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ASTRONOMY AND ASTROPHYSICS

On steady shell formation in stellar atmospheres

I. Analytical 2-D calculations under an optically thin thermo-radiative mechanism

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Abstract. In this work we study a possible physical mechanism which is able to create steady state shells around astrophysical objects. This mechanism is *thermo - radiative* and it is applied to *optically thin* stellar atmospheres. An outflow deceleration region separates the rest of the stellar envelope into inner and outer acceleration regions. The shell is formed in the supersonic region of the outflow. Studying the dynamic nature of the shells, it is found that the shell distance depends on the thin spectral line opacity and the number of thin lines. The shape of the shell depends on the differential rotation of the fluid. It is found that the mass concentration may look like a shell or a double blob over the poles of the central object. The present *thermo - radiative* mechanism is based on the analytical 2 – D, hydrodynamic solution of Kakouris & Moussas (1997) and the analysis of Chen & Marlborough (1994) for the thin radiative force as well as the work of Lamers (1986) for the acceleration mechanism in the envelope of *P Cygni*. The shell characteristics are deduced through applications to superluminous early type supergiants. Applications to late type supergiants as well as *P Cygni* are shown. It is found that superluminous supergiants are expected to form steady shells and an example for *P Cygni* illustrates such a shell at ~ 5 stellar radii. The present three - zone envelope for *P Cygni* resembles that of Nugis, Kolka & Luud (1979) but uses different driving outflow mechanisms. The parameters used in this article are in accordance with previous works and several observational data are reproduced successfully by the model.

Key words: hydrodynamics – methods: analytical – stars: atmospheres – supergiants – stars: mass-loss

1. Introduction

Mass loss from early type stars is thought to be of radiative origin. In many cases, the spectral lines of such stars indicate emission and form P Cygni profiles. This phenomenon is found in main-sequence stars and giants (Be stars) as well as supergiants (B[e] stars) (Doazan 1982). Moreover, IUE observations have shown very clearly that ”shell” or ”blob” characteristics appear frequently in spectral analyses of early type stars (e.g. Cassatella et al. 1979, Lamers et al. 1985, Lamers et al. 1988, Scuderi et al. 1994). The effects of these structures on the stellar spectrum have been reported by Lamers (1994). These shells are considered either stable or variable and in many cases the respective stars exhibit variability in spectral features. The formation of shells, shown in the spectrum, is also known for late-type supergiants (Weymann 1963, Bernat 1977). In these stars the radiation pressure effect is also important.

In this work we study the shell formation dynamics in a stellar atmosphere as a steady state hydrodynamic (HD) procedure. In fact, we apply the analytical 2-D solutions for thermally and radiatively driven stellar winds of Kakouris & Moussas (1997) assuming the stellar atmosphere optically thin. In addition, we used the analyses of Chen & Marlborough (1994) (hereafter CM) and Lamers (1986) for the radiative force based on optically thin spectral lines.

Kakouris & Moussas stellar wind solutions are *thermo-radiative* (i.e. thermally and radiatively driven). In Kakouris & Moussas (1997) (hereafter Paper II) the thermally driven solutions of Kakouris & Moussas (1996) (hereafter Paper I) were generalized to incorporate the differential rotation of the fluid and the optically thin radiative force (also Kakouris 1997). These solutions are global 2-D and self-consistent to HD, and θ- self similar (Tsinganos & Sauty 1994).

The rest of this article is organized as follows: In Sect. 2 we introduce physical considerations for a HD steady state shell. In Sect. 3 we present briefly both thermal and radiative mechanisms which drive stellar winds and the Paper II model which is applied. In Sect. 4 we present solutions with shells for a hypothetical early type superluminous object, for a hypothetical late-type supergiant and for hypergiant P Cygni. Finally, in Sect. 5 we discuss the results and physical characteristics of the model.

2. Steady shell: physical considerations

What we call a *steady shell* in this work is a concentration of the expanding stellar wind plasma. This is also called a *density fluctuation*. The concentration of fluid is formed in the stellar envelope and at a certain distance R_{shell}. The simplest shape for

