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Velocity structures from sunspot statistics in cycles 10 to 22

I. Rotational velocity

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Abstract. Data from the whole era of modern sunspot observations (1853-1996) from four different sources is considered. We find that rotation is continuously changing, not only in the course of a cycle but also in a longer time period. We also find strong fluctuations in the equatorial rotation in the course of a sunspot cycle which can be connected with the observed north/south asymmetry of rotation. The importance of age dependence on rotation is emphasized, both for non-recurrent and recurrent sunspots as the slowing down of rotation along with the age is confirmed.

Key words: Sun: activity – Sun: photosphere – Sun: rotation – sunspots

1. Introduction

The modern era of sunspot observations was started by Carrington in 1853 lasting about 7 years. With no gap in between, Spörer began his sunspot observations in 1861 and carried them out for more than 30 years. In the meantime, the Royal Observatory in Greenwich had started the long-lasting Greenwich Photoheliographic Results (GPR) (1874-1976). After that, one of the still continuing records of sunspots has been made by the Solar Optical Observing Network (SOON) of the US Air Force together with the US National Oceanic and Atmospheric Administration (NOAA). These GPR and SOON/NOAA data sets can now also be found in the internet (see the NASA site: http://wwwssl.msfc.nasa.gov/ssl/pad/solar/ greenwich.htm).

The sunspot measurements of the GPR have been utilized as tracers of rotation by Newton and Nunn (1951), Ward (1966), Balthasar and Wöhl (1980), Arévalo et al. (1982), and Balthasar et al. (1984,1986). Important contributions from other sunspot data were carried out by Howard et al. (1984) and Gilman & Howard (1984). And besides sunspots, other sources such as Doppler shifts, magnetic and coronal hole tracers, and helioseismology have been successfully used (see e.g. Howard et al. 1983, Gough 1985, Stenflo 1989).

The qualitative behaviour of solar differential rotation was already discovered by Carrington and since that it has been confirmed many times. However, quantitative differencies occur because of the selection of the data or the tracers of rotation. Based on sunspots, the mean equatorial rotation is about 14.54 degrees/day (siderial), but variations exist at different cycles and phases inside the cycles, as shown by Balthasar et al. (1986) and Gilman & Howard (1984).

A crucial parameter for classifying sunspots has turned out to be their age. J. Tuominen (1962) noted that rotation derived from birth places of sunspots was about 7% more rapid than that from older sunspots. Tuominen and Virtanen (1988), and Balthasar et al. (1982) pointed out the possibility that the younger sunspots could be anchored to deeper parts of convection zone where rotation would be more rapid than at the photosphere. Nesme-Ribes et al. (1993) studied sunspots in cycle 21 and found the same decreasing tendency in rotation rate in the course of time. The increase of rotational velocity below the photosphere is recently supported also through helioseismology by both ground-base and satellite missions (Thompson et al., 1996 and Kosovichev et al., 1997).

The present work is divided in two parts. This first paper (Paper I) considers only rotation presenting the changes in velocity structures taken place in the 143-year time span of modern era sunspot observations. The second one (Paper II) considers the latitudinal velocity and correlation functions.

2. The data sets and methods

The Carrington and Spörer data contain only the date and coordinates of measurements where the heliographic longitude must be calculated from the Carrington longitude. Due to historical reasons, the rotation rates on which the Carrington longitude is based are different: for the Carrington data the siderial period is the standard 25.380 days (corresponds to 27.2753 days synodic), but for the Spörer data the respective values are 25.234d and 27.107d. In the GPR and SOON/NOAA data, apart from

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Table 1. The four data sets: given are the beginning and ending months of each as well as the number of measurements and velocity values derived from them, with cut-off longitude of 60 degrees

data	begin	end	meas.	vel. val.
Carrington	11/1853	3/1861	2997	1697
Spörer	1/1861	12/1893	21021	13011
GPR	4/1874	12/1976	183244	122052
SOON/NOAA	1/1977	12/1996	51289	34247

heliographic coordinates, also the areas of the sunspots are included.

The Spörer data have been compared with other sunspot measurements by Balthasar & Fangmeier (1988) and Wöhl & Balthasar (1989). They both find the very good quality and applicability of Spörer data compared with the GPR although some differences occur. Also for the Carrington data we find their quality good. Moreover, the distribution of Carrington and Spörer data sets along longitude is homogeneous and along latitude similar to the GPR.

