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Tidal disruption Eddington envelopes around massive black holes

Andrew Ulmer[1], **Bohdan Paczyński**[2], **and Jeremy Goodman**[2]

[1] Max-Plank-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85740 Garching, Germany
[2] Princeton University Observatory, Peyton Hall, Princeton, NJ 08544, USA

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Abstract. Optically-thick envelopes may form following the tidal disruption of a star by a massive black hole. Such envelopes would reprocess hard radiation from accretion close to the black hole into the UV and optical bands producing AGN-luminosity flares with duration ~ 1 year. We show that due to relativistic effects, the envelopes are convective. If convection is efficient, then the structure of the envelopes is similar to that described in previous work; however, the photospheric radius is shown to be very sensitive to the luminosity at the envelope base, suggesting that either the envelope collapses or the envelope expands to a maximum radius at which point a wind may set in. For an envelope without winds, we find a maximum photospheric radius of $\sim 10^{16}$ cm (i.e. minimum effective temperature $\sim 6,000$ K). The evolution of the envelopes is described based on simple energy arguments.

Key words: black hole physics – galaxies: active – galaxies: nuclei – quasars: general – ultraviolet: galaxies

1. Introduction

One likely outcome of the tidal disruption of a star by a massive black hole is a bright flare with duration of a few months to years (e.g. Rees 1988, Ulmer 1998a; hereafter U98). The flares are generally thought to be quite hot. For example, the temperature associated with an Eddington luminosity emitted from a spherical photosphere at the tidal radius is

$$T_{\rm eff} \approx \left(\frac{L_{\rm E}}{4\pi R_{\rm t}^2 \sigma}\right)^{1/4} \tag{1}$$

$$\approx 2.5\times 10^5 M_6^{1/12}\left(\frac{R_\star}{R_\odot}\right)^{-1/2}\left(\frac{M_\star}{M_\odot}\right)^{-1/6} \ {\rm K}, \tag{2}$$

where M_6 is the black hole mass in units of a million solar masses and $M_\star$ and $R_\star$ are the mass and radius of the star. Consequently, the discussions of the spectra and observability of flares have focused on hard emission (e.g. Sembay & West, 1993) or the extreme, ~ 7.5 magnitude, bolometric corrections to optical (e.g. U98). Loeb & Ulmer (1997, hereafter LU97) make the interesting suggestion that if part of the tidal debris forms an extended envelope around the black hole, then the light may be largely reprocessed down to optical with effective temperatures of

$$T_{\rm eff} \approx 1.3\times 10^4 \left(\frac{M_{\rm BH}}{10^6 M_\star}\right)^{1/4} \ {\rm K}. \tag{3}$$

In the model, an accretion disk forms close to the black hole, and the energy released from the disk is reprocessed by the extended envelope which connects in an unknown way to the disk.

In Sect. 2, we show that without knowledge of the base luminosity which is provided by disk accretion onto the black hole, it is impossible to specify uniquely the resulting envelope structure. In Sect. 3, we discuss the small, but important, redshift effects from the Schwarzschild geometry and show that the envelopes are convective. The system's time scales are investigated in Sect. 4, and using simple energy arguments, we make predictions regarding the evolution of the envelope. A discussion of these results is presented in Sect. 5.

2. Lack of closure relations

The envelope density profile derived in LU97 (Eq. 3) results in a logarithmically diverging mass, so an artificial cutoff of the atmosphere was taken in LU97 (Eq. 8). A related problem is that the pressure close to the photosphere is not handled in a self-consistent manner. In this section, we address these problems and discuss their implications.

We operate in a Newtonian potential, assume that the envelope is radiative, and link the envelope model to the atmosphere using the Eddington approximation in which the surface temperature, $T_0 = (1/2)^{1/4} T_{\rm eff}$, where $T_{\rm eff}$ is the effective temperature. At the photospheric radius, $r_{\rm phot}$, where the optical depth is 2/3, $T = T_{\rm eff}$. The Eddington approximation requires that the atmosphere be thin so that the surface is not much larger than $r_{\rm phot}$. The hydrostatic equation is:

