

Period changes of δ Scuti stars and stellar evolution

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Abstract. Period changes of δ Scuti stars have been collected or redetermined from the available observations and are compared with values computed from evolutionary models with and without convective core overshooting.

For the radial pulsators of Pop. I, the observations indicate $(1/P) dP/dt$ values around $10^{-7} \text{ year}^{-1}$ with equal distribution between period increases and decreases. The evolutionary models, on the other hand, predict that the vast majority should show increasing periods. This increase should be a factor of about ten times smaller than observed. For nonradial δ Scuti pulsators of Pop. I, the discrepancies are even larger. The behavior suggests that for these relatively unevolved stars the rate of evolution cannot be deduced from the period changes.

The period changes of most Pop. II δ Scuti (SX Phe) stars are characterized by sudden jumps of the order of $\Delta P/P \sim 10^{-6}$. However, at least one star, BL Cam, shows a large, continuous period increase. The variety of observed behavior also seems to exclude an evolutionary origin of the changes.

Model calculations show that the evolutionary period changes of pre-MS δ Scuti stars are a factor of 10 to 100 larger than those of MS stars. Detailed studies of selected pre-MS δ Scuti stars are suggested.

Key words: δ Sct – stars: oscillations – stars: evolution – stars: pre-main sequence – stars: individual: AI Vel

1. Introduction

δ Scuti variables are short-period pulsators situated inside the classical instability strip on and near the main sequence (hereafter called MS). They represent stars in the hydrogen-burning stage with masses from $1.5 M_{\odot}$ to about $2.5 M_{\odot}$ for the more evolved variables.

The majority of δ Scuti stars pulsates with a multitude of nonradial p- and g-modes (often of mixed character) and low amplitudes near the detection limit of 0.5 millimag. A numerically small subgroup has exceptionally low rotational velocities of $v \leq 30 \text{ km s}^{-1}$ with pulsation properties resembling the classical cepheids and RR Lyrae stars: radial pulsation in the fundamental or first overtone mode with high amplitudes between 0.3

and 1 mag. In the past, the stars in this high-amplitude (HADS) subgroup were sometimes referred to as Dwarf Cepheids or AI Vel stars. They are observationally favored for the study of period stability because of large amplitudes and the presence of only a few excited modes (usually one or two).

The period changes caused by stellar evolution in and across the Lower Instability Strip permit an observational test of stellar evolution theory, provided that other physical reasons for period changes can be excluded. The period – mean density relation for pulsating stars predicts a period-luminosity-color-mass relation of the form

$$\log P = -0.3M_{\text{bol}} - 3 \log T_{\text{eff}} - 0.5 \log M + \log Q + 12.708$$

where P is the radial period and M is the stellar mass in solar units. For the δ Scuti variables with known radial pulsation, the observed coefficients are in almost exact agreement with those predicted by the period–mean density relation (Breger 1980).

This confirmation of the theoretical relation is very important: since the period-luminosity-relation holds for stars in different evolutionary stages, it should also hold for a single star at different evolutionary stages. For individual stars, evolutionary period changes must occur, at least over long time scales.

An evolutionary change in T_{eff} and M_{bol} leads to a period change of size

$$\frac{1}{P} \frac{dP}{dt} = -0.69 \frac{dM_{\text{bol}}}{dt} - \frac{3}{T_{\text{eff}}} \frac{dT_{\text{eff}}}{dt} + \frac{1}{Q} \frac{dQ}{dt},$$

where the coefficient of 0.69 has been derived by multiplying the coefficient of 0.3 by $\ln 10$ from the differentiation.

In the Lower Instability Strip, where the δ Scuti stars are found, stellar evolution leads to increasing periods in the vast majority of stars, with predicted increases of $(1/P)dP/dt$ from $10^{-10} \text{ year}^{-1}$ on the MS to $10^{-7} \text{ year}^{-1}$ for the longer-period evolved variables. (More exact values will be computed in a later section of this paper.) Such period changes should be observable.

There have been several comparisons between theoretical and observed period changes. The careful compilation by Percy et al. (1980) led to inconclusive results. They state that ‘the study of period changes has had a long and sometimes dubious history’ and find that ‘in most cases is a source of confusion and frustration.’

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Breger (1990a) had more data at his disposal and noted that for the four evolved δ Scuti stars of Pop. I the observed period decreases were in contradiction to the radius increases predicted from stellar evolution. Guzik & Cox (1991) have examined possible explanations for the period decreases seen in evolved δ Scuti stars. While their models indeed predicted smaller period increases, period decreases were not found. The fact that all four evolved δ Scuti stars show period decreases may conceivably be a reflection of an additional period changing mechanism (leading to random changes) coupled with small-number statistics. The recent results for the star AI Vel (Walraven et al. 1992) support such a view. They have found that only one of the two radial frequencies shows a period increase, while the second frequency was essentially constant. The example of AI Vel shows that (slow) stellar evolution is not the only mechanism to generate changes in the periods of pulsation. This means that for an individual star the conversion of observed period changes into stellar evolution rates (e.g. radius changes) has to be applied with caution. The nonevolutionary period changes should cancel out only for a larger group of stars in the same stage of evolution.

An important update and analysis was provided by Rodriguez et al. (1995), who confirmed the observed period decreases for evolved δ Scuti stars. They also found relatively large period increases in excess of the expected values for the less evolved Pop. I δ Scuti variables and decreasing periods for Pop. II variables. Furthermore, their study provided further evidence for the earlier suspicions that the observed period changes are not caused by evolution alone.

The present paper examines period changes in δ Scuti stars from a theoretical point of view, provides a review of the observational status, and compares the two.

2. Predicted period changes due to stellar evolution

2.1. Standard evolutionary models and their oscillations

The models of $1.5\text{--}2.5 M_{\odot}$ in MS and post-MS evolutionary stages were constructed using a standard stellar evolution code which was developed in its main parts by B. Paczyński, M. Kozłowski and R. Sienkiewicz (private communication). The same code was used in our recent studies of FG Virginis (Breger et al. 1995) and XX Pyxidis (Handler et al. 1997a,b; Pamyatnykh et al. 1997). The computations were performed starting from chemically uniform models on the Zero-Age Main Sequence (ZAMS), assuming an initial hydrogen abundance $X = 0.70$ and a heavy element abundance $Z = 0.02$. The standard mixing-length theory of convection with the mixing-length parameter $\alpha = 1.0$ was used. The choice of the mixing-length parameter has only a small effect on our models because they are too hot to have an effective energy transfer by convection in the stellar envelope. For the opacities, we used latest version of the OPAL tables (Iglesias & Rogers 1996) supplemented with the low-temperature data of Alexander & Ferguson (1994). In all computations the OPAL equation of state was used (Rogers et al. 1996).