An important factor in observing sunspots is to follow their life between multiple solar rotations. The long-living recurrent sunspots have been found to rotate differently compared to non-recurrent ones. Unfortunately, the classification between non-recurrent and recurrent sunspots is listed only in the GPR.

We considered different cut-off longitudes between 40 and 90 degrees for the velocity measurements. Similarly with the analysis of Balthasar et al. (1986), we find the velocity profile being rather sensitive with cut-off longitudes more than about 60 degrees not only in the GPR, but in the other data sets as well. Therefore, we discard all velocity values that are more than 60 degrees away from the central meridian. The time ranges of different data sets with the restrictions presented above are listed in Table 1. The quickest way to estimate the relevance of this large data set of ours is to look at the distribution of rotational velocities at different latitudes (this is done for cycle 19 in Fig. 1). The mean differential rotation is surrounded by a wide distribution of turbulent fluctuations but hardly any one of the measurements escapes from the general pattern. Therefore, we used no cut-off values for velocities.

Balthasar and Wöhl (1980) remeasured positions from some of the original plates and found an absolute difference greater than 1 degree both in latitude and longitude, so the 0.1° accuracy of the original measurements is certainly optimistic. However, because our preliminary calculations with the plain GPR did not differ practically at all with those of e.g. Balthasar et al. (1986), we do not find it necessary to import any further criteria for the data selection.

The data in the GPR were not recorded in a uniform manner during the whole 103-year period. Between years 1911 and 1955, or sunspot cycles 15 to 18, a special section called Ledgers of sunspots were made in which only the recurrent sunspots or those prominent non-recurrent sunspots with 6 or more measurements were traced. This selection criterion created an anomaly in the distribution of heliographic longitudes of measurements, because it rejected all those sunspots whose

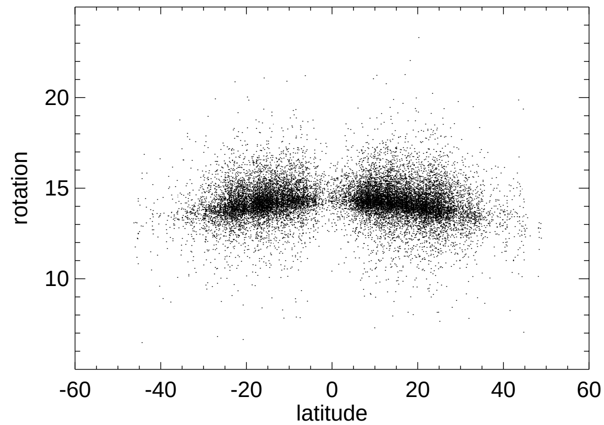

Fig. 1. Values of rotational velocity (in deg/day) for the GPR during cycle 19 with cut-off longitude of 60 degrees

first measurement had longitudes over 20 degrees to the east of the central meridian. Therefore, we use here the combined data of the general catalogue (the internet version) and Ledgers (which contain the recurrency classification) having a total of 183224 sunspot measurements, of which 19743 or 10.8% are of single spots (i.e. sunspots separated by a significant distance from their mother groups). From every pair of measurements of the same sunspot at the same rotation we calculated rotational and latitudinal velocities with the mean values of original coordinates taken as new ones thus getting 122052 velocity values (14096 of single spots) from the GPR. Single spots are measured quite unevenly in the GPR, the ratio between single spots and sunspot groups varies from almost zero in cycle 13 to 32% in cycle 16. Cycles with significant ratio of single spot measurements are shown in Table 3.

The surface rotation rate is traditionally described with the cosine expansion of the polar angle θ:

$$\Omega = A + B \cos^2\theta, \tag{1}$$

where A is the rotation at the equator and B describes the steepness of the profile. In this work, the velocity unit is always 1 deg/day which corresponds to 0.202 μrad/s. The rotational velocity values are always the siderial values.

There are a number of important parameters according to which the sunspot measurements can be classified. The phase of cycle (ϕ_c) for a measurement is defined simply by $\phi_c = (t - t_b)/(t_e - t_b)$, where t_b and t_e are the beginning and ending times of the cycle in which t belongs (see Table 2 for the approximate time ranges of the cycles). Hence ϕ_c varies between 0 and 1. Because the lengths of solar cycles vary from 10 to 12 years in this data, a fraction of a cycle may be a more appropriate short time period than the commonly used one year. For convenience, we may also define a *phase index* C_i which varies between 0 and 9 so that $C_i = 0$ means $0.0 \leq \phi_c < 0.1$ etc. Further, it is very important to find out the birth times of sunspots. If the first measurement is at longitude greater than -60 degrees the sunspot can be assumed to have been born less than about 24 hours before, provided that observations have been made

