity* method. It is found to be rather sensitive to model parameters and blends. Furthermore, there are large discrepancies between the effective temperatures adopted by these two studies (mean discrepancy close to 260 K for the 20 stars in common). That, together with uncertainties due to model atmospheres, to carbon enrichments and effects of the blends, could explain most of the discrepancies between these two studies. The LGEH86 analysis is, on the other hand, rather insensitive to model parameters. We therefore conclude that large $^{12}\text{C} / ^{13}\text{C}$ ratios in cool Galactic carbon stars should be favoured.

Key words: stars: abundances – stars: AGB and post-AGB – stars: carbon – molecular processes

1. Introduction

Carbon stars play an important role in the chemical evolution of galaxies because of their production of heavy elements and their large mass loss rates, but quantitative estimates of their contribution to the galactic enrichment are still far from known (see Gustafsson & Ryde, 1996, for a review, and Prantzos et al., 1996). Another important problem not completely solved today is the way and rate of formation of these stars. Resolutions of these problems could be obtained by determining the chemical compositions of these carbon stars. Of considerable interest in this respect are the abundances of ^{13}C . An important neutron source in Asymptotic Giant Stars (AGB) is the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction. High ^{13}C abundances could enable a large production of *s*-elements and high $^{12}\text{C} / ^{13}\text{C}$ ratios indicate that carbon stars are mainly the results of helium burning producing ^{12}C . Determinations of carbon isotopic ratios thus contain important information about mixing and nucleosynthesis in C-stars interior.

When the star ascends the red giant branch the first dredge-up brings the products of the H burning to the surface, leading to a decrease of the $^{12}\text{C} / ^{13}\text{C}$ ratio from the solar value of ~ 90 to post dredge-up values around 20 (see Charbonnel, 1994). The second dredge-up occurring only in the more massive stars ($M \gtrsim 4 M_{\odot}$) seems to have little effect on the carbon isotopic ratio. However, when the star experiences thermal pulses on the AGB, H burning phases between these instabilities convert some ^{12}C into ^{13}C and mixing events occurring during the thermal pulses bring ^{12}C to the surface. The star may evolve from oxygen-rich to carbon-rich and its $^{12}\text{C} / ^{13}\text{C}$ ratio at the surface increases (see Forestini & Charbonnel, 1997, Marigo et al., 1997 and Busso et al., 1995). This scheme is observationally confirmed. Smith & Lambert (1985, 1990) report values around 10-20 for M thermally pulsing-AGB stars and from 5 up to 50 for MS and S-type stars. Observations of SC stars (Dominy et al., 1986 and Dominy & Wallerstein, 1987) are also consistent. Moreover, the chemical composition of 30 Galactic N-type carbon stars were determined by Lambert et al. (1986, LGEH86 hereafter). They found that the stars show relatively modest carbon enrichments, while nitrogen and oxygen abundances are slightly subsolar. Low ^{13}C abundances were also obtained. These abundances were determined from high-resolution FTS spectra in the $1.6 \mu\text{m}$ and $2.3 \mu\text{m}$ regions. Several different lines of $^{12}\text{C}_2$ and $^{12}\text{C}^{13}\text{C}$, of ^{12}CO and ^{13}CO $\Delta v = 2$ and $\Delta v = 3$ bands and of ^{12}CN and ^{13}CN red system were used for the $^{12}\text{C} / ^{13}\text{C}$ determination and gave consistent results. The resulting high carbon isotopic ratios ($30 < ^{12}\text{C} / ^{13}\text{C} < 100$ with a pronounced peak around 50) are consistent with the addition of pure ^{12}C , produced by helium burning, to the envelope of a typical oxygen-rich star, ^{13}C being left from the first dredge-up. Kahane et al. (1992) also found $^{12}\text{C} / ^{13}\text{C}$ ratios larger than 30 in five C-rich circumstellar envelopes of high mass loss AGB stars, representing later stages of stellar evolution, and Kahane et al. (1988) found $^{12}\text{C} / ^{13}\text{C} = 47$ in the envelope of the dust enshrouded carbon star IRC+10216. Knapp & Chang (1985) report values in the range 5-20 for oxygen-rich Miras and 30-100 for carbon stars from millimeter CO observations. Bakker et al. (1997) also found $^{12}\text{C} / ^{13}\text{C} > 20$ in sixteen post-AGBs. As for the planetary nebulae towards which these stars evolve Bachiller et al. (1997) found carbon isotopic ratios within the large range found in AGB envelopes. A certain agreement be-

tween observation and theory of carbon isotope abundances in AGB stars therefore exists.

However, a significant lower range of $^{12}\text{C} / ^{13}\text{C}$ ratios (average around 27) in 62 Galactic N-type carbon stars has recently been obtained by Ohnaka & Tsuji (1996, OT96 hereafter). This unexpected result is interpreted by assuming that the oxygen abundance is not modified by the third dredge-up and that the carbon abundance is doubled or tripled by the addition of fresh ^{12}C to the envelope of an oxygen-rich star.

In the present paper, we discuss this recent determination by computing synthetic spectra of cool carbon stars (Sect. 2). We then estimate the accuracy of the method used by OT96 to derive the $^{12}\text{C} / ^{13}\text{C}$ ratios (Sect. 3) and explore the effects of uncertainties in the structure of model atmospheres and in the determination of effective temperatures of these stars in Sect. 4. The precision of the work of LGEH86 is also discussed in this section. Final conclusions are drawn in Sect. 5.

2. Calculation of synthetic spectra

OT96 collected spectra of C-rich stars around 8 000 Å and they mainly analyzed the ranges 7 810-7 840 Å and 8 000-8 030 Å. In order to discuss their results we first generated synthetic spectra in the same spectral regions by using the set of programs written at Uppsala Astronomical Observatory.

2.1. Model atmospheres

Continuous opacities were derived from model atmospheres of carbon stars. We used the grid of MARCS models described in Eriksson et al. (1984) and LGEH86. These models are based on the usual assumptions of hydrostatic and local thermodynamic equilibria, plane-parallel stratification, line-blanketing, conservation of radiative flux and convective flux in the mixing-length approximation. Opacities from usual continuous sources and lines of the carbon molecules C_2 , CN, CO, HCN and C_2H_2 , represented by opacity distribution functions, are considered (see Gustafsson et al., 1975, and Eriksson et al., 1984). Effects of spherical extension were found to be unimportant (LGEH86 and Jørgensen et al., 1992). All the models were calculated for $^{12}\text{C} / ^{13}\text{C} = 10$ and for a microturbulence parameter $\xi_{\text{micro}} = 2 \text{ km.s}^{-1}$. LGEH86 have shown that minor changes in the model atmosphere and thence in the derived abundances occur if $^{12}\text{C} / ^{13}\text{C}$ varies from 3 to 99 (see their Fig. 10f). This was already stressed by Fujita & Tsuji (1977). Following the chemical analysis of cool carbon stars by LGEH86, we assumed solar chemical composition for all the elements except carbon and C/O ratios larger than unity.

2.2. Line list

We have collected a list containing atomic lines and C_2 and CN molecular lines found around 8 000 Å. Partition functions of atoms and molecules were derived from the relations given by Irwin (1981) and Sauval & Tatum (1984), respectively. Isotopic

effects on the partition functions of the molecules were also estimated from the relation given in Sauval & Tatum (1984).

