

The relation between line ratio and emission measure analyses

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Abstract. Spectroscopic diagnosis of the temperature and density structure of hot optically thin plasmas from emission line intensities is usually described in two ways. The simplest approach, the ‘line ratio’ method, uses an observed ratio of emission line intensities to determine a ‘spectroscopic mean’ value of electron temperature $\langle T_e \rangle$ or electron density $\langle n_e \rangle$. The mean value is chosen to be the theoretical value of T_e or n_e which matches the observed value. The line ratio method is stable, leading to well defined values of $\langle T_e \rangle$ or $\langle n_e \rangle$ for each line pair but, in the realistic case of inhomogeneous plasmas, these are hard to interpret since each line pair yields different mean parameter values. The more general ‘differential emission measure’ (*DEM*) method recognizes that observed plasmas are better described by distributions of temperature or density along the line of sight, and poses the problem in inverse form. It is well known that the *DEM* function is the solution to the inverse problem, which is a function of T_e , n_e , or both. Derivation of *DEM* functions, while more generally applicable, is unstable to noise and errors in spectral and atomic data. The mathematical relation between these two approaches has never been precisely defined. In this paper we demonstrate the formal equivalence of the approaches, and discuss some potentially important applications of methods based upon combining the line ratio and *DEM* approaches.

Key words: Sun: corona – Sun: UV radiation – Sun: particle emission – accretion, accretion disks – atomic data

1. Introduction and motivation

Knowledge of the densities and temperatures of space plasmas is essential if we are to understand their most basic structure. Without this knowledge, almost nothing can be said regarding the generation and transport of mass, momentum and energy. Thus, since early in the era of space-borne spectroscopy we have faced the task of inferring plasma electron densities, n_e , and temperatures, T_e , for hot solar and other astrophysical plasmas from optically thin emission line spectra (e.g. Gabriel &

Jordan 1969; Munro et al. 1971; Gabriel & Jordan 1971; Dere & Mason 1981; Doschek 1987; Mason et al. 1994).

A fundamental property of hot solar plasmas is their basic inhomogeneity. This is obvious from direct images of the Sun’s corona and transition region which show a wealth of fine scale structure down to the observable limits of resolution (e.g., Golub & Pasachoff 1997). It is confirmed by less direct spectroscopic work which reveals differing values of n_e , T_e for different line ratios (see e.g. discussions by Doschek 1984, 1987). Strong inhomogeneity is expected also from physical considerations (a particularly interesting perspective, addressing why the plasmas do not appear to be even more inhomogeneous than already observed, is given by Litwin & Rosner 1993).

The emergent intensities of spectral lines from optically thin plasmas are determined by integrals along the line of sight through plasma parameters (that depend mostly on temperature and density) through the plasma. There are two common approaches to inferring plasma properties from observed spectral line intensities. Consider the case in which the temperature of the plasma is desired. The simplest approach, the ‘line ratio’ or ‘spectroscopic mean’ method, involves finding the single value of the electron temperature from a theoretical calculation of the ratio of carefully selected emission lines, that is in agreement with the observed ratio. A spectroscopic mean value of the temperature is derived for each line pair. If the plasma were truly isothermal, then the derived spectroscopic mean values for all line pairs would coincide with the actual plasma temperature, to within observational and theoretical errors. This approach was applied as early as 1941 to planetary nebulae by Menzel et al. (1941), and is reviewed by Gabriel & Jordan (1969) and Mason & Monsignori-Fossi (1994). The other method is to recast the above mentioned line integrals into suitable form for ‘inversion’, in which one solves for a function, $\xi(T_e)$, which is a source term that describes the distribution of material as a function of temperature along the line of sight. $\xi(T_e)$ is called the ‘Differential Emission Measure’ (*DEM*) Function¹. This gives a general characterisation of the distribution of the plasma with respect to temperature.

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¹ Misconceptions as to its definition persist in the literature, for example Tandberg-Hanssen (1995), pg. 177.