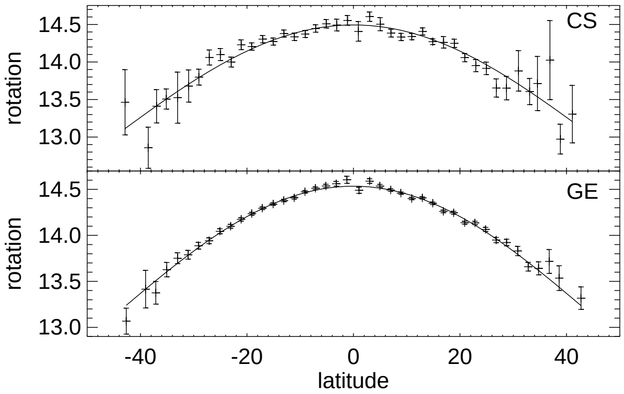

Fig. 2. The calculated differential rotation profile for the CS (upper panel) and GE data (lower panel) in 2° wide bins as well as the fits to Eq. 1

Table 2. The sunspot cycles 9 to 22 included in the data sets. Given are the months of beginning (minimum) and maximum phases of each cycle and the length of cycle in years

cycle #	beg	max	length
9	7/1843	2/1848	12.4
10	12/1855	2/1860	11.4
11	3/1867	8/1870	11.6
12	12/1878	12/1883	10.7
13	3/1890	1/1894	12.1
14	1/1902	2/1906	11.9
15	8/1913	8/1917	10.0
16	8/1923	4/1928	10.2
17	9/1933	4/1937	10.4
18	2/1944	5/1947	10.1
19	4/1954	3/1958	10.6
20	10/1964	11/1968	11.6
21	6/1976	12/1979	10.3
22	9/1986	7/1989	10.2
23	1/1997		

daily. We have divided the data into many age groups, the most important ones are quoted in the text.

As can be understood from Fig. 1, the velocity distributions inside latitude bins are wide, the wider the more measurements. For the errors of calculations we use the mean error of the mean, i.e. the standard deviation divided by the square root of the number of measurements inside the bin.

Table 3. Results of fittings for various subgroups of the data to rotational velocity profile Eq. 1. Only those latitude bins with more than 2 measurements are included. The columns are: data unit, coefficients A and B of the fit with their errors, and number of velocity values in the unit

unit	$A \pm \sigma(A)$	$B \pm \sigma(B)$	no
Carrington	14.254 ± 0.047	-2.496 ± 0.547	1697
Spörer	14.493 ± 0.011	-2.640 ± 0.170	13011
CS	14.475 ± 0.011	-2.710 ± 0.165	14708
CS north	14.479 ± 0.024	-3.018 ± 0.259	6963
CS south	14.476 ± 0.022	-2.510 ± 0.215	7745
GE	14.531 ± 0.003	-2.747 ± 0.048	156299
GE north	14.526 ± 0.007	-2.697 ± 0.065	79238
GE south	14.536 ± 0.007	-2.805 ± 0.070	77061
GE (groups)	14.535 ± 0.003	-2.728 ± 0.051	141647
GE (s/spots)	14.495 ± 0.006	-2.973 ± 0.100	14652
cycle 10	14.409 ± 0.040	-3.278 ± 0.427	2732
cycle 11	14.452 ± 0.035	-2.933 ± 0.310	3494
cycle 12 (CS)	14.537 ± 0.020	-2.776 ± 0.253	5647
cycle 12 (GE)	14.616 ± 0.018	-2.810 ± 0.211	7342
cycle 13 (CS)	14.451 ± 0.034	-2.005 ± 0.301	2660
cycle 13 (GE)	14.636 ± 0.017	-2.553 ± 0.208	10434
cycle 14	14.511 ± 0.018	-2.100 ± 0.261	8956
cycle 15	14.532 ± 0.014	-2.528 ± 0.171	12556
groups	14.538 ± 0.016	-2.528 ± 0.193	10476
s/spots	14.509 ± 0.018	-2.909 ± 0.264	2080
cycle 16	14.531 ± 0.013	-2.890 ± 0.142	13276
groups	14.545 ± 0.017	-2.762 ± 0.179	9793
s/spots	14.495 ± 0.013	-3.390 ± 0.145	3483
cycle 17	14.552 ± 0.012	-2.958 ± 0.111	16555
groups	14.559 ± 0.014	-2.925 ± 0.135	12999
s/spots	14.513 ± 0.017	-2.971 ± 0.157	3556
cycle 18	14.529 ± 0.012	-2.875 ± 0.125	18107
groups	14.531 ± 0.014	-2.762 ± 0.143	14884
s/spots	14.515 ± 0.016	-3.254 ± 0.151	3223
cycle 19	14.505 ± 0.012	-2.783 ± 0.101	18956
groups	14.512 ± 0.013	-2.798 ± 0.105	17803
s/spots	14.402 ± 0.031	-2.496 ± 0.265	1153
cycle 20	14.522 ± 0.014	-2.593 ± 0.138	14894
cycle 21	14.492 ± 0.020	-2.696 ± 0.172	18057
cycle 22	14.436 ± 0.018	-2.286 ± 0.146	16333
youngest (nrc)	14.713 ± 0.017	-2.725 ± 0.158	16453
youngest (rec)	14.852 ± 0.041	-2.508 ± 0.441	2025
young (nrc)	14.618 ± 0.013	-2.677 ± 0.117	28736
young (rec)	14.672 ± 0.022	-2.562 ± 0.228	5184
old (nrc)	14.482 ± 0.007	-2.823 ± 0.074	66370
old (rec)	14.552 ± 0.010	-3.190 ± 0.106	13287
oldest	14.380 ± 0.007	-2.635 ± 0.080	24244

