Astron. Astrophys. 333, 199–204 (1998)

ASTRONOMY
AND
ASTROPHYSICS

Discovery of apsidal motion in α Coronae Borealis by means of ROSAT X-ray eclipse timing

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Received 12 August 1997 / Accepted 3 December 1997

Abstract. Four ROSAT X-ray observations of the secondary optical minimum of the eclipsing binary system α CrB taken in 1992, 1993 and 1997 are presented. Because of the totality of the X-ray eclipse, the times of mid eclipse can be accurately determined from the ROSAT data. The period between secondary minima P_s is found to be significantly different from the optically well determined period between primary minima P_p, thus indicating apsidal motion. The observed value $P_s - P_p$ = 4.8 $\pm$ 2.1 seconds is, first, shown to be almost exclusively due to the primary component, and second, consistent with our current knowledge of α CrB A. The relativistic contribution to the observed value is 0.95 seconds or 17 % of the total effect.

Key words: stars: coronae – stars: activity – binaries: eclipsing – stars: fundamental parameters — stars: individual: α CrB

1. Introduction

Among the brightest eclipsing binaries and in fact the only known totally eclipsing binary among the "α" stars is the system α Coronae Borealis (HR 5793, HD 139006). Tomkin & Popper (1986) review previous optical observations and provide a comprehensive summary of our current knowledge of α CrB. The system consists of an A-type and G-type dwarf star, orbiting each other in a little over 17 days in a rather excentric orbit (excentricity e = 0.37) and large angle of inclination (i = 88°). Consequently one sees a relatively deep primary minimum ($\Delta m \sim$ 0.12), when the G-type star partially occults the A-type star, and a rather shallow secondary minimum ($\Delta m \sim$ 0.02), when the A star totally eclipses the G type star. Since lines from both stars can be detected in the spectrum, α CrB is also a spectroscopic binary and therefore a full solution of the binary orbit is possible as well as a determination of individual stellar masses and radii. Tomkin & Popper (1986) also derive accurate system parameters which are summarized in Table 1 and 4; the internal structure constants are taken from Hejlesen (1987). Specifically they conclude that the primary of α CrB, i.e., α CrB A, is a slightly evolved star of spectral type A0, while α CrB B is a G5 star essentially on the zero age main sequence.

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Table 1. System parameters for α CrB

Parameter	Primary	Secondary
Mass ($M_\odot$)	2.58 $\pm$ 0.045	0.92 $\pm$ 0.025
Radius ($R_\odot$)	3.04 $\pm$ 0.30	0.90 $\pm$ 0.04
Rotation period (days)	1.5	5 (assumed)
Internal structure constant k_2	0.049	0.021

These properties make the α CrB system extremely interesting from the X-ray point of view. Young solar-like stars are known to be strong X-ray sources, while, on the other hand, A-type stars are (usually) X-ray dark (Schmitt 1997). Therefore, if α CrB B is indeed an X-ray source, one expects a total X-ray eclipse at the time of optical secondary minimum, while no effect should be observable at optical primary minimum. This expectation was verified by the observations of Schmitt & Kürster (1993), who indeed observed a total eclipse and constructed an eclipse map of α CrB B from the observed ROSAT X-ray light curve.

Finally, the substantial excentricity of its orbit makes the α CrB system interesting from the point of view of apsidal motion. A positive detection of apsidal motion offers one of the few possibilities and at the moment in fact the only possibility to study the interior structure of stars by determination of the so-called internal structure constants (Sect. 3.1). The number of binary systems with reliably known system parameters and accurately determined apsidal motion rates is still quite small; for example, Claret & Giménez (1993) list only 14 binary systems with well determined parameters, and 10 further systems with less reliably determined parameters. Most of the systems considered by Claret & Giménez (1993) have rather massive primaries, and only 2 out of their 24 primaries are less massive than α CrB A. α CrB has also been proposed as a candidate system for the detection of relativistic apsidal motion (Koch 1973), but Koch (1977) failed to find any evidence for a difference between the periods of primary and secondary minima. Later Volkov (1993) claimed to have detected apsidal motion in α CrB, however, his derived apsidal motion rate is almost a factor three smaller than

