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About statistical-parallax algorithms

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Abstract. The results obtained by two algorithms of statistical parallaxes applied to a large sample of RR Lyrae stars are compared. They show no significant difference and confirm the validity of the earlier approach based on the principle of maximum likelihood.

Key words: RR Lyrae stars – stars: distances

1. Introduction

In a recent paper, Fernley et al. (1997) have studied the absolute magnitude of RR Lyrae stars by using a statistical-parallax algorithm developed by Hawley et al. (1986). This method has been introduced as an improvement over earlier algorithms such as:

- the maximum-likelihood algorithm developed by Clube and Jones and applied to RR Lyrae stars in several studies (Clube & Jones 1971, 1974; Clube & Dawe 1980); this method will be called herafter the CJD algorithm;
- another maximum-likelihood method introduced by Rigal (1958), subsequently improved by Jung (1968, 1970) and generalized by Heck (1975a); this method, called hereafter the RJH algorithm, has been applied to numerous categories of stars (see, for instance, the URL: http://vizier.u-strasbg.fr/∼heck/p-md.htm) including RR Lyrae stars (Heck 1972, 1973, 1975b; Heck & Jung 1975; Heck & Lakaye 1978).

The CJD and RJH algorithms have already been confronted on synthetic data (Jones et al. 1980) and, notwithstanding their different formulations, both gave the correct solution.

The publication of an exhaustive sample of data (including Hipparcos proper motions) for RR Lyrae stars by Fernley et al. (1997) made it interesting to test on it the RJH algorithm and to compare the results with those obtained with the algorithm by Hawley et al. (1986).

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2. The data and the calibrations

The data used here have been exactly those of Fernley et al. (1997), thus guaranteeing the validity of the comparative study. A total of 144 stars had both the proper motions and radial velocities required by the algorithm.

Several calibrations have been performed as described hereafter. The RJH algorithm occasionally rejects stars on the basis of their individual space velocities as not belonging to the parent statistical population.

2.1. Calibrations with individual reddening corrections

1. For a resulting global sample of 143 stars (rejection of RV Cap), $\overline{M}_v = 0.78 \pm 0.13$ with $\sigma_M = 0.13 \pm 0.13$. This result is to be compared with Fernley et al. (1997)'s value of $\overline{M}_v = 0.76 \pm 0.13$;
2. For a resulting sample of 122 RRab stars (rejection of RV Cap), $\overline{M}_v = 0.67 \pm 0.11$ with $\sigma_M = 0.19 \pm 0.13$. There is no equivalent in Fernley et al. (1997).
3. For a resulting sample of 20 RRc stars (rejection of RZ Cep), $\overline{M}_v = 1.03 \pm 0.31$ with $\sigma_M = 0.27 \pm 0.26$. The small size of this sample contributes of course to the poorer precision of the results. There is no equivalent calibration in Fernley et al. (1997).

2.2. Calibrations with an average reddening correction

The average reddening correction applied here has been the cosecant law introduced by Parenago (1940):

$$A_v = 0.14[1 - exp(-0.01r \mid \sin b \mid)] / \mid \sin b \mid$$

where r is the distance (in pc) and b the galactic latitude. There is no equivalent calibration in Fernley et al. (1997).

1. For a resulting global sample of 142 stars (rejection of RV Cap and RZ Cep), $\overline{M}_v = 0.67 \pm 0.12$ with $\sigma_M = 0.15 \pm 0.13$.
2. For a resulting sample of 122 RRab stars (rejection of RV Cap), $\overline{M}_v = 0.56 \pm 0.12$ with $\sigma_M = 0.22 \pm 0.19$.
3. For a resulting sample of 20 RRc stars (rejection of RZ Cep), $\overline{M}_v = 0.90 \pm 0.32$ with $\sigma_M = 0.40 \pm 0.30$. The small size

of this sample contributes of course to the poorer precision of the results.

Compared to the previous one, this set of calibrations confirm that an average correction overestimates slightly the reddening as already pointed out in Heck (1973). It should be avoided as far as possible. Fernley et al. (1997) quite correctly used individual reddening corrections in their study.