2.2.1. Atomic data

Atomic lines of spectroscopic importance in the spectral regions studied have been searched using the "Vienna Atomic Line Data Base" (VALD, see Piskunov et al., 1995). All the lines were selected for a typical N-type model atmosphere and have a depth larger than 1%. Variations of the model parameters do not affect the line list contents. Most of these ~ 250 lines are transitions of Ti I, V I, Cr I and Fe I. VALD provided the gf -values and the excitation energies of the lower levels of these lines. The broadening by radiation damping was calculated as in Edvardsson et al. (1993), when not explicitly given by VALD. Finally, the classical van der Waals damping constant of the atomic lines was modified following Edvardsson et al. (1993). Some gf -values and/or radiation damping constants (actually for less than 25 lines) were, however, corrected after comparison with the observed solar spectrum (see Sec. 2.3).

2.2.2. Molecular data

a) The C_2 molecule: Several lines from different bands of the C_2 Phillips red system are found in the wavelength ranges studied. We used the molecular constants derived by Chauville et al. (1977) to predict all the lines of $^{12}\text{C}_2$. Calculated line positions were then checked with those observed by Ballik & Ramsay (1963). We thus included in our line list all the lines of the (3-0), (4-0), (4-1), (5-1), (6-2) and (7-3) bands. Bands originating from higher vibrational levels were not included since no observed wavelengths have been found for their lines (or no band oscillator strengths are available, see below). However, most of them lying in the spectral range studied have a excitation energy due to very high value of the rotational quantum number J , and should therefore be very weak in cool star spectra.

Line positions of the $^{12}\text{C}^{13}\text{C}$ and $^{13}\text{C}_2$ bands were also predicted by using isotopic substitution relations to calculate the molecular parameters. Since no observed line positions have been identified for the Phillips red system of these two molecules in this spectral region, we assumed that the errors made in calculating these wavelengths are the same as the ones made for the $^{12}\text{C}_2$ corresponding lines. We therefore corrected them by the same factor. We have checked this procedure with the observed line positions of the Ballik-Ramsay system of $^{12}\text{C}^{13}\text{C}$ and $^{13}\text{C}_2$ (Amiot & Verges, 1983). Altogether we considered 126 lines of C_2 and its isotopically substituted molecules.

The gf -values for all the individual lines are calculated by

$$gf = f_{v'v''} \frac{S_{J'J''} \lambda_{v'v''}}{2 \lambda_{J'J''}} \quad (1)$$

where we ignore the small rotational dependence of the Franck-Condon factor (Dwivedi et al., 1978). $S_{J'J''}$ is the Hönl-London factor, $\lambda_{J'J''}$ and $\lambda_{v'v''}$ are the wavelengths of the individual line and of the band origin, respectively, and $f_{v'v''}$ is the band

oscillator strength. The oscillator strengths of the Phillips system are still under discussion. Erman et al. (1982) measured radiative lifetimes of vibrational levels of $^{12}\text{C}_2$ and deduced the $f_{v',v''}$ -values of all the bands we study. More recently Lambert et al. (1995) have derived the value of $f_{2,0}$ from interstellar $^{12}\text{C}_2$ lines. They showed that their result is in good agreement with modern *ab initio* calculations. We used their value to normalize the $f_{v',v''}$ determined by Erman et al. (1982) with a constant electronic transition moment R_e^2 (in better agreement with solar $f_{v',v''}$ values). We then adopted the same band oscillator strengths for the $^{12}\text{C}^{13}\text{C}$ and $^{13}\text{C}_2$ molecules as the corresponding ones of $^{12}\text{C}_2$.

As for the dissociation energy D_0 of the $^{12}\text{C}_2$ molecule Bauschlicher & Langhoff (1987) have shown that theoretical determinations are in excellent agreement with the experimental value reported by Huber & Herzberg (1979). When determining the solar carbon abundance Grevesse et al. (1991) also agreed on the same value. We adopted this determination of $D_0(^{12}\text{C}_2) = 6.21$ eV in our analysis and assumed the same dissociation energy for $^{12}\text{C}^{13}\text{C}$ and $^{13}\text{C}_2$.

b) The CN molecule: An impressive amount of lines of the red system of the CN molecule are found around 8 000 Å, most of them belonging to $\Delta v = 2$ and $\Delta v = 3$ bands. Davis & Phillips (1963) analyzed most of these lines of the ^{12}CN molecule and provided accurate position for them. However, the red system of ^{13}CN was rather poorly investigated. Very few bands have been analyzed by Wyller (1960) and Hosinsky et al. (1976 and 1982) and only few lines of the (2,0) band lie in the spectral ranges synthesized in this work. Brault & Engleman (private communication) have recorded the different isotopic species of the red CN bands from a source consisting in a sealed quartz tube excited by a microwave generator. The tube contained a mixture of argon, nitrogen and methane. They used the McMath Solar Fourier transform spectrometer at Kitt Peak National Observatory. Their measurements provide line-positions accurate to about 0.01 Å for several lines used in this paper. Kotlar et al. (1980) performed a detailed analysis on the red system of ^{12}CN by combining all the existing experimental data. More recently, Jørgensen & Larsson (1990) presented a tape (SCAN-CN) with identifications, gf -values and lower excitation energies for a great number of ^{12}CN and ^{13}CN lines. However, these authors gave priority to a large number of lines rather than accurate line positions.

We first selected all the lines of ^{12}CN and ^{13}CN found in the 7 810-7 840 Å and 8 000-8 030 Å regions from SCAN-CN. The oscillator strengths calculated individually for each isotope were multiplied by a factor 0.734 as suggested by Jørgensen & Larsson (1990). We then generated a complete set of line positions of ^{12}CN with a program based on the work of Kotlar et al. (1980). Most of the line positions predicted from their set of molecular constants are quite accurate (specially for low rotational quantum numbers) and we then used them instead of the positions found in SCAN-CN. We next made some minor corrections of the Kotlar et al.'s positions using the observed wavelengths tabulated in Davis & Phillips (1963). Only lines of bands with large vibrational quantum number ($v' \geq 7$) were cor-

rected by more than 0.1 Å. Some smaller corrections were also made, using the observations of Brault & Engleman. As for the ^{13}CN line positions found in SCAN-CN only some lines of the (2-0) band have been observed by Wyller (1966). We corrected all the others by assuming that the error made by SCAN-CN for a given ^{13}CN line is proportional to the disagreement between SCAN-CN and Kotlar et al.'s positions for the corresponding ^{12}CN line. This procedure seemed efficient since the standard deviation with respect to the 34 (2,0) lines observed by Wyller was reduced by a factor ~ 3 , to less than 0.023 Å. Larger deviations should, however, be suspected for bands with higher vibrational quantum numbers since no observed positions exist for them and the Kotlar & Field predictions are much less accurate. This was partially improved with the Brault & Engleman's line positions. Finally, 316 lines of the (2-0), (4-1), (5-2), (6-3), (7-4), (8-5) and (9-6) bands of the ^{12}CN molecule and 230 lines of ^{13}CN from the same bands were included in our line list.

The dissociation energy of the CN molecule is still under discussion (see e.g. Lambert, 1993, and Costes & Naulin, 1993). The most recent experimental values are close to 7.75 eV (Huang et al., 1992, and Costes et al., 1990) but *ab initio* calculations suggest lower values around 7.65 eV (Bauschlicher et al., 1988). However, Costes et al. (1990) pointed out that their experiment would lead to a lower limit of $D_0(\text{CN})$ when possible errors are taken into account. Therefore, theoretical determinations seem too small. Following the discussion of Lambert (1993) we thus adopted $D_0 = 7.75$ eV and assumed the same dissociation energy for the ^{12}CN and ^{13}CN molecules.

2.3. Comparison with observations

2.3.1. The Sun

The consistency of our line list has been checked by comparing solar observations (Delbouille et al., 1973) and our synthetic spectrum of the Sun. For this purpose we used the Holweger & Müller (1974) semi-empirical solar model. We adopted solar chemical abundances from Grevesse et al. (1996) and a micro-turbulence parameter $\xi_{\text{micro}} = 0.85 \text{ km.s}^{-1}$ (Blackwell et al., 1984) together with a carbon isotopic ratio equal to 90. The agreement between the observed and synthetic solar spectra was found very satisfactory.