The integral equation formalism for temperature sensitive lines was first discussed by Pottasch (1964) and put on a rigorous mathematical basis for arbitrary geometry by Craig & Brown (1976). The formulation was later extended to a *DEM* function $\zeta(n_e)$ differential in n_e for isothermal plasmas (Almleaky et al. 1989, and references therein). The concept was also generalised to the bivariate case of $\mu(n_e, T_e)$ by Jefferies et al. (1972a, 1972b) although their definition contained an error corrected in the independent formulation by Brown et al. (1991). Formulation of how this general bivariate case could be cast as an inverse problem and in principle solved eluded these earlier authors and was finally established by Hubeny & Judge (1995) and elaborated by Judge et al. (1997).

Although more general than the line ratio method, it is well known that the *DEM* formulation is prone to errors in the solution arising from the ill-posed nature of the inverse operator - i.e. instability of the solution to errors in the spectral and atomic data (Craig & Brown 1976; Judge et al. 1997). This is intrinsic to the nature of the inverse problem, in which a continuous distribution function (or discretization thereof) is sought from a finite number of data points. Furthermore, there is significant linear dependence between the ‘kernels’ in the integral equations (see Sects. 3, 4 and 5). There is thus no single mathematical solution to the *DEM* problem, and the intrinsic ill-posedness must be addressed from the outset, essentially by smoothing the desired *DEM* function so that, in a loose sense, the number of independent *DEM* values does not exceed the number of measurements (e.g. Craig & Brown 1986). There are, as well as these fundamental limitations of the *DEM* method, practical problems concerning the nature and magnitude of errors in the theoretical calculation of the intensities of emission lines. For example, Judge et al. (1995) showed that the $\xi(T_e)$ problem also has large sources of systematic error in excess of known errors in line intensities, which they suggested are due to the breakdown of the fundamental assumption of ionisation equilibrium made in formulating the problem. In addition Judge et al. (1997) concluded that systematic errors in the atomic physics, and in the ionisation balance, make straightforward inversion for $\mu(n_e, T_e)$ very difficult or intractable.

The ‘mean value’ or ‘line ratio’ approach on the other hand gives well defined results which are appealing because they are simple to derive, and they can remove, through careful choice of lines, large sources of uncertainty arising from errors in ionization balance. However, they have the serious drawback that the results are not easy to interpret for inhomogeneous plasmas, different line ratios for example giving different mean densities even for lines peaking in the same temperature range because of their different detailed sampling of the temperature distribution (cf. Almleaky et al. 1989 and Brown et al. 1991).

The exact relationship between the two approaches has never been explored in depth, although particular situations were discussed by Brown et al. (1991). Motivated by this, by the advent of new data from the CDS and SUMER instruments on the SOHO spacecraft, and by the desire to remove the large sources of systematic error that plague inversions of emission line data (e.g., Judge et al. 1995; Judge et al. 1997), we study the relation-

ship between these two methods. We show that there is a precise correspondence between *DEM* functions and a suitable complete set of mean spectroscopic densities and/or temperatures in situations where these can be defined. We suggest methods that should improve the stability of inversions for $\xi(T_e)$, $\zeta(n_e)$ and $\mu(n_e, T_e)$, by using hybrid schemes. Such schemes combine line ratio techniques with the *DEM* formulation to minimize systematic errors that can arise from inaccurate or invalid ionisation balance calculations.

2. Mathematical issues and background

The total power P_i radiated by a particular spectral line labelled i , from an optically thin plasma occupying a volume V is:

$$P_i = \iiint_V h\nu_i A_i n_{u(i)} dV \quad \text{erg s}^{-1} \quad (1)$$

where h is Planck’s constant, ν_i is the frequency of the line, A_i (s^{-1}) is the Einstein A-coefficient, and $n_{u(i)}$ (cm^{-3}) is the population density of the upper level $u(i)$. Following standard practice, we define a line emission coefficient, $K_i(n_e(\mathbf{r}), T_e(\mathbf{r}))$, normalised to the electron density squared as

$$K_i(n_e(\mathbf{r}), T_e(\mathbf{r})) = \frac{h\nu_i n_{u(i)} A_i}{4\pi n_e^2} \quad \text{erg cm}^3 \text{ sr}^{-1} \text{ s}^{-1}, \quad (2)$$

then Eq. (1) becomes:

$$P_i = 4\pi \int_V K_i(n_e(\mathbf{r}), T_e(\mathbf{r})) n_e^2(\mathbf{r}) d^3\mathbf{r} \quad \text{erg s}^{-1}. \quad (3)$$