3. Results

In the following we consider the Carrington/Spörer (CS) data as one set and the GPR with SOON/NOAA, or the extended Greenwich (GE) data as another one. When sunspot cycles are considered separately, cycles 10 and 11 are taken from CS data and cycles 12 to 22 from the GE data unless otherwise mentioned. The subtitles tell only the emphasis of the subsections. Due to the connection of dependencies of different parameters the contents are mixed.

3.1. General behaviour

The differential rotation profile for many cases can be found in Table 3. Our results coincide with those by Balthasar et al. (1986), except with a small difference for the equatorial velocity of the whole GPR. This is due to the larger relative weight of the slower cycles 15 to 18 and the inclusion of cycles 21 to 22.

The difference between CS and GE data is noticeable. Although the quality of the CS data is good in terms of rather small errors they are more limited data than GE because of more

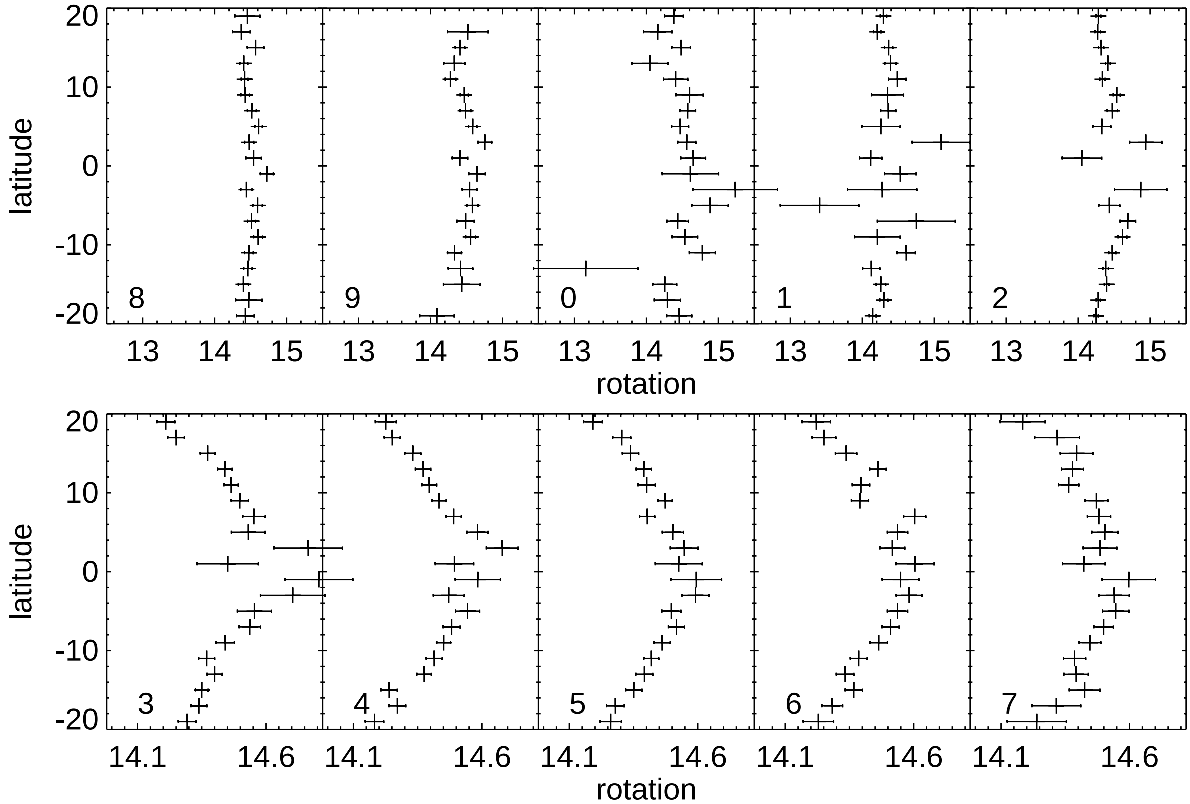

Fig. 3. Rotation profiles of the GE data at the equatorial range between latitudes ± 20 degrees at different phases of cycle (phase index is written in the lower left corner of each panel). Note the two different scales of the abscissa in minimum (indices 8 to 2) and maximum (indices 3 to 7) phases

days with no observations due to climatological circumstances. On the other hand, as pointed out by Balthasar and Fangmeier (1988) and Arévalo et al. (1982), systematical effects may be present in the old GPR up to cycle 13.

The overall differential rotation profile is presented in Fig. 2, both for CS and GE data. In three important cases the actual profile deviates from the fitted one:

– a dip just north of equator is noticeable
– at high latitudes in the northern hemisphere rotation is more rapid than in its surroundings
– in the GE data, the southern hemisphere lays much more smoothly to the fit than the northern one

The dip is in fact the only detectable sign of the strong variation of equatorial rotation visible in shorter time scales. If one looks at this rotation profile of the GE data with 2 degrees wide bins at different cycle phases, large fluctuations around the equator can be seen as is shown in Fig. 3. At the beginning of cycle, equatorial rotation between −20 and 