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The relative frequency of type II and $I_{b,c}$ supernovae and the birth rate of double compact star binaries

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Abstract. Using a population number synthesis code, we estimate the relative rates of different type (II and $I_{b,c}$) of supernovae for regions of constant star formation. We combine a large set of massive single star and massive close binary evolutionary computations and allow for black hole formation. If a neutron star forms, the binary system is followed through a supernova explosion where we account for asymmetries needed to explain the observed space velocity distribution of a large sample of pulsars. We also predict the theoretical formation rate and period distribution at birth of double compact star binaries. Finally we give the distribution of the binary parameters of the progenitors of the double neutron star systems.

Our calculations reveal that the number ratio (II/$I_{b,c}$) strongly depends on the massive binary formation rate, on the binary mass ratio and period distribution. As consequence when average relative rates, obtained from observed supernova events in a sample of different galaxies, are used to predict the relative rates in a particular galaxy, this is meaningful only if the massive binary formation rate, the binary mass ratio and period distribution are the same in all these galaxies.

Interestingly, the observed average number ratio from 2461 galaxies of different types can be reproduced assuming an average (cosmological) massive binary formation rate of ∼40-50% which may differ by a factor two from the massive binary formation rate in one particular galaxy.

The theoretically predicted formation rate of close double neutron star systems in the galaxy, (10^{-6}-10^{-5})/year, is in agreement with the observed values. Taking 40 $M_\odot$ as limiting mass for black hole formation in binaries, we find that double black hole systems form at a very high rate.

Key words: stars: binaries – stars: supernovae: general – stars: pulsars: general – black hole physics

1. Introduction

From theoretical point of view, population number synthesis (PNS) is defined as the study of the number of stars in their different evolutionary phases predicted by stellar evolution. The PNS model for massive stars with a realistic massive binary formation rate, used in Brussels, has been described by Vanbeveren et al. (1997, hereafter Paper I.).It uses

- a model and a detailed set of massive star evolutionary computations of single stars and of interacting close binaries,
- stellar distribution functions describing the formation of stars, i.e. the massive binary formation rate f, the initial mass function Ψ (M) of single stars and of primaries, the mass ratio distribution of binaries Φ (q) and the period distribution of binaries Π (P),
- a description of the evolution of the binary period where we distinct:
  - the effect of stellar wind mass loss
  - mass loss from the binary during Roche lobe overflow (RLOF), the common envelope (CE) phase and the spiral-in phase
  - the effect of an asymmetric SN explosion

In Paper I, the PNS model was used to study the effect of binaries on the Wolf-Rayet star (WR) population in regions where star formation is a continuous process and in star burst regions. The population of O and WR runaways (where the peculiar runaway velocity is due to the SN explosion of a former companion) was investigated by De Donder et al. (1997) whereas the formation of Be stars due to binary interaction was studied by Van Bever and Vanbeveren (1997). In this paper we will discuss:

- the dependency of the theoretically predicted number ratio (II/$I_{b,c}$) on the considered model parameters (Sect. 3),
- the theoretically predicted number of double compact star binaries and the progenitors of double neutron star systems (NS+NS) (Sect. 4).

In Sect. 2 we first introduce some refinements of the PNS model.

2. The PNS model

In the PNS model a lot of parameters appear which are needed to follow the evolution of single and binary stars. A detailed discussion of the complete list of parameters is given in Paper I. For the scope of the present paper only the following parameters are of particular importance.

2.1. *The distribution functions*

2.1.1. The initial mass function: Ψ (M)

For the distribution of primary masses and single star masses we adopt a power law of the form Ψ (M)= $\Psi_0 M^{-\gamma}$.We consider the canonical γ = 2.7 (Scalo, 1986) and γ =2.

2.1.2. The initial mass ratio distribution: Φ (q)

The observed distribution of the mass ratio q (=M_2/M_1) is severely affected by selection effects, especially at low q values. To investigate the influence of different distribution forms we use:

– Φ_1 (q): flat distribution
– Φ_2 (q): Hogeveen distribution (Hogeveen, 1991)
– Φ_3 (q): $\propto$ $q^{0.5}$ (Garmany, 1986)

2.1.3. The period distribution: Π (P)

From observational studies of nearby binaries it seems that the distribution of orbital periods is flat in logP for periods between a few days and several thousand days (Popova et al., 1982; Abt, 1983). We take this distribution for our considered period range of one day to 10 years.