2.3.2. Cool carbon stars

We also checked our calculations by fitting the spectrum of a cool carbon star where much more molecular lines are detected. For this purpose, K. Ohnaka kindly provided us with the observed spectrum of the N-type carbon star ST Cas (see OT96, Fig. 7). Our fit of the spectral region where the OT96 ^{13}CN indicators (see Sect. 3) are found is shown in Fig. 1. We used a model atmosphere defined by the parameters proposed by OT96 for this star: $T_{\text{eff}} = 3\,306$ K, $\log g = 0.0$, solar chemical abundances except $\text{C/O} = 1.1$. Spectra with $^{12}\text{C} / ^{13}\text{C}$ of 9, 18 and 36 were synthesized while OT96 derived $^{12}\text{C} / ^{13}\text{C} = 18$ from their analysis. To match the resolution of the observations the synthetic spectra have been convolved with an instrumental

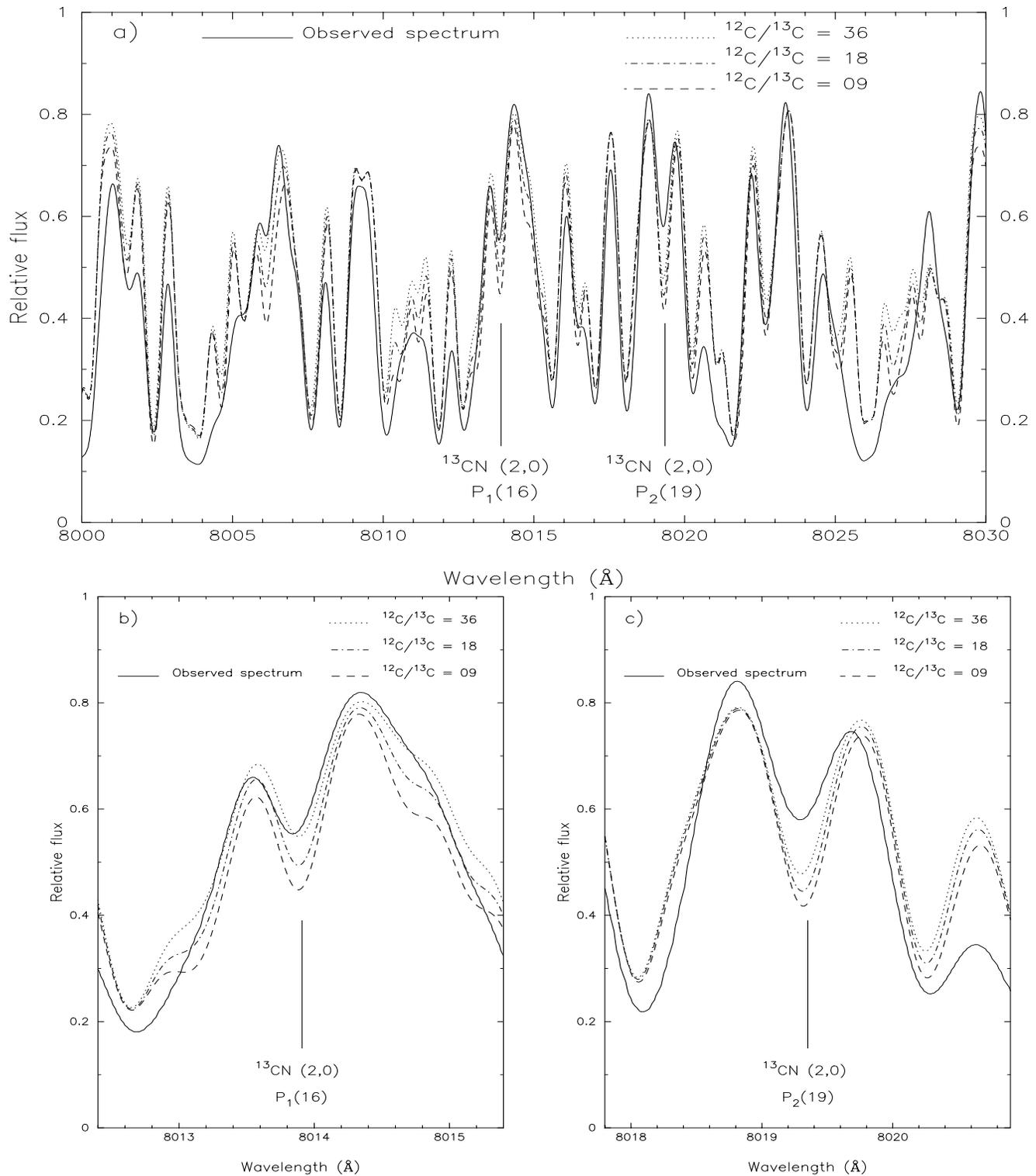


Fig. 1a–c. Observed and synthetic spectra of the N-type carbon star ST Cas ($T_{\text{eff}} = 3306$ K, $\log g = 0.0$, $[\text{Fe}/\text{H}] = 0.0$, $\text{C}/\text{O} = 1.1$ and $\xi_{\text{micro}} = 2 \text{ km.s}^{-1}$). Three synthetic spectra with different carbon isotopic ratios are plotted. The two ^{13}CN indicators used by OT96 in their analysis are also pointed out. The fit of these two lines (bottom panel) seems better for higher $^{12}\text{C}/^{13}\text{C}$ ratio (given the uncertainty of the continuum level in the observed spectrum) in contrary to the value ($^{12}\text{C}/^{13}\text{C} = 18$) proposed by OT96 for this star.

broadening function assumed to be a Gaussian profile with a FWHM of 0.4 Å and with a stellar rotation of 1 km.s⁻¹. The shape of the instrumental profile was actually not described by OT96 (they only give an estimate of the FWHM) but a Gaussian profile gives a reasonable agreement. As shown in Fig. 1, the synthetic spectra agree rather well with the observed one for whatever value of ¹²C / ¹³C. The other spectral region (7 810-7 840 Å) is as well fitted but not shown since the depths of the ¹²CN lines selected by OT96 (see Sect. 3) are almost insensitive to the ¹²C / ¹³C ratios. The two ¹³CN lines used by OT96 in their analysis are best fitted by larger values of ¹²C / ¹³C than by the one derived by OT96. However, there are certainly some uncertainties in our molecular data and these lead to uncertainties in the ¹²C / ¹³C ratios derived with this spectral synthesis method. No modifications of the *gf*-values of the CN lines within their errors were found to improve the fit. We conclude that accurate determinations of the ¹²C / ¹³C ratio by synthetic spectrum analysis can not be easily made in this spectral interval with the present low spectral resolution ($\lambda/\Delta\lambda \sim 20\,000$) of the observations. We have checked that higher resolution spectra would provide less blended lines and line profiles could then be compared with synthetic spectra. Derivation of carbon isotopic ratios from synthetic spectra should then be more accurate.

3. ¹²C / ¹³C ratios from the iso-intensity method

OT96 determined the carbon isotopic ratios in their sample of 62 carbon stars from the *iso-intensity* method and line-blanketed model atmospheres. The *iso-intensity* method uses the central depths of the lines instead of their equivalent widths; since lines are very crowded in the observed spectra equivalent widths can actually not be measured accurately. They isolated the less blended lines of the CN molecule in their analysis and finally selected rather few lines. From their Fig. 11 we can see that the ¹²C / ¹³C ratio is determined from 2 or 3 lines of the ¹²CN (7,4) band (2 of them, Q₁(35) and P₁(29), being blended by the weak lines ¹²CN (8,5) R₂(9) and R₂(10), respectively) and 2 lines of ¹³CN (2,0) only. They plotted the calculated strengths of these lines versus their observed central depths and then derived the carbon isotopic ratio by determining the horizontal shift between the curves of intensity growth formed by the ¹²CN and ¹³CN lines.