20 degrees is almost rigid except for some dips in the southern hemisphere. Then, at phase indices 1 and 2 we note strong fluctuations around the equator which become smaller but do not vanish at cycle maximum (indices 3 to 5). At phase indices 6 and 7 the equatorial rotation is smoother but the more rapid rotation of the southern hemisphere is noticeable. Finally, towards the end of cycle rotation becomes more rigid. Similar variations as described above can be found also in each cycle separately. Deubner and Vázquez (1975) found from Doppler measurements, that the maximum of rotation is not at equator but about 13 degrees away. Their measurements were done at October 1974 which took place at phase index 8 in cycle 20. When we plot this phase index we see a similar pattern: equatorial rotation is about 2% slower in the northern hemisphere and 4% slower in the southern hemisphere than the maximum value which occurs for both hemispheres at about 7 degrees away from the equator.

The second dilemma of Fig. 2 can be at least partially explained by the age distribution of sunspots at high latitude bands. They have relatively many more rapidly rotating young sunspots compared to the other bins. In fact, the oldest sunspots are almost totally missing in highest latitudes. This seems natural, because the weak activity at the beginning of cycle cannot produce long-living sunspots. The absence of old sunspots is seen also at the equatorial region at cycle phase 0. These spots still belong to the previous cycle which is very weak in terms of magnetic activity. A consequence of this feature is discussed with the phase dependence.

The dip at north of the equator may also be related to the asymmetry of rotation between hemispheres. In the GE data it is of the same magnitude than what was found by Balthasar et al. (1986). But a similar behaviour can be found in the CS data. And if we look at the cycles separately, we note that the southern

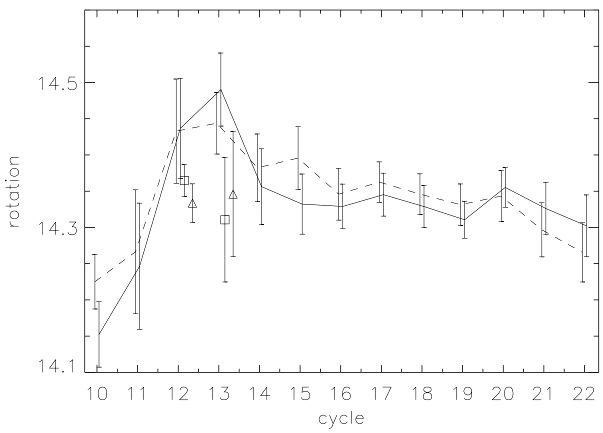

Fig. 4. The asymmetry of rotation between northern and southern hemispheres at different cycles. The solid (dashed) line shows the rotation rate at 15 degrees north (south) of the equator. Cycles 12 and 13 are also calculated based on the Spörer data the triangles representing northern and squares southern hemisphere

