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Letter to the Editor

Chinese models of solar and lunar motions in the 13th century

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Abstract. In the 13th century, a new Chinese astronomical calculation was just put to use and it was almost entirely included in the calendar of *Shoushi* (formulated by Guo Shou-jing and his colleagues, epoch of AD 1281, adopted from AD 1281−1644). This calendar was one of the most famous and accurate in the history of China. The perfectly systematic theoretical models at that time had been developed for thousands of years, such as those for solar and lunar motions. In this paper, these models and their accuracy within their periods of validity are inferred. The results are also compared with contemporary astronomy.

Key words: Ephemerides – history and philosophy of astronomy

1. Introduction

The calendar of *Shoushi* is one of the most famous calendars in ancient China (Needham 1959), which was recorded in the *Lizhi* (ancient calendar book of China) of *Yuanshi* (annals of the *Yuan* Dynasty from AD 1279 to 1367). Parts were translated by Gaubil (1732). Almost every part of this calendar has a corresponding section in the modern astronomical year book. It was the last calendar formulated only by Chinese astronomers and its period of usage (AD 1281−1644) was the longest one in ancient China. It was also a new calendar system which cast off the traditional method that computed from the epoch of the distant past. For counting, the basic constants were obtained by meticulous observations. The mathematical method adopted in this calendar came long before the foundation of Newton's mechanical system, and summed up voluminous real measuring results. Based upon our work concerning it (Li and Zhang 1996a,b,c & 1997a; Li 1997), in this paper the systematic mathematical models of the solar and lunar motions have been reduced from the original. Then these results were compared with present theories. Finally we give the accuracy in their applicable period. In the system of ancient Chinese calendars, studies of the calculations of other parts of the calendars, such as eclipses, were all based upon them. By the method of the *Shoushi* calendar, we have computed all real new moons from AD 1280 to 1645 and give the distributions of their mean values. The average value of the differences is 0.9 minute of time and the mean of the absolute difference is 21.0 minutes. Among all lunar eclipses, the average is 0.8 minute and the absolute is 19.4 minutes. For solar eclipses, there are only 114 ones computed by us that corresponded to modern calculations. The average is -0.6 minute and the absolute is 23.9 minutes. Because ancient calendars are also thought of as one kind of original material that includes a great number of actual ancient celestial records, these observations could help to identify the reality of historical events, to recover even some omitted data. If we could recover these records (usually long before the publishing period of the *Shoushi* calendar), the data would be very useful to contemporary studies especially of the secular variation of the earth's rotation (Li and Zhang 1997b).

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2. Solar motion

According to the *Shoushi* calendar, the correction for solar motion is called *Richan.* It has functions $M(C)$ and $N(C)$ which were established for this calculation; they are denoted by:

$$M(C) \;=\; 10^{-8} \times 5133200 - (31 \times C + 24600) \times C) \times C \;;$$
$$N(C) \;=\; 10^{-8} \times 4870600 - (27 \times C + 22100) \times C) \times C \;,$$

where C is a parameter, and the unit of $M(C)$ and $N(C)$ is the degree (the *Shoushi* calendar takes the circle as $365^\circ.2575$,, so 1 *Shoushi* degree equals $0^\circ.9856$ used now, but in this paper, we have changed the unit of the *Shoushi* degree to the contemporary one).

The fluctuation function of solar motion was $T(t)$ defined by this calendar (Li and Zhang 1996a, b & 1997a; Li 1997), its unit is the degree, the parameter t is days to the winter solstice just preceding it ($t \leq A'$). A' is the length of a tropical year, $A' = A + INT(\frac{year-1281}{100}) \times 10^{-4}$, $A = 365^d.2425$, and INT means the integer function:

$$T(t) = \quad (1)$$
$$\begin{cases} M(C), & C = t, \quad (0 \leq t < 0.25A' - 2.4014); \\ N(C), & C = 0.5A' - t, \quad (0.25A' - 2.4014 \leq t < 0.5A'); \\ -N(C), & C = t - 0.5A', \quad (0.5A' \leq t < 0.75A' + 2.4014); \\ -M(C), & C = A' - t, \quad (0.75A' + 2.4014 \leq t \leq A'). \end{cases}$$

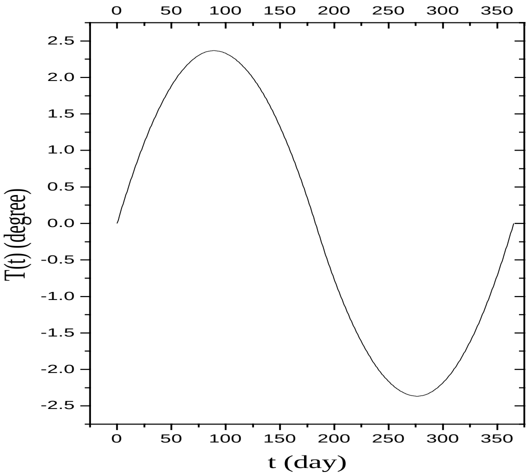

Fig. 1. The solar fluctuation function $T(t)$ of the *Shoushi* calendar within the period (A') from the winter solstice of AD 1280