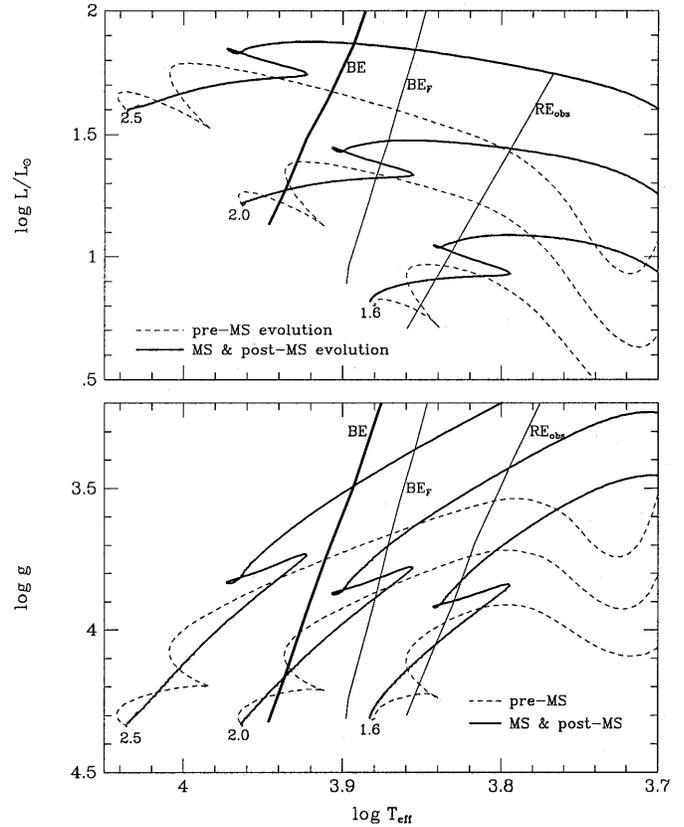


Fig. 1. Evolutionary tracks of 1.6 , 2.0 and $2.5 M_{\odot}$ models. Solid lines correspond to our computations of MS and post-MS evolution for initial hydrogen abundance $X = 0.70$ and heavy element abundance $Z = 0.02$. Dashed lines correspond to the pre-MS evolution as computed by D’Antona & Mazzitelli (1994) for the ‘MLT Alexander’ set of input parameters (see text). The symbols BE, BE_F and RE_{obs} mark boundaries of the instability strip: the general theoretical Blue Edge for radial overtones, the theoretical Blue Edge for the radial fundamental mode, and the empirical Red Edge.

Some δ Scuti stars may be very young evolving towards the MS. We did not compute pre-MS evolution explicitly. Instead we used the evolutionary tracks computed by D’Antona & Mazzitelli (1994) for the ‘MLT Alexander’ set of input parameters¹, which are close to those used in our computations.

The pre-MS and post-MS tracks are plotted in Fig. 1 for three values of stellar mass. There is good agreement between the positions of ZAMS models constructed by D’Antona & Mazzitelli (1994) and our ZAMS models. Therefore we can conclude that both series of the computations are consistent with each other.

For MS and post-MS models we performed a linear non-adiabatic analysis of low-degree oscillations ($\ell \leq 2$) using the code developed by W. Dziembowski (for a general description see Dziembowski 1977). In Fig. 1 we have also shown the the-

¹ Mixing-Length Theory of convection with $\alpha = l/H_p = 1.4$; OPAL opacities due to Iglesias & Rogers (1993) supplemented with low-temperature opacities due to Alexander & Ferguson (1994); initial hydrogen abundance $X = 0.70$, heavy element abundance $Z = 0.019$.

oretical blue edges of the δ Scuti instability strip which were computed for radial oscillations of the models with initial hydrogen abundance $X = 0.70$ and heavy element abundance $Z = 0.02$. The general Blue Edge (BE) is the hottest envelope of blue edges for overtones, from p_8 at the ZAMS to p_4 in the upper parts of both panels. There are no unstable modes in models to the left of this line. BE_F is the blue edge for the radial fundamental mode. Note that in the region under consideration the onset of the instability is practically insensitive to the spherical degree of a mode, so that the position of the blue edges will be unchanged when studying nonradial oscillations. RE_{obs} is the empirical Red Edge transformed from the photometric data to the theoretical parameters (we used simplest approximation about constancy of the convective flux during the oscillation cycle, therefore we were not able to determine theoretical red edge of the instability strip).

To study oscillations in the pre-MS evolutionary stage, we constructed envelope models using global stellar parameters (mass, effective temperature and luminosity) along the evolutionary tracks computed by D'Antona & Mazzitelli (1994), and then computed radial oscillations of these models. The envelopes were integrated from the surface to a layer with temperature $\log T = 7.2$ which corresponds to relative radius $r/R = 0.03 - 0.13$ in all models considered. For such deep envelopes, the periods of radial oscillations are practically the same as for full stellar models: in a $1.8 M_{\odot}$ model near the ZAMS where the errors are largest, the difference in the low-overtone periods between envelope and full models does not exceed 0.07 %. These errors have no influence on the determination of evolutionary period changes. To calculate period changes we used one more result of D'Antona & Mazzitelli: the age of the corresponding pre-MS model.

Note that our evolutionary tracks and periods of oscillations agree with those from a recent paper by Petersen & Christensen-Dalsgaard (1996).

2.2. Evolutionary period changes

The rates of evolutionary period changes were derived using computed values of the oscillation periods and the ages of the corresponding models. The results are presented in Fig. 2 separately for four consecutive evolutionary stages.

The periods of radial oscillations decrease during the pre-MS and the second overall contraction stages, and increase during the MS and post-MS expansion stages. The rate of these changes is determined entirely by the rate of evolution. The typical time spent by a star in the instability strip is quite different for different evolutionary stages: depending on stellar mass, this time is about 0.001–0.01 Gyr for the pre-MS stage, 1–1.5 Gyr for the MS stage, 0.02–0.05 Gyr for the second overall contraction stage and 0.02–0.1 Gyr for the post-MS expansion stage. Therefore, values of period changes can differ one from another by three orders of magnitude. The fastest period changes, which can be easier detected, occur during pre-MS evolution. Therefore, pre-MS variables seem to be the best candidates to test the stellar evolution rate empirically.

