Astron. Astrophys. 333, 374–378 (1998)

ASTRONOMY AND ASTROPHYSICS

The ecliptic in general relativity

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Received 13 August 1997 / Accepted 18 September 1997

Abstract. This article deals with relativistic effects on the motion of ecliptic. It is pointed out that the traditional definition of ecliptic can be used in the framework of general relativity: in general the definition of ecliptic is coordinate dependent and proper quasi-Cartesian coordinates should be adopted to avoid ambiguity of the definition. We derive equations that determine the motion of the ecliptic and have found the magnitude of relativistic effects is rather tiny and can be completely neglected at present and in the near future.

Key words: relativity – reference systems – astrometry

1. Introduction

The ecliptic is one of the fundamental reference planes in dynamics and measurements of the solar system. It is usually defined as the mean orbital plane of the barycenter of the Earth-Moon system (EM) relative to the barycenter of the solar system. One can compute the orbital angular momentum of EM and determine its normal plane, i.e. the osculating plane. It has tiny periodic and secular variations generated by various perturbations. Taking the time-average of the osculating plane, its periodic variation can be eliminated. The time-average orbital plane is exactly the ecliptic, in which there still exist secular variations.

Standish (1981) pointed out that there are two different definitions of the ecliptic in use, which are Le Verrier's ecliptic and Newcomb's ecliptic, respectively. Le Verrier's ecliptic is a mean orbital plane determined by the secular part of the orbital angular momentum of EM in an inertial reference frame, e.g. the equator of epoch. Newcomb's ecliptic is determined in a slowly rotating reference frame, e.g. Le Verrier's ecliptic. Standish (1981) has shown that there is a tiny difference between these two ecliptics. The details concerned with these two ecliptics can be found in papers of Standish (1981, 1982), Kinoshita and Aoki (1983).

Le Verrier and Newcomb's ecliptics are both defined in the framework of classical mechanics. In the last three decades, a lot of new and high-precision observational techniques, e.g. LLR, SLR, VLBI, GPS and so on, appeared and have been extensively used. Observational data have been accumulated with great rapidity. These make relativistic effects to be seriously considered in astronomical practice. Theoretically, astrometry and celestial mechanics should be based on general relativity. For this grand goal, much work has been accomplished.

As early as 1983, Murray (1983) introduced general relativity in his book "Vectorial Astrometry". Even earlier, Brumberg (1972) summarized and developed relativistic celestial mechanics as a branch of astronomy. Ashby and Bertotti (1986), Fukushima (1988), Kopejkin (1988), Brumberg and Kopejkin (1989) and others proposed various ways to construct astronomical reference systems. From 1991 to 1994, Damour, Soffel and Xu (1991, 1992, 1993, 1994) presented a new and systematic post-Newtonian (1PN) theory of systems of N arbitrarily composed and shaped, weakly self-gravitating, rotating, deformable bodies (DSX theory in short). This theory is rigorous and self-consistent at the 1PN level. All these developments make it possible for us to define and discuss the ecliptic at the 1PN level.

Brumberg et al. (1996) wrote that:"..., in accordance with the physical meaning the equator should be defined in GRS (geocentric RS) and, in contrast to Newtonian mechanics, its extension in GRT to BRS (barycentric RS) may be realized in different ways. On the contrary, the ecliptic should be originally defined in BRS and its GRT extension to GRS is not unique. Hence, it is evident, first of all, that the Newtonian definitions of the equinox and the obliquity need to be made more precise." The problem presented by them is significant. But it is not going to be discussed in this paper.

The ecliptic in use is the ecliptic employed in the JPL DE series ephemerides, which adopts a PPN metric but constructs the ecliptic by a traditional and Newtonian way (Standish, 1982). In the framework of general relativity, three questions can be raised: (1) Is the DE ecliptic a proper one in relativity? (2) Is the ecliptic coordinate dependent by considering that so many quasi-Cartesian coordinates exist in general relativity? (3) What is the magnitude of relativistic effects on the motion of ecliptic? This paper tries to answer these questions.