By writing the equations in this form, we note that several implicit assumptions have been made (see, e.g., Judge et al. 1997 for details), including that of ionisation equilibrium. $K_i(n_e(\mathbf{r}), T_e(\mathbf{r}))$ is almost independent of density n_e for collisionally excited permitted transitions decaying to the ground state of a given ion. This equation, with full dependence on n_e and T_e included in the emission coefficient, was studied by Jefferies et al. (1972a, 1972b), by formulating the integrand in terms of a function of electron density and temperature. This function was later identified (see Brown et al. 1991) as the bivariate *DEM* function of n_e and T_e , namely $\mu(n_e, T_e)$. Following the derivation of Brown et al. (1991) we make the following change of integration variable in Eq. (3):

$$d^3\mathbf{r} = \frac{dn_e dT_e}{|\nabla n_e| |\nabla T_e| \sin\theta_{n_e, T_e}} dL_{n_e, T_e} \quad \text{cm}^3 \quad (4)$$

Hence, transforming the volume integral of Eq. (1) to a path integral of the emissivity along a path of constant n_e, T_e . Here θ_{n_e, T_e} (assumed to be non-zero) is the local angle between vectors ∇n_e and ∇T_e normal to surfaces S_{n_e}, S_{T_e} of constant electron density and temperature respectively. The reader is referred to Fig. 1 in Brown et al. (1991) for a perspective of the geometry of the system and a discussion of the degenerate case $\theta_{n_e, T_e} = 0$. So for every spectral line i we have

$$P_i = 4\pi \int_{T_e} \int_{n_e} K_i(n_e, T_e) M(n_e, T_e) dn_e dT_e \quad \text{erg s}^{-1} \quad (5)$$

where, from Brown et al. (1991), $M(n_e, T_e)$ (cm^{-3}) is defined as

$$M(n_e, T_e) = \oint_{L_{n_e, T_e}} \frac{n_e^2}{|\nabla n_e| |\nabla T_e| \sin \theta_{n_e, T_e}} dL_{n_e, T_e} \quad (6)$$

Usually, one does not directly observe the total radiated power P_i , but the intensity ($\text{erg cm}^{-2} \text{st}^{-1} \text{s}^{-1}$), $I_i = P_i/(4\pi S)$, where S is the area of the projected volume V . Defining $\mu = M/S$, which has units of cm^{-5} , we find

$$I_i = \int_{T_e} \int_{n_e} K_i(n_e, T_e) \mu(n_e, T_e) dn_e dT_e \quad (7)$$

We are now in a position to define the differential emission measure in n_e , $\zeta(n_e)$, as the reciprocal density-gradient-weighted mean square electron density and, correspondingly the differential emission measure in T_e , $\xi(T_e)$ as the reciprocal temperature-gradient-weighted mean square electron density, obtained from Eq. (6) as follows:

$$\zeta(n_e) = \int_{T_e} \mu(n_e, T_e) dT_e \quad (8)$$

$$\xi(T_e) = \int_{n_e} \mu(n_e, T_e) dn_e \quad (9)$$

Thus, in terms of physical interpretation of a set of frequency integrated line intensities I_i alone, the differential emission measures in n_e and T_e must form the spectroscopic basis for further interpretation of the raw data, such as determining the energy balance or determining whether the data are compatible with an atmosphere at constant pressure (see Craig & Brown 1976; Judge et al. 1997). Formulation of the relationship between these functions and the ‘mean’ observed quantities is therefore of benefit to the solar physics community.