2.1.4. The kick velocity distribution: f(v_p)

A combination of new measurements of pulsar proper motions (Harrison et al., 1993) with a new pulsar distance scale (Taylor and Cordes, 1993) leads to the following 3-D pulsar velocity distribution f(v_p), discussed by Lyne and Lorimer (1994) and that can be expressed by the relation:

$$f(v_p) = 1.96 \cdot 10^{-6} v_p^{3/2} e^{-3v_p/514}\,, \tag{1}$$

We assume that the kick velocity v_k that a compact star may get during an asymmetric SN explosion follows the same probability distribution.

Relation (1) predicts an average runaway velocity of $\sim$ 500 km/s which has been confirmed by Lorimer et al. (1997) in a detailed statistical study. However, in order to account for non zero probability of large measurement errors, we will also present the results for a distribution

$$f(v_p) = 2.70 \cdot 10^{-5} v_p^{3/2} e^{-v_p/60}\,, \tag{2}$$

which predicts an average v_k of $\sim$ 150 km/s.

2.2. *The massive binary formation rate*

Garmany et al. (1980) studied the binary frequency among all known O type stars brighter than m_v = 7 and north of -50° (a total of 67 O type single stars or primaries of binaries). Deleting the O type high-mass-X-ray binaries from the sample, one concludes that 33% of O stars are the primary of a massive close binary (±13% accounting for small number statistics) with mass ratio q $>$ 0.2 and period P $\leq$ 100 days.

Table 1. The formation rate of case B_r binaries with q $\geq$ 0.2 and of binaries with period P $\leq$100 days and q $\geq$ 0.2 in a population of binaries with periods up to 10 years; we separately consider binaries with initial primary masses between 8 $M_\odot$ and 40 $M_\odot$ and between 17 $M_\odot$ and 40 $M_\odot$.

q-distribution	8-100 $M_\odot$		17-100 $M_\odot$	
	case B_r	P≤100d	case B_r	P≤100d
Φ_1 (q)	0.72	0.45	0.75	0.45
Φ_2 (q)	0.6	0.38	0.63	0.38

The binary frequency among the massive B0-B3 stars has been discussed by Vanbeveren et al. (1997). It was concluded that $\sim$ 32% are primary of an interacting close binary. Also here most of them have periods smaller than $\sim$ 100 days. Using an IMF with γ = 2.7 and the binary orbital period distribution of Sect. 2.1.3, Table 1 gives the formation rate of case B_r binaries with q $\geq$ 0.2 and of binaries with P $\leq$ 100 days and q $\geq$ 0.2 in a population of binaries with periods up to 10 years. The numbers are given relative to the total massive binary formation rate f (binaries with periods up to 10 years).

As can be noticed, in order to meet the observations of binaries with q $\geq$ 0.2 and P $\leq$ 100 days, the total binary formation rate has to be very large (f $\geq$ 0.75).

2.3. *Evolutionary parameters*

2.3.1. The β law

The fraction of mass, lost by the primary during Roche lobe overflow (RLOF) and that is accreted by the secondary, depends on the initial mass ratio and period of the binary. For primary masses below 40 $M_\odot$ the following β law is applied:

– systems with an initial mass ratio q $<$ 0.2: β=0 and most of them merge
– late period systems (case B_c/C): β=0
– case A/B_r systems: β=β_{max}·(5q-1) for 0.2$\leq$ q$<$0.4 and β=β_{max} for q$\geq$0.4

For β_{max} we consider the values 0.5 and 1.

Systems with primary masses above 40 $M_\odot$ follow the Luminous Blue Variable (LBV) scenario (Vanbeveren, 1991). No RLOF occurs and no mass is accreted by the secondary. A detailed description of the period evolution during RLOF is given in the appendix.

2.3.2. The limiting mass for BH formation

When a massive star has consumed all thermonuclear fuel it collapses either to a NS producing a SN, either to a BH. If the core mass is too large (M $>$ 2$M_\odot$) the shock wave is not strong enough to expel the surrounding mass shells (Burrows 1992). Accretion by the so called massive transient NS leads to a very fast collapse to a BH. It is very likely that practically all remaining matter is swallowed by the BH and that no SN explosion occurs. In our model we assume that when a BH is formed no kick is imparted to the formed compact object and that the binary parameters by consequence stay unaffected. The initial mass a massive star must have to become a BH may be different for a single star, $M^{BH}_{min,single}$, than for an interacting binary component, $M^{BH}_{min,binary}$. From detailed data-analysis of the observed X-ray pulsar 4U 1223-62 a minimum value of 40$M_\odot$ is estimated for BH formation in binaries (Brown et al. 1996). We computed our results for $M^{BH}_{min,binary}$ = 40$M_\odot$. For single stars the limiting mass depends on the mass loss rates by stellar wind during the RSG phase. In case of a low mass loss rate an underlimit for BH formation is about 15$M_\odot$ corresponding with a neutron star mass of ∼1.6$M_\odot$. For high mass loss rates (comparable with the mass loss through RLOF) the limiting mass may go up to 40$M_\odot$ and even higher. Since mass loss by stellar wind is function of the metallicity the limiting mass may be smaller in regions of different metallicity. For a detailed discussion we refer to the book "The Brightest Binaries" of Vanbeveren et al. (1997). In our model we consider a mass interval 15-40$M_\odot$ for single stars to become BHs.