We suggest that a relativistic 1PN definition of ecliptic should have the following properties: (1) In the Newtonian approximation, it reduces to the standard Newtonian ecliptic. (2) In

Send offprint requests to: T.-Y. Huang the spherically symmetric field, it reduces to the Schwarzschild ecliptic, which will be discussed in the next section. (3) It should be identical or close to the DE ecliptic to keep continuation. (4) There are only observables in the expressions determining the 1PN ecliptic and its motion.

This paper is arranged as follows: In Sect. 2, the Newtonian ecliptic is reviewed, and the definition of ecliptic in the Schwarzschild field is discussed. In Sect. 3, the definition of the 1PN ecliptic and its equations of motion are presented, after we take up the so-called monopole-spin-quadrupole (MSQ) truncation and some reasonable simplifications in the solar system are assumed. In Sect. 4, we estimate the magnitude of relativistic effects on the ecliptic and conclude that all of them can be neglected at the present precision and in the near future.

2. Newtonian and Schwarzschild ecliptic

After a reference frame is taken, the ecliptic is fully determined by the orbital angular momentum of EM. Since the mass is an invariant, we only need to calculate the orbital angular momentum per unit mass of EM, i.e.

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \tag{1}$$

where $\dot{\mathbf{r}} = d\mathbf{r}/dt$, $\mathbf{r}$ is the coordinate vector of the EM barycenter in the barycentric coordinate system of the solar system, which is usually taken as the equator system of epoch.

The longitude of the ascending node Ω and the inclination I of the EM barycenter can be determined from

$$\begin{aligned} h_x/h &= \sin I \sin\Omega \\ h_y/h &= -\sin I \cos\Omega \\ h_z/h &= \cos I \end{aligned} \tag{2}$$

where $h = |\mathbf{h}| = (h_x^2 + h_y^2 + h_z^2)^{1/2}$. Ω and I determine the instantaneous orbital plane. They can be separated into $\Omega = \Omega_s + \Omega_p$ and $I = I_s + I_p$, where the subscripts s and p stand for the secular part and the periodic part of each element, respectively. Taking the time-average of Ω and I, one can then remove the periodic parts, Ω_p and I_p. Ω_s and I_s define a moving reference plane, which is Le Verrier's ecliptic.

Even in Newtonian mechanics, the orbital angular momentum is a pseudo-vector in Euclidean space. The vector $\mathbf{h}$ would be changed if we translate or rotate the reference frame, but is invariant under fixed rigid rotations. If one computes $\mathbf{h}' = \mathbf{r}' \times \dot{\mathbf{r}}'$, where the coordinate vector $\mathbf{r}'$ is now referred to a rotating frame, e.g. Le Verrier's ecliptic. The secular parts, Ω_s' and I_s', of $\mathbf{h}'$ in the equator system of epoch define Newcomb's ecliptic (Standish called it the ecliptic in the rotating sense). Due to the extensive use of Newcomb's ecliptic in history, Standish(1981) considered that it might seem prudent to adopt this definition as a standard for the sake of continuity.

The Newtonian theory is only a primary approximation of relativity. As a simple and instructive example, consider the Schwarzschild field approximation, to which Murray (1983) gave a detailed description on its application in astronomy.

Assume that the Sun is not in rotation and its mass density is spherically symmetric, EM is simplified as a point mass, and masses of other planets are negligible, then the EM barycenter moves along a geodesic in the spherically-symmetric gravitational field of the Sun.