The selected lines of ¹²CN and ¹³CN are separated by ~ 200 Å and OT96 had to assume that the level of the continuum flux is constant over the studied region to derive the line-depths. We checked with our models that this assumption is reasonable and found the continuum levels at 7 800 Å and 8 000 Å to agree within about 0.1%.

In order to check the accuracy of this *iso-intensity* method we computed synthetic spectra of cool carbon stars, derived the curves of growth of central depth intensity (instead of equivalent widths) and studied their relative position for different parameters of the models. OT96 derived a mean value ¹²C / ¹³C = 27 for 62 N-type stars. We started from a synthetic spectrum characterized by $T_{\text{eff}} = 3\,000$ K, $\log(g) = 0$, $[\text{Fe}/\text{H}] = 0$, $\text{C}/\text{O} = 1.1$, $\xi_{\text{micro}} = 2$ km.s⁻¹ and ¹²C / ¹³C = 30. The curves of intensity

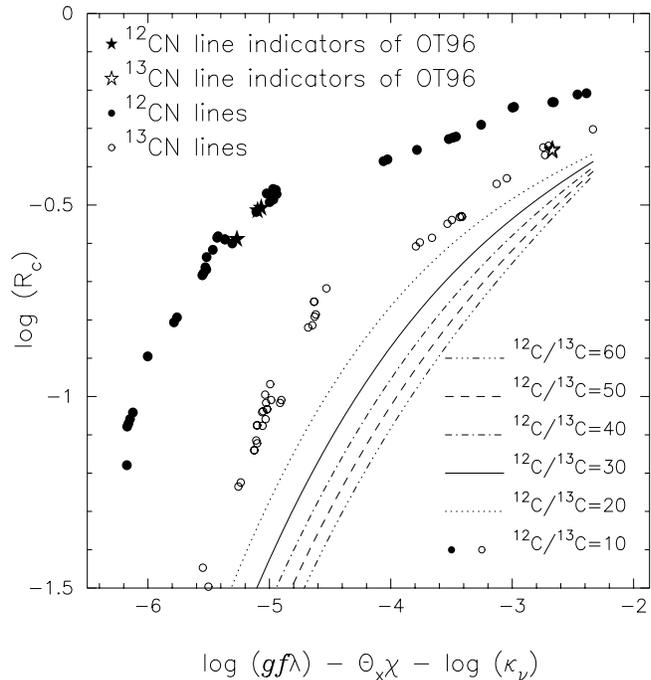


Fig. 2. Calculated curves of intensity growth from the synthetic spectra. The abscissae correspond to the strength of the lines (Θ_X and κ_ν are evaluated at $\tau_{\text{line center}} = 0.1$ for each individual lines). The ordinates are logarithm of the central depth of the lines normalized by the continuum. Model parameters are $T_{\text{eff}} = 3\,000$ K, $\log(g) = 0$, $[\text{Fe}/\text{H}] = 0$, $\text{C}/\text{O} = 1.1$, $\xi_{\text{micro}} = 2$ km.s⁻¹. The filled and open circles represent the ¹²CN lines and ¹³CN lines respectively for ¹²C / ¹³C = 10. The five CN lines used by OT96 are represented by stars (the two ¹³CN lines are superposed because blends are neglected). ¹³CN curves of intensity growth for ¹²C / ¹³C = 20 to 60 are also shown. These curves are fits to ¹³CN line depths calculated individually, as for ¹²C / ¹³C = 10. The solid line represents the reference value: ¹²C / ¹³C = 30. Changing ¹²C / ¹³C from 10 to 60 has no effect on the ¹²CN curves of growth and is therefore not shown for clarity.

growth were well represented by more than 60 calculated lines of different bands for each species and the horizontal shifts (proportional to ¹²C / ¹³C) at the depth of the two ¹³CN indicators of OT96 were easily derived. We show in Fig. 2 such theoretical curves of growth for carbon isotopic ratios varying from 10 to 60. The scattering of the points defining a given curve of intensity growth is actually rather small. However, the curves of intensity growth are constructed from the central depths of isolated lines (i.e. non-blended lines, all the others being neglected). The width of the instrumental broadening function of the spectrometer used by OT96 is indeed so large (FWHM = 0.4 Å) that blends change the line depths. It seems very difficult to select enough unblended lines in a real spectrum to reconstruct well defined curves of intensity growth. Furthermore, most blends are due to lines with different excitation temperature. This certainly leads to large uncertainties in the study of OT96; even their five selected lines are blended. We present in Table 1 the central depth of these lines measured in a real spectrum con-

Table 1. Central depths ($\log R_c$) of the five lines used by OT96, measured in (i) a real full synthetic spectrum and (ii) assuming that the lines are isolated. Parameters of the model are: $T_{\text{eff}} = 3000$ K, $\log(g) = 0.0$, $C/O = 1.1$, $[Fe/H] = 0$, $^{12}C / ^{13}C = 30$, $\xi_{\text{micro}} = 2 \text{ km.s}^{-1}$ and Gaussian instrumental profile with $FWHM = 0.4$.

	^{12}CN (7,4) lines			^{13}CN (2,0) lines	
	$Q_1(34)$	$Q_1(35)$	$P_1(29)$	$P_1(16)$	$P_2(19)$
Full spectrum	-0.43	-0.27	-0.41	-0.39	-0.32
Isolated lines	-0.50	-0.51	-0.58	-0.45	-0.44

taining all the blends and compare it to their central depth when all the possible blends are removed. The lines then appear to have similar central depths or not, depending on the blends. Thus, in our synthetic spectra, the two ^{13}CN indicators have the same central depths when blends are neglected while there is a spread in their calculated depth because of the blends. The curve of intensity growth of ^{13}CN is therefore less well defined from these two lines, leading to an uncertainty in the derived $^{12}C / ^{13}C$ ratio. These blends are due to lines from bands with higher excitation energy. The *iso-intensity* method is therefore sensitive to the temperature distribution in the atmosphere and its accuracy, when used for crowded spectra, can be questioned since finding real *iso-intensity* lines is not an easy task. If models are like stars, we estimate that the uncertainty in the distance between the ^{12}CN and ^{13}CN curves of growth due to blends leads to an error of $^{12}C / ^{13}C$ of about 0.15 dex.

We now discuss the effects of stellar parameters errors on the results of the *iso-intensity* method, neglecting the effects of the blends. We compare the distance variations between the curves of intensity growth of ^{12}CN and ^{13}CN (from which OT96 derived their $^{12}C / ^{13}C$ ratios) due to uncertainties of the model parameters with those induced by (i) an increase of $^{12}C / ^{13}C$ from 30 to 60 and (ii) a decrease of $^{12}C / ^{13}C$ from 30 to 10. For such $^{12}C / ^{13}C$ ratios and a given set of model atmosphere parameters, the depth of the ^{12}CN lines stays almost constant, and the distance between the two curves of intensity growth results only from a variation of the line depth of the more or less abundant ^{13}CN molecules. We note that the shift of the ^{13}CN curve is significantly greater when $^{12}C / ^{13}C$ increases from 10 to 30 than when it increases from 30 to 60 (see Fig. 2).

