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Fundamental physical parameters of RRab stars

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Abstract. The intrinsic mean colours and the metallicity of RRab stars calculated from the light curve parameters are shown to be consistent with the Kurucz model atmosphere results if the zero points of the $B-V$ and $V-I$ colours are corrected. This consistency justifies the application of bolometric correction and temperature transformations derived from the Kurucz models in determining the corresponding static values of the luminosity and effective temperature of the variables. Since we can also infer the mass by using the fundamental pulsation equation, all the basic physical parameters of any variable with known light curve can be determined. The accuracy of the calculated $\log L$, $\log T$, $\log M$ and [Fe/H] values are estimated to be 0.009, 0.003, 0.026 and 0.14 dex, respectively.

The method enables us to derive the above parameters for a large sample of RRab stars, thus the basic relations among the physical parameters can also be given. Although the zero points of the physical parameters are somewhat ambiguous, the slopes of these relations are quite precise. The comparison with pulsational and evolutionary model predictions leads to the following conclusions: (a) there is no pulsational model calculation which explains the strong dependence of the temperature on the luminosity although this is one of the most significant relations defined by the empirical data; (b) the slopes of the different relations between the fundamental physical parameters agree with the results of horizontal branch models well; (c) determining the metallicity dependence of $\log L$, $\log T$ and $\log M$, the various contributions which induce the observed shift of the average periods of RR Lyrae stars with metallicity (Sandage period-shift), can be correctly separated; (d) the data show that the more evolved, post zero-age horizontal branch objects are typically found amongst the most metal-poor variables.

Key words: stars: fundamental parameters – stars: horizontal-branch – stars: RR Lyrae

1. Introduction

In a recent series of papers (Jurcsik and Kovács 1996 (JK); Kovács and Jurcsik 1996; 1997 (KJ96, KJ97)) it was shown that the metallicity, the absolute magnitudes and the intrinsic colours of RRab stars can be expressed as linear combinations of the period and the low order amplitudes and phases of the Fourier decomposition of their V light curve. These quantities, if combined with model atmosphere and pulsational results, can be used to determine the fundamental physical parameters. It was shown by using hydrodynamical models that such relations between the mass, temperature and luminosity *vs.* the period and Fourier phase-parameter (φ_{31}) of RRc stars do exist (Simon & Clement 1993). As RR Lyrae stars are one of the most important test objects of both horizontal branch evolutionary studies (e.g., Sweigart, Renzini and Tornambè 1987; Lee and Demarque 1990; Lee, Demarque and Zinn 1990; Catelan 1993) and pulsational model calculations (e.g., Bono et al. 1997; Feuchtinger and Dorfi 1997; Kovács and Kanbur 1997), therefore, it has great importance to correctly determine the basic physical parameters for not only some individual objects but for a large sample of the variables.

In the present paper the consistency of our previous results with the synthetic colours and the metallicity of the Kurucz atmosphere models is proved. Utilizing this consistency we can determine the temperature and luminosity of the stars, thus the pulsational equation can be used to calculate the mass, too. It is shown that the relationships among the basic physical properties are in convincing quantitative agreement with the predictions of horizontal branch evolutionary models.

2. Comparison of the empirical colours and metallicity with Kurucz model atmosphere results

The luminosity and temperature of RR Lyrae stars can be determined by using the absolute magnitude, intrinsic colours and metallicity calculated from the light curve parameters (called 'empirical' data) and bolometric correction and temperature transformations determined from model atmosphere results (Kurucz 1993, hereafter referred to as K93). This method leads to correct solutions only if there is no intrinsic, built-in inconsistency between the empirical and the K93 data. Therefore, first it is necessary to check the compatibility of the two types of data. This test is performed by determining interrelations of the various parameters given in the K93 tables and by checking their validity using the empirical quantities.

Table 1. Standard deviations of the interrelations of the $B-V$, $V-I$, $V-K$ and [Fe/H] values of the Kurucz tables. Two samples of the models are tested; all the models within the RRab parameter regimes (sample a) and a subsample with temperature and metallicity in such combination as RRab stars actually define (sample b)

sample	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
fit	linear		quadratic		linear		quadratic		linear		quadratic		linear		quadratic	
N_t	σ_{B-V}				σ_{V-I}				σ_{V-K}				$\sigma_{[\mathrm{Fe/H}]}$			
1	0.050	0.027	0.050	0.027	0.012	0.010	0.012	0.010	0.022	0.019	0.021	0.016	0.820	0.459	0.819	0.442
2	0.017	0.011	0.015	0.011	0.006	0.006	0.006	0.006	0.014	0.012	0.012	0.012	0.311	0.201	0.311	0.159
3	0.016	0.010	0.010	0.005	0.006	0.005	0.004	0.004	0.014	0.012	0.008	0.009	0.310	0.201	0.241	0.126
4			0.006	0.004			0.004	0.003			0.008	0.009			0.218	0.112
5			0.006	0.004			0.003	0.003			0.008	0.009			0.187	0.109
.			...				...				...				...	
9			0.005	0.004			0.003	0.003			0.008	0.009			0.186	0.112

2.1. The empirical data

In the course of the present study we use a data base which consists of the Fourier decompositions of 272 RRab V light curves. The sample comprises 90 galactic field stars, 97 globular cluster stars and 85 variables from the Sculptor dwarf galaxy. The references of the light curves were given in JK, KJ96 and KJ97. Altogether, Fourier decompositions of 278 stars, which did not show any evidence of light curve variability, were collected in our previous works but 6 of them deviate significantly from the relations given in the present paper. Omitting these stars the remaining 272 variables still can be regarded as a sample which is large and heterogeneous enough to represent the whole RRab population in most respects.

Using the above data set the metallicity, the intensity averaged M_V and the intrinsic mean colours are determined according to the formulae given in JK, KJ96 and KJ97, respectively. The colour indices are calculated as the differences of the magnitude averaged absolute brightnesses. For easier application the following equations list the formulae we use:

$$[\mathrm{Fe/H}] = -5.038 - 5.394P + 1.345\varphi_{31} \quad (1)$$

$$M_V = 1.221 - 1.396P - 0.477A_1 + 0.103\varphi_{31} \quad (2)$$

$$B-V = 0.308 + 0.163P - 0.187A_1 \quad (3)$$

$$V-I = 0.327 + 0.578P - 0.273A_1 - 0.046\varphi_{41} \quad (4)$$

$$V-K = 1.585 + 1.257P - 0.273A_1 - 0.234\varphi_{31} + 0.062\varphi_{41}. \quad (5)$$

These quantities calculated from the V light curves of the 272 RRab stars are the fundamental empirical data the present work is based on.

2.2. Interrelations arising from the Kurucz models

In order to derive precise but also simple relations on the K93 tables, we select model atmosphere parameters as close to the ones corresponding to the static equivalents of RRab stars as possible. For this reason, only models within the following parameter ranges are concerned: $6000K \leq T_{\mathrm{eff}} \leq 7000K$, $-2.5 \leq [\mathrm{Fe/H}] \leq +0.3$ and $2.5 \leq \log g \leq 3.0$. The T_{eff} interval is assigned by the models which have any synthetic colour within the ranges of the empirical values. There are five different temperature and two different surface gravity model sets of the grid which meet these requirements. We shall refer to this sample of the K93 models as 'sample a'. Altogether 120 different models belong to this sample. A subsample of this set of models, referred to as 'sample b', will be defined later.