Table 2. Journal of ROSAT PSPC observations of α CrB

ROR number	Date	Observing time (ksec)
201103	July 12 1992	21.7
201557	July 28-29 1993	39.9
201558	Aug 15-16 1993	39.1
180168	Feb 19-20 1997	17.0

the rate derived in this paper and the theoretically expected rate. Hence his results appear somewhat suspicious.

In the optical, observations of secondary minimum are difficult to carry out. First of all, the minimum is quite shallow, and second, an accurate timing of the minimum requires observations of both eclipse ingress **and** egress, which are almost nine hours apart. Therefore from any given site coverage of ingress and egress of the same eclipse is extremely difficult to achieve. X-ray observations do not suffer from these problems: First, the X-ray eclipse is total and can be easily detected, and second, X-ray observatories tend to operate 24 hours per day and can in fact cover ingress and egress of the same eclipse.

The plan of this paper is as follows: In Sect. 2 the X-ray light curves, and the derivation of mid eclipse times are presented with a detailed discussion of the possible sources of error as well as the derivation of the period between secondary minima again including error discussion. The theoretical formalism to interpret the observations is briefly sketched in Sect. 3.1 and applied to the α CrB X-ray data in Sect. 3.2, where also an extensive discussion of the theoretically expected periastron advance in this system is given, and the final Sect. 4 contains my conclusions.

2. Observations and data analysis

2.1. X-ray light curves

X-ray observations of the optical secondary minimum of α CrB were obtained with the position sensitive proportional counter (PSPC) on board the ROSAT X-ray satellite. A detailed description of ROSAT and its scientific instruments has been given by Trümper (1983). A list of these observations with the observation dates, the length of the useful data intervals and the respective ROSAT observation request sequence numbers (ROR) is given in Table 2. The X-ray data for α CrB were analyzed following standard procedures (Zimmermann et al. 1994). In order to construct an X-ray light curve, the source photons were extracted within an extraction radius of 1.5 arc minutes around the apparent position of α CrB; the obtained count rates were corrected for background and detector dead time. The background was estimated from a control area in the image close to α CrB but without any other conspicuous X-ray source. The bin times (and not the individual photon arrival times) were corrected from universal time to heliocentric time. In Fig. 1a-d I show the obtained PSPC light curves for the four ROSAT observations with a temporal binning of 200 seconds. It is apparent from these figures, that the X-ray coverage of α CrB by ROSAT

