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Nonlinear coupling of MHD waves in inhomogeneous steady flows

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Abstract. The nonlinear coupling of MHD waves in a cold ($\beta = 0$) compressible plasma with a smoothly inhomogeneous low-speed steady flow directed along the magnetic field is considered. The effect is similar to Alfvèn wave phase mixing in a static, inhomogeneous medium and leads to the production of steep transversal gradients in the plasma parameters, which increases dissipation. Transversal gradients in the total pressure, produced by phase mixing, lead to the secular generation of obliquely propagating fast magnetosonic waves, at double the frequency and the wavenumber of the source Alfvén waves. The efficiency of the generation is defined by the Alfvén wave amplitude and the transversal spatial scale of the flow inhomogeneity. The secular growth of density perturbations, connected with fast waves, takes place for flow speeds that are considerably below the thresholds of the Kelvin - Helmholtz and negative energy wave instabilities. The initial stage of the nonlinear generation of the fast waves is considered analytically and illustrated by numerical simulations.

Key words: MHD – waves – Sun: corona

1. Introduction

The heating of the coronae of the Sun and late-type stars remains one of most intricate problems in modern astrophysics. Detailed reviews of this problem are given in Narain & Ulmschneider (1990, 1996), Browning (1991) and Zirker (1993). One mechanism for coronal heating is based on the dissipation of MHD waves in the upper atmospheric layers of the Sun and stars. Although the efficiency of the dissipation of the MHD waves due to viscosity or resistivity is rather small in the rarefied plasma of the corona, the well-observed inhomogeneity of the solar coronal plasma (e.g. active region loops, plumes in coronal holes, etc.) may strongly increase the efficiency of the dissipation through effects of resonant absorption and phase mixing (see e.g. Ionson 1978; Rae & Roberts 1981, 1982; Goossens 1991).

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Alfvén wave phase mixing describes the appearance of sharp transversal gradients in the Alfvén wave propagating along a magnetic field in a plasma that is inhomogeneous in a direction transversal to the magnetic field and homogeneous along the field (Heyvaerts & Priest 1983). After a time, two Alfvén waves propagating at different surfaces with the local Alfvén speed become uncorrelated with each other; their phases become mixed. The sharp gradients due to phase mixing may dramatically increase Alfvén wave dissipation due to viscosity and resistivity, since the dissipation rate is proportional to the transversal spatial scale of the wave.

Compressibility of the plasma adds to the phenomenon of phase mixing effects connected with the interaction of Alfvén and magnetosonic waves. Nonlinear coupling of Alfvén and magnetosonic waves propagating in the longitudinal direction was considered by Nocera, Priest & Hollweg (1986). An evolutionary nonlinear equation describing the propagation of a nonlinear Alfvén wave has been derived. It has been shown in numerical simulations by Malara, Primavera & Veltri (1996) that the nonlinear interaction of magnetosonic and Alfvén waves leads to an energy transfer to smaller and smaller spatial scales. This phenomenon may lead to the formation of shock waves and, therefore, to an increased heating of plasma by viscosity and resistivity. Nakariakov, Roberts & Murawski (1997) considered the nonlinear generation of fast magnetosonic waves by Alfvén wave phase mixing and showed that transversal gradients in the Alfvén wave, produced by phase mixing, lead to the secular generation of obliquely propagating fast waves. In the collisionless plasma of the solar corona, obliquely propagating fast waves are subject to strong dissipation due to Landau damping (Wentzel 1989). This phenomenon may be considered as indirect heating of the coronal plasma by phase mixing (Nakariakov, Roberts & Murawski 1997).

Both effects of direct dissipation of Alfvén waves and excitation of fast magnetosonic waves are defined by production of transversal gradients through phase mixing. Whereas the direct dissipation is proportional to the plasma viscosity and resistivity, the nonlinear excitation is proportional to the Alfvén wave amplitude. Therefore, the competition between these two effects is defined by the ratio of the viscosity (or resistivity) to the wave amplitude. Qualitatively, we may suppose that both effects take place simultaneously in a compressible plasma, slightly decreasing the efficiency of each other (but increasing the total efficiency of the heating). A quantitative comparison of these effects remains to be considered in future.

All theories of coronal heating involving the dissipation of MHD waves in an inhomogeneous medium are based on the observational fact that the solar corona is highly inhomogeneous in both plasma density and magnetic field and therefore in the Alfvén velocity. Also, there are a number of observational confirmations of the presence of a variety of plasma flows in the upper levels of the solar atmosphere (spicules and macrospicules, reconnection outflows, jets in coronal holes, etc.) Often, the plasma flows are directed along the magnetic field and are compact in the transversal direction. Note that the inhomogeneous flows do not always coincide with the inhomogeneities in the plasma density and magnetic field. Moreover, even in a homogeneous plasma with a constant Alfvén velocity there can be inhomogeneous flows, corresponding to the shifting of one plasma layer relative to another, along the magnetic field.

The propagation of MHD waves may be strongly affected by inhomogeneous plasma flows. Super-Alfvénic transversal shifts in the flow speed provoke the Kelvin - Helmholtz instability (KHI) or may lead to the spontaneous generation of fast waves from a level with a flow shift, due to the effect of over-reflection (McKenzie 1970; Nakaryakov & Stepanyantz 1994). Slower steady flows may lead to negative energy effects in magnetoacoustic waves (Ryutova 1988; Joarder, Nakariakov & Roberts 1998). For the appearance of such effects, the flow speed has to exceed a certain threshold value; for the low-β plasma of the solar corona, this threshold value is about the tube speed in the plasma (Joarder et al. 1997). Slower plasma flows (more typical of coronal conditions) give rise to slight changes in the dispersive characteristics of magnetoacoustic waves (Nakariakov & Roberts 1995a; Nakariakov, Roberts & Mann 1996), with no dramatic changes in wave propagation.

The aim of this work is to investigate the effects of phase mixing of Alfvén waves propagating in a medium with a smoothly inhomogeneous steady flow directed along a magnetic field. The steady flow velocity is considered to be less than the threshold values of KHI and negative energy instabilities. The plasma is supposed to be cold, $\beta = 0$. The temporal development of an initially plane Alfvén wave leads to the inclination of the wave profiles due to the mutual shift of different transversal layers (magnetic surfaces) by the steady flow. After a certain time, Alfvén perturbations (i.e. perturbations of components of the magnetic field and the velocity perpendicular to the magnetic field and the steady flow gradient) of different magnetic surfaces will be uncorrelated with each other. This is phase mixing of the Alfvén waves in a medium with an inhomogeneous steady flow. As in the classical case of phase mixing at an Alfvén speed inhomogeneity, phase mixing in the inhomogeneous steady flow leads to a secular growth of transversal gradients in the Alfvén waves. These gradients increase dissipation of the wave and may nonlinearly generate an obliquely propagating fast magnetosonic wave with frequency and wavenumber that is twice that of the Alfvèn wave.