We search for interrelations between the $B-V$, $V-I_c$, $V-K_J$ and [Fe/H] values of the K93 tables. As these are the quantities we can also determine from the light curve parameters, these relations are suitable for checking the compatibility of the empirical and the K93 data. Each of the above parameters are fitted with linear and quadratic combinations of the other three. Table 1 summarizes the fitting accuracy of the interrelations obtained by using the two samples of the K93 models. The standard deviations of the best linear and quadratic fits using N_t number of terms are given. For the linear solutions N_t equals to the number of parameters but for the quadratic ones the number of the actually used parameters can be smaller than N_t. The standard deviations listed in Table 1 show that the accuracies of the two-parameter linear formulae which predict the colours are already close to the typical errors of absolute photometry. However, the addition of quadratic terms decrease the standard deviations significantly, especially in the case of $B-V$ and [Fe/H], indicating nonlinearity of these relations.

In Fig. 1 some of the interrelations between the colours and metallicity of the K93 tables (sample a) are shown. The best one- and two-parameter linear, and quadratic formulae with 3 terms are plotted. The terms of the fitting formulae are given in the upper left corners of the figures. The boxes in the last panels of the colour fits show the observed ranges of the colours as determined from the light curve parameters. The different sizes and positions of the boxes in comparison with the intervals defined by the K93 values indicate that, on the one hand, only a subsample of the models of sample a refers to the empirical data and, on the other hand, zero point differences exist between the empirical and the synthetic colours. As zero point ambiguity distorts the nonlinear formulae while affects only the constant terms of the linear ones, we give preference to linear solutions if they are still sufficiently accurate.

In Sect. 4.1.2 it will be shown that the temperature and the metallicity of RRab stars are correlated. The standard deviations

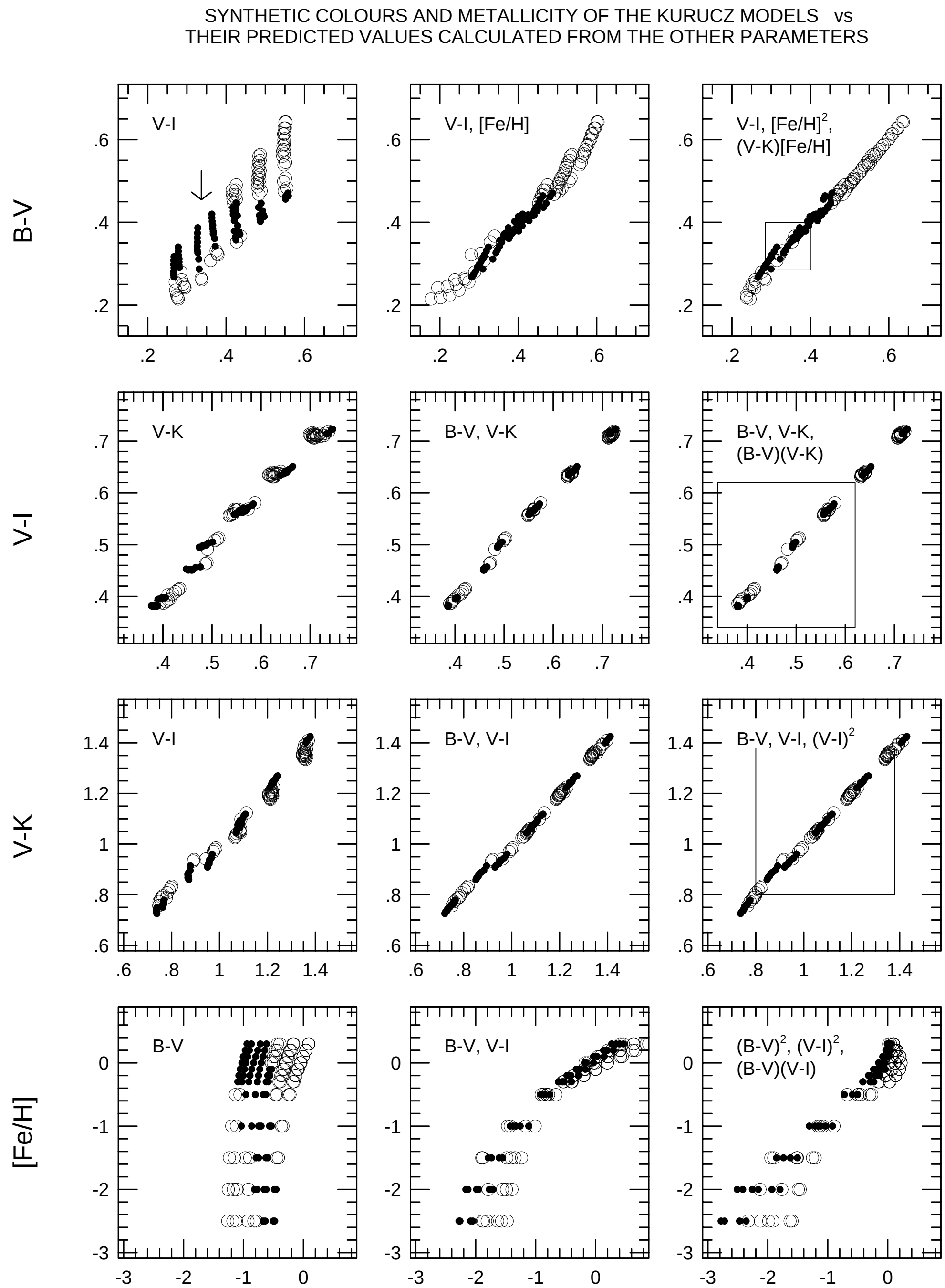

Fig. 1. Interrelations of the $B-V$, $V-I$, $V-K$ and [Fe/H] values of Kurucz models with parameters typical for RRab stars (sample a). In the left and middle panels the best linear fits using 1 and 2 parameters, in the right quadratic formulae with 3 terms are shown. The terms of the fitting formulae are given in the upper left corners of each plot. In the last panels of the $B-V$, $V-I$ and $V-K$ fits the boxes show the observed ranges of the colours. Filled circles indicate models with [Fe/H] and $T_{\rm eff}$ values which fulfil the overall relation found between these physical parameters (sample b), whereas open circles are for all the other models in the parameter ranges of RRab stars. It can be seen that selecting models according to the $[{\rm Fe/H}]-T_{\rm eff}$ relation both explains the observed narrow $B-V$ range and gives better solutions for the [Fe/H] fits. The observed range of the colours cover different models, therefore zero point differences between the Kurucz colours and the empirical values can be suspected. In the first panel of the $B-V$ plots the arrow indicates discontinuity of the K93 data. In this plot the vertical groups of the points represent the different temperature models with metallicity increasing upwards. The arrow pointing to the $6750K$ models shows that the synthetic $B-V$ colours and their predicted values using $V-I$ differ significantly for the $\log g = 2.5$ and $\log g = 3.0$ models exclusively at this temperature. This discontinuity can also be seen in most of the other panels.

listed for sample b in Table 1 refer to formulae defined by using a subsample of sample a of the K93 models which have [Fe/H] and $T_{\rm eff}$ values in such combinations as the overall relation found between these parameters predicts (Eq. (21), see also Fig. 5). There are 58 models in the originally selected sample which fulfil this requirement. In Fig. 1 these models are shown by filled circles while open circles denote all the other models of sample a. It can be seen that selecting models according to the $[{\rm Fe/H}]-T_{\rm eff}$ relation explains the narrowness of the observed $B-V$ range. Moreover, for this subsample of the grid, it is also possible to fit the [Fe/H] values more precisely.

The two-parameter linear formulae defined by sample b are already precise enough for all the colours and also for the metallicity to test the empirical data (see Table 1). The formulae of these interrelations are:

$$(B-V) = \underset{(+0.002)}{-0.016} + 0.814(V-I) + 0.049[{\rm Fe/H}] \quad (6)$$

$$(V-I) = \underset{(-0.093)}{-0.021} + 0.263(B-V) + 0.443(V-K) \quad (7)$$

$$(V-K) = \underset{(+0.203)}{+0.045} - 0.548(B-V) + 2.229(V-I) \quad (8)$$

$$[\mathrm{Fe/H}] = +0.727 + 17.358(B-V) - 15.198(V-I) \quad (9)$$
$$(+0.321).$$

The constants given in parentheses are the terms of the respective formulae of the zero point shifted version of the Kurucz tables. The transformation of the K93 data, that is necessary in order to match with the empirical quantities, will be determined in the next section.