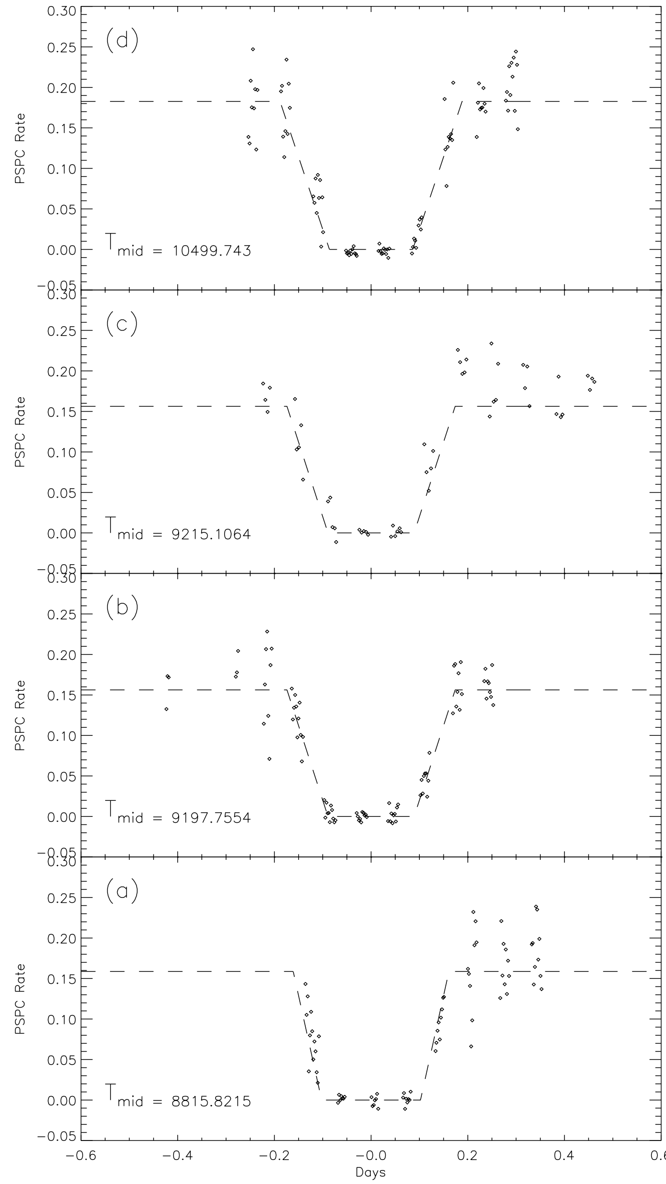

Fig. 1a–d ROSAT PSPC light curves of α CrB during the optical secondary eclipse; the derived central eclipse times are indicated on the plots. The dashed lines are the best fit models used to derive the eclipse parameters in Tab. 3. **a** Observation on July 12 1992; **b** Observation on July 28-29 1993; **c** Observation on Aug 15-16 1993; **d** Observation on Feb 19-20 1997.

has not been continuous; the data come in groups interrupted by periods when the satellite traversed a region of high particle background and the detector high voltage had to be turned down or when α CrB was Earth blocked with respect to ROSAT. Nevertheless one immediately recognizes that in all four cases a total secondary minimum is observed; in all cases the count rate drops to zero during the optical secondary minimum.

2.2. Timing of the X-ray eclipses

In order to address the issue of apsidal motion in α CrB, the midpoint of the observed X-ray eclipses (Figs. 1) has to be determined as accurately as possible. Unfortunately there is no reasonable *a priori* model of the spatial structure of the corona around α CrB; in fact, from the observed X-ray eclipse ingress and egress it is possible to infer such structuring (Schmitt & Kürster 1993). The X-ray luminosity of α CrB B exceeds that of the Sun at solar maximum by more than an order of magnitude; it appears quite likely that the X-ray emission of α CrB comes from a number of active regions, distributed somehow over its visible surface. If one were to model – for the sake of simplicity – each of these active regions as a point source, one would obtain a stepwise decreasing (increasing) light curve during eclipse ingress (egress). The eclipse duration of each region need not be the same, but depends on its location with respect to the rim of the occulting A-type star. In any case, whatever model one chooses, one expects a **symmetry** with respect to time in the ingress and egress light curves, as long as the X-ray emission from α CrB does not vary intrinsically during the observations. Of course, the center of the X-ray eclipse need not necessarily coincide with the center of the optical eclipse. The solar corona is known to be highly structured, and depending on the actual coronal configuration, the intensity-weighted center of its X-ray corona need not be identical with the center of the optical disk. By analogy, the same should apply to a stellar corona. If, in the worst case, the corona of α CrB was concentrated in a single point close to the limb, the eclipse curve would of course still be symmetric, however, the time of mid X-ray eclipse would obviously be shifted with respect to the time of mid optical eclipse by half the time it takes the A-type stars to traverse the apparent radius of the G-type star (8.62 hours). As a consequence I decided to model the PSPC light curves in such a way that the X-ray emission outside eclipse (i.e., before first and after fourth contact) is assumed to be constant, and between second and third contact is assumed to be identically zero. Between first and second contact and between third and fourth contact the X-ray emission is assumed to vary linearly with time from the out-of-eclipse rate to zero and back up again. Thus I assume that there are only four parameters describing the overall light curve: the out-of-eclipse count rate r, the duration of eclipse ingress and egress (assumed to be identical) t_{ing}, the duration of total eclipse t_{dur}, and the eclipse midpoint t_{mid} . For the purpose of determining apsidal motion, one is interested only in the time of the eclipse midpoint t_{mid} ; all other parameters are "uninteresting". The values of t_{mid} for the four observed eclipses (as well as the parameters r, t_{dur}, and t_{ing}) were determined by varying the four model parameters and finding those parameter values resulting in a best fit through χ^2-minimization and are listed in Tab. 3; for comparison I also give the expected times of secondary eclipse, using Volkov's (1993) times of secondary minimum but the period between primary minima, i.e., assuming no apsidal motion. The best fit models are also shown in Fig. 1 with dashed lines. In Fig. 2 I plot the value of the χ^2 test statistic as a function of t_{mid}, obtained by minimizing χ^2 with respect to all parameters but t_{mid} for the observations taken in February 1997, shown in panel (d) in Fig. 1. As is obvious from Fig. 2, a well-determined time of mid eclipse t_{mid} is indeed obtained by this procedure; further, errors on t_{mid} can be easily calculated by computing that time interval $(t_1 < t < t_2)$ such that $\chi^2(t) < \chi^2(t_{mid}) + 1$.