2.3. Effects of stellar rotation and convective overshooting

Some indications in favor of significant overshooting from convective cores of δ Scuti stars were obtained from a comparison of evolutionary tracks with calibrated photometric data (e. g. Napiwotzki et al. 1993). An asteroseismological test, based on sensitivity of some nonradial mode frequencies to the size of mixed stellar core, was proposed by Dziembowski & Pamyatnykh (1991). Another important parameter which must influence stellar evolution and pulsations, is rotation. Detailed studies of non-evolutionary rotational effects on the oscillation frequency spectrum are beyond the scope of the present paper. Some estimates of the second order rotational corrections to period ratios were obtained by Pérez Hernández et al. (1994); the complexity of the mode identification problem in the presence of rotation is demonstrated by Pamyatnykh et al. (1997) in an attempt to find a seismic solution to 13 observable frequencies of XX Pyxidis (for a short discussion see Pikall et al. 1998). However, rotation significantly modifies the structure and evolution of a star. It may be desirable to compare the rate of period changes in such models with that of standard non-rotating models.

To test the effects of overshooting and rotation explicitly, we computed two new families of evolutionary tracks of $1.5 - 2.5 M_{\odot}$: models with overshooting from the convective core and rotating models without overshooting.

The overshooting distance, d_{over} , was chosen to be $0.2 \cdot H_p$ where H_p is the local pressure scale height at the edge of the convective core. A similar value of the overshooting parameter was used by many authors (see, for example, Schaller et al. 1992, Napiwotzki et al. 1993, Claret 1995). The tracks with and without overshooting are shown in Fig. 3.

The overshooting results in an extension of the MS-stage in the HR Diagram due to an enlargement of the mixed core: more hydrogen fuel is available for nuclear burning. The displacement of the TAMS-points is about $\Delta(\log T_{\text{eff}}) = -0.02$, $\Delta(\log g) = -0.1$ to -0.15^2 . At a fixed effective temperature, a model with overshooting is slightly more luminous than a standard model without overshooting. Stellar lifetimes in the MS-stage are increased due to overshooting by 12–14 percent. At the TAMS, hydrogen is slightly more exhausted in models with overshooting: the central hydrogen abundance is about 4.0% (in mass) as compared with 4.5% in the case without overshooting.

It is interesting to compare stellar lifetimes in different evolutionary stages for the models with and without overshoot-

² According to our computations with a given choice of d_{over} , overshooting influences the TAMS position 1.5–2 times smaller than according to Schaller et al. (1992), Napiwotzki et al. (1993) or Claret (1995). The disagreement may be caused by numerical effects as a test shows. To achieve a sufficient accuracy in the oscillation computations we need to rely upon more detailed stellar models than one uses usually in the evolutionary computations. For example, our typical $2.0 M_{\odot}$ MS model consists of approximately 1300 layers, and there are 135 models between the ZAMS and TAMS. If we increase both space and time step sizes by factor of about 7–8, we can reproduce the corresponding evolutionary track of Schaller et al. (1992) almost precisely.

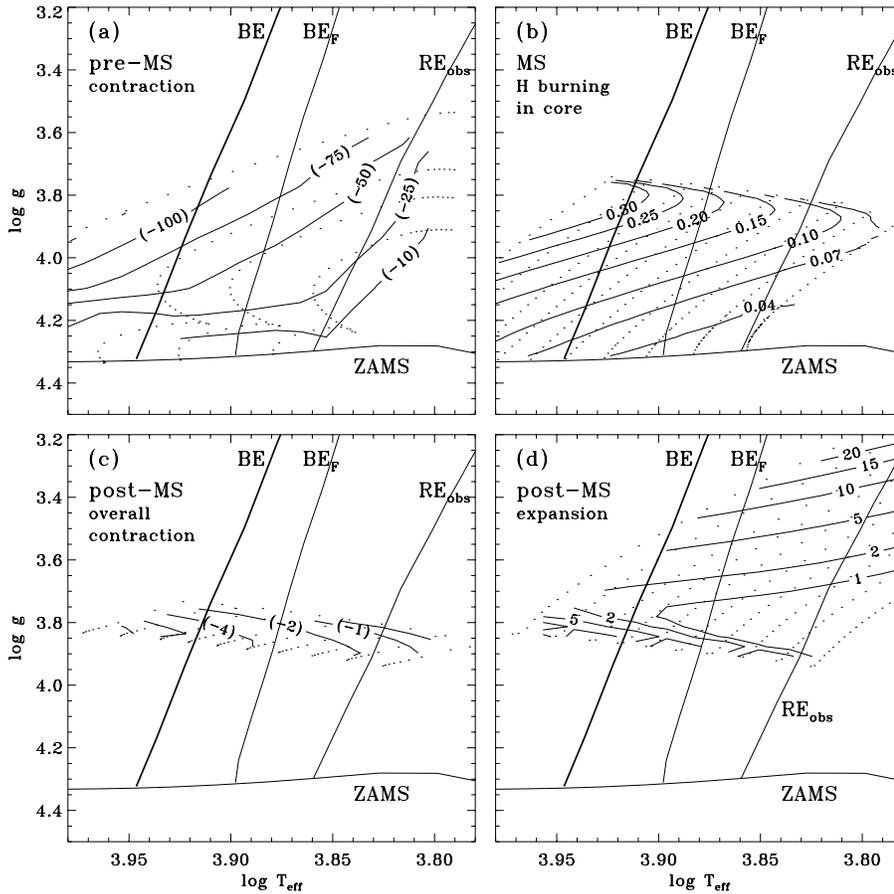


Fig. 2a–d. Lines of constant period changes in $\log g - \log T_{\text{eff}}$ diagram. The numbers give values of relative period changes, $(1/P_F)dP_F/dt$, in units of $10^{-8} \text{ year}^{-1}$, where P_F is the period of the radial fundamental mode. The four panels correspond to consecutive evolutionary stages: **a** pre-MS contraction stage, **b** MS stage of hydrogen burning in the stellar core, **c** second overall contraction stage of effective hydrogen exhaustion in the core, **d** post-MS expansion stage of the hydrogen burning shell source. Points mark the position of the models in the corresponding evolutionary stages (compare with Fig. 1). The boundaries of the instability strip are also shown with coding as in Fig. 1.

ing. Let us consider segments of the evolutionary tracks of the $1.8 M_{\odot}$ model in the effective temperature range between the TAMS and the leftmost point of the second contraction stage. The star crosses that region three times in its life on and beyond the MS. In the overshooting case ($\log T_{\text{eff}} = 3.8081 - 3.8671$, see Fig. 3) these times are equal to 0.35, 0.034, and 0.019 Gyr for MS, second contraction and post-MS expansion stages, correspondingly. In the standard case without overshooting ($\log T_{\text{eff}} = 3.8260 - 3.8769$) these times are equal to 0.35, 0.038, and 0.056 Gyr. We see that in both cases the second contraction times are similar and are one order of magnitude shorter than in the final part of the MS–evolution. On the other hand, post-MS expansion in the overshooting case is a factor of 1.8 faster, and in the standard case a factor of 1.5 slower than the corresponding second contraction. Such a significant difference in the post-MS expansion times between evolution with and without overshooting seems to be important for statistical investigations of the δ Scuti stars.