It should be noted that the colour formulae (Eqs. (6)-(8)) defined by sample b are not significantly different from the respective formulae obtained from sample a. The substitution of the empirical data into the two sets of equations yields results which are identical within less than 0.005 mag standard deviation.

2.3. *Consistency of the empirical data with their counterparts in the Kurucz tables*

The correctness of Eqs. (6)-(9) when using the empirical data provides answer to the question of consistency between the empirical parameters calculated from the light curve parameters and the data of the K93 models. Fig. 2 shows the result of this test. In the left panels the empirical colours and metallicity of the 272 RRab stars calculated from Eqs. (1),(3)-(5) (observed values) are compared with their predicted values, obtained by substituting the other empirical values into the interrelations of the K93 data (Eqs. (6)-(9)). Although, as it was shown in Table 1 and Fig. 1, the interrelations of the K93 tables are nonlinear, neither the displacement nor the scatter of these plots are smaller if quadratic formulae are used. The application of the 3-term quadratic $B-V$ and [Fe/H] formulae even results in definitely larger scatter of the plots.

The left panels in Fig. 2 already show that the predicted values strongly correlate with the empirical ones but constant shifts do appear supposedly due to zero point (ZP) differences. As the K93 colours need to be recalibrated for stars differing significantly from the calibrating objects of the Kurucz models (the Sun and Vega), the predicted values are matched with the empirical ones by determining ZP corrections of the K93 colours. Gratton, Carretta and Castelli (1996) and also Clementini et al. (1995) showed that the K93 $V-K$ colours equal to recent empirical calibrations based on infrared flux method and interferometric diameter measurements for dwarfs and differ only slightly for giants. The sample used for the empirical calibration of giants was, however, very poor (see Fig. 1c. in Gratton, Carretta and Castelli 1996). Thus, assuming that the K93 $V-K$ colours are correct, the ZP differences of the $B-V$ and $V-I$ colours are determined as follows.

First the displacements between the observed and the predicted values of the colour indices and [Fe/H] are determined, then using standard least squares method, it is solved, what transformations of the K93 $B-V$ and $V-I$ colours would eliminate these shifts. As a result, if only constant shifts between the observed and the predicted values are considered, the following ZP corrections are obtained:

$$(B-V)_{emp} = (B-V)_{K93} - 0.051 \qquad \text{and}$$

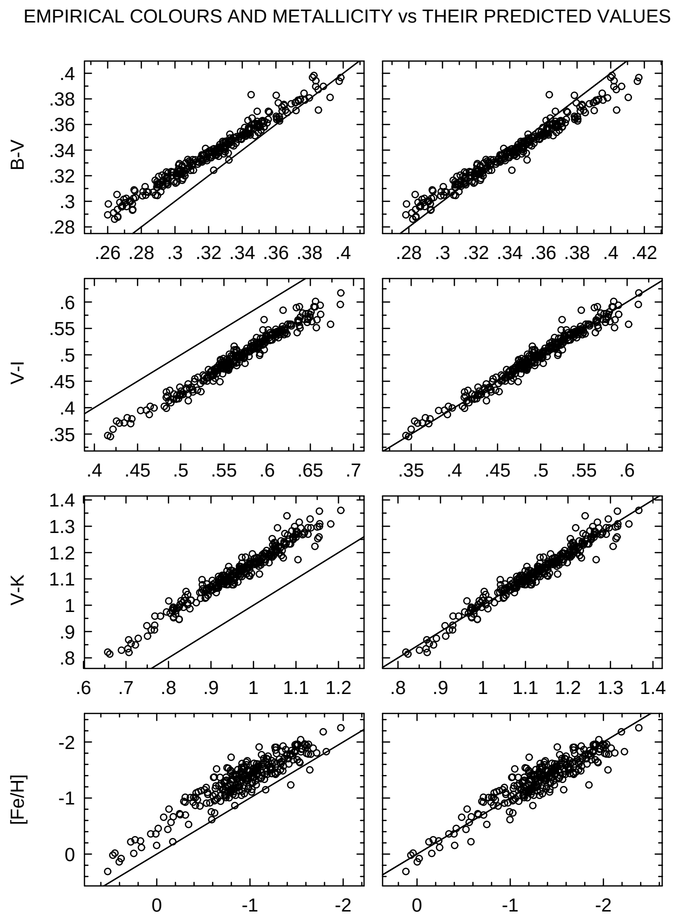

Fig. 2. $B-V$, $V-I$, $V-K$ and [Fe/H] calculated from light curve parameters compared with their estimations using formulae defined on the K93 tables. The 45° lines are drawn for reference. On the left panels the direct application of the formulae (Eqs. (6)-(9)) are plotted. Correcting the zero points of the K93 $B-V$ and $V-I$ colours the fits are shown in the right. The standard deviations of the latter plots are 0.008, 0.010 and 0.022 mag and 0.17 dex for $B-V$, $V-I$, $V-K$ and [Fe/H], respectively.

$$(V-I)_{emp} = (V-I)_{K93} - 0.085. \quad (10)$$

If interrelations of sample a of the K93 models are used instead of Eqs. (6)-(9), these results change only slightly, the similarly determined ZP shifts in this case are -0.057 and -0.085, respectively. Similar displacements between the colour-temperature calibrations based on empirical results and on the K93 data can be seen in Fig. 3 of Clementini et al. (1995) confirming the zero point transformations we have obtained.

If ZP difference between the empirical and the K93 [Fe/H] values is also allowed then the set of linear equations which is solved to obtain the ZP shifts becomes underdetermined and thus gives unrealistic results. Moreover, the inclusion of the [Fe/H] transformation results in no changes in the final agreement between the empirical and the predicted values when linear formulae are used. Consequently, although we do not rule out the possibility that ZP difference between the K93 and the empirical [Fe/H] values may also exist, we only consider the ZP corrections of the $B-V$ and the $V-I$ colours as given in

Eq. (10). Hereafter, the K93' abbreviation is used to refer to the ZP corrected version of the Kurucz tables.

A slight inclination of the $B-V$ plot in Fig. 2 can also be seen but its effect is only about ± 0.01 mag over the whole $B-V$ range. Although this inclination can be corrected by changing the coefficients of the Fourier parameters in Eq. (3) within their error ranges, we also checked whether zero point and/or scale differences between the two types of data can also be an explanation. Using linear expression for the deviation between the empirical and the predicted $B-V$ colours, both linear and nonlinear interrelations, for both constant and linear transformations of the K93 colours (including also $V-K$) and [Fe/H], were tested. We have found that neither constant, nor linear transformation between the K93 and the observed metallicities can be the cause of the inclination between the observed and predicted values of $B-V$, in spite of the fact that the interrelations which express $B-V$ are nonlinear in [Fe/H]. The metallicity transformation which corrects the inclination also results in very pronounced overall increase of the scatters. Reliable solution to correct the inclination can only be obtained when using linear transformation between the two $B-V$ scales. It was already mentioned that the empirical $B-V$ may have such uncertainty, but the problems of model atmospheres as discussed by Kurucz (1996) and also by Castelli, Gratton and Kurucz (1997) do not rule out the possibility that besides the zero point, the scale of the K93 $B-V$ colours is also a bit ambiguous. Since the effect of the inclination is small, and we cannot decide which $B-V$ values need to be transformed, only the constant ZP shift is adopted. The $B-V$ colour, however, will not be used when deriving the physical parameters of RRab stars, thus this unsolved problem has no influence on the $\log L$, $\log T$ and $\log M$ values calculated later. This problem, however, if derives from defects of the Kurucz models also warns of the possible errors of $\log L$, $\log T$ transformations based on the $B-V$ colours of the K93 tables.