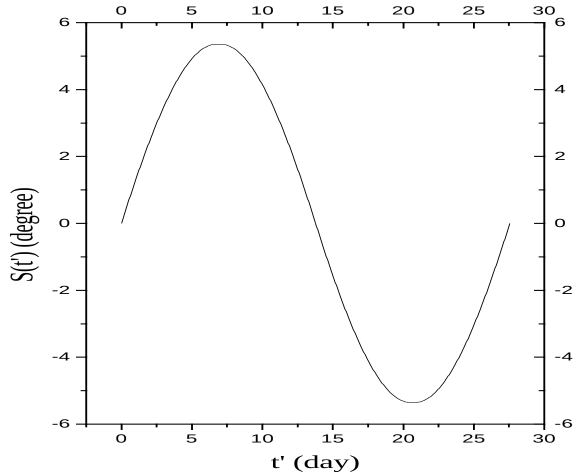

Fig. 2. The lunar fluctuation function $S(t')$ of the *Shoushi* calendar within the period (B') from perigee just before the winter solstice of AD 1280

The function $T(t)$ is taken directly from the equations of *Richan* of the *Shoushi* calendar as determined by the ancient astronomers.

From this figure, the plot of $T(t)$ resembles that of a trigonometric function. Its amplitude is $2^\circ.3668$. When $t = 0$ (winter solstice), $0.5A'$ (Summer Solstice), A' (winter solstice), $T(t) = 0$, the sun is at its mean position, there are no deviation. When $t = 0.25A' - 2.4014$, $T(t) = 2^\circ.3668$ (maximum); when $t = 0.75A' + 2.4014$, $T(t) = -2^\circ.3668$ (minimum).

The apparent longitude $L_S(jd)$ of the sun is given by ($t = jd - 2188925^d.2267$, jd is the Julian Day):

$$L_S(jd) = 270^\circ + 0^\circ.985646522 \times t + 1''.3299 \times 10^{-8} \times t^2 + T(MOD(t, A')). \quad (2)$$

According to the *Shoushi* calendar, the Julian Day of the winter solstice in AD 1280 (Dec. 14, 1^h26^m, apparent solar time of 120°E) is: $jd = 2188925^d.2267$. Where $MOD(X,Y) = mod(X,Y)$, when $mod(X,Y) \geq 0$ & $MOD(X,Y) = (mod(X,Y) + Y)$, when $mod(X,Y) < 0$; the function:

$$mod(X,Y) = (X - INT(\frac{X}{Y}) \times Y)\,; \text{if } Y \neq 0\,.$$

In Eq. (2), $270^\circ + 0^\circ.985646522 \times t$ represents the linear terms of the apparent longitude (average motion of the sun is $360^\circ/A = 0^\circ.985646522$ per day); $1''.3299 \times 10^{-8} \times t^2$ is its secular term; $T(MOD(t, A'))$ is the periodical term.

3. Lunar motion

In this calendar, the correction for lunar motion is called *Yueli*. The $P(D)$, function for this computation, was made up for this purpose:

$$P(D) = 10^{-8} \times (11110000 - (325 \times D + 28100) \times D) \times D,$$

where D also is a parameter, the result's unit of $P(D)$ is degree.

The function of fluctuation of lunar motion was $S(t')$ defined by this calendar (Li and Zhang 1996a, b & 1997a; Li 1997), its unit is the degree. The parameter t' is the days to the start of the anomalistic month (perigee) just preceding it ($t' \leq B'$), $B' = 27^d.5546$ (days of anomalistic month) and the coefficient $K = 1/0.082$:

$$S(t') = \begin{cases} -P(D), & D = K \times t', & (0 \leq t' < 0.25B'); \\ -P(D), & D = K \times (0.5B' - t'), & (0.25B' \leq t' < 0.5B'); \\ P(D), & D = K \times (t' - 0.5B'), & (0.5B' \leq t' < 0.75B'); \\ P(D), & D = K \times (B' - t'), & (0.75B' \leq t' \leq B'). \end{cases} \quad (3)$$

The function $S(t')$ is also obtained directly from the equations of *Yueli* of the *Shoushi* calendar.

The function $S(t')$ also resembles a trigonometric function. Its amplitude is $5^\circ.3507$. When $t' = 0$ (perigee), $0.5B'$ (apogee), B' (perigee), $S(t') = 0$, the moon is at its mean position, there are no deviation. When t' is around $0.25B'$, $S(t') = 5^\circ.3507$ (maximum); when t' is about $0.75B'$, $S(t') = -5^\circ.3507$ (minimum).

The apparent longitude $L_L(jd)$ of the moon is given by ($t' = jd - 2188905^d.38472$, jd means the Julian Day):

$$L_L(jd) = 259^\circ.6642 + 13^\circ.17632081 \times t' + 1''.7778 \times 10^{-7} \times t'^2 - S(MOD(t' - 7^d.1845, B')). \quad (4)$$

In Eq. (4), according to the *Shoushi*, the average motion of the moon is $13^\circ.36875 \times 0.9856 = 13^\circ.17632081$ per day. The Julian Day of the time of real new moon just before winter solstice in AD 1280 is $jd = 2188905^d.38472$; and the apparent longitude then is about $259^\circ.6642$. In this equation, $259^\circ.6642 + 13^\circ.17632081 \times t'$ represents the linear terms of the apparent longitude of the moon (average motion of the moon); $1''.7778 \times 10^{-7} \times t'^2$ is the secular term; $-S(MOD(t' - 7^d.1845, B'))$ is the periodic term.