Taking quasi-Cartesian coordinates which keep spherical symmetry, t and $\mathbf{x}$, the metric of the Schwarzschild field is

$$ds^2 = -c^2 e^{\lambda} dt^2 + e^{\iota}(\mathbf{u}\cdot d\mathbf{x})^2 + e^{\kappa} d\mathbf{x}^2 \tag{3}$$

where $\lambda,\ \iota,\ \kappa$ are functions of $|\mathbf{x}|$ and

$$\begin{aligned} e^{\lambda} &= 1 - 2\mu e^{-\kappa/2}/c^2|\mathbf{x}| \\ 1 + e^{\iota-\kappa} &= e^{-\lambda}(1+\kappa_1|\mathbf{x}|/2)^2 \\ \kappa_1 &= d\kappa/d|\mathbf{x}|, \end{aligned} \tag{4}$$

in which μ is a constant, an arbitrary function κ shows the freedom of coordinate choice. One of the following three choices is usually adopted:

$$\kappa = \begin{cases} 0, & \text{(standard coordinate)} \\ 4\ln(1+\mu/2c^2|\mathbf{x}|), & \text{(isotropic coordinate)} \\ 2\ln(1+\mu/c^2|\mathbf{x}|), & \text{(harmonic coordinate)} \end{cases} \tag{5}$$

It is easy to get the orbital angular momentum

$$\mathbf{x} \times d\mathbf{x}/dt = (d\tau/dt) e^{-\kappa} H \mathbf{h} \tag{6}$$

Here τ is the proper time, H a scalar constant and $\mathbf{h}$ a constant unit vector. Eq. (6) clearly shows that the ecliptic in the Schwarzschild field is coordinate-independent among the quasi-Cartesian coordinates that keep spherical symmetry. It would not be the case in a more practical model.

3. The 1PN ecliptic

According to the first three properties presented in the introduction section which a 1PN ecliptic should have, we define the orbital angular momentum per unit mass of EM in a solar system barycentric coordinate system as

$$s_i^{EM} \equiv \epsilon_{ijk} z_{EM}^j v_{EM}^k \tag{7}$$

where ϵ_{ijk} is a Levi-Civita pseudo-tensor, $\mathbf{z}_{EM}$ the coordinate vector of the EM barycenter, and $\mathbf{v}_{EM} \equiv d\mathbf{z}_{EM}/dt$ its coordinate velocity. Here one should adopt some quasi-Cartesian coordinates. The ecliptic is determined by the secular part of $\mathbf{s}_{EM}$.

It is evident that $\mathbf{s}_{EM}$ is coordinate dependent, therefore more restrictions should be added on the coordinates. We suggest to adopt so-called DSX coordinates, which satisfy the post-Newtonian assumption and the algebraic coordinate condition for the metric (Damour et al., 1991). The family of DSX coordinates is wide enough to include various coordinates appeared in the references of relativistic celestial mechanics and astrometry. DSX coordinates have made the metric and the equations of the gravitational field quite simple. There is a gauge freedom in DSX coordinates (Damour et al., 1991), that is

$$t = t' - c^{-4}\lambda(t', x'^j), \quad x^i = x'^i \tag{8}$$

where λ is an arbitrary function. It is easy to see that $\mathbf{s}_{EM}$ in Eq. (7) is gauge-invariant in the 1PN approximation. Hereafter in this section, we will derive the equation of variation of $\mathbf{s}_{EM}$ and show that it can be expressed by observables, which meets the fourth requirement of an 1PN ecliptic. In our derivation, we will follow the symbols and conventions in Damour et al. (1991).

The equation of variation of $\mathbf{s}_{EM}$ is

$$\dot{s}_i^{EM} = \epsilon_{ijk} z^j_{EM} a^k_{EM} \tag{9}$$

where $\mathbf{a}_{EM} = d^2\mathbf{z}_{EM}/dt^2$ is the coordinate acceleration of the EM barycenter.

From Eq. (6.29) in Damour et al. (1991), we can compute the accelerations of the Earth and Moon,

$$\begin{aligned} a^k_A &= (1 + \bar{w}_A/c^2 - v_A^2/c^2)(\delta_{bc} - \\ &- V_b^A V_c^A/2c^2)\delta_c^k A_b^A + O(4), \quad A = E, M \end{aligned} \tag{10}$$

where indices E and M mean the Earth and Moon respectively, V_b^A and A_b^A are roughly the velocity and acceleration of body A's barycenter respectively, resolved in its local coordinate system, $\bar{w}_A$ the external potential of body A, generated by other bodies in the global coordinate system. A_b^A is determined by the translational equations of motion (Damour et al., 1992).