The right panels in Fig. 2 show the empirical vs. predicted values of the colours and [Fe/H] using interrelations derived from the K93' data. As linear formulae are used, these interrelations differ from Eqs. (6)-(9) only in the constant terms. The constants of these formulae are given in parentheses below the respective equations. The ZP corrected formulae can also be applied to control multicolour photometric data and reddening corrections.

The standard deviations of the zero point corrected fits seen in Fig. 2 are 0.008, 0.010, 0.022 mag and 0.17 dex for $B-V$, $V-I$, $V-K$ and [Fe/H], respectively. These values are close to the 0.011, 0.006, 0.012 mag and 0.20 dex accuracies of the applied formulae (see Table 1) and also to the 0.010, 0.013, 0.028 mag and 0.14 dex estimated errors of the empirical values. The error estimations of the empirical colours and metallicity are to be explained in more details in the next paragraph and in Sect. 3.1. Such an agreement in the different accuracies shows that the two types of data can be regarded as identical within the limits set by the intrinsic errors of the quantities and the formulae.

We estimate the accuracy of the empirical colours by using the following arguments. The $B-V$ formula fits the original calibrating values with 0.010 mag standard deviation (KJ97). Assuming that this is the typical uncertainty of the empirical $B-V$ values, this is also the error of the calculated reddenings. Since the uncertainty of the distance estimation does not count when colour indices are calculated, their accuracy exclusively depends on the error of the calculated E_{B-V}. Consequently, it seems to be a reliable estimation of the accuracy of the $V-I$ and $V-K$ values to multiply the 0.01 mag error of $B-V$ by the coefficients of the $E_{V-I} = 1.26E_{B-V}$ and $E_{V-K} = 2.75E_{B-V}$ formulae, i.e., $\sigma_{V-I} = 0.013$ and $\sigma_{V-K} = 0.028$ mag. These values do not differ significantly from the original 0.011 and 0.024 mag fitting accuracies of the I_0 and K_0 formulae (KJ97).

It is important to emphasize that the fits seen in the right panels of Fig. 2 are obtained when combining two intrinsically independent data sets, namely the Kurucz model atmosphere grid and the empirical colours and metallicity of RRab stars. *The applicability of the formulae defined on the K93 tables to the same parameters but calculated from light curve characteristics with such precision is a very strong proof of the reliability and accuracy of both the K93 and the empirical data.*

However, the agreement could only be reached if the zero points of the K93 colours were corrected. We have found that, on the one hand, previous colour-temperature calibrations based on empirical temperature determinations and on the K93 data led to similar displacements of the $B-V$ and $V-I$ colours as we have determined and, on the other hand, scale or ZP transformation between the two types of the metallicity values is not obviously necessary. We therefore think that the colour and metallicity scales what we use (defined in KJ97 and Jurcsik (1995)) are quite correct. Still, we do not rule out the possibility that our solution, shifting only the $B-V$ and $V-I$ colours of the Kurucz models, is not the absolutely correct one. Thus even the K93' values may lead to incorrect results when absolute quantities are concerned. By using only linear expressions, however, the corrections will be simple, if the ZP transformations are solved later differently.

The K93 models correspond to population I composition and the empirical temperature calibrations are also based on mostly population I stars. RR Lyraes are, however, typical population II objects. Therefore, as a consequence of the similarity of the ZP displacements of the K93 colours obtained by the empirical temperature calibrations and by using the empirical colours of RRab stars, it can be also concluded that the differences between the chemical compositions of population I and population II objects cannot have a larger effect on the $B-V$, $V-I$ and $V-K$ colours than $0.01-0.02$ mag.

The fact that we use the magnitude mean colours of the variables and these colours can be brought into agreement with the K93' data, also means that they have to correspond to the static values quite well. A similar conclusion was drawn by Sandage (1993b), too. However, this is in contrast with the result of Bono, Caputo and Stellingwerf (1995) who claimed that 'over the whole instability strip neither the blue nor the infrared mean colours are representative of the equivalent static value' and 'the discrepancy between the synthetic mean colours and the static colour is a function of the amplitude of the light curve'. The most probable reason why our empirical data and the hydrodynamical model calculations lead to such different conclusions is that the parameters (L, $T_{\rm eff}$, M and [Fe/H]) of the models investigated by Bono, Caputo and Stellingwerf (1995) were not chosen according to the general relations of the basic physical parameters of RRab stars (see Sect. 4). Consequently, their sample of model stars cannot be regarded as an equivalent of the empirical sample of the variables. Some defects in the treatment of convection in the hydrodynamical models should also lead to disagreement.

3. Physical parameters of RRab stars

3.1. Temperature and luminosity

After showing the consistency of the parameters obtained from light curve characteristics with their counterparts given in the Kurucz tables, in the following we determine $\log T_{\rm eff}$ and $\log L/L_{\odot}$ by using the empirical parameters of the variables calculated from Eqs. (1)-(6) and formulae defined on the K93' tables. First we express the $\log T_{\rm eff}$ and the bolometric correction (BC) values of the selected models with the colours and the metallicity. Now we use sample a of the Kurucz models. The application of formulae defined by sample b would yield, however, negligible differences. The calculated $\log T_{\rm eff}$ and BC values would remain practically the same, the differences have 0.0003 and 0.0025 rms scatters, respectively. According to the recommendation of IAU Commission 25 (Roger Cayrel, 1997, XXIII General Assembly, Kyoto), when calculating $\log L/L_{\odot}$, $M_{bol_{\odot}} = 4.75$ mag and BC$_{\odot} = -0.08$ mag values are used. To match the bolometric corrections of the Kurucz grid to the BC scale defined by the above BC$_{\odot}$ value, the K93 bolometric corrections are shifted by 0.114 mag (BC$_{\odot}$(K93)$= -0.194$ mag).

Table 2 lists the standard deviations of the best linear and quadratic fits of $\log T_{\rm eff}$ and BC of the K93' tables with the colours and metallicity. N_t denotes the number of terms the given formula contains. Again, although the unbiased estimations of the standard deviations of the quadratic formulae are smaller, we accept the two parameter linear fits. These formulae, considering the other sources of uncertainties, can be regarded as sufficiently correct. The fact, that the best quadratic fits with two terms are the linear formulae both for $\log T_{\rm eff}$ and BC, shows the stability of the two-parameter linear solutions. The 0.0018 and 0.0100 standard deviations of these formulae correspond to ± 27 K and ± 0.004 errors of the calculated $T_{\rm eff}$ and $\log L$ values, respectively. Since the inherent uncertainties in the Kurucz models (see Castelli, Gratton and Kurucz 1997) are of similar order, these accuracies are acceptable. The two-parameter linear expressions of $\log T_{\rm eff}$ and BC are as follows:

$$\log T_{\rm eff} = 3.9291 - 0.1112(V-K) - 0.0032[{\rm Fe/H}] \quad (11)$$

$${\rm BC} = 0.2033 - 0.1736(V-K) + 0.0395[{\rm Fe/H}]. \quad (12)$$

Using these formulae and Eqs. (1),(2),(5) it is already possible to calculate the mean effective temperature and luminosity (referring to the intensity averaged value) of any variable, if the light curve parameters are known.