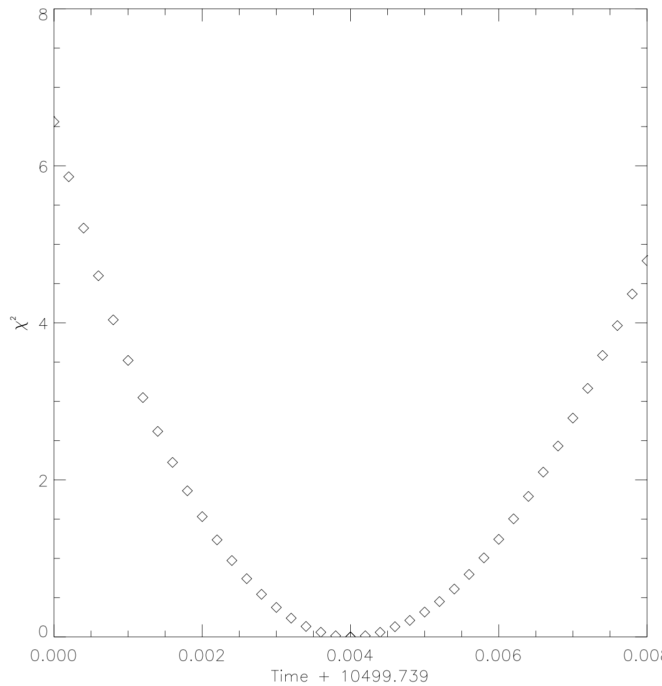

Fig. 2. Test statistic χ^2 vs. t_{mid} (with χ^2 minimized with respect to uninteresting parameters for the same data shown in Fig. 1d.

2.3. Calculation of observed X-ray period

With the help of the observed times of X-ray mid eclipse, which I assume to be identical to the times of optical secondary minimum, I can compute the period P_s between two consecutive secondary minima, which in turn is related to the rate of apsidal motion (Sect. 3.1). Of course, as already discussed, assuming the times of mid eclipse to be identical at optical and X-ray wavelengths, is a potential cause of systematic error. However, the derived X-ray eclipse centers agree rather closely with the optical ephemeres (see Table 3), and further, the observed length of eclipse ingress and egress (Table 3) argues against a point-like concentration; note that in the worst case scenario, the transition between out-of-eclipse to eclipse would occur instantaneously, and finally, the total eclipse durations shown in Table 3 compare well to the optical eclipse duration (between first and fourth contact) of 0.35925 days. Also, hopefully, given a sufficiently large number of X-ray observations, the centroid of averaged spatial emission is expected to coincide with the centroid of the optical disk, and hence - at least asymptotically - the effect should cancel out. At any rate, there is nothing one can reasonably do about this except note that systematic errors of the times of mid eclipse may be present which exceed the statistical errors quoted in Table 3.