The expression of A_b^A is an infinite series in bilinear terms of relativistic multipole moments and tidal moments. It is rather complicated and a reasonable truncation is necessary. For the purpose of this paper, we will do the following simplification: (1) We will only keep the terms containing the mass M^A, spin S_a^A and mass quadrupole moments M_{ab}^A of body A. It is called the MSQ truncation model. (2) In the post-Newtonian terms, we will only keep the terms containing the mass M^A and spin S_a^A. The largest error would be caused by neglecting the Earth's mass quadrupole moments in the 1PN terms, which is estimated to be less than 3×10^{-15}.

Under the above simplification and by the aid of Eqs. (6.15)-(6.17) in Damour et al. (1992), the expression of A_b^A is simplified as

$$A_a^A = \sum_{B \neq A} [G_a^{B/A} + M_{bc}^A G_{abc}^{B/A}/2M^A] \tag{11}$$

where $G_a^{B/A}$ and $G_{abc}^{B/A}$ are the gravitoelectric tidal moments contributed by body B to body A. Eq. (10) can be rewritten as

$$\begin{aligned} a^k_A &= A_k^A + \tfrac{1}{c^2}[(\sum_{B \neq A} W^{B/A} - v_A^2)A_k^A - \\ &- v_A^k(\mathbf{A}_A \cdot \mathbf{v}_A)/2] + O(4), \quad A = E, M \end{aligned} \tag{12}$$

Inserting Eq. (11) into (12), we have immediately

$$\begin{aligned} a^k_A &\equiv \sum_{B \neq A} a_k^{B/A}, \quad A = E, M \\ a_k^{B/A} &= G_k^{B/A} + M_{bc}^A G_{kbc}^{B/A}/2M^A + \\ &+ \tfrac{1}{c^2}(GM^B/r_{AB}^2) \times \{(v_A^2 - \sum_{C \neq A}(GM^C/r_{AC}))n_{AB}^k + \\ &+ v_A^k(\mathbf{v}_A \cdot \mathbf{n}_{AB})/2\} + O(4) \end{aligned} \tag{13}$$

where only $G_k^{B/A}$ needs to reach the 1PN precision; other physical quantities only need to reach the Newtonian precision. The expression of $G_k^{B/A}$ and $G_{kbc}^{B/A}$ can be found in Xu et al. (1997).

Finally, we obtain the 1PN expression of a_A^k under MSQ truncation

$$a_A^k = \sum_{B \neq A} a_k^{B/A}, \; a_k^{B/A} = a_{k,EIH}^{B/A} + a_{k,SQ}^{B/A}, \; A = E, M \tag{14}$$

where the subscript EIH means the EIH case (Einstein et al., 1938), i.e. the monopole truncation in the DSX framework, SQ means correction terms produced by spins and mass quadrupole moments. Their expressions are

$$\begin{aligned} a_{k,EIH}^{B/A} &= -\frac{GM^B}{r_{AB}^2} n_{AB}^k \{1 + c^{-2}[2v_{AB}^2 - v_A^2 - \frac{3}{2}(\mathbf{n}_{AB} \cdot \mathbf{v}_B)^2 \\ &- 4\sum_{C \neq A} \frac{GM^c}{r_{AC}} - \sum_{C \neq B} \frac{GM^c}{r_{BC}}(1 - (\mathbf{n}_{AB} \cdot \mathbf{n}_{BC})\frac{r_{AB}}{2r_{BC}})]\} \\ &+ c^{-2}\frac{GM^B}{r_{AB}}[-\frac{7}{2}\sum_{C \neq B} \frac{GM^c}{r_{BC}^2} n_{BC}^k \\ &+ \tfrac{\mathbf{n}_{AB} \cdot (4\mathbf{v}_{AB} + \mathbf{v}_B)}{r_{AB}} v_{AB}^k] + O(4) \end{aligned} \tag{15}$$