4. Precision

From Fig. 3, According to our earlier work (Li and Xu 1995) on Newcomb's Tables of the sun (Newcomb 1898), the fluctuations

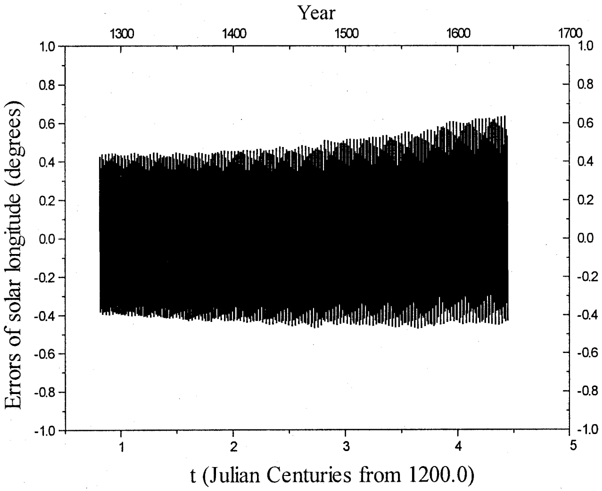

Fig. 3. The errors of solar apparent longitude (functions $L_S(jd)$)of the *Shoushi* calendar for its publishing period from AD 1281 to 1644

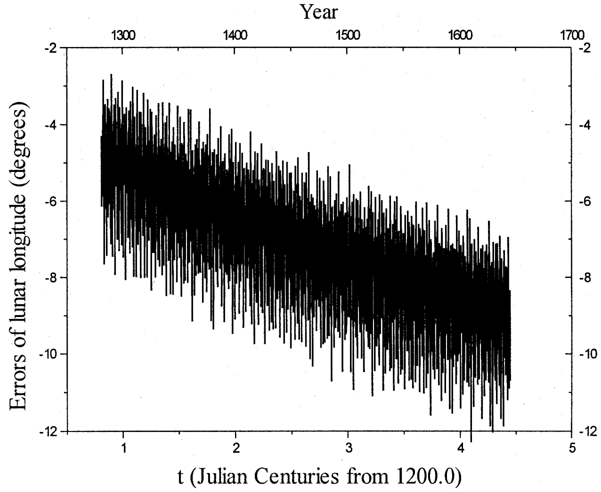

Fig. 4. The errors of lunar apparent longitude (functions $L_L(jd)$)of the *Shoushi* calendar for its publishing period from AD 1281 to 1644

($> 1''$) of the solar longitude are collected to compare with $L_S(jd)$. By comparing them, the two models are approximately in good correspondence, the difference between $L_S(jd)$ and Newcomb's solar theory is less than $0^\circ.62$ in more than 350 years. The mean velocity error of the sun is -7.1×10^{-7} degrees per day. So the formula $L_S(jd)$ applied by ancient Chinese astronomers in the 13th century approximately represents the real motion of the sun.

In Fig. 4, we compare the contemporary lunar longitude model of Chapront-Touzé and Chapront (1988) with $L_L(jd)$. In this paper, we only discuss those terms with amplitude $> 1''$. The plot shows the two models differ somewhat. Because the lunar motion is more complex than that of the sun, the main reasons for the errors are due to errors in the zero point of the longitude and errors of mean velocity of the lunar motion. The mean speed error of the *Shoushi* calendar is -3.8×10^{-5} degrees per day. This difference between two speeds produces the long-term variation. The results are less accurate than those of the sun.

Also in Fig. 3 and Fig. 4, the mean value of the absolute values of the differences of the solar and lunar longitude calculated by the *Shoushi* calendar from AD 1280 to 1644 are $0^\circ.3322$ (solar), $7^\circ.1433$ (lunar) and the mean value of the differences are $0^\circ.0240$ and $-7^\circ.1433$. Removing the errors in the zero point of the longitude and the errors of the mean velocities, the results of the mean value of the absolute are $0^\circ.3319$ and $1^\circ.0453$ and the mean ones are $0^\circ.0$ and $-0^\circ.0005$.

The precision is obtained by comparing the difference between calculating methods of the *Shoushi* calendar and contemporary astronomy. The accuracy of the calculations is based upon the precision of these models, the functions $T(t)$ and $S(t')$ mainly represent the solar and lunar equations of the center. The results computed by us show that these calculations of solar longitude are more precise than that of the moon.

Although this calendar was only used for about 350 years because of the problem of precision, actually a lot of useful historical survey records are just enclosed by it, especially those records long before the period of use of the *Shoushi* calendar. The use of the *Shoushi* calendar to identify and recover those data of celestial phenomena which were omitted has great potential.

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