This problem was first considered by Ryutova & Habbal (1995), who examined the case of cylindrical geometry and restricted their attention to the dynamics of Alfvén waves only. They have shown that inhomogeneous steady flows can change the efficiency of plasma heating by phase mixing. In contrast to their work, we consider here the case of planar geometry and a compressible plasma. Consideration of the planar case allows us to emphasise the main features of wave-flow interaction connected with the inhomogeneity of the flow, using a simple analytical approach. Also, a planar geometry allows us to compare our results directly with these of Heyvaerts & Priest (1983) for the static case. The introduction of compressible effects brings our model nearer to realistic conditions in the solar corona. As in the static case (Nakariakov, Roberts & Murawski 1997), compressibility changes strongly the effect of Alfvén wave phase mixing.

In the next section, the governing set of equations and the geometry of the problem considered are discussed. In the third section, phase mixing of an Alfvén wave in a medium with an inhomogeneous steady flow of matter is considered. In the fourth section, the dynamics of fast magnetosonic waves in the presence of the inhomogeneous steady flow and Alfvén wave phase mixing is considered. In the fifth section, we introduce the numerical methods used for illustration of the analytical results obtained in Sects. 3 and 4. Numerical results are discussed in the sixth section. The last section contains a summary of the results obtained and our conclusions.

2. Governing equations

We consider the dynamics of MHD perturbations of the following stationary state. The ambient magnetic field $B_0\mathbf{e}_z$ is taken to be uniform and directed along the z-axis, along which there is a steady flow $U_0(x)\mathbf{e}_z$ inhomogeneous in the x-direction. The stationary plasma density $\rho_0(x)$ is also inhomogeneous in the x-direction. We consider perturbations with $\partial/\partial y = 0$, but the y-components of the perturbed flow $\mathbf{V}$ and the magnetic field $\mathbf{B}$ will be taken into account. The plasma configuration considered is homogeneous in the y-coordinate, and this allows as to consider the effects of phase mixing; thus the condition of phase mixing, discussed by Parker (1991), is fulfilled.

The governing set of equations is that of cold (plasma $\beta = 0$) dissipative magnetohydrodynamics,

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right] =$$

$$-\frac{1}{4\pi}\mathbf{B}\times \mathrm{curl}\ \mathbf{B} + \rho\nu\left[\nabla^2\mathbf{V} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{V})\right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{curl}\ (\mathbf{V}\times\mathbf{B}), \qquad \mathrm{div}\ \mathbf{B} = 0, \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\mathbf{V}) = 0,$$

where ν is the kinematic viscosity. In the following consideration, the dissipation is supposed to be weak; since our aim is to analyse the effects of the dissipation only qualitatively, we neglect the resistivity of the plasma but retain viscosity. The viscosity of the plasma can be connected, for example, with plasma turbulence.

Considering small perturbations of the stationary state, one obtains

$$\rho_0(x)\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)V_x+\frac{B_0}{4\pi}\left(\frac{\partial B_z}{\partial x}-\frac{\partial B_x}{\partial z}\right)=$$
$$\rho_0(x)\nu\left[\left(\frac{4}{3}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)V_x+\frac{1}{3}\frac{\partial^2 V_z}{\partial x\partial z}\right]-\frac{1}{4\pi}B_y\frac{\partial B_y}{\partial x}, \quad (2)$$

$$\rho_0(x)\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)V_y-\frac{B_0}{4\pi}\frac{\partial B_y}{\partial z}=$$
$$\rho_0(x)\nu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)V_y, \quad (3)$$

$$\rho_0(x)\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)V_z+\rho_0(x)\frac{dU_0}{dx}V_x=$$
$$\nu\frac{d^2U_0(x)}{dx^2}\rho+\rho_0(x)\nu\left[\left(\frac{\partial^2}{\partial x^2}+\frac{4}{3}\frac{\partial^2}{\partial z^2}\right)V_x+\frac{1}{3}\frac{\partial^2 V_x}{\partial x\partial z}\right]$$
$$-\frac{1}{4\pi}B_y\frac{\partial B_y}{\partial z}, \quad (4)$$

$$\frac{\partial B_x}{\partial t}+U_0(x)\frac{\partial B_x}{\partial z}-B_0\frac{\partial V_x}{\partial z}=0, \quad (5)$$

$$\frac{\partial B_y}{\partial t}+U_0(x)\frac{\partial B_y}{\partial z}-B_0\frac{\partial V_y}{\partial z}=0, \quad (6)$$

$$\frac{\partial B_z}{\partial t}-\frac{\partial}{\partial x}\left(U_0(x)B_x\right)+B_0\frac{\partial V_x}{\partial x}=0, \quad (7)$$

$$\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)\rho+\frac{\partial}{\partial x}\left(\rho_0(x)V_x\right)+\rho_0(x)\frac{\partial V_z}{\partial z}=0. \quad (8)$$

We have neglected all nonlinear terms except those in the x- and z-components of the momentum equation (Eqs. (2) and (4)). The terms retained are responsible for the nonlinear generation of fast waves. This occurs through the development of transversal and longitudinal gradients in the magnetic pressure, in the Alfvèn wave (see Nakariakov & Oraevsky 1995; Nakariakov, Roberts & Murawski 1997; Nakariakov, Roberts & Petrukhin 1997 for details of the treatment of the nonlinear terms). The omitted quadratic nonlinear terms are shown in the Appendix.

Eqs. (2)-(8) form two linearly decoupled subsystems, which describe two different waves. Eqs. (2), (4), (5), (7) and (8) describe the dynamics of variables V_x, V_z, B_x, B_z and ρ in a fast magnetosonic wave; Eqs. (3) and (6) describe an Alfvén wave perturbing variables V_y and B_y. Note that, according to Eq. (4), the fast wave perturbs also the plasma in the longitudinal direction (V_z). The linear perturbations of the plasma in the longitudinal direction by the fast wave are proportional to the transversal gradient in the steady flow speed, $dU_0(x)/dx$. Note that in the homogeneous case (U_0 = const) the fast wave perturbs the plasma in the longitudinal direction only nonlinearly (Nakariakov, Roberts & Murawski 1997).

We note also that the Alfvén waves are linearly decoupled from the fast magnetoacoustic waves only in the plane (x, z), formed by the gradient of density and flow speed inhomogeneities and the direction of the magnetic field. Dependence on the y-coordinate, neglected in this paper, leads to linear coupling of the Alfvén and fast waves and their dynamics becomes more complicated.