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \tag{4}$$

Send offprint requests to: A. Ulmer

where M, is the mass of the black hole alone. We neglect the envelope mass since $M_{\rm env}/M_{\rm BH} \sim 10^{-6}$. The equation of radiative transfer is

$$\frac{dP_{\rm rad}}{dr} = -\frac{\kappa L\rho}{4\pi c r^2}, \tag{5}$$

where the luminosity, L, and absorption coefficient, κ, are assumed to be constant throughout the envelope. Dividing the previous two equations yields the familiar result:

$$1 - \frac{dP_{\rm gas}}{dP} = \frac{dP_{\rm rad}}{dP} = \frac{\kappa L}{4\pi c G M} = \frac{L}{L_{\rm E}}, \tag{6}$$

where $L_{\rm E}$ is the Eddington luminosity. The right side of Eq. 6 is constant, because the mass is dominated by the mass of the black hole, the luminosity is wholly supplied by the accretion disk at the base of the envelope, and κ is dominated by electron scattering. Integrating, we find

$$P_{\rm gas} = (P - P_0)\left(1 - \frac{L}{L_{\rm E}}\right), \tag{7}$$

where the surface pressure, P_0, is, in the Eddington approximation, equal to $P_{\rm rad,0} = L/6\pi c r_{\rm phot}^2$. This surface term was omitted in LU97.

Because pressure goes as the fourth power of temperature, throughout most of the envelope the surface pressure term in Eq. 7 is unimportant, and

$$P_{\rm gas} = P\left(1 - \frac{L}{L_{\rm E}}\right) = P\beta, \tag{8}$$

where β is a constant. It then follows from the equation of state of the gas ($P_{\rm gas} = \rho k_{\rm B} T/\mu m_{\rm h}$) and the radiation ($P_{\rm rad} = aT^4/3$) that everywhere in the envelope except at low optical depth,

$$P = A\rho^{4/3} = \left(\frac{k_{\rm B}}{\beta\mu m_{\rm h}}\right)^{4/3}\left(\frac{3(1-\beta)}{a}\right)^{1/3}\rho^{4/3}. \tag{9}$$

This relation was assumed in LU97.

It is now possible to solve the hydrostatic equation (Eq. 4) for temperature or, equivalently, density by using the approximate relation between P and ρ (Eq. 9):

$$T = \frac{\beta\mu m_{\rm H} G M}{4k_{\rm B}}\left(\frac{1}{r} - \frac{1}{r_0}\right) \tag{10}$$

$$\rho = \left(\frac{GM}{4A}\right)^3\left(\frac{1}{r} - \frac{1}{r_0}\right)^3, \tag{11}$$

where r_0 is a constant of integration which was neglected in LU97. The constant is determined by the requirement that at the photospheric radius, $r_{\rm phot}$, $T = T_{\rm eff}$.

Following LU97, we find a relationship between the envelope mass and β. The envelope mass is found by integrating Eq. 11 from the base radius,

$$r_{\rm b} \sim R_{\rm T} = R_\odot (M_{\rm BH}/M_\odot)^{1/3} \sim 10^{13}\ {\rm cm} \tag{12}$$

to $r_{\rm phot}$ (The exact location of base radius is an unknown; it is the poorly understood interface between the disk and envelope. The base radius, as argued in LU97, should be close to the tidal radius, because that is where the dynamical effects of angular momentum are expected to become important.)

$$M_{\rm env} = \int_{r_{\rm b}}^{r_{\rm phot}} \rho 4\pi r^2 dr \tag{13}$$

$$\approx 4\pi\left(\frac{GM}{4A}\right)^3 [\ln(r_{\rm phot}/r_{\rm b}) - 1.8] \tag{14}$$

$$\approx 1.9\times 10^{15} M_6^3 \beta^4 [\ln(r_{\rm phot}/r_{\rm b}) - 1.8]\, M_\odot. \tag{15}$$

An envelope which contains no more than a solar mass must be extremely close to the Eddington limit with $\beta \lesssim 10^{-4}$. Eq. 15 shows that the photospheric radius is exponentially sensitive to β, suggesting that very small changes in base luminosity, which is controlled by a poorly understood accretion in the disk, may create large changes in the photospheric radius. For example, in this Newtonian approximation, the steady state radius would have to change from 10^{14} to 10^{15} cm when $(1 - L/L_{\rm E})$ changes from $\sim 8\times 10^{-5}$ to $\sim 5\times 10^{-5}$. This result illustrates the extreme fine tuning (to three parts in 10^5) which was required in LU97 to reach the steady state solution. As we describe in Sect. 3, the fine tuning required in the Schwarzschild case is less, but the level is $\sim 2\%$ *above* the local critical luminosity.