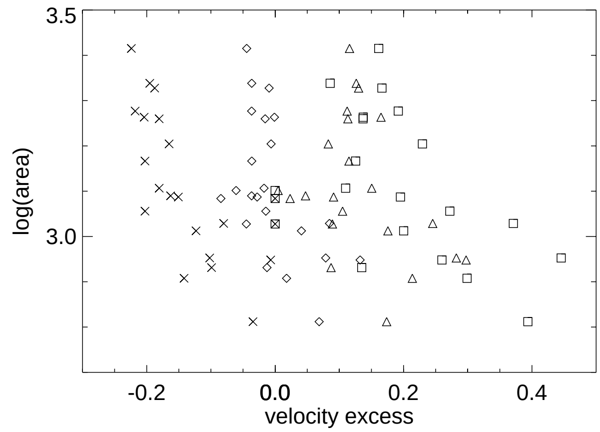

Fig. 5. The relative activity of cycles vs. velocity excess. The both hemispheres of each cycle is considered separately. The diamonds denote full data, triangles young non-recurrent, squares young recurrent, and crosses oldest recurrent

hemisphere is at most cycles more rapid than the northern one. This is illustrated in (Fig. 4).

A major importance must be given to single spots versus sunspot groups. When single spots are first measured, they are separated from sunspot groups that are already a few days old. Therefore, the age distribution of single spots is different from that of sunspot groups and we expect single spots to have slower rotation rate compared to sunspot groups. This is what happens, as can be seen in Table 3. Only among the old sunspots rotation is of the same magnitude than for single spots. From this we may draw the conclusion that single spots are representatives of older sunspots than what their age would tell. Another explanation is connected to the different proper motion of single spots compared with sunspot groups. This is studied in Paper II.

3.2. Cyclic behaviour

In addition to the asymmetry variations between different cycles, Fig. 4 shows also the slightly slowing trend of rotation during the 20th century. Cycles 12 and 13 show a high peak, but as mentioned above, the old GPR may contain some systematical effects (Arévalo et al., 1982). However, for these two cycles, the Spörer data may be more reliable giving values closer to the overall averages. Looking back we see that cycle 11 has about the same rotation rate as cycle 22 and that cycle 10 is even slower, but it consists mainly of Carrington's more uncertain measurements.

The activity of a sunspot cycle is loosely related to its length — the more active a cycle the shorter it is. Another relation for the cycle activity can be found with the rotation rate. In Fig. 5 the activity, or amplitude of a cycle is obtained by calculating the total area of sunspots during each cycle. A negative correlation is apparent — the more active the cycle the slower the rotation. As is seen, this happens in the full data as well as in all age groups.

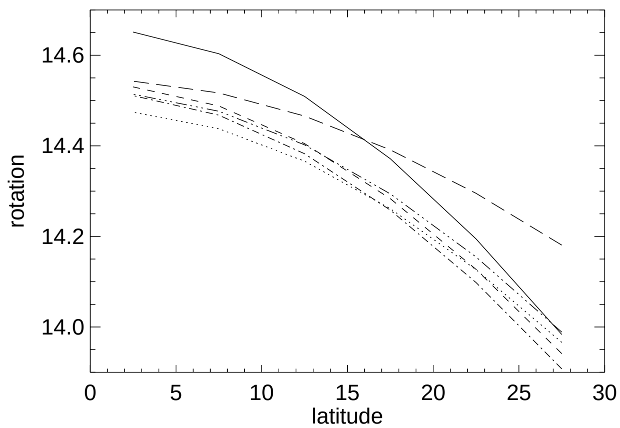

Fig. 6. The differential rotation profiles obtained from the GE data in cycle phases 0 (solid), 1 (dotted), 2 (dashed), 4 (dash-dotted), 7 (dash-dot-dotted), and 8 (long-dashed line)

3.3. Phase dependence

As was seen in Fig. 3, the differential rotation profile changes a lot in the course of the cycle. The qualitative differences can be seen better in Fig. 6. At the beginning of the cycle, the rotation profile is rapid and steep (solid line), but quickly slows down in phase 1 (dotted line). During the maximum phases rotation is more or less stationary but then at phase 8 the profile becomes flatter (long-dashed line). For individual cycles, the number of data points is often too small to study this phenomenon, but for the most active cycles the behaviour shown in Fig. 6 does happen. For sunspots of different age, the above effect is seen in many age groups, especially among youngest sunspots.