3. Relationship between $\xi(T_e)$ and $\langle T_e \rangle$

Consider a line i for which $K_i(n_e, T_e)$ is a weak function of density, such as a resonance line. $K_i(n_e, T_e)$ can then be replaced by $K_i(T_e)$. In this case we have the spectral line intensity

$$I_i = \int_{T_e} \xi(T_e) K_i(T_e) dT_e. \quad (10)$$

For two such lines i, j , the ratio of the two line intensities is

$$R_{ij} = \frac{I_i}{I_j} = \frac{\int_{T_e} \xi(T_e) K_i(T_e) dT_e}{\int_{T_e} \xi(T_e) K_j(T_e) dT_e}, \quad (11)$$

and if the emission coefficients are different, then the ratio depends on T_e . If the plasma is homogeneous in temperature, i.e. isothermal, we could express the DEM = $\xi(T_e)$ function as $\xi(T_e) = \xi_0 \delta(T_e - \langle T_e \rangle)$ such that, on substituting this expression into to Eq. (11) and integrating over the whole temperature domain, we have

$$R_{ij} = \frac{\xi_0 K_i(\langle T_e \rangle)}{\xi_0 K_j(\langle T_e \rangle)} \quad (12)$$

and on dividing throughout by ξ_0 we may express R_{ij} in terms of the ‘mean’ spectroscopic temperature, $\langle T_e \rangle_{ij}$, for the particular line pair (i, j) , i.e.

$$R_{ij} = \frac{K_i(\langle T_e \rangle)}{K_j(\langle T_e \rangle)} = S_{ij}(\langle T_e \rangle_{ij}) \quad (13)$$

where $S_{ij}(T_e) = \frac{K_i(T_e)}{K_j(T_e)}$ is assumed to be a monotonic, bijective (invertible) function which has a unique inverse on the temperature domain considered when we restrict our study to resonance lines, i.e. different T_e and *no* dependence on n_e , only. For these conditions the relationship is almost always satisfied. Therefore, on inspection, the relation between $\langle T_e \rangle_{ij}$ and the observed line ratios R_{ij} is given by

$$\langle T_e \rangle_{ij} = S_{ij}^{-1}(R_{ij}) \quad (14)$$

To formulate an expression for $\xi(T_e)$ in terms of the ‘mean’ spectroscopic temperatures we must return to Eq. (10). On dividing through Eq. (10) by any other line intensity $I_{j(i)}^{obs}$, ($i \neq j$), known to depend differently on T_e from line i (hence the notation $j(i)$), we obtain

$$R_{i,j(i)}^* = \frac{I_i}{I_{j(i)}^{obs}} = \int_{T_e} \xi(T_e) K_i'(T_e) dT_e \quad (15)$$

with $K_i'(T_e) = \frac{K_i(T_e)}{I_{j(i)}^{obs}}$. This expression thus gives the ratio of the *theoretical* intensity for line i to the *observed* intensity of line $j(i)$. At this stage I_i , and hence $R_{i,j(i)}^*$, are not known quantities. If we set $R_{i,j(i)}^* = R_{i,j}$, the observed line ratio, then Eq. (15) becomes an integral equation with known LHS, and known kernel $K_i'(T_e)$, in which $\xi(T_e)$ is the quantity to be determined. Consider forming n ratios of the intensities of a set of emission lines to form a vector \underline{R} :

$$\underline{R} = (R_{1,j(1)}, R_{2,j(2)}, \dots, R_{n,j(n)}) \quad (16)$$

If we discretize Eq. (15) with respect to T_e , then the equation becomes a matrix equation of the form:

$$\underline{R} = \underline{K}' \underline{\xi}. \quad (17)$$

The rows of \underline{K}' are simply rows of kernels of Eq. (10) divided by observed line intensities. This has the (poorly conditioned, see Craig & Brown 1986) analytical solution:

$$\underline{\xi} = \underline{K}'^{-1} \underline{R}. \quad (18)$$

This equation for n ratios permits ξ to be determined at up to n discrete temperatures. The above illustrates that the equations for line ratios can be simply re-written in a standard form, which can thus be used in numerical algorithms and will be discussed below. But we have not yet written the formal equivalence between the $\xi(T_e)$ functions and a set of line ratios, and their corresponding mean derived temperatures. From the above, this is clearly just

$$\underline{\xi} = \underline{K}'^{-1} \{S_{ij}(\langle T_e \rangle_{ij})\}, \quad (19)$$

where $\{S_{ij}(\langle T_e \rangle_{ij})\}$ denotes the array of line ratios indexed by i . This expression relates the DEM to the set of spectroscopically derived temperatures through the inverse of the matrix \underline{K}' .

4. Relationship between $\zeta(n_e)$ and $\langle n_e \rangle$

Here we use an approach analogous to that of Almléaky et al. (1989). Consider an optically thin plasma that is isothermal with $T_e = T_0$. The total emission of a line labelled i , given by Eq. (7) and Eq. (8) is

$$I_i = \int_{n_e} \zeta(n_e) K_i(n_e) dn_e, \quad (20)$$

since $K_i(n_e) = K_i(n_e, T_e = T_0)$. Since the plasma has no unique n_e , we can nevertheless define a spectroscopic ‘mean’ electron density for the any ratio of lines displaying some degree of density sensitivity, for instance using a resonance line and an intersystem line from a common ionisation stage of a particular ion. For this pair (i, j), we seek the electron density of a homogeneous plasma that would yield the same line ratio, R_{ij} , as the inhomogeneous plasma under observation. To achieve this we define $\zeta(n_e) = \zeta_0 \delta(n_e - \langle n_e \rangle)$, where $\langle n_e \rangle$ is the ‘mean’ spectroscopic electron density as defined earlier.

By direct analogy to the steps producing Eqs. (11) through (19) we can construct a relationship for the discretized differential emission measure in n_e , ζ , in terms of a set of ‘mean’ spectroscopic densities $\langle n_e \rangle_{ij}$, and the operator $H_{ij}(n_e) = \frac{K_i(n_e)}{K_j(n_e)}$. For purposes of writing expressions formally equivalent to those above, this operator must now be *assumed* to be unique (monotonic, bijective). Thus,

$$\zeta = \underline{K}'^{-1} \{ H_{ij}^{-1}(\langle n_e \rangle_{ij}) \}, \quad (21)$$

where \underline{K}'^{-1} is to be understood as the equivalent (but clearly not identical) matrix to that in Eq. (19). While this expression assumes that the inverse operator $H_{ij}^{-1}(\langle n_e \rangle_{ij})$ has a unique solution, notice that a numerical solution for ζ , analogous to Eq. (18), makes no such assumption. In fact, it removes ambiguities that can arise from the non-unique inverse operator $H_{ij}^{-1}(\langle n_e \rangle_{ij})$ for certain line ratios in important ions. This is because, in a numerical implementation, this operation is not performed. The vector element is instead set to the observed ratio $R_{i,j(i)}$. An example of non-unique inverse operators occurs for certain ratios of intersystem lines in the boron isoelectronic sequence (e.g., see Brage et al. 1996, Fig. 2).

5. Relationship between $\mu(n_e, T_e)$ and $\langle n_e \rangle, \langle T_e \rangle$ pairs

In the general case we wish to obtain information about the form of the bivariate differential emission measure, $\mu(n_e, T_e)$ from a set of ‘mean’ spectroscopic densities, $\langle n_e \rangle$, and temperatures, $\langle T_e \rangle$, discussed above. These ‘mean’ values are usually derived individually, as described earlier, by looking at line pairs that are mostly sensitive to T_e , or n_e , but not both.