2.3.3. The minimum mass for single stars to lose all hydrogen layers

We define $M^{X=0}_{min,single}$ as the minimum initial mass for a single star to lose all its hydrogen layers by a stellar wind during core hydrogen burning (CHB), the RSG phase and during the blue core helium burning (CHeB) phase. According to Vanbeveren (1996) and Vanbeveren et al. (1996) the minimum mass is about 15-20$M_\odot$ for the Galaxy and the Large Magellanic Cloud (LMC) and larger than 40$M_\odot$ for the Small Magellanic Cloud (SMC). It can be expected that $M^{X=0}_{min,single} \leq M^{BH}_{min,single}$ in regions of small metallicity. Also for the Galaxy, accounting for the uncertainty in the mass loss rate during the RSG phase, this situation cannot be excluded. It's obvious that in this case there will be no contribution of the single stars to the SNI population.

2.3.4. The spiral-in process

When after the first SN explosion the binary is not disrupted, the system may undergo a spiral-in phase during expansion of the OB secondary. Hereby the compact companion (CC) comes into the envelope of the OB star, possibly removing the outer layers of the OB star. Whether the system merges or not depends on the efficiency α_{sp} of the conversion of orbital energy into potential energy of the envelope of the OB star. We treat α_{sp} as a model parameter and consider the values 0.5 and 1.

If the system merges, a Thorne-Zytkow object (TZO) (Thorne & Zytkow, 1997; Gamow & Teller, 1983) is formed, i.e. a RSG with a neutron core. The fate of such stars is unknown. A possibility is that the TZO loses its complete envelope by a stellar wind and that a rapidly rotating NS is left, producing millisecond radio pulses.

If the system does not merge a small period CHeB+CC binary is formed (CC=compact companion, e.g. either a NS or a BH).

3. The SN-rates

3.1. The theoretical predictions

Before discussing our results we give an overview of which systems contribute to which type of SN. In our classification we make no distinction between type I_b's and I_c's but take them together as $I_{b,c}$'s.

- if $M^{X=0}_{min,single} < M^{BH}_{min,single}$ then single stars with $M^{X=0}_{min,single} \leq M_{ZAMS} \leq M^{BH}_{min,single}$ produce type $I_{b,c}$'s, else no type $I_{b,c}$'s are produced by the single stars
- if $M^{X=0}_{min,single} < M^{BH}_{min,single}$ then single stars with 8$M_\odot \leq M_{ZAMS} \leq M^{X=0}_{min,single}$ contribute to type II's, else type II's are produced by single stars with 8$M_\odot \leq M_{ZAMS} \leq M^{BH}_{min,single}$
- all massive primaries with 10$M_\odot \leq M_{ZAMS} \leq M^{BH}_{min,binary}$ and member of interacting binaries that do not merge during evolution, explode as type $I_{b,c}$'s
- merged binaries are treated as single stars with a mass equal to the sum of the masses of both components and follow the same rules for SN classification as the single stars
- secondaries coming from disrupted binaries are also followed as single stars
- secondaries of binaries that have survived spiral-in after the first SN explosion contribute to type $I_{b,c}$'s if their post-mass transfer mass was smaller than $M^{BH}_{min,binary}$
- TZOs are assumed to not produce any type of SN

Starting from a population of single stars and binaries we have calculated the number ratio (II/$I_{b,c}$) for different combinations of the model parameters.

The results are presented in Table 2. The theoretically predicted number ratio (II/$I_{b,c}$) obviously depends more or less on the adopted evolutionary parameters. Moreover, it is important to realize that

- *the number ratio (II/$I_{b,c}$) strongly depends on the adopted massive binary formation rate, the binary mass ratio and period distribution.*

3.2. Comparison to observations

SN statistics are severely affected by uncertainties inherent to observation and coming from small number statistics. To deal with the latter Cappellaro et al. (1993a,b) combined the data files of two independent SN searches and calculated new estimates of the observed rates for each different type of SN as a function of galaxy type. In order to predict the ratio of one particular galaxy (the Galaxy as an example), they use the average ratio holding for the whole set of galaxies of the same type. Since our PNS results indicate that the ratio depends critically on the massive binary formation rate, the binary mass ratio and period distribution, we can state that