Table 3. Eclipse parameters for α CrB

t_{mid} (HJD)	$t_{mid,error}$ (days)	t_{dur} (days)	t_{ing} (days)	$t_{dur} + 2t_{ing}$ (days)	$t_{mid,opt}$ (days)
8815.8253	0.0022	0.2045	0.0588	0.3221	8815.8216
9197.7551	0.0018	0.1782	0.0848	0.3477	9197.7394
9215.1025	0.0032	0.1366	0.1197	0.3761	9215.0993
10499.7430	0.0017	0.1723	0.1021	0.3766	10499.7321

In order to compute P_s, I carry out a linear least square fits of the observed times of secondary minima $T_{min,i}$, i=1,4, against a reference time T_0 and the period P_s through minimizing the expression

$$\sum_{i=1}^{4}(T_{min,i} - T_0 - n_{orbit,i}\, P_s)^2; \qquad (1)$$

$n_{orbit,i}$ is the number of elapsed binary orbits since T_0. This process results in $P_s - P_p$= 4.8 seconds. Alternatively, one can use – as an additional measurement – the ephemeris given by Volkov (1993) for the secondary minimum, i.e,. JDH$_{sec}$ = 2447010.3923, which results in $P_s - P_p = 5.0$ seconds. Since, however, first, no error is quoted by Volkov (1993) for his result, and second, this number is derived using an – in my opinion – incorrect value for P_s, I refrain from using this value in the following. The same procedure can be carried out by using the Kron & Gordon (1953) measurement of the secondary minimum (as quoted by Koch 1977), i.e., JDH$_{sec}$ = 2432410.695, which results in $P_s - P_p$= 2.66 $\pm$ 0.25 seconds. In this case one obtains a highly significant difference between primary and secondary period with a much smaller error because of the extended baseline, but the value of $P_s - P_p$ is considerably smaller than those derived by omitting Kron & Gordon's (1953) measurements. I have no real explanation for this discrepancy, except noting that the error quoted by Koch (1977) for the timing of the secondary minimum from the Kron & Gordon (1953) measurements seems to me unrealistically low, given the photometric quality of these data (cf., Fig. 4 in Tomkin & Popper 1986).

As discussed above, each of the four estimated mid eclipse times has an error of approximately 170 seconds (Table 3). In order to assess the influence of these individual measurement errors on the deduced value of P_s, I repeated the least square estimation from Eq. 1 many times with values for T_1 and T_{min} chosen randomly assuming a Gaussian error distribution. The resulting bootstrapped error distribution is shown in Fig. 3. From this I estimate the error in the derived $P_s - P_p$ to be 2.1 seconds. Also, the significance of $P_s - P_p$ being greater than zero, is – from the bootstrap distribution – found to be 0.984. Repeating the calculations using Volkov's (1993) time of the secondary minimum (assuming - unrealistically - no error) I find an error of 0.7 seconds for $P_s - P_p$ and an overall significance of the existence of apsidal motion in excess of 0.9999. Therefore the two derived values for $P_s - P_p$ are consistent, the error is however substantially reduced when including Volkov's (1993) time of secondary minimum. In Fig. 4 I plot analogously the bootstrapped distribution of the derived epoch of the ROSAT secondary minima and find T_{sec} = 8815.8281 $\pm$ 0.002.