We now estimate the maximum radius at which the Eddington approximation is valid. The approximation requires that the atmosphere be thin, or equivalently, that the gas pressure scale height be much less than the radius at the photosphere. This limit is also a physical dividing line, because for further extended atmospheres, one expects high mass loss as the atmosphere is nearly isothermal at $\tau \ll 1$, so the escape velocity falls faster than the thermal velocity.

The gas pressure scale height at the photosphere can be evaluated using Eqs. 4 and 8 as:

$$h_{\rm Pg}^{-1} = -\frac{1}{P_{\rm gas}}\frac{dP_{\rm gas}}{dr} \tag{16}$$

$$\frac{h_{\rm Pg}}{r} = \frac{k_{\rm B}(L_{\rm E}/4\pi\sigma r_{\rm phot}^2)^{1/4}}{\mu m_{\rm h}}\frac{r_{\rm phot}}{\beta G M} \tag{17}$$

At $r_{\rm phot}$, the gas pressure scale height must be much less than the photospheric radius:

$$1 \gg \frac{h_{\rm Pg}}{r_{\rm phot}} = \frac{k_{\rm B}(L_{\rm E}/4\pi\sigma)^{1/4}}{\mu m_{\rm h}}\frac{r_{\rm phot}^{1/2}}{\beta G M} \tag{18}$$

$$r_{\rm phot} \ll 1.5\times 10^{16}\left(\frac{M_{\rm env}}{0.5 M_\odot}\right)^{1/2} M_6^0\ {\rm cm}. \tag{19}$$

where we have written β in terms of the envelope mass (Eq. 15). Note that result is independent of the black hole mass. The effective temperature therefore has the same scaling as in LU97:

$$T_{\rm eff} \gg 4,500\left(\frac{M_{\rm BH}}{M_{\rm env}}\right)^{1/4}\ {\rm K} \tag{20}$$

These limiting results are surprisingly close to those of LU97 who found $r_{\rm phot} \sim 2\times 10^{15}$ cm and $T_{\rm eff} \sim 15,000$ K. The difference, of course, is that our results show that smaller photospheric radii could exist. The exact photospheric radius cannot be determined without knowledge of the inner luminosity source.

An additional way to see the wind constraint on the radius is to consider r_0^{-1} in Eq. 10, which enforces the condition that $T = T_{\rm eff}$ at the photosphere (which radiates at the Eddington limit). The constant is zero when $r_{\rm phot} \sim 10^{15}$, and becomes negative for larger values of $r_{\rm phot}$. Negative values of r_0^{-1} give (see Eq. 10) regions at the top of the envelope which approach an isothermal state and which would likely drive winds.

Even before the radius expands so far that the pressure scale height becomes comparable with the radius, winds could begin to play an important role–either in altering the envelope structure or in removing much the envelope mass. An estimate of the importance of the winds may be found by connecting, at the photospheric radius, isothermal wind solutions to the envelope solutions. Because of the extremely low densities ($\sim 10^{-15}$g/cm^3), the envelope is ionized and the cross-section is largely provided by electron-scattering, so an isothermal wind powered by the continuum cross-section (rather than line transitions), may be most appropriate. In this case, the following equation describes the outflow (e.g. Kudritzki, 1988):

$$(1-\frac{v_{\rm s}^2}{v^2})v\frac{dv}{dr} = \frac{2v_{\rm s}^2}{r} - \frac{GM_{\rm BH}}{r^2}(1-\frac{L}{L_{\rm E}}), \quad (21)$$

where $v_{\rm s}$ is the sound speed (in our case, isothermal so that $v_{\rm s} = (k_{\rm B}T_{\rm eff}/\mu m_{\rm h})^{1/2}$ and the sonic point, $r_{\rm s} = GM(1 - L/L_{\rm E})/2v_{\rm s}^2$. Dimensionless radius, $\eta = r/r_{\rm s}$, and dimensionless velocity, $u = v/v_{\rm s}$, allow Eq. 21 to be neatly integrated to obtain:

$$\ln u - \frac{u^2}{2} = -\frac{2}{\eta} - 2\ln\eta + 1.5. \quad (22)$$

Connecting such outflows to the Eddington envelopes allows for determination of mass loss as a function of envelope photospheric radius. Generally, the time for such a wind to significantly reduce the envelope mass is at least 1000 years—many times longer than the other relevant time scales for the problem (results are shown in Fig. 1). For photospheric radii larger than $\sim 3\times 10^{15}$ cm, the sonic point occurs below the photosphere, showing, in agreement with the limiting radius found above, that the hydrostatic solution (Eqs. 10 and 11) is no longer valid.