For another family of models we assumed uniform (solid-body) stellar rotation and conservation of global angular momentum during evolution from the ZAMS. These assumptions were chosen due to their simplicity. The same approach was used in our recent studies of XX Pyxidis (Handler et al. 1997a,b; Pamyatnykh et al. 1997) The initial equatorial rotational velocity was assumed to be 150 km/s. The tracks for rotating and non-rotating models are shown in Fig. 4.

Rotation results in a shift of the tracks to smaller T_{eff} and gravity. The displacement of the TAMS mimics the overshooting effect, with $\Delta(\log T_{\text{eff}})$, $\Delta(\log g)$ smaller by a factor 1.5 in comparison with those for the overshooting case (compare Figs. 3 and 4). The tracks of rotating models lie slightly above the tracks of non-rotating models in $\log g - \log T_{\text{eff}}$ diagram, and below the corresponding tracks in $\log L - \log T_{\text{eff}}$ diagram which can be easily explained by decreasing effective gravity due to centrifugal force. The MS lifetime for rotating models is only 0.5–1.0 percent longer than that for non-rotating models. With our assumption on the global angular momentum conservation the equatorial rotational velocity is decreasing during the MS–evolution from 150 km/s at the ZAMS to about 120 km/s at the TAMS.

On the MS, the effects of the overshooting and rotation on period changes are determined practically entirely by mentioned changes in positions of the tracks. In a plot similar to Fig. 2b, lines of constant period changes for the overshooting case will be prolonged to smaller gravities due to the TAMS displacement. For the rotating models we will obtain a small shift of the isolines to lower gravities according to both the ZAMS and the TAMS displacements. We disregard the rotation effects on evolutionary period changes in the rest of this paper because they mimic the overshooting effects but are less pronounced than these.

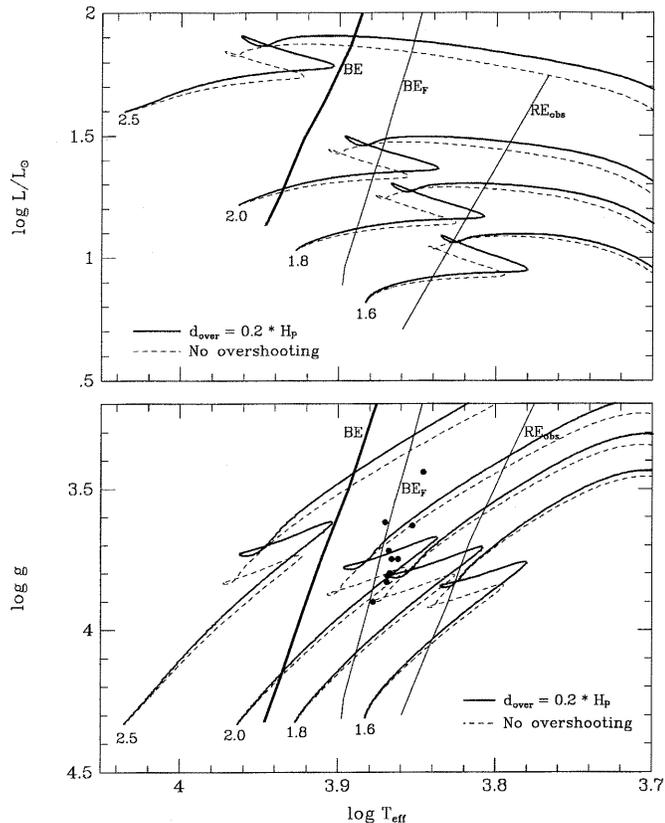


Fig. 3. Evolutionary tracks of 1.6, 1.8, 2.0 and 2.5 M_{\odot} models with (solid lines) and without convective overshooting (dashed lines). The ZAMS models are practically identical in both cases because they are chemically uniform. The instability boundaries are the same as in Fig. 1. In the lower panel, the location of 10 Pop. I stars from Table 1 is shown by filled circles. The values are based on the $uvby\beta$ data from the catalogue of Rodriguez et al. (1994) transformed to $\log g$, $\log T_{\text{eff}}$ using the calibration of Moon & Dworetzky (1985).

2.4. Period changes during fast phases of post-MS evolution

We will show in this subsection that fast period changes, more pronounced for the overshooting case, can occur during very short phases of the post-MS evolution. As it can be seen from Fig. 3, there is a noticeable difference in the morphology of the tracks with and without overshooting at the end the second overall contraction stage. Here a negative and a positive peak in the period changes are produced. Physically, at this stage a significant reorganization of stellar interior structure occurs: towards the end of the second overall contraction stage the convective core begins to decrease very quickly due to progressive inefficiency of the nuclear-energy generation in the center, which, in turn, is caused by hydrogen exhaustion. This process occurs more suddenly in the overshooting case: for example, in the 1.8 M_{\odot} model the decrease of the convective core from the value of 0.005 (mass fraction) to 0.0 lasts for about 0.9 million years which is three times faster than in the standard case. At the end

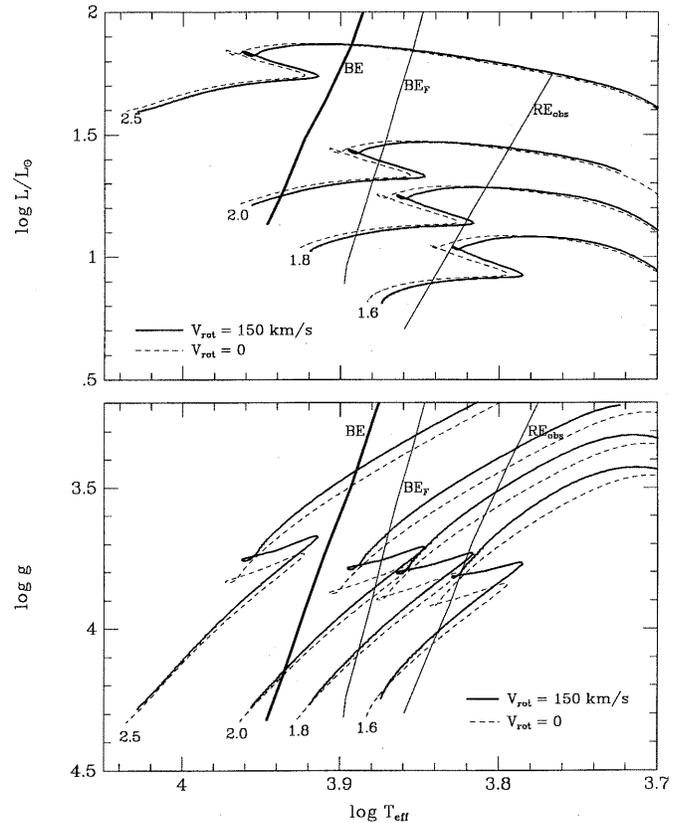


Fig. 4. Evolutionary tracks of uniformly rotating (solid lines) and non-rotating (dashed lines) stellar models with masses 1.6, 1.8, 2.0 and 2.5 M_{\odot} . (See text for details.) The instability boundaries are the same as in Fig. 1.