Although both of the above formulae use $V-K$ and [Fe/H], the ones which utilize $V-I$ or $(V-I)^2$ and [Fe/H] have similar accuracy. The strong dependence of $\log T_{\rm eff}$ on $V-I$ can also be seen in Table 2, as this is the colour the best one-parameter formulae use. Table 2 also shows that the more precise expressions include both colour and [Fe/H] terms. Therefore, the above formulae are more correct for RR Lyrae stars than those which exclusively use the colour dependence of the temperature and/or the metallicity dependence of BC (see e.g., Clementini at al. 1995; Gratton Carretta and Castelli 1996; Sandage 1993b; Fernley 1993). When comparing Eqs. (11) and (12) with other similar formulae it is important to remember that our results are valid only for stars in very narrow parameter ranges. However, it is these limitations that make it possible to reach the required accuracy already with simple linear formulae.

It is also important to estimate the errors of the such obtained temperature and luminosity values. The formal errors of the intensity averaged M_V and [Fe/H] can be calculated according to the error formulae given in KJ96 and JK. These formulae yield $\sigma_{M_V} = 0.014$ mag and $\sigma_{\rm [Fe/H]} = 0.14$ dex, if the typical errors of the A_1 and φ_{31} Fourier parameters are estimated to be about 0.01 mag and 0.10 rad, respectively. If the accuracy of the $V-K$ colour is 0.028 mag, as argued in Sect. 2.3, then the formal errors of the calculated $\log T_{\rm eff}$ and BC are 0.0031 and 0.0075, respectively. Concerning $\log L$, even the unrealistic case of full correlation between the errors of M_V and BC gives only 0.009 uncertainty, as an upper limit. Since all these errors strongly depend on the the accuracy of the Fourier parameters, using better quality light curves would decrease them significantly. E.g., if $\sigma_{A_1} = 0.005$ mag and $\sigma_{\varphi_{31}} = 0.05$ rad then $\sigma_{\rm [Fe/H]} = 0.075$ dex, $\sigma_{M_V} = 0.009$ mag, $\sigma_{\rm BC} = 0.005$ mag, $\sigma_{\log T_{\rm eff}} = 0.003$ and $\sigma_{\log L} = 0.006$. The accuracy of the temperature depends mostly on that of the calculated $V-K$. We have shown, however, that this basically depends on the correctness of the expression of $B-V$ (Eq. (3)). Consequently, further improvement of the accuracy of the temperature estimation can be achieved if a more precise formula is derived to calculate $B-V$. With the inclusion of new, good quality multicolour cluster observations this will become possible probably in the near future.

3.2. Mean surface gravity and mass

If the $\log g$ values of the K93' tables could also be expressed with the other parameters, then it would be possible to determine the surface gravity of the stars similarly as we determine $\log T_{\rm eff}$ and BC. Unfortunately, all attempts has failed to obtain such an expression of $\log g$ which would be accurate enough, even if the $U-B$ colour is also involved. Most probably, this is not because of the sparse sampling of the models in $\log g$, but is the consequence of the discontinuities in the K93 synthetic colours. The recent discussion of the ATLAS9 code given by Castelli, Gratton and Kurucz (1997) points to the fact that the treatment of convection used in the model calculations led to such a short-

Table 2. Fitting accuracy of the formulae which express the effective temperature and the bolometric correction of the K93' tables with the colours and the metallicity

	N_t	linear fits		quadratic fits	
		st. dev.	terms	st. dev.	terms
$\log T_{\rm eff}$					
	1	0.0030	$V-I$	0.0024	$(V-I)^2$
	2	0.0018	$V-K$, [Fe/H]	0.0018	$V-K$, [Fe/H]
	3	0.0017	$V-I$, $V-K$, [Fe/H]	0.0017	$V-I$, $V-K$, [Fe/H]
	4	0.0016	$B-V$, $V-I$, $V-K$, [Fe/H]	0.0014	$V-I$, $V-K$, [Fe/H], $[\text{Fe/H}]^2$
	5			0.0013	...
	.			...	...
	14			0.0009	...
BC					
	1	0.0363	$V-K$	0.0323	$(B-V)$[Fe/H]
	2	0.0100	$V-K$, [Fe/H]	0.0100	$V-K$, [Fe/H]
	3	0.0094	$B-V$, $V-I$, [Fe/H]	0.0076	[Fe/H], $(V-I)^2$, $[\text{Fe/H}]^2$
	4	0.0093	$B-V$, $V-I$, $V-K$, [Fe/H]	0.0068	$B-V$, [Fe/H], $[\text{Fe/H}]^2$, $(B-V)$[Fe/H]
	5			0.0065	...
	.			...	...
	14			0.0043	...

coming. We think that this is the explanation of the discontinuity indicated by an arrow in the first panel of Fig. 1, too. In this plot the vertically grouped points correspond to the five different temperature model grids. The arrow points to the 6750 K models showing that the $B-V$ synthetic colours and their predicted values using $V-I$ of the $\log g = 2.5$ and $\log g = 3.0$ models differ much more significantly at this temperature than at any of the others. This effect can also be seen in most of the panels of Fig. 1. Probably, at present these discontinuities limit the accuracy of all the formulae defined on the K93 tables and they may also be responsible for artificial nonlinearities. This possibility supports our preference for using linear formulae.

Alternatively, we determine the pulsational mass of the stars and we calculate their mean surface gravity from:

$$\log g = \log M - \log L + 4 \log T_{\rm eff}. \quad (13)$$

In Eq. (13) all the quantities are expressed in solar units. In the following this simplified notation means solar units exclusively for the luminosity and mass.

To obtain the pulsational mass, the fitting formula of the fundamental period given by Kovács and Buchler (1994, Eq. (1)) is used, after expressing its coefficients with cubic functions of the metallicity. The so obtained formula, when using parameters ($P, T_{\rm eff}, L$ and [Fe/H]) in such combinations as the RRab sample defines, can be substituted with the following linear equation:

$$\begin{aligned}\log P \ = \ & 11.904 - 0.570 \log M + 0.842 \log L - \\ & -3.575 \log T_{\rm eff} + 0.021[\text{Fe/H}], \end{aligned} \quad (14)$$

with $\sigma_{\log P} = 0.0011$ accuracy.

The most important difference between Eq. (14) and the generally used van Albada and Baker (1971) period formula is the extra [Fe/H] term. In a recent paper Bono, Incerpi and Marconi (1996) also showed that the fundamental period depends on the metallicity, too. According to their result the nonlinear periods strictly increase with metallicity in agreement with the positive coefficient of [Fe/H] in Eq. (14).

To test the accuracy of the mass determination, noisy data and formulae are generated taking into account the uncertainty of the $V-K$ and [Fe/H] values and also the errors of the coefficients of the BC and $\log T_{\rm eff}$ formulae. The uncertainty of the period fitting formulae is simulated by taking also some artificial noise of the periods. Using the 0.0003 day correct period fitting formula of Kovács and Buchler, or its linear approximation (Eq. (14)), the rms errors of the calculated masses for individual stars are 0.026 and $0.030\ M_\odot$, respectively.

Using Eq. (13), now we can determine the mean surface gravity of the variables in the sample. All the calculated $\log g$ values are within the $2.6-3.0$ interval, validating our original selection of the models. This result is not the consequence of our preliminary restriction in the model parameters. If the BC and $T_{\rm eff}$ formulae are defined by models of a broader, $2.0 \le \log g \le 3.5$, surface gravity range, then it has practically no effect on the resultant $\log g$ values.

The $\log g$ of RRab stars calculated in this way – which is in fact some combination of the period and the Fourier parameters – can be expressed as a simple linear function of $\log P$ with 0.004 accuracy:

$$\log g = 2.473 - 1.226 \log P. \quad (15)$$

Eq. (15) is in good agreement with the general period – gravity relation of radially pulsating variables (Fernie 1995). Without interpreting this relation, we emphasize its usefulness e.g., when choosing model atmosphere parameters.