The phase dependence of rotation can also be studied through plotting the excess of rotational velocity vs. time (Fig. 7). This has been done also by Gilman & Howard (1984) for the Mount Wilson data and Balthasar et al. (1986) for the GPR. The highest peak is found just at the beginning of every

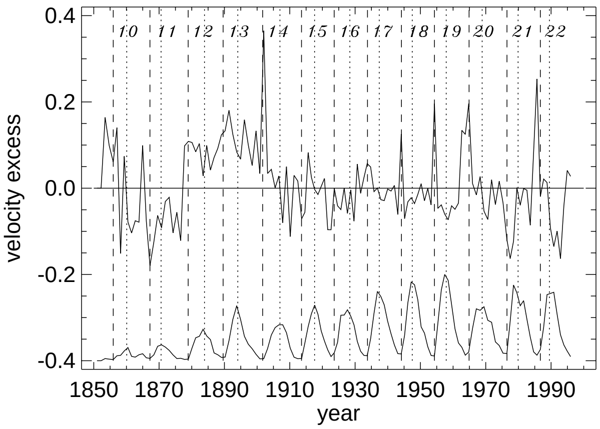

Fig. 7. Temporal development of the excess of mean rotation through cycles 10 to 22. The vertical dashed (dotted) lines denote the minimum (maximum) phase of the cycle. Number of velocity measurements is plotted in arbitrary units in the bottom line

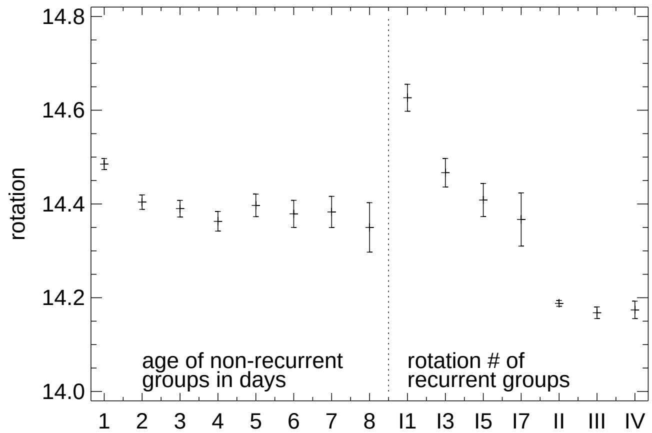

Fig. 9. Development of rotational velocity as a function of sunspot age. On the left side are non-recurrent, and on the right side recurrent sunspots at 4 first rotations. The first rotation has been divided into 4 parts (1, 3, 5, or 7 days old)

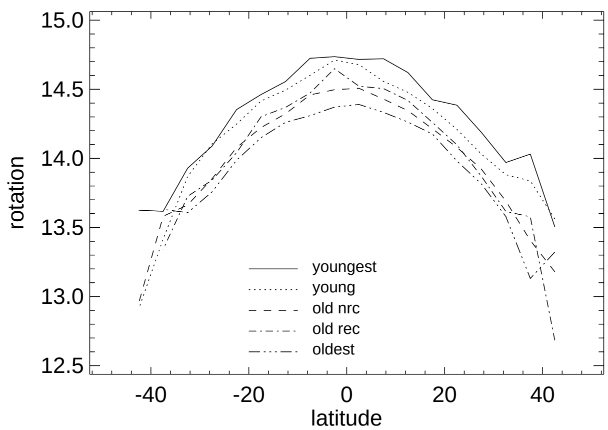

Fig. 8. Differential rotation for different-aged groups. The solid, dotted, dashed, dash-dotted, and dash-dot-dot-dotted lines correspond to the “youngest” (age less than 1.5 days), “young” (age 1.5 to 7.5 days), “old non-recurrent” (age over 7.5 days), “old recurrent” (age over 7.5 days), and “oldest” (recurrent, in 2nd or later rotation), respectively