and

$$\begin{aligned} a_{k,SQ}^{B/A} \; = \; & -\frac{15G}{2r_{AB}^4} n_{AB}^{<kbc>}(M_{bc}^B + \frac{M_{bc}^A M^B}{M^A}) \\ & + \frac{6G}{c^2 r_{AB}^3} v_{AB}^d S_b^B (\epsilon_{bcd} n_{AB}^{<ck>} + \epsilon_{kcb} n_{AB}^{<cd>}) \\ & + O(4) \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B, \; \mathbf{r}_{AB} = \mathbf{z}_A - \mathbf{z}_B, \\ &r_{AB} = |\mathbf{r}_{AB}|, \; \mathbf{n}_{AB} = \tfrac{\mathbf{r}_{AB}}{r_{AB}}, \\ &n_{AB}^{<ab>} = n_{AB}^{<a} n_{AB}^{b>}, \; n_{AB}^{<abc>} = n_{AB}^{<a} n_{AB}^b n_{AB}^{c>}, \end{aligned} \tag{17}$$

The angular bracket denotes the symmetric and trace-free (STF) part (Damour et al., 1991). We can also include higher-mass multipole moments in Newtonian terms without difficulty, if necessary.

Eqs. (14)-(16) show the 1PN expressions of the accelerations of the Earth and Moon under MSQ truncation. They are not enough to determine the motion of ecliptic. We have to calculate the coordinate, velocity and acceleration of the EM barycenter. For an isolated N-body system, there exists 1PN energy conservation (Damour and Vokrouhlicky, 1995)

$$\sum_A M^A(1+f_A) = \text{constant} \tag{18}$$

and 1PN momentum conservation

$$\sum_A M^A z^i_A(1+f_A) = 0 \tag{19}$$

where

$$f_A = \frac{1}{2c^2}(v^2_A - \sum_{B\neq A}\frac{GM^B}{r_{AB}}) \tag{20}$$

when the space origin is properly chosen. Let M^A, z^i_A, v^i_A and a^i_A denote the mass, coordinate, velocity and acceleration of the EM barycenter respectively, then they can be computed by the following equations without difficulty

$$\begin{aligned}
&M^A(1+f_A) = M^E(1+f_E) + M^M(1+f_M) + O(4)\\
&M^A z^i_A(1+f_A) = M^E z^i_E(1+f_E) + M^M z^i_M(1+f_M)\\
&\quad + O(4)\\
&M^A[v^i_A(1+f_A) + z^i_A\dot{f}_A] = M^E[v^i_E(1+f_E) + z^i_E\dot{f}_E]\\
&\quad + M^M[v^i_M(1+f_M) + z^i_M\dot{f}_M] + O(4)\\
&M^A[a^i_A(1+f_A) + 2v^i_A\dot{f}_A + z^i_A\ddot{f}_A] =\\
&\quad M^E[a^i_E(1+f_E) + 2v^i_E\dot{f}_E + z^i_E\ddot{f}_E]\\
&\quad + M^M[a^i_M(1+f_M) + 2v^i_M\dot{f}_M + z^i_M\ddot{f}_M] + O(4)
\end{aligned} \tag{21}$$

4. Discussion and conclusion

The ecliptic should be defined as the mean orbit of the EM barycenter relative to the solar system barycenter and be determined in the framework of general relativity. This section will concentrate on estimating the magnitude of various relativistic effects.

(1) Choice of the space origin

The space origin should be the solar system barycenter rather than the Sun. The largest difference between the positions of the barycenter and the Sun is due to the giant planets, Jupiter and Saturn. The deviation of the inclination of the EM orbit caused by choosing the space origin at the Sun can be estimated by

$$\Delta I \approx i_P a_P \frac{M_P}{M_S} \tag{22}$$

where i_P, a_P and M_P are the inclination, semi-major axis (in astronomical units) and mass of planet P respectively. They are about 23″ and 24″ for Jupiter and Saturn, respectively.