4. Relations among the physical parameters

We have shown that it is possible to calculate L, $T_{\rm eff}$, M and [Fe/H] from the light curve parameters with satisfying accuracy.

Earlier similar attempts to derive the physical parameters for larger samples of the variables used multicolour photometric observations and the Kurucz (1979) models (e.g., Lub 1987). The present data define all the overall empirical relations among the fundamental physical parameters well, thus we can also give the metallicity dependence of the physical parameters precisely. This was previously defined only by using much more limited samples of RRab stars (e.g., Sandage 1993b).

When comparing observations with evolutionary and pulsational results it has crucial importance to know the physical parameters of the variables and their global interdependences correctly. Right conclusions can only be drawn if such problems as selecting models with the same parameter combinations as real stars have, or selecting stars which have the same temperature, are properly solved. Any systematic errors in these selections may lead to artificial disagreement. E.g., the interpretation of the Sandage period shift has been thought to be in conflict with evolutionary models for long, as it needs the quantitative measures of the metallicity dependence of the luminosity, temperature and the mass of the stars and also the right selection of the same temperature objects.

In the following subsections we give some of the important linear relations of the physical parameters as defined by the 272 RRab stars. Those formulae which also contain the temperature correspond to the respective relations at constant temperatures, too. The standard deviations of the regressions indicate the significance of the correlation of the concerned parameters.

Since only linear fits are examined, our conclusions are not affected by the possible errors of the zero points. This would only result in some changes of the constants of the formulae below but the coefficients what we compare with evolutionary results would remain the same. Considering the zero points we recall that the luminosity calibration relies on the Baade-Wesselink magnitudes, while the temperature scale is defined by the Kurucz models with the assumptions that the K93 $V-K$ colours are correct and that they equal to the empirically determined values. The relative values of the different empirical data purely depend on the assumption that the physical parameters of the variables are unique functions of their light curve parameters. In our previous papers we proved that regarding the intrinsic colours, the absolute magnitudes and the metallicity, this was true. Because it was also shown (Sect. 2) that the scales of those parameters of the K93 tables what we used are possibly correct, therefore the calculated fundamental physical parameters cannot have major systematic errors, besides zero point ambiguities. Consequently, the slopes of the various regressions can be defined very precisely, and are determined independently from any previous estimations.

4.1. The metallicity dependence of the physical parameters

4.1.1. Luminosity – [Fe/H]

The comparison of the well known luminosity – metallicity relation predicted by horizontal branch (HB) calculations with the observations has great importance in checking the correctness of the models and in determining the evolutionary state of the variables. Fitting the luminosity of the stars with [Fe/H], and in order to see the variation at fixed temperature with [Fe/H] and $\log T_{\rm eff}$, yields the following formulae:

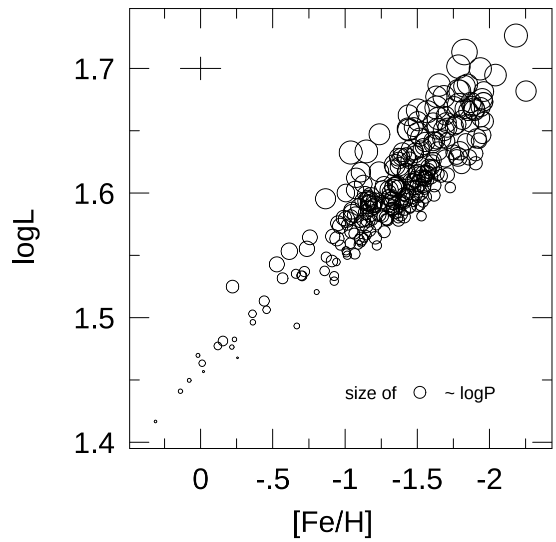

Fig. 3. Log$L$ *vs.* [Fe/H] of 272 RRab stars. The size of the circles is scaled by $\log P$, i.e. the smaller circles denote stars with shorter periods. The estimated $\pm 1\sigma$ accuracies of the data are shown in the upper left corner. The variation of the luminosity and the period of the same metallicity stars indicates evolutionary effects.

$$\log L \;=\; 1.464 - 0.106[\mathrm{Fe/H}] \qquad (16)$$

$$\log L \;=\; 10.260 - 0.062[\mathrm{Fe/H}] - 2.294\log T_{\rm eff}. \qquad (17)$$

The standard deviations of the above regressions are 0.020 and 0.013, respectively.

For comparison, the metallicity dependence of the luminosity of zero-age horizontal branch (ZAHB) models ($\log T_{\rm eff} = 3.85$, $Y_{\rm MS} = 0.25$) has $\frac{\mathrm{d}\log L}{\mathrm{d}\log Z} = -0.07$ slope according to the calculations of Sweigart, Renzini and Tornambè (1987, hereafter SRT). Dorman (1993) obtained similar, -0.07 and -0.08, values ($\log T_{\rm eff} = 3.84, Y_{\rm MS} = 0.24$) using oxygen enhanced and solar-scaled composition models, respectively. Synthetic horizontal branch (SHB) simulations, which also considered the evolution away from the ZAHB, led to slightly different values. At $\log T_{\rm eff} = 3.83$, $\frac{\mathrm{d}\log L}{\mathrm{d[Fe/H]}}$ was between -0.07 and -0.09 for α-enhanced ($Y_{\rm MS} = 0.23$) models while it was between -0.10 and -0.15 for the solar-scaled ($Y_{\rm MS} = 0.20$) ones (Catelan 1993). From Eq. (17) it follows that at constant temperature $\frac{\mathrm{d}\log L}{\mathrm{d[Fe/H]}} = -0.062$. If we compare the above theoretical results with this empirical value, the agreement is reassuring, indeed. It can be also concluded that the SHB simulations slightly overestimate the luminosity variation and that the predictions of evolutionary calculations using α-enhanced models are in better agreement with the empirically determined coefficient of [Fe/H] than in the case of the solar-scaled ones.

In KJ96 it was shown that besides [Fe/H], the absolute magnitude of RRab stars also depends on the period (or on φ_{31}). The same is true for the luminosity, while the accuracy of Eq. (16)

is 0.020, the

$$\log L = 1.611 - 0.067[\mathrm{Fe/H}] + 0.384 \log P \qquad (18)$$

relation is valid with 0.009 standard deviation. To see the variation at constant temperature again, we fit $\log L$ with [Fe/H], $\log P$ and $\log T_{\mathrm{eff}}$:

$$\log L = -9.499 - 0.082[\mathrm{Fe/H}] + 0.789 \log P + 2.938 \log T_{\mathrm{eff}}, \qquad \sigma = 0.007. \qquad (19)$$

According to this equation there are also stars in the sample which have already evolved away from the ZAHB. Eq. (19) shows that at fixed temperature and metallicity, different luminosity objects exist, the more luminous they are the longer periods they have. These conclusions are in perfect agreement with both horizontal branch evolutionary models and with the requirements of the period fitting formula, none of them being applied in any respect in the course of the derivation of Eq. (19).

The luminosity – metallicity relation as defined by the empirical data and the variation of the period with luminosity at a given metallicity can be seen in Fig. 3.

4.1.2. Temperature $-$ [Fe/H]

One of the most important consequence of the determination of the physiacal parameters for a large sample of RRab stars is that the luminosity dependence of the temperature can be clearly shown:

$$\log T_{\mathrm{eff}} = 4.119 - 0.194 \log L, \qquad \sigma = 0.0046. \qquad (20)$$

To see the significance of this relation the HR diagram of the RRab stars is shown in Fig. 4. Comparing the morphology of the fundamental mode instability strip as defined by the empirical data with hydrodynamical model calculation results, it can be found that neither the inclination nor the width of the instability strips agrees (see also Jurcsik 1997). However, the theoretical edges of the instability strip (e.g., Bono et al. 1995; 1997) are determined by using models with parameter combinations different from what the evolutionary model calculations would demand. Therefore, it is possible that if the physical parameters of the models are chosen according to the $\log L - [\mathrm{Fe/H}]$ and $\log M - [\mathrm{Fe/H}]$ relations defined by horizontal branch models, then a similar luminosity dependence of the temperature along the theoretically defined instability strip will also be found. Another source of the discrepancy may arise from the defects of the convection treatment that the hydrodynamical models apply.