- *estimating the SNII/SNI$_{b,c}$ number ratio in a particular galaxy by using the average ratio of a large set of galaxies of the same type, is meaningful only if the massive binary*

Table 2. The theoretical predicted number ratio ($II/I_{b,c}$) for a large set of model parameters.

f	γ	β_{max}	$\Phi(q)$	α_{sp}	α_{CE}	$M^{BH}_{min,single}$	$M^{BH}_{min,binary}$	$< v_k >$	$M^{X=0}_{min,single}$	($II/I_{b,c}$)
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	2.0136
0.8	2.7	0.5	Φ_2 (q)	0.5	0.5	25	40	500	20	2.7657
0.8	2.7	0.5	Φ_3 (q)	0.5	0.5	25	40	500	20	1.7258
0.8	2.7	0.5	Φ_1 (q)	1	0.5	25	40	500	20	1.9145
0.8	2.7	0.5	Φ_1 (q)	0.5	1	25	40	500	20	1.9901
0.8	2.7	1	Φ_1 (q)	0.5	0.5	25	40	500	20	1.6206
0.8	2	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	1.5544
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	150	20	1.8446
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	>25	2.5339
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	40	40	500	30	2.4361
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	15	40	500	>15	1.7648
0.6	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	2.7006
0.4	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	3.8673
0.2	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	6.2854
0.08	2.7	0.5	Φ_1 (q)	0.5	0.5	25	40	500	20	9.5721
0.6	2.7	0.5	Φ_2 (q)	0.5	0.5	25	40	500	20	3.6095

Table 3. The birth rate per year of NS+NS, BH+NS and BH+BH systems. The results are presented for different combinations of the model parameters. All rates are given for a Galactic massive star formation rate of one massive star per year.

f	γ	β_{max}	$\Phi(q)$	α_{sp}	α_{CE}	$< v_k >$	NS+NS	BH+NS	BH+BH
0.8	2.7	1	Φ_1 (q)	0.5	0.5	500	0.00048	0.00252	0.14451
0.8	2.7	1	Φ_2 (q)	0.5	0.5	500	0.00029	0.00178	0.06200
0.8	2.7	1	Φ_3 (q)	0.5	0.5	500	0.00056	0.00297	0.18213
0.8	2.7	1	Φ_1 (q)	1	0.5	500	0.00137	0.00282	0.14481
0.8	2.7	1	Φ_1 (q)	0.5	1	500	0.00048	0.00252	0.14451
0.8	2.7	0.5	Φ_1 (q)	0.5	0.5	500	0.00038	0.00124	0.06722
0.8	2.7	1	Φ_1 (q)	0.5	0.5	300	0.00140	0.00456	0.14451
0.8	2.7	1	Φ_1 (q)	0.5	0.5	150	0.00379	0.00846	0.14451
1	2.7	1	Φ_1 (q)	0.5	0.5	500	0.00060	0.00315	0.18064
0.6	2.7	0.5	Φ_2 (q)	1	0.5	300	0.00113	0.00205	0.01705
0.4	2.7	1	Φ_1 (q)	0.5	0.5	500	0.00024	0.00126	0.07226
0.8	2	1	Φ_1 (q)	0.5	0.5	500	0.00044	0.00480	0.25432

formation rate, the binary mass ratio and period distribution are the same in all the galaxies where the average value is based on.

From our theoretical calculations (with f $\geq$ 0.75) we conclude that in the Galaxy, type II's are 2-3 times more frequent than the type $I_{b,c}$'s.

For all the galaxies in their sample, Cappelaro et al. derive an average ratio of ∼4. Since the sample of observed galaxies is large (2461) and since all morphological types are encountered, this value could be considered as some cosmological average. If we try to recover this rate, we have to adopt an average massive binary formation rate between 40 and 60%, i.e.

- *the cosmological massive binary formation rate may be of the order of 50%.*

Cappellaro et al. distinguished early type and late type spiral galaxies. The early types seem to have a ratio of ∼2.3 whereas the ratio in the late types ≈ 5.5. If this difference is due to a different massive massive binary formation rate, we may conclude that

- *the massive binary formation rate in late spirals is ∼factor 2 smaller than in early type spirals.*

4. Double compact binaries

4.1. Theoretical predictions

Double neutron star systems (NS+NS) arise from CHeB+NS systems that have survived the SN explosion of the CHeB star. Assuming that BHs form above 40$M_\odot$ (25$M_\odot$ for not interacting binaries) the formation of BH+NS and BH+BH systems is possible as well.