of the overall contraction stage the convective core disappears³. As hydrogen is effectively exhausted in the center, this region rapidly cools and contracts. A significant part of the luminosity is provided with release of thermal and gravitational energy. Simultaneously with the cooling at the center (in the inner 6–7% of stellar mass) the heating outside this region occurs where hydrogen is more abundant. As a consequence, a nuclear hydrogen-burning shell is established outside the hydrogen-exhausted core. The stellar luminosity adjusts quickly to the energy production by the nuclear shell-burning source. In the HR diagram, such ‘non-explosive hydrogen ignition’ occurs on the ascending part of the evolutionary track soon beyond a small loop.

Details of post-MS stages of standard stellar evolution were described more than thirty years ago by Iben (1967 and references therein). When discussing a very fast stage of the convective core disappearance and of the hydrogen-burning shell development, the overshooting *versus* non-overshooting analysis seems to be similar to the analysis of the standard evolution of models of different mass: for a higher mass we proceed to construct the models with more extended convective cores. Therefore, as was noted by Iben (1966) in his discussion of

³ Of course, convective overshooting also disappears, but we keep the term ‘model with overshooting’ or ‘the overshooting case’ for later evolutionary stages to distinguish this case from the standard one.

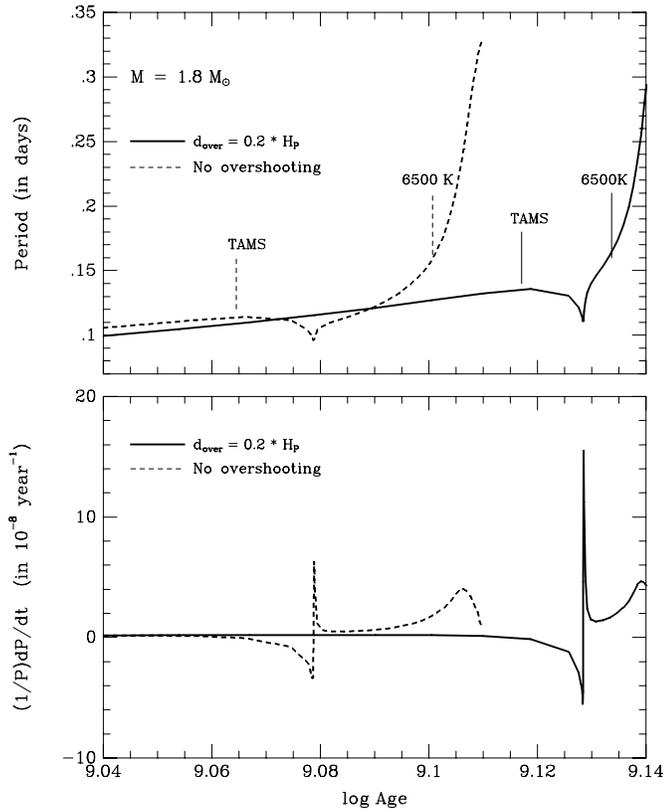


Fig. 5. Period of radial fundamental mode and relative period change for late MS and post-MS (right of the TAMS) evolution of $1.8 M_{\odot}$ model with and without overshooting. Short vertical lines in the upper panel mark ages of the TAMS models and those of the models with $T_{\text{eff}} = 6500$ K which is close to the Red Edge of the instability strip (see Fig. 3).

the $5M_{\odot}$ versus $3M_{\odot}$ evolution, the nuclear fuel disappears suddenly over a somewhat larger fraction of the interior. As a consequence, ‘semidynamic’ effects of the stellar structure reorganization are manifested more clearly (see small loops in our tracks).

The time spent by a $1.8 M_{\odot}$ star in the small loop near the end of overall contraction stage is equal to 0.0015 and 0.0014 Gyr for the overshooting and the standard case, correspondingly. It is 25 times shorter than the total time of overall contraction.

The periods of the radial fundamental mode and their changes during late MS and post-MS evolution of a $1.8 M_{\odot}$ star are shown in Fig. 5. Note that the MS–evolution with overshooting lasts for about 13 % longer than in the standard case (1.31 and 1.16 Gyr, correspondingly). At the same time, post-MS evolution occurs more rapidly in the overshooting case. The dips in the period behaviour and corresponding peaks in the rate of the period change are seen clearly⁴

⁴ The lower panel of Fig. 5 is similar to plots presented by Heller & Kawaler (1988) who studied evolutionary period changes in rapidly oscillating Ap stars. We compared our standard results (without overshooting) for 1.6, 2.0 and $2.5 M_{\odot}$ with those of Heller & Kawaler and found agreement in the MS stage. However, our peak values of period changes are three times larger for negative peaks and six times

larger for positive peaks. Also, the positions of our peaks are shifted to higher ages. These differences can be explained by differences in the evolutionary tracks. Somewhat higher new OPAL opacities in the interior result in an enlargement of the convective core and in a corresponding extension of MS evolutionary stage. The situation is similar to our comparison of the overshooting case with the standard case (see Fig. 5).

3. Observed period changes

We have collected, examined, and in some cases recalculated the observed period changes for δ Scuti stars. Most available data refer to stars with radial pulsation and large amplitudes. The period changes are usually derived from a collection of observed times of maximum light. Regrettably, a potentially confusing variety of different units can be found in the literature. In a few cases, these have lead to errors. Because of these problems it appears prudent to list the equations and units used in this paper:

Let (O-C) refer to the difference in the observed and computed times of maxima measured in days. If the adopted period, P , is correct and refers to the beginning of a long series of observations covering a time t (also in days), then

$$(O - C) = 0.5 \frac{1}{P} \frac{dP}{dt} t^2.$$

This equation is similar to an expression derived by Percy et al. (1980), where phase units are used. It is customary to express $(1/P)dP/dt$ in units of year^{-1} , so that the value of $(1/P)dP/dt$ found above has to be multiplied by a factor of 365.25.