It was already mentioned in Sect. 2.2 that a $\log T_{\mathrm{eff}} - [\mathrm{Fe/H}]$ relation explains why the observed $B-V$ range is much smaller than all the possible Kurucz models in the parameter ranges of RRab stars assign. The strong correlation found between $\log L$ and $\log T_{\mathrm{eff}}$ (Eq. (20)), together with the empirically known and theoretically explained $\log L - [\mathrm{Fe/H}]$ relation (Eq. (16)), predicts correlation between $\log T_{\mathrm{eff}}$ and [Fe/H], too. Linear regression between the temperature and the metallicity yields the following formula:

$$\log T_{\mathrm{eff}} = 3.834 + 0.019[\mathrm{Fe/H}], \qquad \sigma = 0.0068. \qquad (21)$$

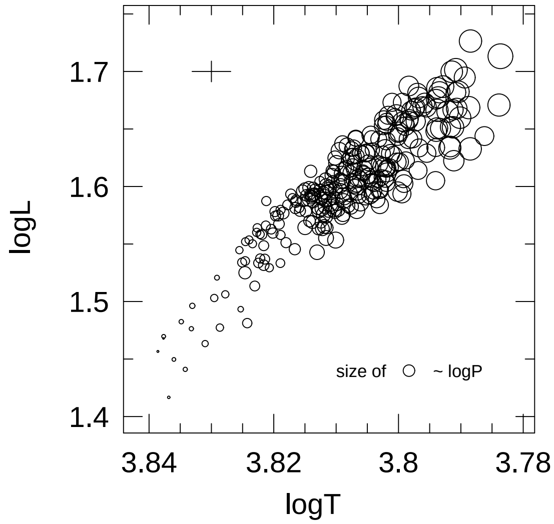

Fig. 4. Log$L$ *vs.* $\log T_{\mathrm{eff}}$. The estimated $\pm 1\sigma$ accuracies of the data are shown in the upper left corner. The fundamental mode instability strip defined by this sample of variables is very narrow and has considerable inclination, too. The strong dependence of the temperature on the luminosity has to be taken into account when interpreting the period shift of RR Lyraes.

This relation is shown in Fig. 5.

In Fig. 1 Kurucz models with temperatures satisfying the condition given in Eq. (21) within $\pm 3\sigma$ were denoted by filled circles. The $B-V$ range of this subset of the models agrees with the observed range much better than in the case of the originally selected models (sample a), giving a further proof of the correctness of our results.

The actual value of the coefficient of [Fe/H] in Eq. (21) has great importance in the explanation of the period shift of RR Lyrae stars. Sandage (1993a) obtained 0.012 and 0.018 values of this slope at the blue and red edge of the fundamental instability strip by using the data of Blanco (1992). Considering that Sandage's result was based on other methods and data the somewhat different slope we have obtained is not surprising.

4.1.3. Mass $-$ [Fe/H]

The horizontal branch models also indicate dependence of the stellar mass on the metallicity within the instability strip (see Fig. 6 in Castellani, Chieffi and Pulone 1991). Their quantitative estimation on the slope of this relation for ZAHB models was $\frac{\mathrm{d}\log M}{\mathrm{d}\log Z} = -0.058$. The ZAHB models of SRT led to values between -0.06 and -0.11 at $\log T_{\mathrm{eff}} = 3.85$, whereas Sandage (1993b), reading $\frac{\mathrm{d}\log M}{\mathrm{d}\log Z}$ at the blue edge of the fundamental mode instability strip, got -0.059 using the oxygen enhanced ZAHB models of Dorman (1992). The SHB simulations resulted in slightly less negative ($-0.045 \div -0.052$) values (Sandage 1993b). For comparison, the $\log M - [\mathrm{Fe/H}]$ relation defined by the RR Lyrae sample is:

$$\log M = -0.328 - 0.062[\mathrm{Fe/H}], \qquad \sigma = 0.019. \qquad (22)$$

The agreement with the HB models is very good again.

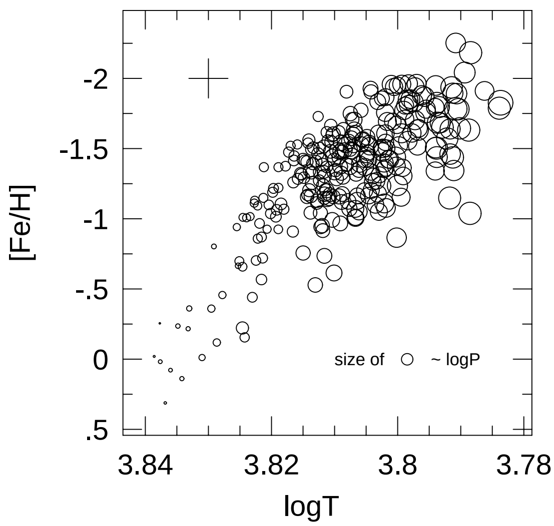

Fig. 5. [Fe/H] *vs.* $\log T_{\rm eff}$. This corellation explains the observed narrow range in $B-V$. It can be also seen that, although the period is increasing with decreasing metallicity, at constant temperature the sign of the period variation is just the opposite.

4.2. Explanation of the Sandage period shift

The above relations make it possible to separate the components of the observed period shift phenomenon, i.e., they can explain why and to what extent the average period of RRab stars increases with decreasing metallicity. The concentrated efforts to solve this problem (e.g., LDZ; Sandage 1993b; Fernley 1993; Catelan 1994) led to the following main conclusions. Firstly, evolutionary effects should play an important role especially in the case of the most metal- deficient objects. Secondly, satisfying explanation of the observations can only be reached if the different metallicity stars on the average are at different temperatures, consequently, the effect of temperature shift on the period variation has to be taken into account, too.

The global period shift defined by the 272 stars is $\frac{\Delta \log P}{\Delta [\mathrm{Fe/H}]} = -0.10$. It is slightly smaller than the usually accepted -0.12 value (Sandage 1993a) due to differences partly in the metallicity scales, partly in the selection of the samples of the stars by which the period shift is defined. While Sandage determined the period shift at the blue edge of the fundamental mode instability strip on the metallicity scale of Butler (1975), we determine the average period-metallicity relation of a large sample of the variables on a metallicity scale, which is consistently valid for both field and cluster variables as defined in Jurcsik (1995).

The linear expression of the fundamental period (Eq. (14)) leads to the following interpretation of the period shift phenomenon:

$$\frac{\mathrm{d}\log P}{\mathrm{d}[\mathrm{Fe/H}]} = -0.570\frac{\mathrm{d}\log M}{\mathrm{d}[\mathrm{Fe/H}]} + 0.842\frac{\mathrm{d}\log L}{\mathrm{d}[\mathrm{Fe/H}]} -3.575\frac{\mathrm{d}\log T_{\rm eff}}{\mathrm{d}[\mathrm{Fe/H}]} + 0.021. \quad (23)$$

From Eqs. (16),(21) and (22) it follows that $\frac{\mathrm{d}\log M}{\mathrm{d}[\mathrm{Fe/H}]} = -0.062$, $\frac{\mathrm{d}\log L}{\mathrm{d}[\mathrm{Fe/H}]} = -0.106$ and $\frac{\mathrm{d}\log T_{\rm eff}}{\mathrm{d}[\mathrm{Fe/H}]} = 0.019$. Thus the contributions of the variation of mass, luminosity, temperature and metallicity to the observed -0.10 period shift are 0.04, -0.09, -0.07 and 0.02, respectively. It can be seen that the effect of the displacement of the instability strip towards higher temperatures with increasing metallicity has really crucial importance in understanding the observed period shift. It is also worth noting, that this result changes only slightly if the original van Albada and Baker (1971) formula is used instead of Eq. (14) to interpret the period shift of RR Lyrae stars.