Similarly as in the previous section we start from the same initial stellar population and apply the same PNS model where the period evolution of each binary is computed in detail. It is obvious to realize that the results here critically depend on

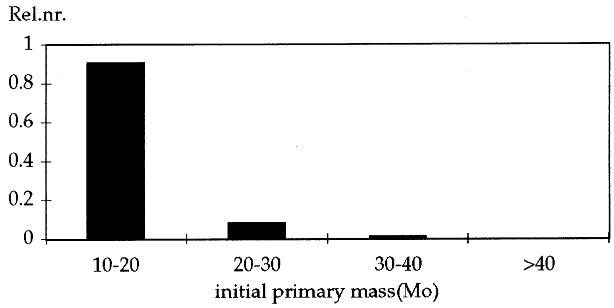

Fig. 1a. The distribution of the primary mass at ZAMS of the progenitors of the NS+NS systems. The calculations are performed for the following model parameters: α_{sp}=0.5, γ=2.7, β_{max}=1,$M^{BH}_{min,binary}$=40$M_{\odot}$ and $< v_k >$=500km/s.

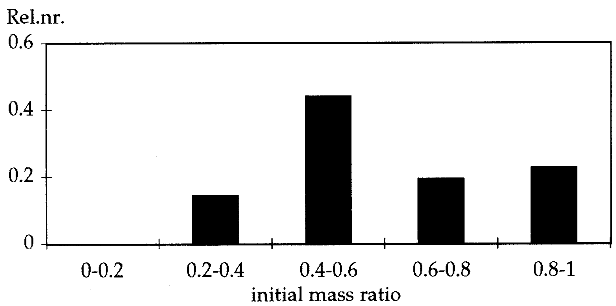

Fig. 1b. The distribution of the mass ratio at ZAMS of the progenitors of the NS+NS systems. The calculations are performed for the same model parameters as in Fig. 1a.

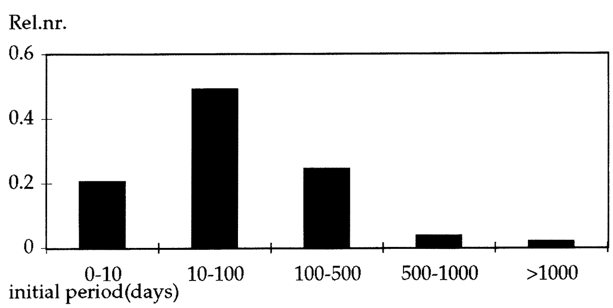

Fig. 1c. The distribution of the period at ZAMS of the progenitors of the NS+NS systems. The calculations are performed for the same model parameters as in Fig. 1a.

the spiral-in parameter α_{sp}, contrary to the results on the SN rates. The CHeB + CC binary is followed through a second asymmetric SN explosion if a NS will be formed whereas the binary stays unaffected in case the CHeB component collapses into a BH. Hereby we investigate the influence of the degree of asymmetry on the number of NS+NS systems by considering different average kick velocities $< v_k >$. In Table 3 we give the formation rate of the considered double compact star binaries for different combinations of the population model parameters. We conclude that:

- *the theoretical birth rate of double NS systems is about (0.0003- 0.004) times the formation rate of massive stars,*

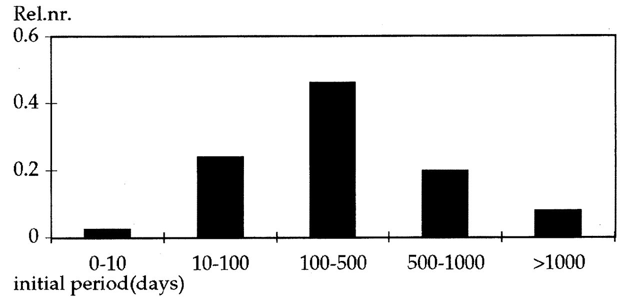

Fig. 1d. The same as Fig. 1c but for β_{max}=0.5.

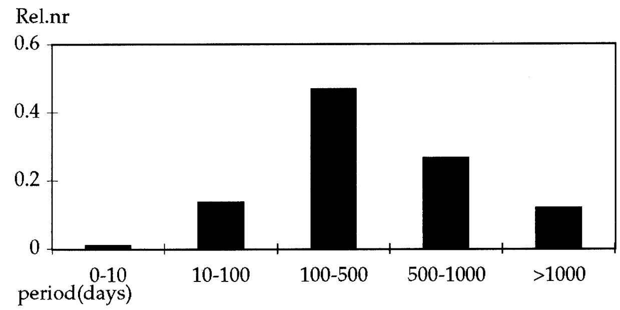

Fig. 1e. The same as Fig. 1d but for $< v_k >$=150km/s.