3. Effects in the Schwarzschild metric

The Schwarzschild geometry has a significant impact on envelopes which radiate near the Eddington limit. At first appearance, the Schwarzschild geometry seems to be an unimportant correction to the problem, because the base of the photosphere is at $\sim 25R_{\rm S}$, and redshift effects are of order 2%, $(1+z) \sim (1 + R_{\rm S}/2r)$. However, because the envelopes are very close to the Eddington limit, as is required to produce an extended photosphere, the envelope structure is very sensitive to $\beta = 1 - L/L_{\rm E}$. In particular for envelopes of $\sim 1M_\odot$, β is $\sim 10^{-4}$, so a 2% reduction in luminosity will significantly alter the envelope structure.

We quote below a number of results which were obtained in the study of x-ray bursts and envelopes around neutron stars (Paczyński & Anderson 1986; Paczyński & Prószyński 1986). For our purposes, the most important feature of the relativistic stellar structure equations is that the local critical luminosity does not scale with radius in the same manner as the local luminosity with the consequence that the structure is convective rather than radiative. Specifically, the local luminosity and critical luminosity determined by the local gravitational forces (equivalent to the Eddington luminosity at large r) are

$$L = L_\infty\left(1-\frac{R_{\rm S}}{r}\right)^{-1} \quad (23)$$

$$L_{\rm cr} = \frac{4\pi cGM}{\kappa}\left(1-\frac{R_{\rm S}}{r}\right)^{-1/2}, \quad (24)$$

where L_∞ is the luminosity as measured far from the black hole. Because of the different scalings, a near critical luminosity at the surface requires a super-critical luminosity at the base. As a consequence the star is convective. Alternatively, the convective nature can be seen by comparing the radiative and adiabatic gradients:

$$\nabla_{\rm ad} = \frac{1}{4}\left[\frac{1-\frac{3}{4}\beta}{1-\frac{3}{4}\beta-\frac{3}{32}\beta^2}\right] \approx \frac{1}{4}\left[1+\frac{3}{32}\beta^2\right] \quad (25)$$

$$\nabla_{\rm rad} = \frac{1}{4}\left[\frac{L_\infty}{(1-\beta)L_{\rm E}}\left(1-\frac{R_{\rm S}}{r}\right)^{-1/2} + \frac{4P}{\rho_0 c^2}\right]\times\tilde{P} \quad (26)$$

$$\approx \frac{1}{4}\left[\frac{L_\infty}{(1-\beta)L_{\rm E}}\left(1-\frac{R_{\rm S}}{r}\right)^{-1/2}\right] \quad (27)$$

$$\tilde{P} = \left[1+\frac{(4-1.5\beta)P}{\rho_0 c^2}\right]^{-1}. \quad (28)$$

Even for a near critical surface luminosity, $\nabla_{\rm rad}$ quickly becomes larger than $\nabla_{\rm ad}$ as the coordinate, r, decreases.

As a consequence, the envelope must be convective. Whether or not the convection is efficient is difficult to determine because in radiation dominated regimes, convection is not fully understood, although progress is being made (e.g., Arons 1992). If convection is efficient, then by definition, $\nabla_{\rm ad}$ is nearly a constant. The equation of the temperature gradient yields:

$$\frac{dP_{\rm rad}}{dP} = \frac{1}{4}\left[\frac{1-\frac{3}{4}\beta}{1-\frac{3}{4}\beta-\frac{3}{32}\beta^2}\right] \approx \frac{1}{4}\left[1+\frac{3}{32}\beta^2\right] \quad (29)$$

so $P_{\rm rad}/P$ is very closely a constant (to ~ 1 part in 10^{-9}), and we recover the polytropic equation of state $P = A\rho^{4/3}$, which was used in the previous section. Assuming that convection is efficient, we can determine the relationship between the photospheric radius, the envelope mass, and the surface luminosity in the same manner as in Sect. 2. Ignoring the relativistic terms in the hydrostatic stellar structure equations, which is a good approximation (to $\sim 2\%$) because the terms enter multiplicatively rather than as differences, we recover eqn. 15. In contrast to the Newtonian case, in the Schwarzschild case, the luminosity at the base must be locally super-critical in order to support an extended envelope with a near-Eddington surface luminosity.