Ambiguities should not be present if the observations are expressed in terms of the times of maximum light (epoch):

$$T_{\text{max}} = T_o + P_o N + 0.5 \beta N^2,$$

where $\beta = P dP/dt$, T_o refers to the epoch and P_o to the period at the time of the listed epoch.

Table 1 lists a summary with some of the recent references and can be regarded as an update of earlier collections given in Breger (1990a) and Rodriguez et al. (1995). We have deliberately not listed the statistical uncertainties of the dP/dt values, because in almost all cases the published or calculated uncertainties are unrealistically small. This can be demonstrated for a number of stars for which new observations have become available and new values of the standard deviation were computed. However, we have omitted the few stars for which we regard the period changes as very uncertain. Although not included in this paper, the calculation of reliable standard deviations as well as upper and lower limits for those stars with small period changes are important.

larger for positive peaks. Also, the positions of our peaks are shifted to higher ages. These differences can be explained by differences in the evolutionary tracks. Somewhat higher new OPAL opacities in the interior result in an enlargement of the convective core and in a corresponding extension of MS evolutionary stage. The situation is similar to our comparison of the overshooting case with the standard case (see Fig. 5).

Table 1. Observed period changes of Delta Scuti stars

Star	Period (days)	(1/P)dP/dt (year ⁻¹ x 10 ⁻⁸)	References	Comments
Population I stars, radial pulsation				
IP Vir	0.067	-0.5	Hintz (priv. comm.)	no jump, Note
GP And	0.079	13	Rodriguez et al. 1993a	
AE UMa	0.086	-48	Hintz, Hintz & Joner (1997)	
EH Lib	0.088	small	Mahdy & Szeidl (1980) Yang et al. 1992	
BE Lyn = HD 79889	0.096	...	Kiss & Szatmary (1995), Rodriguez et al. (1995)	binary light-time effects? Note
YZ Boo	0.104	3	Hamdy et al. (1986) Jiang (1985)	
AI Vel	0.112	0	Walraven et al. (1992)	
	0.086	15		Note
SZ Lyn	0.121	7	Moffett et al. (1988), Paparò et al. (1988)	Note
AD CMi	0.123	4	Fu & Jiang (1996), Rodriguez et al. (1995)	Note
RS Gru	0.147	-11	Rodriguez et al. (1995)	
DY Her	0.149	-4	Szeidl & Mahdy (1981), Yang et al. (1993)	
VZ Cnc	0.178	...	Arellano Ferro, Nuñez & Avila (1994)	Note
BS Aqr	0.198	-5	Yang et al. (1993)	
Population I stars, nonradial pulsation				
XX Pyx	0.026	340	Handler et al. (1997a)	jump possible
	0.028	-910		jump possible
	0.030	small		
τ Peg	0.054	variable	Breger (1991)	Note
4 CVn	0.116	-110	Breger (1990b)	jump possible
	0.171	-300		jump possible
Population II stars = SX Phe stars, radial pulsation				
BL Cam	0.039	29	Hintz et al. (1997)	Note, no jump
SX Phe	0.055	-2	Thompson & Coates (1991), Coates et al. (1982)	jumps likely
	0.043	-16		
CY Aqr	0.061	...	Powell et al. (1995)	4 jumps
show diagram				
DY Peg	0.073	-3	Mahdy (1987), Peña et al. (1987)	
XX Cyg	0.135	positive	Szeidl & Mahdy (1981)	jump in 1942

IP Vir: Pop. I nature suspected (Landolt 1990).

BE Lyn: A parabolic fit would lead to $(1/P)dP/dt = 40 \times 10^{-8} \text{ year}^{-1}$ (Liu & Jiang 1994, Rodriguez et al. 1995), but the scatter is large. The binary hypothesis by Kiss & Szatmary (1995) with an orbital period ~ 2350 d looks promising, but needs to be confirmed.

AI Vel: The value of dP_1/dt given by Walraven et al. (1992) has been multiplied by P_1^2 to correct for an error. The new value can be confirmed by inspection of Fig. 1 of that paper.

SZ Lyn: Light-time corrections for the binary orbit have been applied.

AD CMi: Fu & Jiang (1996) list an additional explanation of the stage shifts in terms of orbital light-time effects.

VZ Cnc: Data cannot reliably distinguish between constant period, abrupt period changes or orbital light-time effects. Some additional data by Arellano Ferro, Avila & Gonzalez (1994) are available.

τ Peg: Period increases and decreases at different times, jumps possible.

BL Cam: Fig. 4 and Eq. 3 of Hintz et al. (1997) lead to a period change of 0.02 s in the last 20 years, not the value of 0.009 s listed in the paper.

The star 28 And was not included in Table 1, because Rodriguez et al. (1993b) have found that the times of maxima can be fit by a constant period, i.e. the star's period changes are

smaller than the uncertainties of determination. Furthermore, the period changes of KZ Hya = HD 94033 (Hobart et al. 1985) are regarded as too uncertain to be included here.

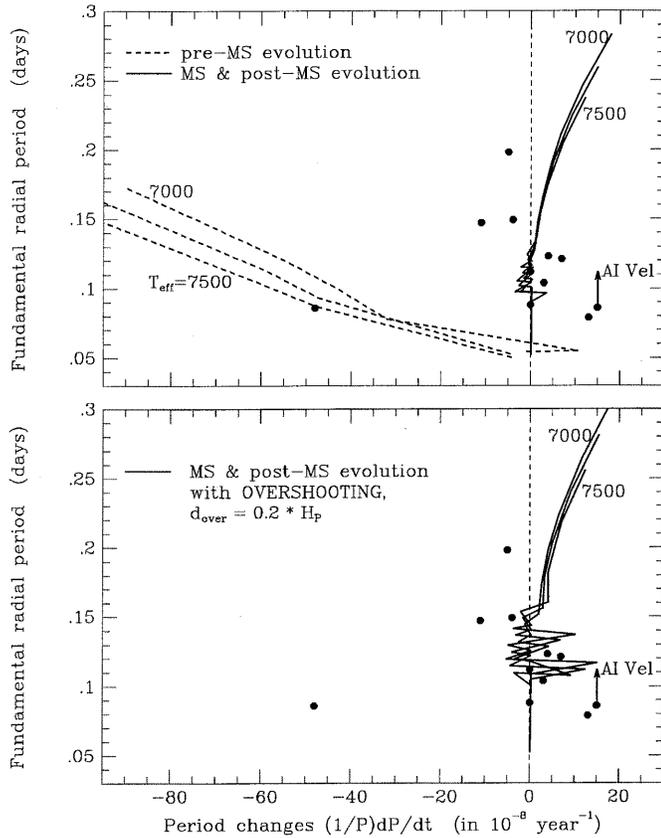


Fig. 6. Comparison between the observed (points) and computed (curves) period changes of Pop. I radial pulsators. The arrow corresponds to the observed period change of the radial first overtone mode of AI Vel corrected to the value of the fundamental mode. The theoretical data for models with (lower panel) and without (upper panel) core overshooting are given for three effective temperatures. The zigzag behaviour of the curves at periods of about 0.1–0.15 days corresponds to the very fast evolution of models of different masses at the end of the second overall contraction stage. During this phase sudden period changes occur (see Fig. 5). The inclusion of the overshooting extends the domain of zigzag behavior.