3. The Alfvén wave

Eqs. (3) and (6) can be combined into one second order equation for V_y,

$$\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)^2V_y-C_A^2(x)\frac{\partial^2}{\partial z^2}V_y=$$
$$\nu\left(\frac{\partial}{\partial t}+U_0(x)\frac{\partial}{\partial z}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)V_y, \quad (9)$$

where $C_A(x)=B_0/(4\pi\rho_0(x))^{1/2}$ is the local Alfvén velocity. The perturbation B_y can be expressed through V_y from either Eq. (3) or (6). One can rewrite the left handside of (9) as

$$\mathrm{LHS}(9)=\left[\frac{\partial}{\partial t}+(U_0(x)-C_A(x))\frac{\partial}{\partial z}\right]$$
$$\left[\frac{\partial}{\partial t}+(U_0(x)+C_A(x))\frac{\partial}{\partial z}\right]V_y. \quad (10)$$

Now, the left handside of (9) describes two waves propagating with speeds $U_0(x)\pm C_A(x)$. When $U_0(x)=0$, the waves are symmetrically forward and backward propagating waves. In the presence of the steady flow, these waves are shifted by the flow. When $|U_0(x)|$ exceeds $C_A(x)$, both waves propagate in the same direction. Note that, due to the transversal inhomogeneity of the steady flow, backward waves can propagate in both positive and negative directions, simultaneously, on different layers. From the point of view of an initial value problem, the left handside of (9) describes D'Alembert's splitting of an initial perturbation.

When the viscosity ν is absent, harmonical perturbations proportional to $\exp(i\omega t-ikz)$ give the dispersion relation for the Alfvén wave,

$$\omega=(U_0(x)\pm C_A(x))\,k. \quad (11)$$

Eq. (11) shows that the Alfvén waves propagate on different magnetic surfaces (corresponding to different values of x) with different phase velocities. This is the effect of phase mixing. When $U_0(x)=0$, Eqs. (9) and (11) lead to the results of Heyvaerts & Priest (1983). When $dU_0(x)/dx\neq 0$, the inhomogeneity in the steady flow leads to phase mixing even if the plasma is otherwise homogeneous (so that $C_A(x)$ is constant).

In the absence of viscosity, a solution of (9) for $C_A\neq 0$ can be presented in the form

$$V_y=\Psi_L(x)f\left[z-(U_0(x)-C_A(x))\,t\right]+$$
$$\Psi_R(x)g\left[z-(U_0(x)+C_A(x))\,t\right], \quad (12)$$

where Ψ_L, Ψ_R, f and g are arbitrary functions. In the absence of a steady flow, the terms on the right handside of (12) correspond to waves propagating in negative and positive directions of z, respectively. The variable B_y has a similar form to (12).

Fig. 1 illustrates the temporal development of an initially plane harmonical Alfvén wave propagating in a medium with an inhomogeneity in the steady flow speed. The flow profile $U_0(x)$ has a maximum gradient in the vicinity of $x = 0$. The inhomogeneity leads to the inclination of the wave front and to phase mixing beginning in the vicinity of the region with sharpest flow speed gradients.

We may estimate the damping of an Alfvén wave with given frequency ω. The local wavenumber $k(x)$ is defined by dispersion relation (11). The development of phase mixing leads to the appearance of sharp transversal gradients in the wave, thus a characteristic spatial scale becomes to be larger than a wavelength $\lambda = 2\pi/k(x)$. Thus, one can neglect the second derivative in z on the right handside of Eq. (9) with respect to the second derivative in x. According to Heyvaerts & Priest (1983), we look for a solution of (9) as

$$V_y(z,x,t) = V(\mu z)\exp(i\omega t - ik(x)z), \tag{13}$$

where μ is a small positive parameter showing how slowly the wave amplitude is changed by the viscosity ν. Expression (13) allows us to examine the damping of the wave as a function of z. The variation of the amplitude is assumed to be a slowing varying function with respect to the oscillatory factor, which can take place when the dissipation is sufficiently weak (the wave amplitude is not strongly changed in a wave period).

Substituting expression (13) in Eq. (9) and collecting terms of the same order, we obtain (in the zeroth order) the dispersion relation

$$\omega^2 - 2\omega k(x)U_0(x) + \left(U_0^2(x) - C_A^2(x)\right)k^2(x) = 0, \tag{14}$$

with solution

$$k(x) = \frac{\omega}{U_0(x) \pm C_A(x)} \tag{15}$$

for given frequency ω.

To first order in μ, we have

$$2i\omega C_A(x)\frac{dV}{dz} = \nu\left\{-iU_0(x)\frac{d^2k}{dx^2} + \right.$$
$$+\left[\frac{\omega C_A(x)}{U_0(x)\pm C_A(x)}\frac{d^2k}{dx^2} - 2U_0(x)\left(\frac{dk}{dx}\right)^2\right]z -$$
$$\left.\frac{i\omega C_A(x)}{U_0(x)\pm C_A(x)}\left(\frac{dk}{dx}\right)^2 z^2\right\}V. \tag{16}$$

Eq. (16) is a linear homogeneous differential equation of the first order and can be solved easily. We restrict attention to a consideration of the behaviour of the Alfvén wave amplitude for large z, when the last term on the right handside of (16) is dominant and Eq. (16) reduces to

$$\frac{dV}{dz} = -\frac{\nu}{2(U_0(x)\pm C_A(x))}\left(\frac{dk(x)}{dx}\right)^2 z^2 V. \tag{17}$$

Eq. (17) yields

$$V(z) =$$
$$V(0)\exp\left\{-\frac{\nu}{6(U_0(x)\pm C_A(x))}\left(\frac{dk(x)}{dx}\right)^2 z^3\right\}. \tag{18}$$

Expression (18) coincides with the formula obtained by Heyvaerts & Priest (1983) when $U_0 = 0$, and is a planar analogue of the formula obtained by Ryutova and Habbal (1995).

According to (18), forward and backward propagating waves are affected by dissipation in different ways. When the steady flow is absent, forward and backward propagating waves dissipate equally. Note that for the backward wave the sign in the exponent becomes positive, but the wave propagates in the negative direction, so formula (18) describes wave decay. Suppose $U(x)$ is positive and less than $C_A(x)$ for all x. In this case, the forward wave dissipates more slowly than the backward wave. If $U_0(x)$ equals $C_A(x)$ for some x, then a resonance takes place and the wave energy dissipates most effectively (but we cannot use Eq. (18) for the description of this phenomenon, because the equation has been derived under the assumption of weak dissipation).

For typical phenomena in the solar corona, $|U_0(x)| \ll |C_A(x)|$ for any x, and one can rewrite solution (18), taking into account dispersion relation (15), as

$$V(z) =$$
$$V(0)\exp\left\{-\frac{\nu\omega^2}{6C_A^5(x)}\left[\frac{d}{dx}\left(U_0(x)\pm C_A(x)\right)\right]^2 z^3\right\}. \tag{19}$$

When the transversal spatial scale of the flow inhomogeneity is much less than the transversal spatial scale of the Alfvén velocity, formula (19) reduces to

$$V(z) = V(0)\exp\left\{-\frac{\nu\omega^2}{6C_A^5(x)}\left[\frac{d}{dx}\left(U_0(x)\right)\right]^2 z^3\right\}. \tag{20}$$

In this case, the efficiency of dissipation is independent of the direction of Alfvén wave propagation.