2.3. Calibrations according to the metallicity

The total sample has been subdivided according to [Fe/H] following Fernley et al. (1997)'s structuring of their data.

Their intention was to consider two groups of stars, respectively with [Fe/H] $\leq$ -1.3 and [Fe/H] $\geq$ -0.8, but, because of the small size of the latter sample, they splitted their total sample at [Fe/H] = -1.3.

We followed suit, but also calibrated the sample of stars with [Fe/H] $\geq$ -0.8. All the calibrations have been performed with individual corrections of reddening.

1. For a resulting sample of 84 stars (no rejection), $\overline{M}_v = 0.81 \pm 0.15$ with $\sigma_M = 0.32 \pm 0.21$. This is to be compared with $\overline{M}_v = 0.77 \pm 0.17$ obtained by Fernley et al. (1997) for their group of 84 stars with an average [Fe/H] of -1.66.
2. Starting with an initial sample of 60 stars with [Fe/H] $>$ -1.3, the tests on the space velocities performed by the RJH algorithm rejected 18 stars out of which 17 had an [Fe/H] between -1.3 and -0.8, the 18th object being S Ara (see Calibration 2.3.4 below). This is typical of an heterogeneous sample and it is interesting how the RJH algorithm can detect it. For the remaining sample of 42 stars, $\overline{M}_v = 0.91 \pm 0.26$ with $\sigma_M = 0.92 \pm 0.30$. Since the size of the sample decreased, the precision became poorer.
3. Starting with the same sample, but with looser tests on the space velocities (factor 3.5 on the semi-axes instead of 3.0), only RY Col was rejected and, for the remaining sample of 59 stars, $\overline{M}_v = 0.73 \pm 0.32$ with $\sigma_M = 0.70 \pm 0.24$. Clearly here, the larger size of the sample compensates for the dispersion resulting from the heterogeneity, but σ_M remains large. This result is to be compared with the value of $\overline{M}_v = 0.69 \pm 0.21$ given by Fernley et al. (1997) for their group of 60 stars with an average [Fe/H] of -0.85.
4. For a resulting sample of 25 stars (S Ara rejected) with [Fe/H] $\geq$ -0.8, $\overline{M}_v = 0.42 \pm 0.33$ with $\sigma_M = 1.20 \pm 0.35$. This calibration has not been done by Fernley et al. (1997) and the small size of this sample contributes of course to the poorer precision of the results. The trend is however clearly towards a brighter magnitude for these metal-rich RR Lyrae stars. Notice also here that S Ara is not considered as belonging to the parent population (as in Calibration 2.3.2).

3. Comments and conclusion

When performed for the same samples and in the same conditions, the calibrations (2.1.1, 2.3.1 & 2.3.3) give results remarkably close to those by Fernley et al. (1997) and definitely within the error bars.

This contradicts Hawley et al. (1986)'s assertion that the RJH algorithm would be 'afflicted with considerable statistical bias'. They actually put this on Clube & Dawe (1980) whose comment was not quite as strong as that and whose opinion was rectified after a comparative study of performance (Jones et al. 1980) as recalled in Sect. 1.

It is not clear either whether the imprecisions given by Hawley et al. (1986)'s method are imprecisions on the numerical values or dispersions (σ_M) of the parent population, two distinct results with the RJH algorithm which provides also the mean solar motion and the corresponding velocity ellipsoid for each calibration (not discussed here since not provided by Fernley et al. 1997).

The tests automatically carried out by the RJH algorithm on the individual space velocities also easily detect foreign members to a parent statistical population (well examplified in Calibration 2.3.2).

The validity of the earlier RJH approach based on the principle of maximum likelihood is confirmed by the convergence of results with a more recent algorithm.

The critical importance of good data at hand is stressed and, in this respect, the study by Fernley et al. (1997) based on a renewed sample of RR Lyrae stars involving Hipparcos proper motions and individual reddening corrections is definitely an important contribution.

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