4. Period changes among Pop. I δ Scuti stars

In Fig. 6 we compare the observed period changes of the Pop. I radial pulsators with the values expected from stellar evolution calculations described in a previous section of this paper. The stars with known period changes are generally those with high amplitudes (HADS), which are restricted to the central part of the instability strip. Consequently, the comparison occurs in a limited temperature range of about 7000 to 7500 K. For the figure we have chosen the size of the radial fundamental period as the indicator of the evolutionary state. For these radial pulsators, this is a better indicator than the position in the HR Diagram, because of the uncertainties of about ± 0.3 mag in the photometric absolute magnitude calibrations. The uncertainties in the values of the measured periods are negligible.

Fig. 6 also shows the computed period changes for three temperatures of 7000, 7250 and 7500 K. Models with and without overshooting were considered. Not surprisingly, the rate of

evolution is not a strong function of temperature. Furthermore, in the case of standard evolution without overshooting, only for the pre-MS and most evolved stars does the predicted rate of period change exceed $(1/P)dP/dt = \pm 10^{-7} \text{ year}^{-1}$. Taking overshooting into account results in large period changes also at the end of the second overall contraction stage.

The expectation of generally positive period changes increasing in size with increasing evolutionary status is not borne out by the observations. In fact, the stars divide equally into groups with increasing and decreasing periods. Furthermore, the observed period changes are generally one order of magnitude larger than expected. The disagreement should not be caused by incorrectly computed rates of stellar evolution, since the situation will not be changed significantly if we take into account some additional effects such as rotation or convective overshooting.

An exception may only occur during the very fast stages of post-MS evolution as has been discussed in subsection 2.4. It can be seen in Fig. 6 that overshooting results in a significant enlargement of the zigzag domain and a shift to somewhat longer periods. In principle, up to half of observed period changes could now be accounted for. Let us examine a possible contender for the zigzag domain in more detail: the star AI Vel. The values of both the observed radial fundamental and first overtone modes place the star inside or near the zigzag domain. The value of the period ratio is accurately known and can be fitted to this fast evolutionary stage without any problems. This is illustrated in Fig. 7, which is similar to Fig. 11 in the paper by Petersen & Christensen-Dalsgaard (1996). It can be seen that a model of $2.0 M_{\odot}$ with the overshooting parameter d_{over} between 0.0 and $0.2 \cdot H_p$ can fit the observational periods of AI Vel very precisely.

Consequently, AI Vel may be an example of a star in the zigzag domain of Fig. 6 so that the observed period change of the first overtone could be explained. However, the radial fundamental mode of AI Vel is also observed and shows no period changes. This is not in agreement with the computations⁵. Furthermore, the agreement between positions of many stars in Table 1 with the zigzag domain may be accidental. Stellar evolution in this region around the end of the second overall contraction stage is very fast, as was already demonstrated in Fig. 5: for a $1.8 M_{\odot}$ star, the stage of the negative and positive evolution peaks in the period change takes not longer than 1.5 million years. The probability of finding such a high fraction of the observed stars in this short evolutionary stage is essentially zero. Consequently, the overall disagreement between the observed and expected period changes is not removed.

Among the Pop. I radially pulsating δ Scuti stars, AE UMa has the fastest measured period change of $-5 \times 10^{-7} \text{ year}^{-1}$.

⁵ There exists a very simple test to check whether observed period changes can be explained by the evolutionary effects. If a star shows multiperiodic behaviour and if the extracted frequencies can be interpreted as pure acoustic modes (for example, the radial fundamental mode and the first overtone in most of double-mode variables), we expect the logarithmic rate of period changes to be the same for all modes because their periods are determined almost entirely by the mean stellar density.

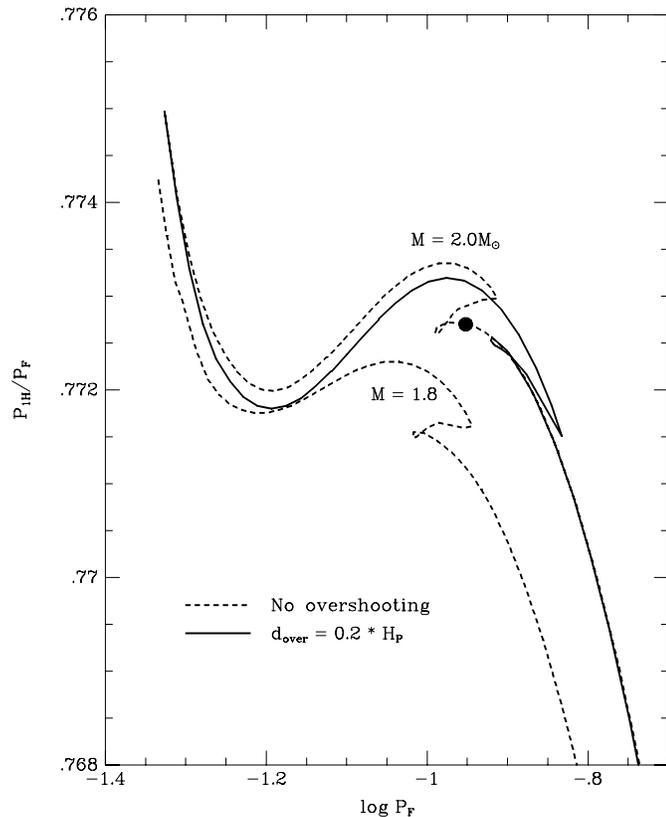


Fig. 7. Period ratio diagram for $2.0 M_{\odot}$ evolutionary models with and without overshooting. The leftmost points correspond to the ZAMS models. For comparison, the standard $1.8 M_{\odot}$ track is shown. The filled circle marks the position of AI Vel with data taken from Petersen & Christensen-Dalsgaard (1996).

The data look reliable. The rate of the period decrease is similar to that expected for pre-MS variables (see Fig. 6). However, there exists no evidence that AE UMa is a pre-MS star.