4. The fast magnetosonic wave

Eqs. (2), (5) and (7) can be combined into one second order differential equation for B_x,

$$\left(\frac{\partial}{\partial t} + U_0(x)\frac{\partial}{\partial z}\right)^2 B_x - C_A^2(x)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)B_x =$$
$$\nu B_0\frac{\partial}{\partial z}\left[\left(\frac{4}{3}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)V_x + \frac{1}{3}\frac{\partial^2 V_z}{\partial x\partial z}\right] -$$
$$\frac{B_0}{8\pi\rho_0(x)}\frac{\partial^2 B_y^2}{\partial x\partial z}. \tag{21}$$

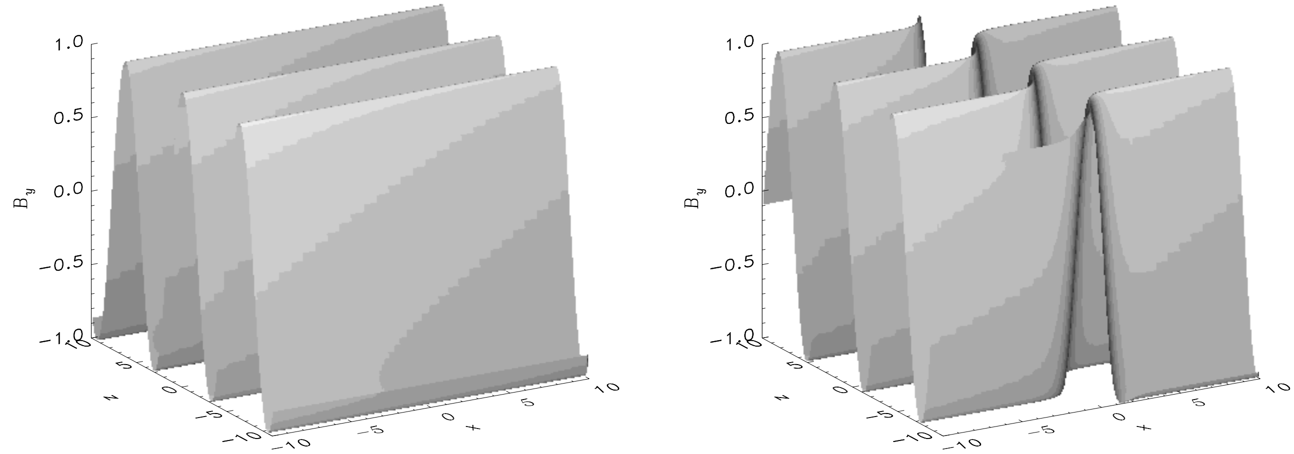


Fig. 1a and b. An initially plane, forwardly propagating, harmonical Alfvén wave $B_y = \cos(z - (U_0(x) + 1)t)$ at **a** (left panel) $t = 0$ and its development at **b** (right panel) $t = 2$. The inhomogeneous steady flow is $U_0(x) = 1 + \tanh(x)$. All variables are dimensionless. Note the phase mixing near $x = 0$.

For harmonical perturbations and in the absence of the Alfvén wave, $B_y = 0$, Eq. (21) reduces to

$$\frac{d^2B_x}{dx^2} + \left[\frac{(\omega - U_0(x)k)^2}{C_A^2(x)} - k^2\right] B_x = ik\frac{\nu 4\pi\rho_0(x)}{B_0}\left[\left(\frac{4}{3}\frac{d^2}{dx^2} - k^2\right)V_x - \frac{ik}{3}\frac{dV_z}{dx}\right]. \quad (22)$$

Consider the propagation of fast waves, described by Eq. (22), in the absence of dissipation,

$$\frac{d^2B_x}{dx^2} + \left[\frac{(\omega - U_0(x)k)^2}{C_A^2(x)} - k^2\right] B_x = 0. \quad (23)$$

In the equilibrium state $U_0 = 0$, the properties of Eq. (23) are well-known (Edwin & Roberts 1988, Roberts 1991, Nakariakov & Roberts 1995b). In particular, a wave front turns always to regions with smaller local Alfvén speed. Consequently, the energy of fast waves is gathered in such regions, which are fast magnetosonic waveguides.

When the Alfvén speed is constant, $C_A(x) = C_A$, Eq. (23) can be re-written as

$$\frac{d^2B_x}{dx^2} - m^2(v_p, k, x)B_x = 0, \quad (24)$$

where

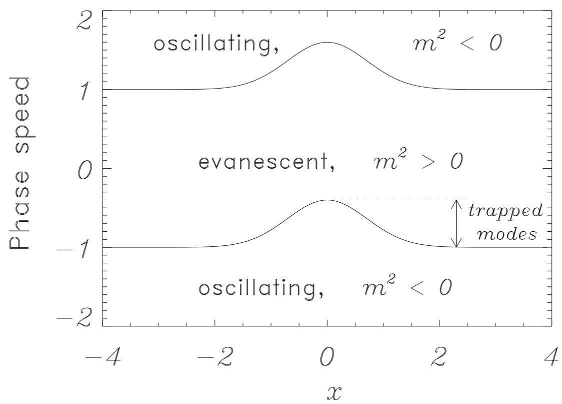
$$m^2 = k^2\left[1 - \left(v_p - M(x)\right)^2\right]; \quad (25)$$

$v_p = (\omega/k)/C_A$ is the phase speed measured in units of the Alfvén speed and $M(x) = U_0(x)/C_A$ is the local Alfvénic Mach number. When $m^2 > 0$, Eq. (24) describes evanescent modes. The growing solution has to be excluded because, in the present consideration, we restrict ourself by taking $|M(x)| < 1$ everywhere; therefore the considered model is always stable with respect to the Kelvin - Helmholtz instability. When $m^2 < 0$, Eq. (24) describes oscillating modes. In a profile of the steady flow speed $U_0(x)$, there is the possibility for the existence of trapped waves having an oscillating structure inside some region and evanescent outside that region. The phase speed of such waves is located in a certain band defined by the shape of the steady flow speed profile. In Fig. 2, we sketch for two different cases of symmetrical profiles of the steady flow speed, namely for a profile with a maximum in $U_0(x)$ and one with a minimum, the existence intervals for trapped fast waves. Under our assumption that $|M(x)| < 1$ everywhere, the profile with the maximum is a waveguide for backward propagating fast magnetosonic waves, and the profile with the minimum is a waveguide for forward propagating fast waves. The physical reason for this phenomenon is the inclination of the front of the fast wave by the steady flow inhomogeneity. Consequently, if the steady flow speed profile contains a minimum or a maximum it becomes a refractive waveguide for either forward or backward propagating fast magnetosonic waves.

In the general case, when inhomogeneities in both the Alfvén speed and steady flow speed occur, refraction of the fast magnetoacoustic waves and, in particular, the possibility for the existence of trapped fast waves is defined by a competition of the two inhomogeneities. For example, trapped waves may exist in a region between non-coinciding and oppositely directed gradients in the local Alfvén speed and steady flow speed. Mathematically, if both $M(x)$ and $C_A(x)$ are monotonic functions, one growing and the other decreasing, then trapped fast magnetosonic waves may exist in the vicinity of a spatial minimum of the function $m^2(x)$.