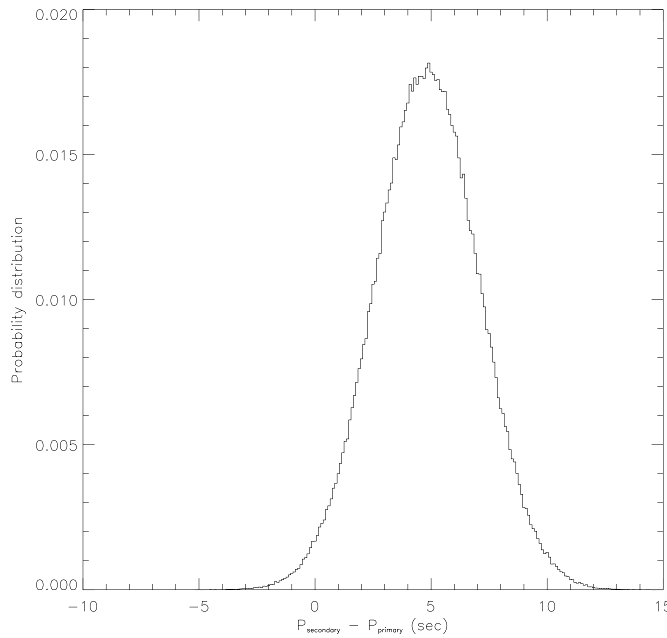

Fig. 3. Probability distribution of the bootstrapped distribution of the $P_s - P_p$ values derived from Eq. 1.

3. Calculation of observed and expected apsidal motion

3.1. Theoretical formalism

Consider a not necessarily synchronously rotating binary system. If P denotes the orbital period and U the period of revolution of the apsidal line, the theoretical ratio P/U for both rotational and second-harmonic tidal distortion of both stars can be expressed as (Kopal 1978, p. 243)

$$P/U = c_1\, k_{12} + c_2\, k_{22}, \qquad (2)$$

where the coefficients c_i (i=1,2) are given by

$$c_i = [(\frac{\omega_i}{\omega_K})^2\, (1 + \frac{M_{3-i}}{M_i})\, \frac{1}{(1-e^2)^2} + 15\, \frac{M_{3-i}}{M_i}\, \frac{8 + 12e^2 + e^4}{8(1-e^2)e^5}] r_i^5. \qquad (3)$$

Table 4. Orbit parameters for α CrB

Parameter	
orbit eccentricity e	0.370 $\pm$ 0.005
ascending node ω (degrees)	131.0 $\pm$ 1.0
Primary period (days)	17.35990016 $\pm$ 0.00000013
Primary radius (relative to semimajor axis) r_p	0.071 $\pm$ 0.007
Secondary radius (relative to semimajor axis) r_s	0.021 $\pm$ 0.001
r_s/r_p	0.295 $\pm$ 0.005

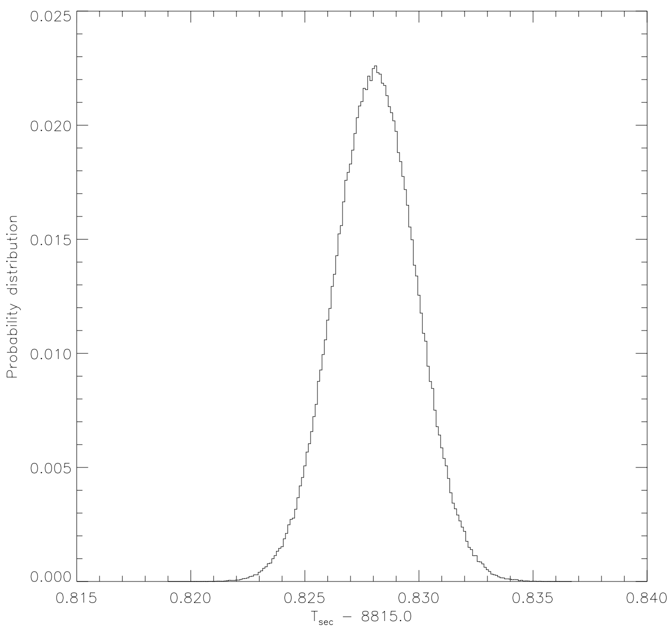

Fig. 4. Probability distribution of the bootstrapped distribution of the T_{sec} values derived from Eq. 1.