In general relativity, the Newtonian law of momentum conservation should be replaced by the 1PN law of momentum conservation, Eq. (19). The position vector of the Sun, $\mathbf{z}_S$, can be computed from those of the planets, $\mathbf{z}_B$, by use of

$$\mathbf{z}_S = -\frac{1}{M_S}\sum_{B\neq S} M^B \mathbf{z}_B(1+f_B-f_S) \tag{23}$$

rather than by the Newtonian law

$$\tilde{\mathbf{z}}_S = -\frac{1}{M_S}\sum_{B\neq S} M^B \mathbf{z}_B \tag{24}$$

The difference is

$$\Delta\mathbf{z}_S = -\frac{1}{M_S}\sum_{B\neq S} M^B \mathbf{z}_B f_B \tag{25}$$

where f_S has been neglected by the fact that $f_S \ll f_B (B\neq S)$.

From Eq. (20), f_B can be estimated as

$$f_B \approx \frac{1}{2c^2}(v^2_B - \frac{GM^S}{r_{BS}}) \leq \frac{GM^S}{2c^2 a_B} e_B \tag{26}$$

where a_B and e_B are the semi-major axis and the eccentricity of planet B respectively. The largest contribution for $\Delta\mathbf{z}_S$ comes from Jupiter, which is estimated to be about 4 cm. Its contribution to ΔI is about 10^{-3} μas and can obviously be dropped.

To investigate if this relativistic effect has an accumulating effect, we did a simple numerical simulation. In a Sun-Earth two-body model, the Earth revolves around the barycenter of the system under Newtonian gravitation, but the Sun's coordinate is computed by the 1PN Eq. (23). Numerical integration shows no observable variation of the elements of the Earth. When Jupiter's relativistic perturbation on the Sun is added into the model, the Earth's apsidal line has a small oscillation and a tiny secular regression of about 3 μas per century. Jupiter's apsidal line has a larger and more evident regression of about 12 μas per century. We have not found any visible secular variation of other elements.

(2) Choice of the reference point

The reference point of the angular momentum for determining the ecliptic should be the EM barycenter rather than the Earth. The difference between the two can be estimated by

$$\Delta I \approx \frac{M_M}{M_E+M_M} a_M i_M \tag{27}$$

where M_M, M_E, a_M and i_M are the mass of the Moon, mass of the Earth, semi-major axis (in astronomical unit) and inclination of the Moon relative to the Earth respectively. ΔI can reach $0\farcs6$.

Similarly, the EM barycenter should be determined by the 1PN law of momentum conservation. If one computes the position and velocity vectors of the EM barycenter, $\mathbf{z}_A$ and $\mathbf{v}_A$, just by the Newtonian law, the deviation would be

$$\Delta\mathbf{z}_A = \tfrac{M_E M_M}{(M_E+M_M)^2}(f_M - f_E)(\mathbf{z}_M - \mathbf{z}_E) \tag{28}$$

and

$$\begin{aligned}\Delta\mathbf{v}_A = &\tfrac{M_E M_M}{(M_E+M_M)^2}[(f_M - f_E)(\mathbf{v}_M - \mathbf{v}_E)\\ &+ (\dot{f}_M - \dot{f}_E)(\mathbf{z}_M - \mathbf{z}_E)]\end{aligned} \tag{29}$$

where

$$\dot{f}_E = -\frac{1}{2c^2}\sum_{B\neq E}\frac{GM_B}{r_{EB}^2}\mathbf{n}_{EB}\cdot(\mathbf{v}_E+\mathbf{v}_B) \tag{30}$$

The expression for $\dot{f}_M$ is similar. It is easy to estimate that

$$|\Delta\mathbf{z}_A|/|\mathbf{z}_A| < 5\times10^{-15}, \quad |\Delta\mathbf{v}_A|/|\mathbf{v}_A| < 7\times10^{-14}$$

This would cause an error of the orientation of ecliptic less than $10^{-3}\mu$as and can be completely neglected.