If the luminosity is $\sim 2\%$ super-critical, and the envelope is able to expand, then photospheric radius is still sensitive to the base luminosity, but not exponentially sensitive as is the case for a Newtonian atmosphere. The luminosity at the photosphere is nearly equal (to better than one part in 10^4) to the critical luminosity (Eqs. 24), so

$$L_{\rm phot} \approx L_{\rm cr} = L_{\rm E} \left(1 - \frac{R_{\rm S}}{r_{\rm phot}}\right)^{-1/2}. \qquad (30)$$

Using Eq. 23 to write the luminosity at the base, $L_{\rm b}$ as a function of photospheric radius, yields the result:

$$r_{\rm phot} \approx \frac{R_{\rm S}}{1-(L_{\rm E}/L_{\rm b})^2}. \qquad (31)$$

In steady state the photospheric radius would increase from 10^{14} to 10^{15} cm with a 0.1% change in base luminosity.

There may be additional effects which compete with the relativistic effects. For neutron stars, the temperatures at the base of the envelope are so high that relativistic corrections to the Thompson cross section become important (e.g. Paczyński & Anderson 1986), but the temperatures around the tidal disruption created envelopes never reach such high temperatures. In our case, slight rotation may serve to reduce the critical luminosity at small radii, in the plane of rotation. Along the rotation axis, the envelope would likely be convective for the reasons described above.

4. Evolution in time

Both the Newtonian and relativistic envelopes require fine tuning of the luminosity in order to produce static extended envelopes, because $r_{\rm phot}$ depends sensitively on the luminosity. In the Schwarzschild case, unless the base of the envelope knows to shine locally at $\sim R_{\rm S}/2r_{\rm b} \sim 2\%$ above the local critical luminosity, and the luminosity is restricted to a narrow regime, it appears difficult to maintain the type of envelopes discussed in Sect. 2. The dependence on luminosity is strong; a fraction of a percent change in base luminosity could increase the steady-state radius from 10^{14} to 10^{15} cm. Since the energy source (the accreting torus) is probably not extremely well-coupled to the envelope, it is unlikely that the base luminosity is so finely tuned. Therefore, we consider how the photosphere will change with time for different base luminosities.

We identify three cases. If the base luminosity is sub-critical, the envelope is not much bigger than the accreting torus. When the luminosity is super-critical, the envelope must expand. If the super-criticality is less than $\sim 2\%$, the expansion is modest and the envelope may remain in hydrostatic and thermal equilibrium. If the base luminosity is super-critical by a significant factor, e.g. 2, then the envelope expands hydrostatically on a thermal time scale, until it expands so much that the dynamical time scale becomes shorter than the thermal time scale. After that, the outer

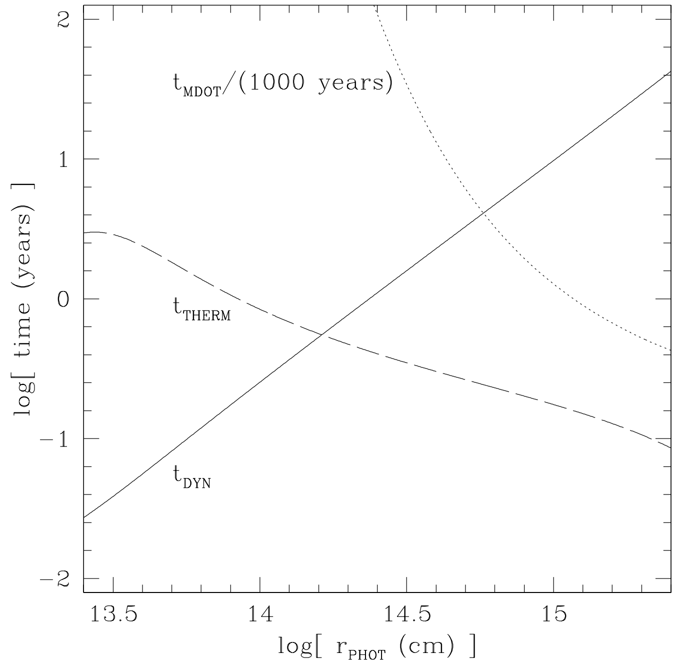

Fig. 1. Time scales for envelopes as a function of photospheric radius: dynamical time, thermal time, and time for wind driven mass loss. The thermal time scale is calculated assuming a energy input of $2L_{\rm E}$ and energy output of $L_{\rm E}$ and $r_{\rm b} = 10^{13}$ cm. The envelopes can stay in hydrostatic equilibrium only as long as $t_{\rm dyn} < t_{\rm therm}$. The mass loss to winds produced by an isothermal outflow are small enough that they do not appear to affect the system.