- *double BH systems are formed at a surprisingly high rate,*
- *mixed systems i.e. consisting of a BH and a NS are (2-5) times more frequent than NS+NS systems,*
- *the two most crucial model parameters are the efficiency parameter during spiral in, α_{sp}, and the average kick velocity $< v_k >$.*

The high birth rate of double BH systems is explained by the fact that although fewer stars become BHs, the binary system is not subjected to any kick shot during its evolution. Beside this the chance for a CHeB+BH system to merge after spiral-in is very small because of a large period. The majority of the mixed systems originates from massive X-ray binaries (MXBs) with a BH as compact companion. Only few systems form in which the primary exploded and where the secondary has accreted sufficient mass to become a BH.

To get some information on the progenitors of NS+NS systems, we kept all data of the different evolutionary phases from birth to final state. In Figs. 1a–e we show the distribution of the binary parameters at the ZAMS of the progenitors of the NS+NS systems and in Figs. 2a–c we give the period distribution of the OB+NS systems that will produce NS+NS systems and this for different values of the most important model parameters. It is clear that the majority of double NS systems comes from case B_r systems that have undergone quasi-conservative mass transfer during the first RLOF. During mass transfer the period has increased such that the system survives the first SN explosion and the following phase of spiral-in during the CE evolution. Large period systems (case B_c/C) on the contrary have a too large period to survive the SN explosion even for kick velocities of the order of 150 km/s. By consequence, the treatment of these systems and the related model parameters (e.g. α_{CE}) are

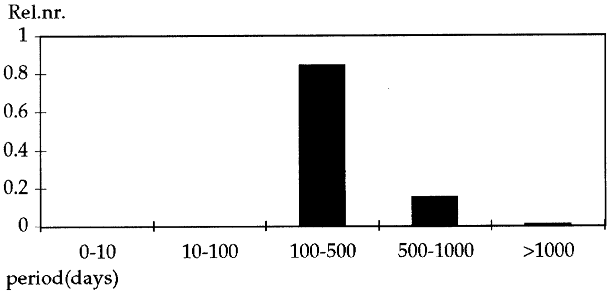

Fig. 2a. The period distribution of OB+NS systems that will produce NS+NS systems. We used the parameter values α_{sp}=1, γ=2.7, β_{max}=0.5, $M^{BH}_{min,binary}$=40$M_\odot$ and $< v_k >$=500km/s.

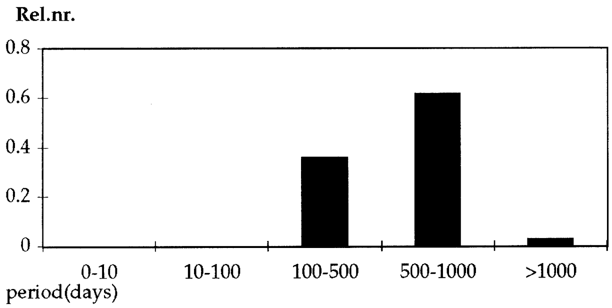

Fig. 2b. The same as Fig. 2a but for α_{sp}=0.5.

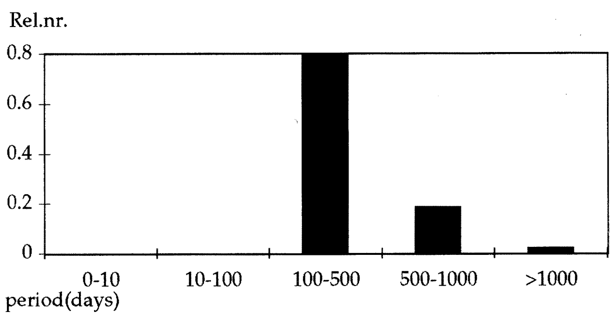

Fig. 2c. The same as Fig. 2a but for $< v_k >$=150km/s.

of minor importance for the production of NS+NS systems. The fraction of accreted matter (β) seems to be the most influencing parameter on the initial period distribution.If the RLOF occurs completely conservative, the period increases (up to a factor 4), lowering the chance to survive the SN explosion and so only the shorter period systems will stay bound. In case of partially conservative RLOF the increament of the period is smaller and there is a redistribution of the relative numbers towards larger initial periods.

In order to produce a NS+NS system, the OB+NS system must survive the spiral-in phase and the second SN explosion. From our figures we conclude that the most crucial parameter is the efficiency parameter α_{sp} during spiral-in. The lesser is the efficiency, the higher the initial period before spiral-in must be in order not to merge.

Since the CHeB+NS system is less strong gravitational bound than the OB+CHeB system and because of the required large period to survive spiral-in, the asymmetry in the SN explosion needed to disrupt the system is smaller. This is reflected in the fact that lowering the kick velocity from 500 km/s to 150 km/s hardly affects the period distribution of the OB+NS systems.