The last term on the right handside of (21) is responsible for the nonlinear generation of fast waves by gradients in the total pressure in the Alfvén wave. According to (12), in the absence of dissipation, the temporal and spatial evolution of an initially plane Alfvén pulse is described by

$$B_y = A\ g(\xi), \quad \xi = z - (U_0(x) + C_A(x))\,t, \quad (26)$$



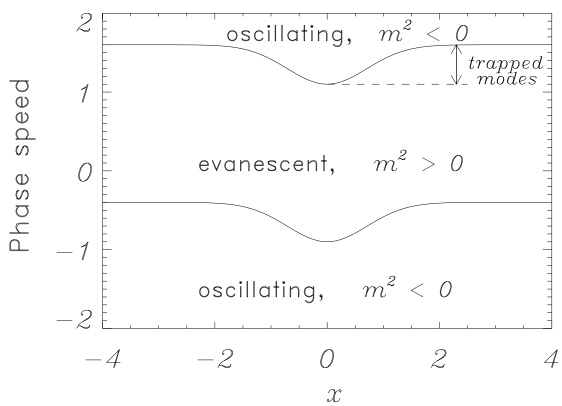


Fig. 2a and b. Sketches of the existence intervals for fast magnetosonic waves trapped by an inhomogeneous steady flow $U_0(x)$ with symmetric profile localised near $x = 0$, containing either **a** (upper panel) a maximum in the flow speed or **b** (lower panel) a minimum in the flow speed. Solid lines separate regions with different signs of the parameter $m^2(v_p, k, x)$, for a fixed wavenumber k. For given phase velocity v_p (measured in units of the Alfvén speed C_A), trapped modes have oscillating ($m^2 < 0$) structure inside the flow inhomogeneity and are evanescent ($m^2 > 0$) outside the inhomogeneity. Arrows show existence intervals in the phase speed. The absolute value of the flow speed $U_0(x)$ is everywhere taken to be less than the Alfvén speed C_A.

where we have taken, for simplicity, $f = 0$ and $\Psi_R(x) = 1$, and A is the Alfvén pulse amplitude. In this case, the forcing term on the right handside of (21) becomes

$$RHS(21) = \frac{B_0}{4\pi\rho_0(x)}\frac{d}{dx}\left[U_0(x) + C_A(x)\right]$$
$$\left[\left(\frac{dg(\xi)}{d\xi}\right)^2 + g(\xi)\frac{d^2g(\xi)}{d\xi^2}\right] A^2t. \quad (27)$$

Thus the right handside grows secularly in time. The rate of growth is defined by the initial parameters of the Alfvén pulse, the amplitude A and the shape of the profile $g(\xi)$, as well as the transversal steepness of the profiles of the steady flow velocity $U_0(x)$ and the Alfvén speed $C_A(x)$. In a homogeneous medium, the secular growth is absent. Secular growth is also absent when the profiles of the steady flow velocity and the Alfvén speed are such that $U_0(x) + C_A(x) = \text{const}$. However, in this case the other solution of Eq. (9), corresponding to a wave propagating in the opposite direction and described by the function f, gives a secularly growing forced term on the right handside of Eq. (21). Consequently, if there are Alfvén waves propagating in both directions, the nonlinear excitation of the fast waves takes place for any profiles of the Alfvén and steady flow speeds.

Consider the harmonical Alfvén wave

$$B_y = A\cos\Theta, \quad \Theta = k\left[z - (U_0(x) + C_A(x))t\right]. \quad (28)$$

Then, the right handside of (21) is

$$RHS(21) =$$
$$-\frac{B_0}{4\pi\rho_0(x)}k^2A^2t\frac{d}{dx}\left[U_0(x) + C_A(x)\right]\cos 2\Theta. \quad (29)$$

Thus, the response of the fast wave equation to the Alfvén forcing term on the right handside of Eq. (21) is on the *second* harmonics of the Alfvén wave, so the forced fast magnetosonic oscillations have frequency and wavenumber *double* that of the Alfvén wave.

When the initial shape of the Alfvén pulse (26) is given and fast magnetosonic perturbations are initially absent from the system, so that

$$B_x(x, z, t = 0) = 0, \quad \frac{\partial B_x(x, z, t = 0)}{\partial t} = 0, \quad (30)$$

then Eq. (21) for the fast magnetosonic waves contains a given source term on the right handside. A forced solution of this equation can be expressed in terms of retarded potentials. To illustrate this, we consider the case $\nu = 0$, $C_A = \text{const}$ and $|U_0(x)| \ll C_A$, when Eq. (21) becomes the usual wave equation in a homogeneous medium, with given source term on the right handside,

$$\left[\frac{\partial^2}{\partial t^2} - C_A^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\right]B_x =$$
$$\frac{B_0}{4\pi\rho_0(x)}\frac{d}{dx}U_0(x)\left[\left(\frac{dg(\xi)}{d\xi}\right)^2 + g(\xi)\frac{d^2g(\xi)}{d\xi^2}\right]A^2t. \quad (31)$$

The forced solution of (31) supplemented by initial conditions (30) is

$$B_x(x, z, t) =$$
$$\frac{1}{2\pi C_A}\int_0^t\int_\Sigma\int\frac{p(\tilde{x}, \tilde{z}, \tau)\, d\tilde{x}d\tilde{z}d\tau}{\sqrt{C_A^2(t-\tau)^2 - (x-\tilde{x})^2 - (z-\tilde{z})^2}}, \quad (32)$$

where

$$p(\tilde{x}, \tilde{z}, \tau) =$$
$$\frac{B_0}{4\pi\rho_0(\tilde{x})}\frac{d}{d\tilde{x}}U_0(\tilde{x})\left[\left(\frac{dg(\tilde{\xi})}{d\tilde{\xi}}\right)^2 + g(\tilde{\xi})\frac{d^2g(\tilde{\xi})}{d\tilde{\xi}^2}\right]A^2\tau,$$

with $\tilde{\xi} = \tilde{z} - C_A\tau$, and Σ is the region of $(\tilde{x}, \tilde{z}, \tau)$-space defined by the inequalities

$$0 \le \tau \le t, \quad (\tilde{x} - x)^2 + (\tilde{z} - z)^2 \le C_A^2(\tau - t)^2.$$

Formula (32) allows us to calculate the magnetic field component B_x, excited by development of an initially plane Alfvén wave described by formula (26) at t = 0. Perturbations of other physical variables can be found from Eqs. (2) - (8) for known $B_y(x, z, t)$ and $B_z(x, z, t)$. The dynamics of the fast wave perturbations generated by Alfvén wave phase mixing in a medium without steady flow, but with a sharp inhomogeneity in the Alfvén speed, is described by an analogous formula (Nakariakov, Roberts & Murawski 1997). (The two formulae differ from each other through expressions for p, corresponding to the different mechanisms of phase mixing.)