The size of the circles refer to the length of the period in Fig. 5. The period clearly increases with decreasing metallicity while at constant temperature period variation in the opposite direction can be seen. Fitting the period of the RRab stars with [Fe/H] and $\log T_{\rm eff}$ this variation can be quantitatively measured. The coefficient of [Fe/H] in this fit is 0.026, thus the period shift at constant temperature defined by our data is very small and of the opposite sign as the global period shift of RR Lyraes.

The comparison of the observations with model predictions needs the correct definition of the instability strip to select the corresponding models or the correct selection of the same temperature stars. In general the latter solution is applied. Considering evolutionary models all the efforts to obtain significant (negative) period shift at constant temperature has failed. The ZAHB models resulted in only about $\frac{\mathrm{d}\log P}{\mathrm{d}[\mathrm{Fe/H}]} = -0.003$ according to the calculations of SRT, whereas SHB simulations led to only slightly more negative $-0.004 \div -0.070$ values (SRT; LDZ; Lee 1990). Comparing these results with the corresponding 0.026 value obtained from the empirical data the following conclusions can be drawn. The physical parameter combinations of RRab stars as determined in the present paper yield period shift at constant temperature which agrees with the results of the ZAHB models within the estimated uncertainties. It seems, however, that the SHB simulations overestimate the period shift. We think that since the aim was to reach as negative value as possible in order to explain the observations – which, however, had systematic errors – the SHB calculations might favour to those simulations which gave larger (negative) period shift values.

4.3. Hints of evolutionary effects

In Sect. 4.2.1, it was already shown that the spread in the luminosity at a given metallicity and temperature can be explained as a consequence of the different evolutionary states of the variables.

It is also possible to test this result by checking the interdependence of the physical parameters. If there were no difference in the evolutionary stages of the stars, i.e., they all were zero-age horizontal branch objects, then both the luminosity and the temperature would have to be unique function of the mass and the metallicity. On the contrary, if the same mass and metallicity objects are not located at the ZAHB position on the HR diagram but along the HB evolutionary tracks then no accurate $L(M, [\mathrm{Fe/H}])$ and $T_{\rm eff}(M, [\mathrm{Fe/H}])$ relations are expected to be found. Therefore, by checking the accuracy of these relations, we also check the spread of the evolutionary state of the stars.

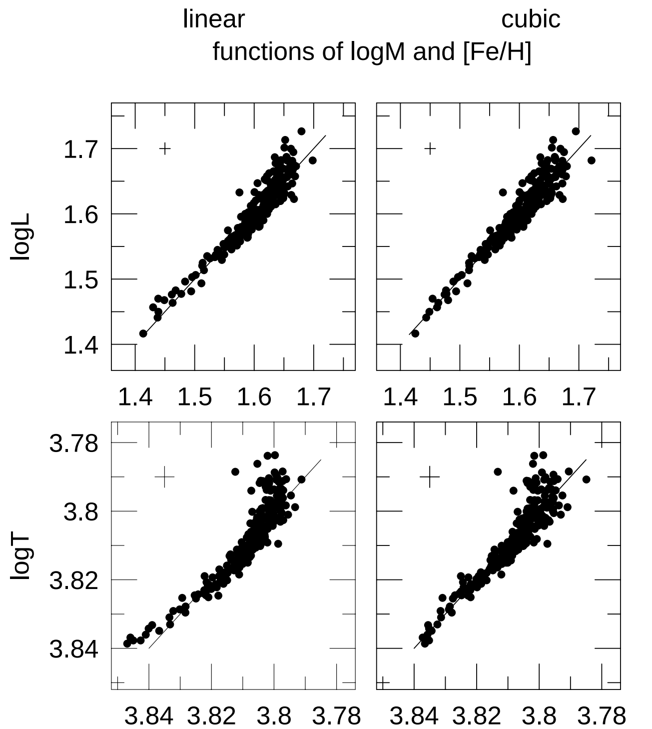

Fig. 6. Linear (left panels) and cubic (right panels) solutions to predict the luminosity and temperature of RRab stars from their mass and metallicity. The increase of the scatter of the plots towards the cooler temperature and larger luminosity stars can be interpreted as evidence of evolutionary effects. The estimated $\pm 1\sigma$ errors of the $\log L$ and $\log T$ values are shown in the upper left corners.

In Fig. 6, the linear and cubic least squares fits of $\log T_{\rm eff}$ and $\log L$ with $\log M$ and [Fe/H] are plotted. The plots show that in the case of the lower temperature and more luminous stars, which are also the most metal-deficient ones, the differences between the 'observed' and predicted values are larger than the estimated errors of the parameters would explain. The increasing dispersion of these plots can be interpreted as sign of evolutionary effects. The figures show that there are stars at the metal-poor end which have already evolved away from the ZAHB. Similar conclusion was drawn by LDZ; they suggested that evolutionary effects have to be considered when interpreting Oosterhoff II clusters. Fernley (1993) also found that a large number of the most metal-poor field stars are highly evolved. The extensive study of SHB simulations given by Caputo et al. (1993) pointed to the fact that the observable effects of post-ZAHB evolution in globular clusters also depend on the HB type, besides the metallicity. According to their models only the bluest HB type, most metal-poor clusters are expected to show evolutionary effects. On the contrary, Sandage (1990) determined the vertical heights of the horizontal branch of 12 clusters and came to the conclusion that evolutionary effects are more pronounced in the less metal-poor clusters. The interpretation of Fig. 6, however, definitely favours to find evolved objects amongst the most metal-poor stars.

It should be also noted that $T_{\rm eff}$, L, M and [Fe/H] are the most important but not the only parameters determining the positions of the stars in the HR diagram. Therefore, it is also possible that besides evolution, subtle differences in chemical composition (Y, α-elements), core mass, rotation, etc., have also some influence on the scatters seen in Fig. 6.

5. Conclusions

In the present paper we have shown that the colours and metallicity of RRab stars calculated from light curve parameters can be brought into perfect agreement with the Kurucz model atmosphere results if the zero points of the $B-V$ and $V-I$ colours are corrected. Consequently, using the Kurucz tables and also the pulsational formula of the fundamental mode period, it became possible to calculate the luminosity, temperature, mass and metallicity of any variable if the light curve parameters are known. As a result, the fundamental physical parameters of a large sample of RRab stars can be determined by using the same technique.

The comparison of the related changes of the above parameters as defined by a large RRab sample with the predictions of zero-age horizontal branch models show very good quantitative agreement. The SHB simulations, however, slightly overestimate the absolute values of the variation of the luminosity and the period with metallicity. Since indications of evolutionary effects were also shown, the overestimations are probably due to the uncertainties of the SHB calculations and do not mean that all the RRab stars were close to the ZAHB position.

There are also arguments that the absolute values of the luminosity and the mass of RR Lyrae stars if calibrated to the Baade-Wesselink data are in conflict with some observational and theoretical results (Cacciari and Bruzzi 1993; Fernley 1993; Castellani and De Santis 1994; Walker 1995). The confrontation between the empirically determined physical parameters of RR Lyrae stars and evolutionary and pulsational model calculations in more details, may also help to solve the very important problem of the zero points.

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