parts of the envelope are no longer in hydrostatic equilibrium, but this does not imply that they are instantly lost. The envelope may continue to expand, perhaps even with a structure similar to the hydrostatic solution, until the scale height of atmosphere becomes comparable to the radius (as discussed in Sect. 2) at which point a wind will set in.

We now examine the expansion of the envelope. The relevant time scales are the dynamical time scale, the thermal time scale, and the radiation time scale, over which time all matter would be accreted. The first two time scales are shown in Fig. 1 as a function of photospheric radius.

The dynamical time scale grows with photospheric radius:

$$t_{\rm dyn} = \left(\frac{GM_{\rm BH}(1-L/L_{\rm E})}{r_{\rm phot}^3}\right)^{-1/2} \qquad (32)$$

$$\approx 13 r_{15}^3 M_6^{-1/2} \text{ years.} \qquad (33)$$

The thermal time scale, defined as the time scale to unbind the envelope assuming the energy in the envelope increases at a rate of $L_+ = L_{\rm in} - L_{\rm out} \approx L_{\rm E}$ (e.g. if the base luminosity were $2L_{\rm E}$ and the surface radiated at $L_{\rm E}$). The total energy of the hydrostatic envelope is $E_{\rm tot} = E_{\rm grav} + E_{\rm internal}$, where the internal energy is

$$E_{\rm internal} \approx 3 \int_{r_{\rm b}}^{r_{\rm phot}} 4\pi r^2 P_{\rm rad} {\rm d}r \qquad (34)$$

$$\approx \frac{3}{4} \frac{GM_{\rm BH}M_{\rm env}}{r_{\rm b}[\ln(r_{\rm phot}/r_{\rm b}) - 1.8]}. \quad (35)$$

Note that the energy is strongly dependent on the location of the inner boundary, $r_{\rm b}$. The gravitational energy, $E_{\rm grav} \approx -(4/3)E_{\rm internal}$, so that the thermal time scale is

$$t_{\rm therm} = \frac{E_{\rm internal}}{3L_+} \approx 0.2 \left(\frac{10^{13}\ {\rm cm}}{r_{\rm b}}\right) \left(\frac{L_{\rm E}}{L_+}\right) \text{ years}, \quad (36)$$

with a complex dependence on photospheric radius and black hole mass. The thermal time scale depends both on the net energy injection rate and on the base radius of the envelope. As both parameters depend on the unknown physics of the torus, and as such, their exact values are uncertain. We believe the base radius to be near the tidal radius $\sim 10^{13}$ cm, where rotational support likely becomes important. Similarly, thick tori may produce luminosities up to a few times the Eddington limit, but the exact value cannot be predicted. These parameters enter into the thermal time scale multiplicatively, and together may lengthen or shorten the thermal time scale by a factor of 2 or 3.

Lastly, the radiation time is the time to radiate, at the Eddington limit, all of the accretion energy of the tidal debris:

$$t_{\rm rad} \approx 20M_6^{-1} \text{ years}, \quad (37)$$

for an accretion efficiency of 10% and an envelope mass of $0.5M_\odot$. This time scale does not depend on the photospheric radius, and is generally longer than the other relevant time scales of interest (for a $10^6 M_\odot$ black hole).

As discussed above, if the luminosity at the base is strongly super-critical, then the envelope will expand and the photosphere will move to larger radii (see Fig. 2). For an envelope with its photosphere below $\sim 10^{14}$ cm, the dynamical time is shorter than the thermal time, so the envelope should expand in hydrostatic equilibrium. When the photosphere grows larger than $\sim 10^{14}$ cm, the envelope can no longer be in hydrostatic equilibrium. However, the material at small radii has a much shorter dynamical time, so may be able to adjust to a near-equilibrium state quickly. The envelope could continue to expand, perhaps with the interior in a near equilibrium state and the outer parts further from equilibrium. If the photospheric radius reaches $\sim 10^{15}$ cm, the thermal time scale is so short and the pressure scale height becomes so large compared to the radius, that it seems most likely either a strong wind would form and carry the excess energy away, or the entire envelope would be unbound.