Double compact star binaries lose orbital energy and angular momentum by emitting gravitational waves. The binary period shrinks and the system finally merges. The time τ (expressed in years) within complete spiral-in occurs, is given by the formula,

$$\tau = \mathrm{K} \cdot \frac{(M_{1c}+M_{2c})^{1/3}}{M_{1c}\cdot M_{2c}} \cdot \frac{P^{8/3}}{f(e)} \text{ with}$$
$$\mathrm{K} = 8.5562 \cdot 10^{17} (\frac{M_\odot}{\text{years}})^{5/3},$$
$$f(e) = \frac{1+(73/24)e^2+(37/96)e^4}{(1-e^2)^{7/2}} \text{ and P in years .}$$

(Wagoner,1975).

The time of complete spiral-in is beside depending on the initial orbital period and on the mass of both components (M_{1c}, M_{2c}), strongly dependent on the eccentricity (=e) of the system. For an eccentricity e of 0.9, the time of complete spiral-in is 1000 times shorter than in the case of circular orbits. Taking an average eccentricity of 0.6 (based on the eccentricities of the observed galactic NS+NS systems) one finds that to complete spiral-in by gravitational wave radiation (GWR) within 6 million years the NS + NS system should have an initial orbital period smaller than 2 hours.

The latter means that the system has a short lifetime and may escape potential detection for observation. In Fig. 3 we give the theoretical predicted period distribution at birth of all double compact star binaries. The shape of the distribution is insensitive to different choices of the model parameters. We made our calculations with a flat q-distribution, α_{sp}=0.5, $M^{BH}_{min,binary} = 40M_\odot$, $M^{BH}_{min,single} = 25M_\odot$, β_{max}=1, $< v_k >$=500km/s and f=0.8. The figure shows that only a tiny fraction (<5%) of the NS+NS systems suffices the previous mentioned condition to complete spiral-in within stellar evolution time.

If we do the same for BH+BH binaries but with circular orbits then we may conclude that their orbital periods are not sufficiently short to completely spiral-in even within the Hubble time. This because BHs come from very massive stars ($> 40M_\odot$) having mass loss by a very strong stellar wind causing a strong increase of the orbital separation in course of the evolution. Still we have to remark here that this picture may change if the limiting mass for BH formation in binaries is smaller. Under assumption that our theoretical period distribution reflects more or less reality we may conclude that only a small fraction of double compact star systems will completely spiral-in within evolutionary time.

4.2. Comparison to the observations

Assuming that type I_a's are coming from intermediate mass binaries, the total rate of SNe in the Galaxy produced by massive stars is approximately 1 -2 per century (Cappellaro et al., 1993). Since this number corresponds to the formation rate of massive

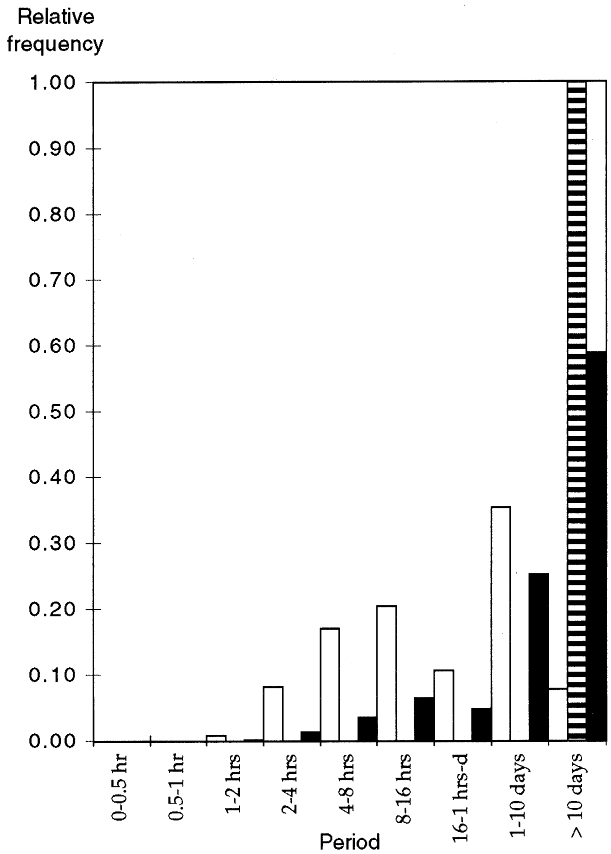

Fig. 3. The theoretical period distribution at birth of the following systems: NS+NS (white bars), BH+NS (black bars), BH+BH (horizontal lined bars).

stars, it follows from Table 3 that our PNS model predicts a Galactic NS+NS formation rate of $\sim (10^{-6} - 10^{-5})/yr$. This is in agreement with the observed formation rate of $\sim 10^{-5}/yr$ (van den Heuvel, 1994) which is based on the observation of 3 NS+NS systems that are located within 7kpc from the Sun.