The effective temperatures are usually determined from the colours of the stars or by the Infrared Flux Method. Both of these methods are uncertain and LGEH86 and OT96 agree that errors as large as ~ 200 K can occur for cool stars (OT96, however, suggest for some stars uncertainties close to 150 K for $T_{\text{eff}} = 3000$ K determined with the Infrared Flux Method). For very few stars angular diameters and bolometric fluxes are available, possibly leading to more accurate T_{eff} determinations. However, large discrepancies in T_{eff} are found between the values adopted by LGEH86 and OT96 (see Sect. 4.2). A 200 K uncertainty of the effective temperature shifts the ^{12}CN and ^{13}CN curves of intensity growth leading to an error of 0.15 dex

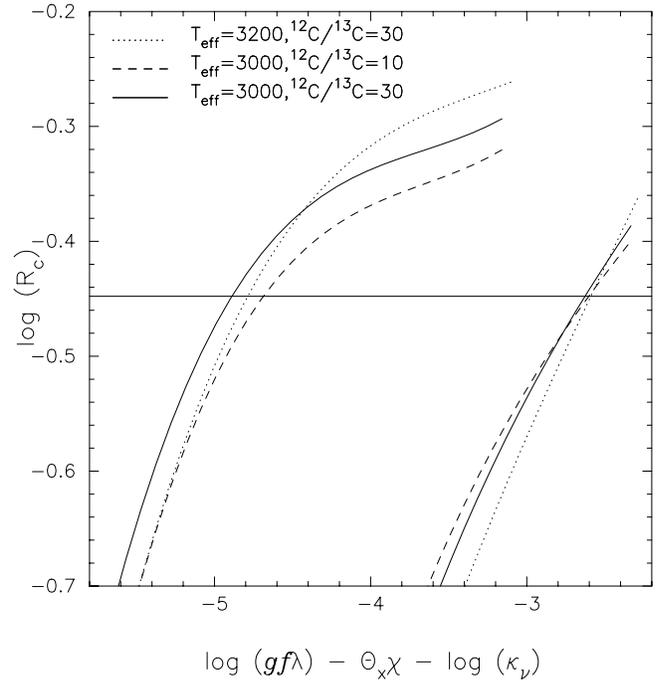


Fig. 3. Effects of uncertainties in the effective temperature when deriving the carbon isotopic ratios. To mimic the procedure of OT96, the depth of their CN lines has been fixed to a given value (as derived from an observed spectrum) when calculating the curves of intensity growths. The reference curve is characterized by $T_{\text{eff}} = 3000$ K, $\log(g) = 0$, $[Fe/H] = 0$, $C/O = 1.1$, $^{12}C / ^{13}C = 30$ and $\xi_{\text{micro}} = 2.0 \text{ km.s}^{-1}$. An uncertainty of $+200$ K in T_{eff} leads to a smaller horizontal shift between the ^{12}CN and ^{13}CN curves of intensity growths at the level of the ^{13}CN lines (horizontal line) and a smaller $^{12}C / ^{13}C$ ratio could be derived if T_{eff} is kept constant at 3000 K. The corresponding error in $^{12}C / ^{13}C$ is 0.15 dex (see also the caption of Fig. 2).

in the $^{12}C / ^{13}C$ ratios derived with the *iso-intensity* method (see Fig. 3). OT96 found a much smaller error (0.02 dex) for the same uncertainty in the effective temperature, but derived an error of 0.10 dex from $\Delta T_{\text{eff}} = 200$ K for the less carbon-rich SC-type stars. This strong disagreement could be due to differences in the calculated opacities due to C-rich molecules. We indeed point out in Sect. 4.2 (see also Fig. 5) that the $^{12}C / ^{13}C$ ratios derived by OT96 and LGEH86 differ much more for the most C-rich N-type stars for which the effects of these molecules are stronger.

OT96 estimated the uncertainty of $\log(g)$ to be 0.5, leading to an error of 0.02 dex. LGEH86 used $\log(g)$ from -1.1 to 0.3 for N-type stars. We found that an increase of $\log(g)$ from -0.3 to 0.5 leads to the larger error of 0.07 dex.

An uncertainty of 1 km.s^{-1} in the microturbulent velocity leads to an error of 0.1 dex as found by OT96.

LGEH86 found C/O ratios in their 30 N-type stars ranging from 1.01 to 1.76. The use of synthetic spectra with $C/O = 1.02$ and 1.35, instead of $C/O = 1.1$, leads to a difference of 0.1 dex

and 0.05 dex, respectively, and close to the values found by OT96.

As for the metallicity, LGEH86 reported approximately solar metal abundances ranging from -0.4 to 0.3. The use of model atmospheres with $[\text{Fe}/\text{H}] = 0.0$ and $[\text{Fe}/\text{H}] = -0.5$ respectively leads to a difference of $^{12}\text{C} / ^{13}\text{C}$ of about 0.18 dex and we estimate the uncertainty due to the range of possible metallicity close to 0.13 dex. This last source of error was neglected by OT96.

Thus, we finally find an upper limit of +0.25 dex in $^{12}\text{C} / ^{13}\text{C}$ ratios derived with the *iso-intensity* method applied to unblended spectra of stars with $^{12}\text{C} / ^{13}\text{C} = 30$ (including the internal errors due to observations, estimated to be about 0.05 dex by OT96). The corresponding lower error is -0.16 dex. Thus, the error range corresponds to $^{12}\text{C} / ^{13}\text{C} = 30_{-10}^{+23}$.

This range is much larger than the one derived by OT96 (their total error is about 0.13 dex, i.e. rather close to our lower limit but much less than the upper one). It is actually as wide as the distribution of the $^{12}\text{C} / ^{13}\text{C}$ ratios derived for their 62 N-type stars. However, our total error estimated for the *iso-intensity* method as applied to N-type stars is very close to their total error for less carbon-rich SC-type stars: 0.28 dex for $^{12}\text{C} / ^{13}\text{C} = 22$ in 15 SC-type stars. Such discrepancies in the error analysis result from several effects:

- OT96 neglected errors due to metallicity uncertainties whereas we find that these are large. We do not see any reason not to take this error into account since the metallicities of the stars have not been derived.
- We also found that the uncertainty of T_{eff} leads to larger errors in the derived $^{12}\text{C} / ^{13}\text{C}$ ratios than claimed by OT96. We propose that this could be explained by some disagreements in the calculation of C-rich molecular opacities.

Finally, we recall that this error analysis does not take into account uncertainties due to the blends of the lines analyzed by OT96. As has already been pointed out, several blends by lines with different excitation energies occur in a real spectrum of a N-type star and the relative position of the curves of intensity growth is not well defined due to these blends.

4. Discussion

4.1. Model atmosphere

In their paper, OT96 proposed that the disagreement between their $^{12}\text{C} / ^{13}\text{C}$ determinations and those of Lambert et al. could be explained by uncertainties in the structure of cool carbon star model atmospheres. They selected for their analysis lines with different lower excitation potential (with a gap around ~ 1.2 eV between the (7,4) ^{12}CN lines and the (2,0) ^{13}CN lines). These lines are therefore formed with different contributions from different depth regions in spite of their similar line depths and the derived $^{12}\text{C} / ^{13}\text{C}$ ratios should be sensitive to the structure of the model atmosphere used. For carbon stars, the realism of a model is not guaranteed and cannot easily be checked.