The presence of secularity shows us that fast magnetosonic waves may be effectively generated by Alfvén wave phase mixing even if the amplitude of the Alfvén wave is small, because the nonlinear term on the right handside of Eq. (21) is inversely proportional to the transversal spatial scale of the Alfvén wave, which tends to zero during the development of phase mixing.

The secular excitation of the fast waves takes place only in the initial stage of nonlinear Alfvén - fast wave coupling. When the fast wave reaches sufficient amplitude, the secular growth of its amplitude saturates as for any other instability. The saturation is connected with either dissipation or nonlinear back-reaction of the fast waves on Alfvén waves. Consequently, our theory describes only the initial stages of Alfvén wave - fast wave coupling.

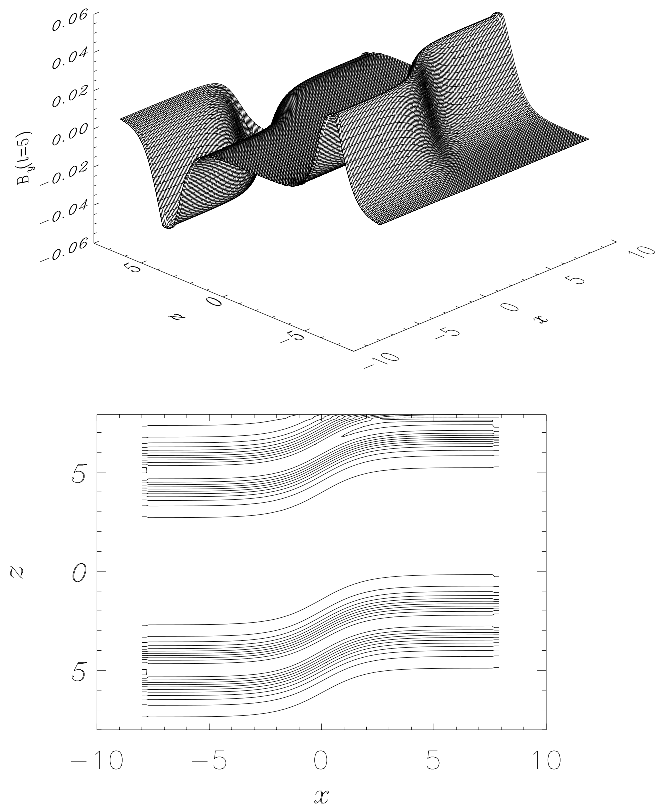


Fig. 3a and b. Spatial distribution at time $t = 5a/C_A$ of the Alfvén wave propagating in a homogeneous plasma with a field-aligned shear flow. **a** (upper panel) surface plot, **b** (lower panel) contours plot. Note that as a consequence of the lack of the flow in the semi-plane $x < 0$, the upwards and downwards propagating waves are symmetrical with respect to the line z = 0. For $x > 0$, the upwards wave is convected by the flow but the downwards propagating wave propagates up-wind of this flow. Consequently, the upwards wave travels in the same time a greater distance than the downwards wave.

5. Numerical aspects

5.1. The numerical scheme

In finite difference methods all spatial derivatives are replaced by their finite difference counterparts. For example, a centred Euler's scheme takes

$$\frac{\partial u}{\partial x} = \frac{u(x + \Delta x) - u(x - \Delta x)}{2\Delta x}.$$

In an ordinary finite difference method, numerically generated local extrema of the function $u(x)$ are pronounced near steep profiles and are a consequence of a dispersion which is particularly enhanced at short wavelengths and as a result of the Gibbs phenomenon. The Gibbs phenomenon, in turn, is caused by a finite numerical grid and appears due to the truncation of the continuous Fourier spectrum. As a result of the truncation, the solution exhibits short ripples. These ripples can lead to an unphysical negative mass density and are undesirable for an adequate representation of this quantity. The flux corrected transport (FCT) method filters these extrema by introducing a diffusion which smooths local oscillations by adding a large amount of numerical diffusion in the neighbourhood of large gradients (Boris & Book 1973). Subsequently, in the anti-diffusion step the known amount of diffusion is removed. At this stage the anti-diffusive fluxes are multiplied by a factor which should not create new extrema or accentuate old extrema. These corrected fluxes remove ripples associated with dispersion and with the Gibbs phenomenon. In consequence, the FCT method is about one-order of magnitude more accurate than other commonly applied methods. Moreover, the FCT method eliminates nonlinear instabilities which reveal themselves in long numerical sessions with some standard algorithms. An advantage of this method over the fast Fourier transform method is it provides an easy way of implementing the boundary conditions. The FCT method is also universal and easily applicable to complex systems. The FCT method suffers from few deficiencies, but one undesirable feature is that in the neighbourhood of extrema of the function $u(x)$ the FCT method leaves some diffusion which eventually replaces a one-point extremum by a few-points plateau. Although the method is very accurate for large gradients, it is less accurate than the fast Fourier transform method for periodic functions. Its detailed description can be found in DeVore (1991). A modified version of this code was recently used by Murawski et al. (1996) to simulate resonant absorption of nonlinear Alfvén waves.

Eqs. (1) are solved numerically using the flux corrected transport method (Boris & Book 1973; Zalesak 1979; Murawski & Goossens 1994). The applied numerical algorithm is a modification of that developed originally by DeVore (1989, 1991). The main modifications consist of an extension of the 2D Cartesian geometry to the 2.5D case.

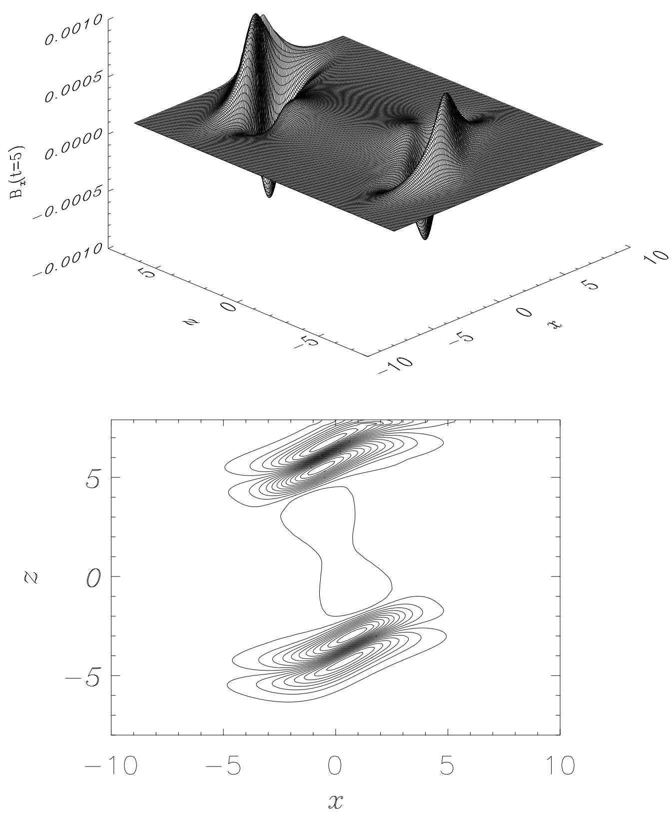


Fig. 4a and b. Spatial distribution at time $t = 5a/C_A$ of the fast wave driven by the Alfvén wave. Note that the wave-front of this wave is oblique to the z-direction. **a** (upper panel) surface plot, **b** (lower panel) contour plot.