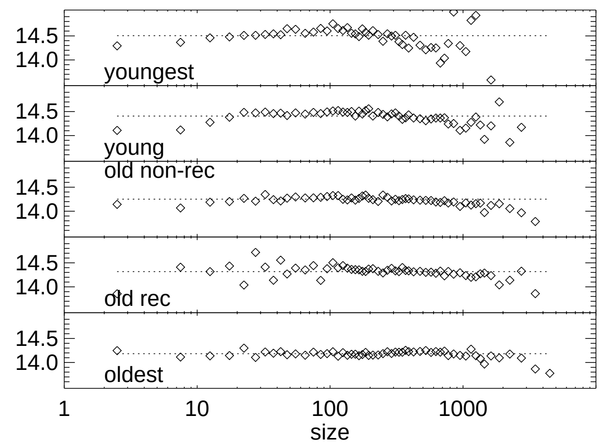

Fig. 10. Rotation for five age groups as a function of the mean size. The dotted lines denote the average of each group

cycle which can be explained by the rapid equatorial rotation at the beginning of the cycle due to the lack of old sunspots, and also by the overoccupation of young sunspots at high latitudes, although their relative weight is too small to be seen in the fits of Fig. 6. Another notion in Fig. 7 is the high jump between cycles 11 and 12.

3.4. Age dependence

When looking from younger to older sunspots one sees a decreasing tendency: the youngest sunspots rotate more rapidly than the older ones. Figs. 8 and 9 show clearly this phenomenon. A dramatic decrease in rotation is seen, especially among recurrent sunspots during their first rotation, but for oldest sunspots rotation settles down to about 14.17 degrees/day.

It may be argued that it is not age that determines the rotation of sunspots but their size. New-born sunspots are small and they grow in size as they get older. A conclusion from this would be that small sunspots rotate more rapidly than larger ones, possibly because of smaller magnetic or thermal diffusion. However, when we plot rotation of the five age groups (youngest, young, old non-recurrent, old recurrent, and oldest, see their definition in Fig. 8) vs. the mean size (Fig. 10), we see that *all* the youngest sunspots rotate more rapidly than young, old, and oldest ones, regardless of size. Although the mean size increases with aging sunspots, there is a wide distribution of different-sized sunspots at any state of their development.

The latest helioseismological measurements from the ground-base network of GONG (Thompson et al., 1996) and satellite observations of SOHO (Kosovichev et al., 1997) reveal the radial rotation profile at some latitudes. The increase of equatorial rotation is found both in the equatorial region and 30 degrees away from it. The magnitude of equatorial rotation varies from about 14.3 deg/day at the photosphere to 14.6 deg/day at 0.95 solar radius. At 30° these two measurements differ slightly, the respective variation from GONG is 13.8 to 13.9 deg/day, and from SOHO 13.3 to 14.0 deg/day. If we compare these with the youngest and oldest sunspots in Table 3, we get very similar variation ranges for rotational velocity as the two helioseismological measurements give.

4. Conclusions and discussion

The relatively large differences in the rotation profiles between data from various sources show that the solar differential rotation is continuously changing. During the present century, the rotation rate seems to have been slowing down, and so the concept of a global mean differential rotation may not be fully applicable. In fact, the data studied here covers so many sunspot cycles that we can even start to look for longer periods of solar activity changes than the 11 years of a solar cycle.

The rapidly rotating cycles 12 and 13 are puzzling because of the large difference between Spörer's data and the GPR (see Fig. 4 and Table 3). This dilemma was studied by Balthasar & Fangmeier (1988) and Wöhl & Balthasar (1989) but they did not find any clear explanation. Due to the compatibility between Spörer's data and the later GPR, it seems perhaps more likely that there is a systematic error in measuring sunspot coordinates during the old GPR.

The North-South asymmetry of rotation is apparently a real phenomenon, see also (Vizoso & Ballester, 1990), and the southern hemisphere is the more rapid one during most of the cycles considered here. Therefore, as the two sunspot regions meet at the end of cycle, the velocity difference may create fluctuations around the equator which then last towards the end of the next cycle.

The age dependence of rotation is significant. The decrease in rotation rate with older sunspots shows how essential it is to record the rotation numbers of recurrent sunspots. Based on the anchoring hypothesis of new-born sunspots carrying information from the inside, it seems possible that the recurrent-to-be sunspots represent deeper layers inside the solar convection zone than shorter living non-recurrent sunspots, this being now indirectly supported by helioseismology. In principle, recurrent sunspots could be used as tracers of the radial gradient for rotation, but such calculations contain too many uncertain factors to be worth reporting.

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