OT96 always included turbulent pressure in computing models whereas LGEH86 did not. The Fig. 1 of OT96 shows

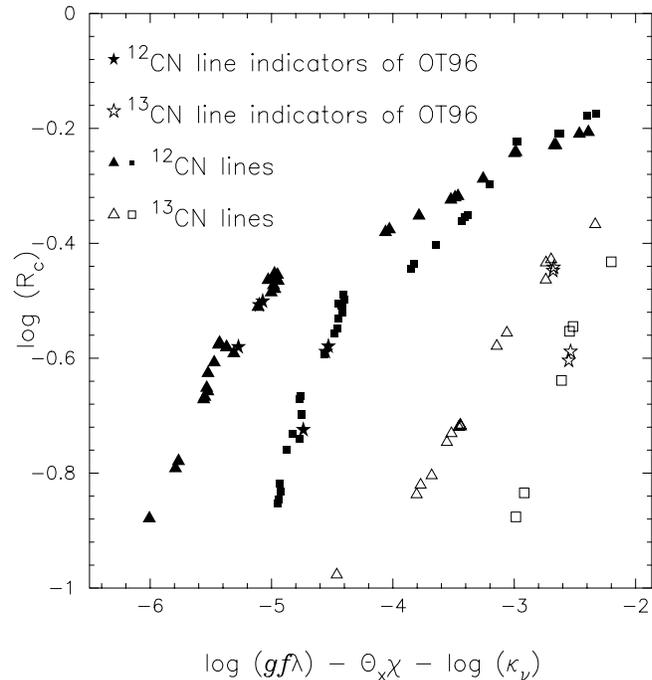


Fig. 4. Curves of intensity growth deduced from models of LGEH86 (triangles) and OT96 (squares) characterized by the same set of parameters: $T_{\text{eff}} = 3000$ K, $\log(g) = 0$, $[\text{Fe}/\text{H}] = 0$, $\text{C}/\text{O} = 1.1$, $^{12}\text{C} / ^{13}\text{C} = 30$ and $\xi_{\text{micro}} = 2.0$ km.s $^{-1}$ (3.0 km.s $^{-1}$ for OT96).

that models without turbulent pressure can be cooler by 100 K to 200 K in the line forming regions. However, Gustafsson et al. (1975, Eq. 15) have shown that adding a turbulent pressure to the pressures of gas and radiation leads to temperature-pressure structures very close to those of corresponding models calculated with a smaller surface gravity: $\log(g_{\text{with turb.}}) = \log(g_{\text{without turb.}}) + \alpha$ (α being always positive). For a cool carbon star we estimate $\alpha \simeq 0.5$ dex. As shown in Sect. 3 such a variation of the gravity leads to rather small uncertainties in the derived $^{12}\text{C} / ^{13}\text{C}$ ratios (less than 0.07 dex) and LGEH86 found similarly small effects on $^{12}\text{C} / ^{13}\text{C}$ in their work. The effects of the turbulent pressure upon the structure of the models are therefore not the cause of the discrepancies between OT96 and LGEH86.

K. Ohnaka also provided us with one of the model atmosphere of OT96 to compare to ours. Their Fig. 19 (temperature versus gas pressure) suggests that a model of LGEH86 is cooler by more than 500 K than their model with the same parameters at a given pressure. However, the comparison of the $T(\tau_{1.04\mu\text{m}})$ relation given by these two models shows a smaller temperature difference close to ~ 300 K between the layers where the CN lines are actually formed. We also note that, in contrast to the LGEH86 model, the OT96 model is much hotter around $\tau = 2/3$ than the effective temperature of the star.

We have explored the effects of such atmospheric structure differences on the determination of the carbon isotopic ratios. First, the synthetic spectra calculated with these two models differ: most of the calculated lines get deeper with the cooler model of LGEH86 but the relative depth of the lines selected

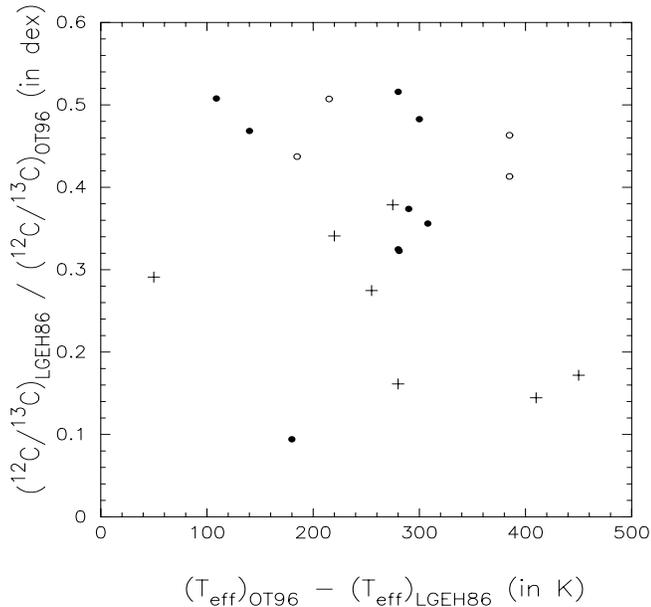


Fig. 5. Comparison between the derived effective temperatures and carbon isotopic ratios of LGEH86 and OT96 for their 20 stars in common. Filled circles, open circles and plus signs represent stars with $C/O \geq 1.1$, $1.1 > C/O \geq 1.05$, and $C/O < 1.05$ respectively (carbon enrichment of these stars from LGEH86).

by OT96 does not change very much. Fig. 4 shows the curves of intensity growth of ^{12}CN and ^{13}CN derived from these models. The shift between them at the depth of ^{13}CN indicators leads to $^{12}\text{C} / ^{13}\text{C}$ corrections of 0.18 dex when one uses the LGEH86 model instead the OT96 one. The differences in the model atmospheres used by LGEH86 and OT96 are large but certainly not enough to cause the disagreement observed between these two studies. Other causes have to be found, which are examined in the following paragraphs.

4.2. Determination of effective temperature

LGEH86 and OT96 have 20 stars of their sample in common. OT96 derived smaller $^{12}\text{C} / ^{13}\text{C}$ ratios for these 20 stars but they also used significantly higher effective temperatures. Fig. 5 displays the differences in the effective temperatures versus those in $^{12}\text{C} / ^{13}\text{C}$. No correlation between the discrepancies in T_{eff} and $^{12}\text{C} / ^{13}\text{C}$ ratios is found, but the mean disagreement in temperature is larger than 260 K and reaches 450 K for AQ Sgr. In the error analysis of Sect. 3, we found that a 200 K cooler effective temperature leads to a derived $^{12}\text{C} / ^{13}\text{C}$ ratio larger by 0.15 dex (see also Fig. 3). Decreasing the effective temperature increases the distance between the ^{12}CN and ^{13}CN curves of intensity growth and thus larger carbon isotopic ratios are derived from the *iso-intensity* method. Most of the disagreement between the LGEH86 and OT96 studies could thus be explained by the hotter effective temperatures used by OT96.

Furthermore, Fig. 5 clearly shows that the larger differences in $^{12}\text{C} / ^{13}\text{C}$ ratios are found for the most carbon-rich stars. Ex-

cept for V Aql, the derived carbon isotopic ratios differ by more than 0.3 dex when $C/O > 1.05$. This may reveal differences in opacities due to carbon-rich molecules (disagreement regarding the actual absorption coefficient of these molecules and/or wrong carbon enrichment considered). OT96 assumed for all their stars the mean value (1.1) of the C/O ratios found by LGEH86 while these showed that lower carbon enrichments are more frequently found in N-type stars (about 50% of the stars in their sample have $C/O < 1.1$). In the error analysis of the preceding section, we found that larger $^{12}\text{C} / ^{13}\text{C}$ ratios are derived from the *iso-intensity* method if smaller carbon enrichments are adopted. Although this effect is weak, it increases the differences in the carbon isotopic ratios derived by OT96 and LGEH86. Furthermore, this could also explain why our error analysis is in better agreement with OT96 for the SC-type stars.

Unfortunately, it was impossible to get a better (or a worse) fit of ST Cas in Fig. 1 by decreasing its effective temperature, lowering C/O and increasing its carbon isotopic ratio.