Here M_i, i=1,2, denote the masses of the two stars, r_i, i=1,2, their radii, e the excentricity of the orbit, and ω_K the so-called Keplerian angular velocity

$$\omega_K^2 \;=\; \frac{G(M_1+M_2)}{A^3}. \tag{4}$$

A is the semimajor axis of the relative binary orbit, and ω_i, i=1,2, denote the actual angular velocity of rotation of the individual components; for the special case of synchronous rotation one obviously has $\omega_i = \omega_K$. The constants k_{i2} denote the so-called second internal structure constants of the two stars, which are related to the internal stellar density distribution (Kopal 1978). The extreme cases apply for a centrally condensed mass, i.e., a point mass, with $k_2 = 0$, and for a star with uniform density distribution with $k_2 = \frac{3}{4}$.

Finally, the relativistic contribution of the periastron advance can be expressed as

$$P/U_{rel} = 6.35\ 10^{-6}\ \frac{M_1\ +\ M_2}{A(1-e^2)}, \tag{5}$$

where both the masses M_1 and M_2 as well as the semimajor axis A of the relative binary orbit are expressed in solar units.

In order to relate the expected rate of apsidal motion to the observable change between period P_p between primary minima and secondary minima P_s, I use the formula derived by Rudkjobing (1959):

$$\frac{P_p - P_s}{P} \;=\; (1-e^2)^{3/2}\frac{4\ e\ sin(\omega)}{(1-e^2 sin^2(\omega))^2}\frac{P}{U}. \tag{6}$$

3.2. Application to α CrB

I now apply Eqs. 2 - 6 to the specific case of α CrB using the numbers quoted in Table 1 and 4, and find - expressing $P_s - P_p$ in seconds - $P_s - P_p$ = 5.67 sec, with a relativistic contribution from Eq. 5 of 0.95 sec or 17%. This theoretical estimate obviously agrees very well with the observed value of 4.8 $\pm$ 2.1 seconds. Nevertheless one has to ask, whether this agreement is fortuitous or whether it indeed indicates a correct theoretical understanding of apsidal motion in α CrB. Putting the question differently, one has to ask what are the errors in the theoretical estimate of $P_s - P_p$.

Looking at Eq. 6, one realizes that there is a variety of possible sources of error for the theoretically expected value of $\frac{P}{U}$, viz., errors in the orbital elements ω and e, errors in the derived radii, masses and periods, and errors in the internal structure constants. As to errors in ω and e, they lead to overall errors of less than 1 percent, and will therefore be ignored in the following. As to the second source of error, I note that the rotation period of the secondary is unknown. Using the nominal system parameters for the primary and secondary, I calculate a 1 percent contribution for the secondary assuming a rotation period of 5 days, and a 6 percent contribution for the secondary assuming a rotation period of 1 day. Given the age of α CrB (it is a member of the Ursa Major stream) and its X-ray luminosity, a rotation period of 1 day is extremely unlikely, and I therefore conclude that the overall contributions to the total apsidal motion and its error budget is essentially determined by the A-type star alone.

How do the parameters M_1, r_1 and ω_1 enter into the expression for P/U ? In the expression 3 one notices that the terms proportional to ω_1 dominate by far; hence the errors in the masses of the stars, basically determined from the photometry and Kepler's law, do not figure significantly. The parameters r_1 and ω_1 are in turn determined from the light curve modeling and the $v\sin(i) = 110\ km/sec$ measurement by Slettebak et al. (1975; unfortunately without error), and the coefficient c_1 turns out be proportional to r_1 through $c_1 \sim r_1^3$; therefore any errors in r_1 propagate and are amplified. Specifically changing the radius of the primary component within the limits quoted by Tomkin & Popper (1986) and keeping all other parameters fixed, changes the expected P/U values in the range 3.43 - 6.31 seconds. Obviously, the radius of primary is the single most important uncertain physical parameter in the α CrB system.