Eqs. (1) are solved numerically on an $x-z$ Eulerian box with the dimension $(-8a, 8a) \times (-8a, 8a)$, where a is a characteristic spatial scale. For all numerical runs 160 grid points were used in both directions. All boundaries of the numerical box are set free to represent the natural extension of the medium outside the computational domain.

5.2. Numerical results

To illustrate the analytical results presented above, we consider a homogeneous plasma with constant equilibrium mass density ρ_0, magnetic field $\mathbf{B} = B_0\mathbf{e}_z$, and inhomogeneous flow $U_0(x)\mathbf{e}$ along the z-axis with

$$U_0(x) = U_{00}\frac{\exp x}{1 + \exp x}; \tag{33}$$

we take $U_{00} = 0.1\ C_A$. Spatial coordinates are non-dimensionalized against the characteristic scale a. The flow is zero at $x \to -\infty$, growing to U_{00} as $x \to +\infty$. The flow is sustained by an inflow at the lower boundary set to $z = -8a$. The stationary flow is perturbed by an Alfvén wave launched initially by perturbing the transversal component of the velocity, *viz.*

$$V_y(x, z, t = 0) = 0.1U_{00}\mathrm{sech}^2 z. \tag{34}$$

This initial pulse evolves in time, splitting into up- and down-going waves, in agreement with formula (10). Since for $x < 0$ the stationary flow is virtually absent, the upwards and downwards propagating waves pass the same distance in the same period of time. However, in $x > 0$, the presence of a stationary flow means that the upwards wave travels a greater distance than the downwards wave. While the upwards wave is advected by the stationary flow, the downwards wave moves up-wind of this flow. This effect is clearly seen in Fig. 3 which displays the spatial profile of $B_y(x, z)$ at time $t = 5\ a/C_A$. At this moment the upwards wave in $x < 0$ reaches the top boundary of the simulation region.

As a consequence of the non-zero gradient of the stationary flow, the wave-front of the Alfvén wave bends at $x = 0$ and large spatial gradients are built up there. These gradients grow in time, allowing the Alfvén waves to phase mix and subsequently dissipate effectively by viscosity.

Eq. (21) shows that the initially absent fast wave is driven by the gradient of the magnetic pressure associated with the Alfvén wave. Fig. 4 shows such a wave at $t = 5\ a/C_A$. According to the discussion in Sect. 4, the wave front of the fast wave is oblique to the vertical z-direction. As B_x is driven by the term

$$\frac{B_0}{8\pi\rho_0(x)}\frac{\partial^2 B_y^2}{\partial x \partial z},$$

its value is largest in the region with the steepest gradient in the inhomogeneous flow, namely at $x = 0$. The field component B_x assumes low values in regions where $\partial B_y^2/\partial x$ is low. As this derivative is zero outside the inhomogeneous region (see Fig. 3), $B_x = 0$ there.

Since the driving force for the fast magnetosonic wave grows secularly in time (see formula (27)), it is expected that the amplitude of this wave grows in time. Fig. 5 displays the energy of the fast wave as a function of time. The energy is calculated inside the simulation region. The energy grows in time until it reaches a maximum and then it declines as a consequence of the fact that the waves leave the simulation region. Smaller amplitude Alfvén waves lead to smaller amplitude fast waves which propagate slower. Consequently, large amplitude waves reach their maxima earlier than low amplitude waves.

6. Conclusions

We have considered the propagation of magnetohydrodynamic waves in a cold plasma with an inhomogeneous steady flow directed along a straight magnetic field, restricting attention to flow speeds that are less than the Alfvén speed in the plasma. Consequently, the effects of negative energy waves and Kelvin - Helmholtz instability are excluded from our investigation. In regions with transversal gradients in the steady flow, phase mixing of Alfvén waves takes place similarly to classical phase mixing in a static medium with an inhomogeneity in the Alfvén speed. Due to phase mixing, sharp transversal gradients appear in the Alfvén wave and in the magnetic pressure.

The sharp transversal gradients lead to the appearance of very small spatial scales in the Alfvén wave (much less than a wavelength). Since dissipation of the Alfvén wave due to magnetohydrodynamic mechanisms (viscosity and resistivity) is proportional to the inverse spatial scale of the wave, the Alfvén waves are effectively dissipated in those regions with the sharpest transversal gradients in the steady flow. According to the theory developed here, the dissipation of an Alfvén wave with a given frequency, due to phase mixing on the inhomogeneous steady flow, is described by an exp($-z^3$)-law, as in the case of an inhomogeneous but static medium. Our formula is a planar analogue of a result derived by Ryutova and Habbal (1995) for the case of cylindrical geometry. The specific coefficients in this dependence are defined by the parameters of the flow inhomogeneity. Alfvén waves propagating in opposite directions (with and against the flow) have different rates of dissipation, due to the presence of a preferred direction in the medium. In the specific case, when inhomogeneities in both the steady flow and Alfvén speed are present, the inhomogeneities compensate each other and the Alfvén wave propagates without strong dissipation. This effect occurs only for a wave propagating in a one direction; the wave propagating in the opposite direction is dissipated.

A smoothly inhomogeneous steady flow can trap fast magnetosonic waves due to refraction, the inhomogeneous steady flow providing a waveguide for the fast waves. In particular, fast waves propagating with the flow may be trapped in regions of minima in the steady flow speed. Waves propagating against the flow may be trapped in regions with maxima in the flow speed.

Sharp transversal gradients in the magnetic pressure, produced by Alfvén wave phase mixing and consequent Alfvén wave dissipation, are also a source of obliquely propagating fast magnetosonic waves. The fast waves are generated nonlinearly with wavenumbers and frequencies that are double the wavenumbers and frequencies of the source Alfvén wave. This effect increases the efficiency of plasma heating by phase mixing. Furthermore, the oblique fast waves generated by phase mixing spread the dissipated energy across the magnetic field and this is important for the thermodynamics of coronal structures. Based on the mechanism considered, we may expect that regions with steady flows should always be surrounded by radiated fast magnetosonic waves, which occupy a region equal to the dissipation length of the wave.