Following the method of the previous sections, we seek mean parameters $\langle n_e \rangle$ and $\langle T_e \rangle$ of the homogeneous plasma that will yield the same line ratio as the observed inhomogeneous plasma. Some care must be taken here, as can be seen by, following earlier sections, assuming that the bivariate DEM function can be approximated by $\mu(n_e, T_e) = \mu_0 \delta(T_e - \langle T_e \rangle) \delta(n_e -$

$\langle n_e \rangle)$. Using Eq. (7) to form the line ratio of two lines with labels i and j , ($i \neq j$):

$$R_{ij} = \frac{I_i}{I_j} = \frac{\int_{T_e} \int_{n_e} K_i(n_e, T_e) \mu(n_e, T_e) dn_e dT_e}{\int_{T_e} \int_{n_e} K_j(n_e, T_e) \mu(n_e, T_e) dn_e dT_e} \quad (22)$$

On substitution of $\mu(n_e, T_e)$ given above into Eq. (22) and performing the double integral we obtain

$$R_{ij} = \frac{K_i(\langle n_e \rangle, \langle T_e \rangle)}{K_j(\langle n_e \rangle, \langle T_e \rangle)} = M_{ij}(\langle n_e \rangle, \langle T_e \rangle) \quad (23)$$

To try to determine $\langle n_e \rangle$ and $\langle T_e \rangle$ does not make sense, since there is just one equation, but two unknowns, $\langle n_e \rangle$ and $\langle T_e \rangle$. Thus it is clear that another equation is needed. One possible solution is to assume that $\langle T_e \rangle = T_{ij}^0$ where T_{ij}^0 is the coronal ionisation equilibrium temperature for the particular ion(s) under study. This is in fact a common assumption made for solar corona lines (e.g., Mason 1991). If this assumption (or something else) is made, then for a set of emission lines of temperature and density sensitivity, we see that the pair $(\langle n_e \rangle, \langle T_e \rangle)$ can be determined provided there exists an inverse function M_{ij}^{-1} , i.e.

$$(\langle n_e \rangle_{ij}, T_{ij}^0) = M_{ij}^{-1}(R_{ij}) \quad (24)$$

Repeating the steps taken to formulate Eq. (15) we divide through Eq. (7) by another line intensity, $I_{j(i)}$, again displaying the required functional (either density sensitive or temperature sensitive) behaviour to produce:

$$R_{i,j(i)} = \frac{I_i}{I_{j(i)}} = \int_{T_e} \int_{n_e} K'_i(n_e, T_e) \mu(n_e, T_e) dn_e dT_e \quad (25)$$

discretizing this with respect to n_e and T_e we have the following

$$R_{i,j(i)} = \sum_{l=1}^m \sum_{q=1}^p \mu(n_q, T_l) K'_i(n_q, T_l) \Delta n_e \Delta T_e \quad (26)$$

Performing an operation described in Hubeny & Judge (1995) we re-index from $l = 1, \dots, m$ and $q = 1, \dots, p$ to $\kappa = 1, \dots, mp$ so that Eq. (26) may be recast in a standard matrix form, where the $\Delta n_e, \Delta T_e$ terms are combined to form a measure of the redimensioned space, namely $\Delta(N_e \otimes T_e)$ and absorbed into the redimensioned form of $K'(n_e, T_e)$. Therefore Eq. (26) becomes:

$$R_{i,j(i)} = \sum_{\kappa=1}^{mn} U_{\kappa} K'_{i\kappa} \quad (27)$$

Where \underline{U} is the 1 dimensional transform of the 2 dimensional function $\underline{\mu}$. This has an analytical solution of the form (c.f. Eqs. (19) and (21))

$$\underline{U} = \underline{K}'^{-1} \{ M_{ij}^{-1}(\langle n_e \rangle_{ij}, T_{ij}^0) \}. \quad (28)$$

Again \underline{K}'^{-1} is equivalent, but not equal to that of Eq. (19). Also, the comments above on the uniqueness of inverse operators in the ζ problem apply equally to the bivariate problem.