In an earlier discussion of three giants of similar luminosity and temperature, Breger (1990a) noted that, at least in principle, the theoretical evolutionary tracks could be adjusted to lead to decreasing radii in this part of the HR Diagram. However, the present analysis also shows disagreements for the less evolved stars. This makes an explanation in terms of completely erroneous evolutionary tracks improbable. An additional, but possibly unneeded argument concerns the observed period–luminosity relation shown by the hundreds of δ Scuti detected so far, indicating a radius increase for the more evolved stars with longer periods (see Introduction).

The disagreement between observed and computed evolutionary period changes is even worse for the stars showing nonradial pulsation. However, the calculation of the nonradial pulsation frequencies and their evolutionary changes is more complicated than for radial pulsation. The reason is that the rotational splitting may change with evolution so that modes with different values of the azimuthal order, m , may show different period changes, even in the same star (as observed).

We conclude that the period changes observed in Pop. I δ Scuti stars are not caused by stellar evolutionary changes. The period changes cannot at this stage be used to check stellar evolution calculations. Of course, the evolutionary changes should still be present in these stars, but are masked by other effects causing period changes. A much larger sample of stars is needed to average out these other effects.

4.1. On non-evolutionary period changes

The physical origin of the observed nonevolutionary period changes is not known. There are several suggestions in the literature:

Stellar companions can produce (O-C) shifts due to the light-time effects caused by the orbital motion. With limited amounts of data the observed shifts can often be matched equally well by both parabolic and trigonometric functions, so that a binary origin of the time shifts cannot be ruled out. The ambiguity has been demonstrated by Fu & Jiang (1996) for the star AD CMi. The situation is, however, not hopeless: the orbital changes reverse themselves over a longer time scale.

It is possible that most of period changes which are observed in δ Scuti stars are caused by nonlinear mode interactions, not by some specific processes in the interiors like considered by Sweigart & Renzini (1979) for RR Lyrae models almost twenty years ago. (The authors proposed random mixing events in a semiconvective zone to be the origin of period changes of both signs, especially sudden period jumps. It was noted by Rodriguez et al. (1995) and others that some of the changes in δ Scuti stars resemble those also seen in RR Lyrae stars. However, there is no semiconvective zone in the less evolved δ Scuti stars.)

The effect of period changes due to nonlinear mode interactions was demonstrated in a completely different context by Buchler et al. (1995). The authors studied nonlinear behaviour of rotationally split dipole ($\ell = 1$) modes in pulsating stars. In particular, when all three components of the triplet are excited and the amplitudes are constant, the direct nonlinear interactions between components result in symmetric frequency spacing. This is contrary to the results from linear theory, in which the second order effects of rotation cause the rotational mode splitting to be highly nonequidistant (see Dziembowski & Goode, 1992).

Another specific case of the nonlinear mode interactions was studied by Moskalik (1985). He considered the problem of resonant coupling of an unstable mode to two lower frequency stable modes. For δ Scuti stars such an approach means that a low-order radial or nonradial unstable mode interacts with two stable gravity modes. The coupling can result in a periodic amplitude modulation. The period of the modulation depends mainly on the excitation rate of the unstable mode and is supposed to be of the order of years in δ Scuti stars. During most of the modulation cycle the amplitude of the unstable mode grows nearly exponentially but finally decreases rapidly due to nonlinear interactions with the two stable modes. The large jump of pulsation phase occurs simultaneously – positive or negative depending on the sign of the frequency mismatch in the res-

onance condition. Before and after this jump one can observe very fast changes of the pulsation period. For δ Scuti models in a wide range of oscillation parameters the rate of these fast period changes $(1/P) dP/dt$ was found to lie, approximately, between $7.0 \cdot 10^{-4}$ and $1.0 \cdot 10^{-3} \text{ year}^{-1}$. These large values can be appropriate to interpret some changes like observed in XX Pyxidis (see Handler et al. 1997a). As mentioned by Moskalik, a very serious limitation of his study is connected with the assumption that only three modes take part in the interaction.

If the radially pulsating Pop. I stars with observed period changes are plotted on the $\log g - \log T_{\text{eff}}$ diagram (Fig. 3), it can be seen that all stars are located close to the theoretical Blue Edge of the fundamental radial mode. This conformity may be accidental because the fundamental Blue Edge lies approximately along the most populated central part of the instability strip. Moreover, the calibration errors may be significant too. But if this agreement is real, then, potentially, it can take a part in the future period change interpretations. At this line the fundamental radial mode instability appears or disappears depending on the direction of evolution. That means that quite significant changes in the excitation or damping rate of different modes may occur. They must influence the efficiency of the nonlinear interactions between different modes which, in turn, may influence the periods more significantly than during stages of more steady oscillations.

The situation for comparing observed period changes with stellar evolution calculations may be brighter for pre-MS stars. The calculations shown in Fig. 6 indicate expected period changes a factor of 10 to 100 times larger than for MS stars. In fact, these expected rates of period change are even larger than the nonevolutionary period changes of MS stars. Consequently, the evolutionary changes might not be hidden among other effects. A few δ Scuti stars with probable pre-MS status are known, e. g. the stars W2 and W20 in NGC 2264. So far, the variability of these stars has not been studied since their discovery as δ Scuti stars (Breger 1972). The study of period changes among the pre-MS stars appears to be very promising, in contrast to MS and post-MS stars.

5. Observed period changes in SX Phe stars

The evidence is accumulating that most of the observed period changes in the Pop. II δ Scuti stars occur in sudden jumps, followed by constant or nearly-constant periods. This is clearly shown in the star CY Aqr (Powell et al. 1995). Furthermore, the abrupt period change of 1989 reversed the decreases in period from the previous two period changes. This convincingly argues against the interpretation of the period changes in terms of long-term stellar evolution (at least in this star).

Other stars tend to show a similar behavior: The star XX Cyg had a sudden period change in 1942 (Szeidl & Mahdy 1991), while the (O-C) variations of SX Phe are best interpreted as two sudden jumps in 1969 and 1975 (Thompson & Coates 1991). In all three stars, the jumps were of the order of $\Delta P/P \sim 10^{-6}$.

The data for the star DY Peg can be interpreted equally well as either a sudden jump in 1961 or a continuous change (see

Mahdy & Szeidl 1980). On the other hand, BL Cam shows a continuously changing period.

The character of the observed period changes in δ Sct stars suggest that they are caused by nonlinear effects in pulsation and not by stellar evolution, at least in most cases. Therefore the main application of the data we collected here could be used in the future to test the theory of nonlinear multimode stellar oscillation.

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