Since the viscosity of the coronal plasma is extremely small, transversal sizes of matter flows can have very small transversal scales and the boundaries of the flows can be very sharp. Thus, Alfvén wave phase mixing is likely to be more important in dynamical coronal structures than in static structures. Moreover, the physical mechanism responsible for driving the steady flows may simultaneously be a source of Alfvén waves propagating along the flows. The dissipation of Alfvén waves due to phase mixing in dynamical structures can lead to heating of the corona. The heated regions are not only located in the regions with sharpest gradients, because fast magnetosonic waves generated by phase mixing spread the heating over a distance equal to the dissipation length from the boundaries of the coronal dynamical structures. Thus, the presence of inhomogeneous dynamical structures in the corona may give an additional impact to the problem of coronal heating by MHD waves, even if the speed of the flow is much less than the thresholds of negative energy waves instabilities and KHI.

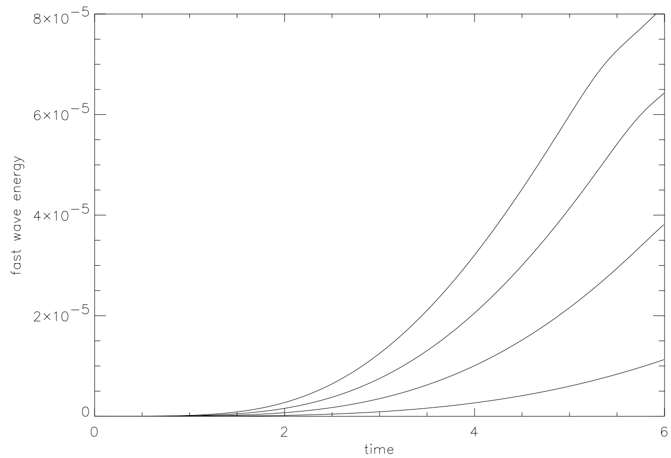


Fig. 5. The kinetic energy of the fast wave as a function of time. The lowest curve corresponds to the initial Alfvén pulse with the amplitude $0.1\ C_A$. The upper curves correspond to $0.2\ C_A$, $0.3\ C_A$, and $0.4\ C_A$, respectively. The energy is calculated in the simulation region only and the fast wave leaves this region after a time $t = 8\ a/C_A$. The kinetic energy is expressed in units $\rho_0 C_A^2 a^2/2$ and the time in the Alfvén transit time, a/C_A.

The generation of magnetosonic waves in phase mixing regions can be used for observational detection of the phenomenon of Alfvén wave phase mixing in the corona. According to the above theory, phase mixing is always accompanied by the appearance of plasma density perturbations in the inhomogeneous regions, and these can be observed as perturbations of intensity.

Also, the secular generation of fast waves by Alfvén wave phase mixing give us a possibility to re-consider some interpretations of coronal phenomena, involving such effects as KHI. In particular, the secular (instability-like) generation of fast waves may lead to the onset of compressible MHD turbulence in inhomogeneous flows with relative shear velocities much less than $2C_A$. The effect considered does not require high-speed shear velocities; only steepness of the flow inhomogeneity is required. The possibility of the onset of MHD turbulence in low-speed flows is interesting for the interpretation of radio-emission from reconnection events and *in situ* observations of MHD perturbations in the solar wind (in particular, in the trailing edges of high-speed streams).

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Appendix A: nonlinearities

The complete expressions for quadratic nonlinear terms in Eqs. (2)-(8) are

$$\mathrm{N}(2) = -\rho \frac{\partial V_x}{\partial t} - \rho_0(x) \left(V_x \frac{\partial}{\partial x} + V_z \frac{\partial}{\partial z} \right) V_x -$$

$$U_0(x)\rho\frac{\partial V_x}{\partial z}+\frac{B_z}{4\pi}\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)-\frac{B_y}{4\pi}\frac{\partial B_y}{\partial x}, \tag{A1}$$

$$\mathrm{N}(3)=-\rho\frac{\partial V_y}{\partial t}-\rho_0(x)\left(V_x\frac{\partial}{\partial x}+V_z\frac{\partial}{\partial z}\right)V_y- U_0(x)\rho\frac{\partial V_y}{\partial z}+\frac{1}{4\pi}\left(B_z\frac{\partial B_y}{\partial z}+B_x\frac{\partial B_y}{\partial x}\right), \tag{A2}$$

$$\mathrm{N}(4)=-\rho\frac{\partial V_z}{\partial t}-\rho_0(x)\left(V_x\frac{\partial}{\partial x}+V_z\frac{\partial}{\partial z}\right)V_z- U_0(x)\rho\frac{\partial V_z}{\partial z}-\frac{dU_0(x)}{dx}\rho V_x- \frac{1}{4\pi}\left[B_x\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)+B_y\frac{\partial B_y}{\partial z}\right], \tag{A3}$$

$$\mathrm{N}(5)=\frac{\partial}{\partial z}(V_xB_z-V_zB_x), \tag{A4}$$

$$\mathrm{N}(6)=\frac{\partial}{\partial z}(V_yB_z-V_zB_y)-\frac{\partial}{\partial x}(V_xB_y-V_yB_x), \tag{A5}$$

$$\mathrm{N}(7)=\frac{\partial}{\partial x}(V_zB_x-V_xB_z), \tag{A6}$$

$$\mathrm{N}(8)=-\frac{\partial}{\partial x}(\rho V_x)-\frac{\partial}{\partial z}(\rho V_z), \tag{A7}$$

where the notation N(i) corresponds to the terms on the right handside of Eq. (i), with i=2,...,8.

A linear Alfvén wave described by Eqs. (3) and (6) perturbs values V_y and B_y, while a linear fast wave described by Eqs. (2), (5) and (7) perturbs values V_x, B_x and B_z and, through Eqs. (4) and (8), values V_z and ρ. Consequently, variables V_y and B_y can be referred to as Alfvén variables, and V_x, V_z, B_x, B_z and ρ as fast magnetoacoustic variables.

The nonlinear terms are not symmetric for fast and Alfvén waves. The nonlinear terms N(3) and N(6) acting on the Alfvén wave are always products of the Alfvén and fast magnetoacoustic variables. The nonlinear terms acting on the fast wave, N(2), N(4), N(5), N(7) and N(8), are products of either two fast magnetoacoustic or two Alfvén variables, and so do not contain products of Alfvén and fast magnetoacoustic variables. Consequently, if the Alfvén wave is initially absent from the system (all the Alfvén wave variables are zero), then it is not excited by the fast wave. A finite amplitude fast wave behaves nonlinearly according to the "fast wave nonlinearity", nonlinear terms containing the products of the fast magnetoacoustic variables. If the fast wave is initially absent from the system (all the fast magnetoacoustic variables are zero), a finite amplitude Alfvén wave propagates linearly, because the nonlinear terms N(3) and N(6) are zero. However, the nonlinear terms N(2) and N(4) are not zero, which leads to the nonlinear generation of the fast wave by the Alfvén wave. As our aim is to investigate this generation, we retain in the governing equations only those nonlinear terms responsible for the effect.

Taking into account cubic nonlinear terms can change this asymmetry, but the cubic nonlinear effects take place either for later times or for higher wave amplitudes, and thus they can be neglected in the investigation of the early stages of nonlinear Alfvén - fast wave coupling.

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