The error in the relativistic contribution to the periastron advance depends only on the total mass, which is very accurately known, and the semimajor axis of the orbit, which again can be considered to be quite well known, since the new *HIPPARCOS* parallax for α CrB confirms the parallax used by Tomkin & Popper (1986). Therefore the error on the calculated relativistic periastron advance of 0.95 sec should be quite small.

Finally, I turn to errors in the structure constants k_{12}. Except for systematic "errors" in the stellar models, which one of course would like to determine from the observations of apsidal motion, the errors in k_{12} depend on the observational uncertainties in mass, age, and chemical composition. I use the grid of models by Hejlesen (1987), who published the internal structure constants for stellar models as a function of mass and age for a few values of chemical composition. Obviously only the primary matters. First of all, varying the mass of the primary within the permissible range (10 %), I find the same uncertainty in the derived internal structure constants k_2. Tomkin & Popper (1986) quote an age of $2 - 4 \times 10^8$ yrs for α CrB; over this time span, the primary has evolved about 0.4 magnitudes away from the zero age main sequence (ZAMS), while the secondary should be on the ZAMS for practical purposes. They further show that a chemical composition of X=0.7 and Z=0.02 yields acceptable fits to effective temperature and surface gravity of both stars.

Obviously, the internal structure constants change rapidly for evolution off the main sequence. Using specifically Hejlesen's (1987) model tracks, one finds for a ZAMS model of mass $\log M = 0.4$ and chemical composition $X = 0.7$ and $Z = 0.02$, $k_2 = 0.063$, while the same model at an age of $3.57\ 10^8$ yrs yields $k_2 = 0.035$, i.e., a reduction of 80 %. Similar changes are found when varying the chemical composition, however, it appears that low metallicity models, for which one obtains the largest internal structure constants are excluded by the photometry and age of the system. Using the nominal system parameters listed in Table 1 and 4, I calculate corresponding periastron advances of 5.3 sec assuming a ZAMS model, and 2.95 sec for a $3.57\ 10^8$ year old model; adding a relativistic contribution of 0.95 sec, one obtains a total periastron advance of 6.25 and 3.91 sec respectively for the two cases. This range fits perfectly to the measured value of periastron advance of 4.8 $\pm$ 2.1 sec. Thus, at the moment, one can only state that the observed periastron advance is certainly consistent with theoretical expectations.

4. Conclusions

This paper presents the first credible detection of apsidal motion in the binary system α CrB. To my knowledge this is the first time that such a classic measurement (through eclipse timing) has been performed in the X-ray domain. Because of the deep X-ray eclipse at the time of secondary optical minimum and the 24 hour availability of satellite-based observatories, the required timing measurements can easily be performed. The significance of the detected periastron advance is quite large; the probability for the periastron advance being greater than zero is 0.986. At the moment the observed time difference between secondary and primary minima is 4.8 seconds with an error of 2.1 seconds. I expect that this value can be significantly improved upon in a few years when new, more sensitive X-ray observatories allow uninterrupted coverage of eclipse ingress and egress; a determination of $P_s - P_p$ with an accuracy of 0.1 seconds, i.e., an order of magnitude better, appears certainly feasible. For the time being the measured periastron advance of 4.8 $\pm$ 2.1 sec is consistent with our knowledge of the primary component of the α CrB system. The relativistic contribution to the total periastron advance, 0.95 seconds, amounts to 17 % of the measured effect, and may amount to 24 % of the total.

Acknowledgements. The ROSAT project has been supported by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie (BMBF/DARA). I particularly thank J. Trümper for supporting the ROSAT PSPC observations carried out in February 1997.

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