Among the stars in common between OT96 and LGEH86, only four have effective temperatures obtained from angular diameter observations (see Table 4 of OT96) and therefore relatively independent of model atmospheres. We summarize in Table 2 effective temperatures derived for these stars from independent angular diameters and the ones used by OT96 and LGEH86. These two groups agree only for TX Psc. For Y Tau, X Cnc and AQ Sgr, OT96 favoured larger effective temperatures derived by the Infrared Flux Method (IFM), even when occultation measurements are available. On the other hand LGEH86 adopted for these stars smaller T_{eff} 's, being much closer to the occultation values. For all the other stars, effective temperatures were derived by OT96 by the IFM, calibrated by their model atmospheres, while LGEH86 adopted IFM-values from Tsuji (1981) for half of their sample and employed (T_{eff} - color index) calibrations for the others. It is not quite clear which set of effective temperatures has to be preferred. But this effective temperatures difference is an important cause of disagreement in the derived $^{12}\text{C} / ^{13}\text{C}$ ratios; it is at least as large as the differences due to model atmospheres and blends. Also note that most of these stars are variables and one could suspect that their effective temperatures may vary during the cycle. For instance, Richichi et al. (1995) proposed that the photosphere of Y Tau may experience variations of ~ 300 K in T_{eff} . The effective temperatures of these stars should therefore be derived at the same epoch as the estimates of the carbon isotopic ratio.

4.3. $^{12}\text{C} / ^{13}\text{C}$ determinations of LGEH86

We have shown that the carbon isotopic ratios in cool carbon stars derived with the *iso-intensity* method are not very accurate, partly because of the uncertainties due to model parameters. In order to further clarify the situation, we now discuss the precision of the method used by LGEH86. We recall that these authors determined $^{12}\text{C} / ^{13}\text{C}$ from three independent analysis considering several tens of unblended lines of the CN $\Delta v = -2$ and CO $\Delta v = 2$ and 3 bands. They furthermore checked the

Table 2. Effective temperatures (in K) derived from independent angular diameter observations versus the values adopted by OT96 and LGEH86.

Star	Angular Diameter	OT96	LGEH86
AQ Sgr	2 684±110 (Walker et al., 1979)	3 100±200	2 650±200
TX Psc	3 172±180 (Lasker et al., 1973)	3 080±154	3 030±200
	3 365±210 (de Veigt, 1974)		
	2 980±380 (Dunham et al., 1975)		
	2 805±126 (Quirrenbach et al., 1994)		
X Cnc	3 075±75 (Richichi et al., 1995)	2 910±146	2 620±200
	2 653 (Bartholdi et al., 1972)		
Y Tau	2 701±100 (Ridgway et al., 1982; T_{eff} derived by OT96)	2 880±144	2 600±200
	2 701±120 (Ridgway et al., 1977)		
	2 540±50 (Schmidtke et al., 1986; T_{eff} derived by OT96)		
	2 574±50 (Schmidtke et al., 1986; T_{eff} derived by OT96)		
	2 878±40 (Schmidtke et al., 1986; T_{eff} derived by OT96)		
	2 810±70 (Dominy & Wallerstein, 1986)		
	2 590±30 (Richichi et al., 1995)		

consistency of these different determinations with the relative intensities of the $^{12}\text{C}_2$ and $^{12}\text{C}^{13}\text{C}$ Phillips system lines.

A key point of the procedure involving the CN lines is that the derived ratio ought to be rather insensitive to the atmospheric models because ^{12}CN and ^{13}CN lines with similar equivalent widths are compared. LGEH86 indeed found that analysis with alternative models lead to errors smaller than ± 0.02 dex in $^{12}\text{C} / ^{13}\text{C}$ ratios for uncertainties in T_{eff} as large as ± 500 K (i.e. more than the largest discrepancy in the effective temperatures used by OT96 and LGEH86), ± 1 for $\log(g)$, ± 1 km.s^{-1} for the microturbulent velocity and carbon enrichments spanning over the complete interval of C/O ratios found for the stars of their sample. Such uncertainties of the model parameters are as large as the ones we considered in Sect. 3 for the *iso-intensity* method but lead to much smaller errors. Rather small systematic errors due to uncertainties in the atmospheric structure are also suspected.

As for the $^{12}\text{C} / ^{13}\text{C}$ ratios derived from the CO lines, the estimated errors are found to be larger for CO $\Delta v = 3$ lines than for CN because of (i) the effects of blends, (ii) the uncertainty in the location of the continuum affects the ^{12}CO and the weaker ^{13}CO lines differently and (iii) the larger sensitivity of the estimated carbon isotopic ratios to the model parameters (the greatest contribution being caused by the effective temperature uncertainty). LGEH86 found that the total uncertainty of the CO $\Delta v = 3$ lines analysis, including measurement errors, is about ± 0.14 dex (OT96 estimated that their total error is of the same order while we found much larger uncertainties for the *iso-intensity* method). On the other hand, the analysis of the CO $\Delta v = 2$ lines was found to be much less sensitive to the estimated uncertainties in T_{eff} , $\log(g)$, ξ_{micro} and chemical composition and insensitive to the atmospheric structure. The total error of this third analysis is therefore dominated by measurement errors (mostly line-to-line scatter in the curve of growth) and is then smaller than in the CO $\Delta v = 3$ analysis. In summary, even this last method (i.e. the less precise one) is at least as accurate as the *iso-intensity* analysis of OT96.

Finally, these three determinations of $^{12}\text{C} / ^{13}\text{C}$ ratios by LGEH86 are very concordant, especially the CN and CO $\Delta v = 2$ analyses (see their Fig. 22). More scatter is, however, found between CN and CO $\Delta v = 3$ reflecting the larger sensitivity of CO lines to model atmospheres and possible contamination with unidentified lines. The final estimate by LGEH86 of the carbon isotopic ratio of a given star is an average of these three measurements with weights decreasing from CN to CO $\Delta v = 2$ and to CO $\Delta v = 3$. The total error includes the uncertainties discussed above plus other possible sources of errors (continuum location, estimation of errors in assumptions and structures of model atmospheres, dust emission...). A total error of ± 0.1 dex was finally found for $^{12}\text{C} / ^{13}\text{C}$ ratios determined from the CO lines, and only ± 0.03 dex for CN determination.

We conclude that the determination of carbon isotopic ratios in N-type stars by LGEH86 is rather accurate and probably more reliable than that of OT96 because (i) three independent and consistent analyses were performed and (ii) they are much less sensitive to model parameters. Therefore, we favour the large carbon isotopic ratios of LGEH86.

5. Conclusion

We have shown that the LGEH86 and OT96 studies disagree due to differences not only in the structure of the model atmospheres but also in the adopted effective temperatures. These two causes work in the same direction and lead to almost equal errors in $^{12}\text{C} / ^{13}\text{C}$ ratios determined by the *iso-intensity* method of OT96. LGEH86 adopted cooler model atmospheres and effective temperatures leading to larger $^{12}\text{C} / ^{13}\text{C}$ ratios while OT96 used hotter models and larger T_{eff} 's in deriving their smaller carbon isotopic ratios. The carbon enrichment of these stars also increases the differences between these two studies. Finally, the *iso-intensity* method is found to be risky, and should actually be avoided, when applied to the crowded spectrum of any cool star due to the effects of the blends. In principle, also the LGEH86 equivalent widths determined from their FTS spectra may be af-

ected by blends. However, the smaller density of spectral lines in their wavelength regions, their use of independent analysis with consistent results, and their precaution in selecting lines (e.g. from plots of equivalent widths versus line depths) suggest to us that effects of blends are less severe.