Fig. 2 illustrates possible evolutionary sequences for different values of the base luminosity. For Fig. 2, we make the assumption that the envelope is in a hydrostatic state (described by eqn. 10), even after the thermal time scale falls below the dynamical time scale. The time to move between envelopes $\Delta t = \Delta E/L_+$, where ΔE is the energy difference between sequential envelopes (A similar method was applied by Ulmer (1998b) to investigate the evolution of thick accretion disks.) Evolutionary sequences show the evolution of both photospheric radius and temperature with time. The curves become nearly vertical as the envelopes evolve to large radii, because the binding energy becomes so small that they can be unbound by very little energy input.

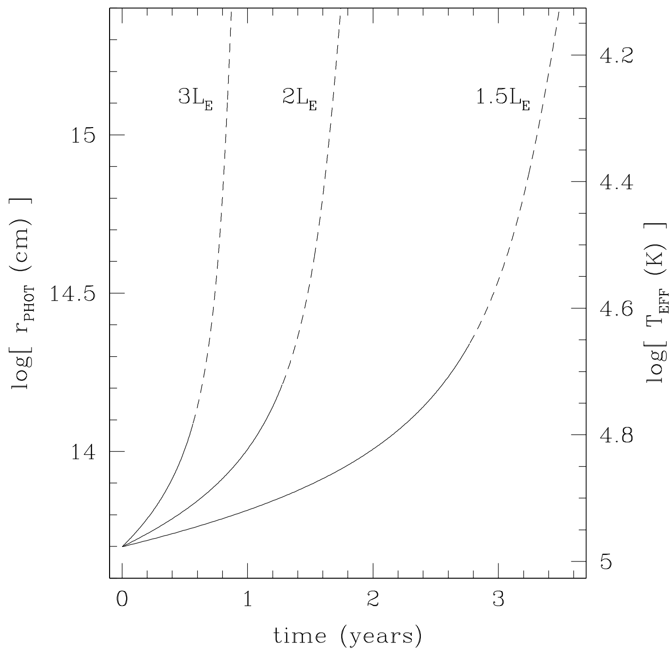

Fig. 2. Radius and temperature for sequences of expanding envelopes labeled with three different energy input rates (each envelope radiates at the Eddington limit) and a base radius of 10^{13} cm. The solid lines are the phase in which the envelopes are in hydrostatic equilibrium. The dashed lines signify that the envelope is not in hydrostatic equilibrium, but for the purposes of this illustration, we assume that the models do stay in hydrostatic equilibrium and calculate the time between models at $\Delta t = \Delta E/(L_{\rm in} - L_{\rm out})$, where ΔE is the energy difference between models. These calculations are described in more detail in Sect. 4.

It seems likely that the envelope will expand on a time scale of months to years, depending on the base radius and the base luminosity, to $\sim 10^{14}$ cm where $T_{\rm eff} \sim 50,000$ K. Subsequently, the envelope will expand, but will not be able to maintain hydrostatic equilibrium throughout.

5. Discussion

Eddington envelopes require fine tuning of the luminosity in order to produce extended envelopes. It appears unlikely that the luminosity is held within such a tight region. More reasonably, either the average luminosity will be sub-critical at the base, and the envelope will collapse, or the luminosity will be super-critical at the base, and the envelope will grow (and thereby radiate at lower temperatures between ~80,000 K and ~20,000 K as shown in Fig. 2) and eventually reach the most extended hydrostatic model or the wind solutions, as described in Sect. 2. This scenario may be valid only if convection is efficient. The stellar structure equations are not as tractable if convection is inefficient, and we have not attempted to solve them here.

We reiterate that the remarkable feature of extended envelopes is that they could reprocess much of the hard radiation produced by a tidal disruption event into the optical band. Bolometric corrections would drop from ~ 7.5 magnitudes to 1–2.

The color temperatures of the objects may be slightly higher than the effective temperatures due to suppression of absorption processes at such low densities, but such changes are expected to be about a factor of two (see the work on neutron stars by Babul & Paczyński (1987)). Even scattering-dominated envelopes retain their important property of producing optically bright flares.

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