(3) Relativistic perturbation on the ecliptic

We now turn to estimate the relativistic effects due to the acceleration in Eq. (14). Only the normal component of $\mathbf{a}_A$, which is perpendicular to the ecliptic, would cause motion of the ecliptic. Let

$$W_A = \mathbf{a}_{A,EIH}^{relativistic}\cdot\mathbf{h} \tag{31}$$

be the normal component of the relativistic acceleration, in which the index $relativistic$ represents choosing the terms in $\mathbf{a}_{A,EIH}$ with the factor c^{-2} only.

W_A can be separated into two parts: W_A^B contributed from planets B and W_A^S from the Sun. For a rough estimate of the magnitude of relativistic effects, we will make the following simplifications: (a) neglecting the second perturbations by planets; (b) taking a Sun-Earth-Jupiter model; (c) neglecting the eccentricities of their orbits; (d) neglecting v_J/v_E, which is about 0.4 and too big to be dropped, but this approximation can greatly simplify the calculation and will not harm the estimate of magnitude. Then we have

$$W_E^J \approx -\frac{3GM^J}{r_{AJ}^3}(\frac{v_E}{c})^2(\mathbf{z}_J\cdot\mathbf{h}) \tag{32}$$

where $\mathbf{h}$ is the unit vector along the normal of the Earth's orbit plane. The derivation of W_A^S is a little more complicated and the law of momentum conservation has to be adopted,

$$W_E^S \approx \frac{3GM^J}{r_{AS}^3}(\frac{v_E}{c})^2(\mathbf{z}_J\cdot\mathbf{h}) \tag{33}$$

The Lagrangian planetary equations for elements I_E and Ω_E are

$$\frac{dI_E}{dt} = \frac{1}{n_E a_E^2\sqrt{1-e_E^2}} r_E \cos u_E W_E \tag{34}$$

and

$$\sin I_E \frac{d\Omega_E}{dt} = \frac{r_E \sin u_E}{n_E a_E^2\sqrt{1-e_E^2}} W_E \tag{35}$$

where u_E is the latitude argument of the Earth. Finally, we have the estimate of the variation of the ecliptic as

$$|\tfrac{dI_E}{n_E dt}| \approx |\sin I_E \tfrac{d\Omega_E}{n_E dt}| < \\ < 3(\tfrac{M_J}{M_S})(\tfrac{v_E}{c})^2(\tfrac{a_J}{a_E})\sin I_J \approx 0.4\ \mathrm{mas/century} \tag{36}$$

This is not a very small quantity to be neglected. But the motion of the ecliptic is represented by the secular part of dI_E/dt and $\sin I_E d\Omega_E/dt$, and it is easy to see that their average values would be zero if one of the eccentricities of the Earth and Jupiter is omitted. Therefore, the magnitude of the relativistic motion of ecliptic is about $0.4e_E e_J$ mas per century, which is roughly 0.4 μas per century. It can certainly be neglected.

From Eq. (16), influence on the motion of ecliptic by the spin of the Sun can be also estimated in a magnitude of microarcseconds per century.

Our main conclusions are: (a) The traditional definition of the ecliptic can be used in the framework of general relativity. (b) The definition of the ecliptic is usually coordinate-dependent. To avoid this ambiguity, we have to fix the adopted quasi-Cartesian coordinates. We suggest to adopt DSX coordinates. The 1PN ecliptic is invariant under gauge transformations among DSX coordinates. The family of DSX coordinates is wide enough to include various coordinates appeared in the references of relativistic celestial mechanics and astrometry. (c) In the 1PN approximation for the ecliptic, the solar system barycenter and the barycenter of the Earth-Moon system can be computed by the Newtonian law. (d) Practically, relativistic effects on the ecliptic can be neglected at present and in the near future, if the coordinate system be properly chosen. (e) The DE ecliptic is a qualified one at present and in the near future.

Acknowledgements. The authors would like to thank Prof. C. Xu and Prof. M. Soffel for helpful discussion. And our special thanks are to Prof. V. A. Brumberg for his critical comments. This research is supported by The National Natural Science Foundation of China.

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