Obviously, the larger $^{12}\text{C} / ^{13}\text{C}$ ratios derived by LGEH86 are not ruled out by the work of OT96. Furthermore, previous studies on $^{12}\text{C} / ^{13}\text{C}$ ratios in AGB stars (see the introduction of this paper and more particularly the analysis of M, MS and S stars progenitors of N-type stars by Smith & Lambert, 1990) are consistent with the carbon isotopic ratios derived by LGEH86.

However, future observational determinations of $^{12}\text{C} / ^{13}\text{C}$ in a carbon star should be based on more accurate stellar parameters (metallicity, C/O ratio and particularly effective temperature) especially if the *iso-intensity* method is used or lines of different depths of formation are exploited. Improvements of model atmospheres are also needed. Finally, spectral regions with minimized risks for blending and high spectral resolution should always be used.

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References

- Amiot C., Verges J., 1983, ApJ, 263, 993
 Bachiller R., Forveille T., Huggins P.J., Cox P., 1997, A&A, in the press
 Bakker E.J., van Dishoek E.F., Waters L.B.F.M., Shoenmaker T., 1997, A&A in the press
 Ballik E.A., Ramsay D.A., 1963, ApJ, 137, 84
 Bartholdi P., Evans D.S., Mitchell R.I., Silverberg E.C., Wells D.C., Wiant J.R., 1972, AJ, 77, 756
 Bauschlicher C.W.Jr., Langhoff S.R., 1987, J. Chem. Phys., 87, 2919
 Bauschlicher C.W.Jr., Langhoff S.R., Taylor P.R., 1988, ApJ, 332, 531
 Blackwell D.E., Booth A.J., Petford A.D., 1984, A&A, 132, 236
 Brault J.W., Engleman R. Jr., private communication
 Busso M., Lambert D.L., Gallino B.R., Raiteri C.M., Smith V.V., 1995, ApJ, 446, 775
 Charbonnel C., 1994, A&A, 282, 811
 Chauville J., Maillard J.P., Mantz A.W., 1977, J. Molec. Spectrosc., 68, 399
 Costes M., Naulin C., 1993, in *Molecules in the Stellar Environment*, ed. U.G. Jørgensen, Springer-Verlag (Berlin), 250
 Costes M., Naulin C., Dorthe G., 1990, A&A, 232, 270
 Davis S.P., Phillips J.G., 1963, *The Red System ($A^2\Pi - X^2\Sigma$) of the CN Molecule*, University of California Press (Berkeley)
 Delbouille L., Neven I., Roland G., 1973, *Atlas photométrique du spectre solaire de $\lambda 3000$ à $\lambda 10000$* , ed. Institut d'Astrophysique de l'Université de Liège
 de Veit C., A&A, 34, 457
 Dominy J.F., Wallerstein G., 1986, PASP, 98, 1193
 Dominy J.F., Wallerstein G., 1987, ApJ, 317, 810
 Dominy J.F., Wallerstein G., Suntzeff B., 1986, ApJ, 300, 325
 Dunham D.W., Ewans D.S., Silverberg E.C., Wiant J.R., 1975, MN-RAS, 173, 61
 Dwidedi P.H., Branch D., Huffaker J.N., Bell R.A., 1978, ApJS, 36, 573
 Edvardsson B., Andersen J., Gustafsson B., Lambert D.L., Nissen P.E., Tomkin J., 1993, A&A, 275, 101
 Eriksson K., Gustafsson B., Jørgensen U.G., Nordlund Å, 1984, A&A, 132, 37
 Erman P., Lambert D.L., Larsson M., Mannfors B., 1982, ApJ, 253, 983
 Forestini M., Charbonnel C., 1997, A&AS, 123, 241
 Fujita Y., Tsuji T., 1977, PASJ, 29, 711
 Grevesse N., Lambert D.L., Sauval A.J., van Dishoek E.F., Farmer C.B., Norton R.H., 1991, A&A, 242, 488
 Grevesse N., Noels A., Sauval A.J., 1996, in *Cosmic Abundances*, eds. S.S. Holt and G.Sonneborn, ASP. conf. ser. Vol.99, 117
 Gustafsson B., Bell R.A., Eriksson K., Nordlund Å, 1975, A&A, 42, 407
 Gustafsson B., Ryde N., 1996, in *The carbon star phenomenon*, ed. R. Wing, in the press
 Holweger H., Müller E.A., 1974, Solar Phys., 39, 19
 Hosinsky G., Klynning L., Lindgren B., 1982, Phy. Scripta, 25, 291
 Hosinsky G., Lindgren B., 1976, A&AS, 25, 1
 Huang Y., Barts S.A., Halpern J.B., 1992, J. Phys. Chem., 96, 425
 Huber K.P., Herzberg G., 1979, *Molecular Spectra and Molecular Structure*, Vol.4, *Constants of Diatomic Molecules*, Van Nostrand Reinhold Company (New York)
 Irwin A.W., 1981, ApJS, 45, 621
 Jørgensen U.G., Johnson H.R., Nordlund Å, 1992, A&A, 261, 263
 Jørgensen U.G., Larsson M., 1990, A&A, 238, 424
 Kahane C., Cernicharo J., Gómez-González J., Guélin M., 1992, A&A, 256, 235
 Kahane C., Gómez-González J., Cernicharo J., Guélin M., 1988, A&A, 190, 167
 Knapp G.R., Chang K.M., 1988, ApJ, 293, 281
 Kotlar A.J., Field R.W., Steinfeld J.I., 1980, J. Mol. Spectrosc., 80, 86
 Lambert D.L., 1993, in *Molecules in the Stellar Environment*, ed. U.G. Jørgensen, Springer-Verlag (Berlin), 1
 Lambert D.L., Gustafsson B., Eriksson K., Hinkle K.H., 1986, ApJS, 62, 373 (LGEH86)
 Lambert D.L., Sheffer Y., Federman S.R., 1995, ApJ, 438, 740
 Lasker B.M., Bracker S.B., Kunkel W.E., 1973, PASP, 85, 109
 Marigo P., Bressan A., Chiosi C., 1997, A&A, in the press
 Ohnaka K., Tsuji T., 1996, A&A, 310, 933 (OT96)
 Piskunov N.E., Kupka F., Ryabchikova T.A., Weiss W.W., Jeffery C.S., 1995, A&AS, 112, 525
 Prantzos N., Aubert O., Audouze J., 1996, A&A, 309, 760
 Quirrenbach A., Mozurkewich D., Hummel C.A., Buscher D.F., Armstrong J.T., 1994, A&A, 285, 541
 Richichi A., Chandrasekhar T., Lisi F., Howell R.R., Meyer C., Rabbia Y., Ragland S., Ashok N.M., 1995, A&A, 301, 439
 Ridgway S.T., Jacoby G.H., Joyce R.R., Siegel M.J., Wells D.C., 1982, AJ, 87, 808
 Ridgway S.T., Jacoby G.H., Joyce R.R., Wells D.C., 1980, AJ, 85, 1496
 Ridgway S.T., Wells D.C., Joyce R.R., 1977, AJ, 82, 414
 Sauval A.J., Tatum J.B., 1984, ApJS, 56, 193
 Schmidtke P.C., Africano J.L., Jacoby G.H., Joyce R.R., Ridgway S.T., 1986, AJ, 91, 961

- Smith V.V., Lambert D.L., 1985, ApJ, 294, 326
Smith V.V., Lambert D.L., 1990, ApJS, 72, 387
Tsuji T., 1981, J. Astr. Ap., 2, 95
Walker A.R., Wild P.A.T., Byrne P.B., 1979, MNRAS, 189, 455
Wyller, A.A., 1966, ApJ, 143, 828