6. Applications

The above formalism suggests new ways in which line ratio and DEM methods may be utilized together in an optimal fashion. Consider Eq. (11), and for illustration let us restrict our discussion to the temperature dependent problem only for a plasma close to coronal ionisation equilibrium. To illustrate the issue, let us split each kernel into the following factors:

$$K_i = \frac{h\nu_i}{4\pi} \cdot A_i \cdot \frac{n_{u(i)}}{n_{ion}n_e} \cdot \frac{n_{ion}}{n_{el}} \cdot \frac{n_{el}}{n_H} \cdot \frac{n_H}{n_e} \text{ erg cm}^3 \text{ sr}^{-1} \text{ s}^{-1}, (29)$$

where $\frac{n_{u(i)}}{n_{ion}}$ is the population density of the upper atomic level of the line divided by that of the ion to which it belongs (the subscript *ion* is a label for the ion charge, including the neutral case), $\frac{n_{ion}}{n_{el}}$, the ionic abundance, $\frac{n_{el}}{n_H}$, the elemental abundance relative to hydrogen and, $\frac{n_H}{n_e}$, the relative abundance of H to electrons respectively. As discussed at length by Judge et al. (1997), errors in the calculation of the number densities $n_{u(i)}$ (or, equivalently, the emission coefficients K_i) can, for this case, be split into two components. First, relatively small errors ($\leq 10\%$) arise in the cross sections for bound-bound processes that affect the factor $\frac{n_{u(i)}}{n_{ion}n_e}$ in Eq. (29). Second, much larger, more systematic errors arise from errors in treatment of bound-free processes affecting the ionisation balance factor $\frac{n_{ion}}{n_{el}}$ in Eq. (29). This immediately suggests that, when trying to solve DEM problems, line pairs i, j can be chosen only to include those of the same ionisation stage. This is of course standard practice for application of the line ratio approach. The proposed algorithm would thus be based upon minimization of the squared difference between observed and computed line ratios, instead of the more standard solution that would minimize the squared difference between observed and computed line intensities. This would remove (to zeroth order) most of the major sources of uncertainty that render the latter approach intractable (e.g., Judge et al. 1997). Some promising initial results have been obtained using such a scheme. Details will be presented elsewhere (M^CIntosh 1998).

7. Summary

We have shown that for an optically thin plasma, there is indeed a unique mathematical relationship between the ‘mean’ spectroscopic quantities $\langle n_e \rangle$, $\langle T_e \rangle$ and the differential emission measure functions ($\xi(T_e)$, $\zeta(n_e)$, and $\mu(n_e, T_e)$). These relationships are true provided certain assumptions hold regarding the nature of the emitting plasma, and the characteristic behaviour of particular line ratios, and show the equivalence between the full inversion and mean value methods.

For an optically thin, homogeneous, plasma and given a set of observed resonance line intensities, we have derived an expression that relates the ‘mean’ spectroscopic temperatures $\{\langle T_e \rangle\}$ and the discretized differential emission measure in temperature ξ (see Eq. (19)). Following a similar method we have obtained, for an isothermal plasma, an expression relating the ‘mean’ spectroscopic densities $\{\langle n_e \rangle\}$ and the discretized differential emission measure in electron density, ζ , which is given

in Eq. (21). In the treatment of the general bivariate DEM function $\mu(n_e, T_e)$, Sect. 4 shows that we can obtain a representation of $\mu(n_e, T_e)$ when the conditions for Sects. 2 and 3 occur simultaneously (*i.e.* situations where ‘mean’ densities and temperatures are actually defined). Essentially this means that for a large enough set of observed lines with different temperature and density characteristics, a relationship of the form of Eq. (28) will hold for the particular set of inferred ‘mean’ $\{\langle T_e \rangle, \langle n_e \rangle\}$ pairs.

Finally, we pointed out some potentially important applications of inversion schemes based upon minimizing differences between observed and computed *ratios*, instead of the more usual *intensities*. These offer the possibility of removing large, systematic errors that may arise from uncertain ionization balance, as have been suggested to explain solar data (Judge et al. 1995), and have been demonstrated to be the dominant source of error in standard inversions of line intensities (Judge et al. 1997).

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