5. Concluding remarks

In this paper we have shown the importance of different evolutionary parameters, the massive binary formation rate, the binary mass ratio and period distribution on SN number statistics. Their influence leads to the conclusion that average SN rates obtained from observed SN events in a sample of galaxies are not necessarily representative for one particular galaxy because of the possibility of dealing with a mix of different values for the above mentioned parameters. Further we have explored all possible outcomes in the late stages of close binary evolution if BH formation occurs. Our theoretical prediction on the birth rate of double NS systems in the Galaxy fairly agrees with the observed formation rate. We have also investigated the possibility that some of these systems may not be observable because of complete spiral-in within stellar lifetime as a consequence of GWR and conclude that they are a minority. Trying to get some information on the progenitors of the NS+NS systems we find that the major part is the end station of case B_r binaries. Finally we found that double BH systems are formed at a much higher rate than double NS systems.

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Appendix A: the evolution of the period during RLOF

A.1. Binaries with q ≤0.2

Systems with a low mass ratio q become tidal instable when the primary evolves and expands. The low mass companion, the secondary, is then dragged into the envelope of the primary and the spiral-in process starts. Most of the systems merge as was outlined in Paper I. Their further evolution is at present unknown. We treat them as single stars with a mass equal to the sum of the masses of both components.

A.2. Case B_c and case C binaries: common envelope evolution

The mass transfer in these late period binaries occurs on a dynamically time scale leading to the formation of a common envelope. Embedded in the common envelope, the orbit of the binary shrinks due to friction. The energy of the orbit is converted with some efficiency α_{CE} into the potential energy of the mass that needs to be removed. Also here we apply the formalism of Webbink (1984) to describe the spiral-in process. The outcome is a rapidly rotating giant when both components have merged. If the energy is sufficient to unbind the envelope, a small period binary is left consisting of a helium star and a main sequence star.

A.3. Case A and case B_r binaries with q>0.2

Case A/Br systems with a primary mass $\leq 40 M_{\odot}$, perform RLOF during core hydrogen/hydrogen shell burning. Above some minimum value q_{min} (=0.4) it is assumed that the mass transfer occurs quasi-conservative for which we take β, the fraction of matter lost by the primary due to RLOF and that is accreted by the secondary, to be constant ($= \beta_{max}$).

To estimate the period variation during non-conservative RLOF for case A and case B_r systems, we assume that matter leaves the binary system through the second Lagrangian point, forming a ring around the binary. Taking the orbit and ring as circular, whereby the ring revolves in the same sense as the binary at a distance A_{ring} from the center of mass of the binary, the angular momentum per unit mass in the ring can be written as:

$$j_{ring} = G\sqrt{(M_1 + M_2)A_{ring}}\,, \quad \text{(A1)}$$

with M_1 and M_2 respectively the mass of the primary and secondary. The variation of orbital angular momentum of the binary is given by

$$dJ = j_{ring}(1-\beta)dM_1\,, \quad \text{(A2)}$$

Using Kepler's law and the formula for orbital angular momentum of the binary, elaboration of Eq. (2) gives the rate of change of the orbital period:

$$\frac{dP}{P} = (1+3\zeta)\cdot\frac{(1-\beta)dM_1}{(M_1+M_2)} - 3\cdot\frac{dM_1}{M_1} - 3\cdot\frac{dM_2}{M_2}\,, \quad \text{(A3)}$$

with $\zeta = \frac{(M_1+M_2)^2}{M_1 M_2}\sqrt{\frac{A_{ring}}{A}}$

For or a large range of binary mass ratio's the L_2 distance from the center of mass is approximately 1.3 times the semi-major axis A. Although A_{ring} is somewhat larger than the L_2 distance we take $A_{ring} = \eta$ A with $\eta = 1.3$. Integration of Eq. (3) results in the following relation between the initial and final period of the system after a non-conservative RLOF:

$$\frac{P}{P_0} = \frac{(M_1+M_2)}{(M_1^0+M_2^0)}\cdot\left(\frac{M_1}{M_1^0}\right)^{3\sqrt{\eta}(1-\beta)-1} \cdot\left(\frac{M_2}{M_2^0}\right)^{-3\left(\sqrt{\eta}\frac{(1-\beta)}{\beta}\right)+1}, \quad \text{(A4)}$$

In the formula the index zero refers to the initial values before RLOF.

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