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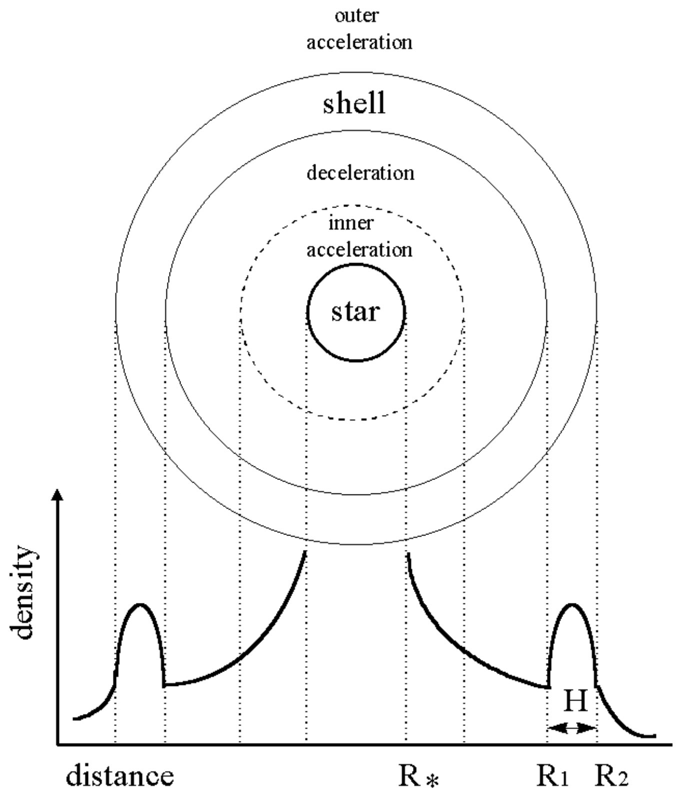

Fig. 1. Schematic view of a spherical steady shell

this shell is spherical and the width of it, H, is determined by the inner R_1 and outer R_2 radii ($H = R_2 - R_1$). The fluid density in the shell region determines the shell mass M_{shell}. The fluid passes through the shell region as it expands and also rotates. A characteristic time period is the time in which a certain fluid element passes through the shell.

Using the single fluid HD theory, it is obvious that in a situation like this there is evidence of deceleration in the stellar envelope. If the deceleration is efficiently strong the conservation of mass leads to an increase in density. So, the fluid must be initially accelerated from low velocities at the stellar surface to higher, then it decelerates and then it accelerates again and escapes to infinitely large distances from the central object. In this way, three domains in the stellar envelope exist: an *inner acceleration region*, a *deceleration region* and an *outer acceleration region*. A schematic view of this case is shown in Fig. 1. The deceleration is due to the gravitational attraction of the central object. The outer acceleration region is probably due to radiative forces since no other mechanism is known to drive the outflow at very large distances. Under discussion is the mechanism that drives the outflow in the inner acceleration region. In this work we adopt a thermal mechanism to drive the outflow in the inner part with a smaller radiative force contributing. We expect the flow in the shell region to be supersonic because in the subsonic region the acceleration must be very high and there is no evidence for deceleration there. The previously described situation is going to be modelled in the next sections. This kind of shell is qualitatively different from dusty shells, from time-dependent travelling shells or from shell-like structures formed by interactions of supernovae progenitor winds with the ambient circumstellar gas. In the first kind the interaction between plasma and grains has to be evaluated while in the other two time-dependent HD must be employed (especially when the structure is formed by an explosion in the star's interior).

3. The thermo-radiative mechanism

Mass loss is a dynamic phenomenon of major importance which is observed in stars of all types (Cassinelli 1979). The first theoretical 1-D description of this phenomenon was given by Parker (1958) who had predicted the *solar wind* as a thermally driven mass outflow from the Sun. Thermally driven winds were described self-consistently (i.e. by solving the HD equations analytically or exactly) in 2-D by Tsinganos & Vlastou (1988), Tsinganos & Sauty (1992), Lima & Priest (1993), Kakouris & Moussas (1996).

On the other hand, the thermal mechanism is considered unimportant in winds from early type stars. From the early times of radiatively driven wind models it has been pointed out that Parker's model fails to describe mass loss from early type stars because it needs an enormously hot corona ($T > 10^7\ K$). In this case, strong high energy emission and/or absence of some ions should appear in the stellar spectrum which is not consistent with observations (e.g. Weymann 1963, Lucy & Solomon 1970). This situation is discussed by Underhill (1982) (p. 242) where the author distinguishes between "truly" coronal region ($T > 10^6\ K$) and "corona-like" layer ($T < 10^6\ K$) for an area above the photosphere.

The existence of a hot corona ($T \geq 10^5\ K$) in early type stars and a thermal mechanism plus a radiative one (based on continuum absorption), has been proposed by Hearn (1975a,b) in order to model the atmosphere of ζ *Ori* which exhibits shell characteristics. The author (1975b) calculated the energy loss of a stellar corona showing that for a given surface pressure there is a surface (coronal) temperature that minimizes the energy loss. This coronal temperature is about a factor of 10 larger than the effective temperature for early type stars with high mass loss rates. Relevant works, based on observations (especially in heavy ion spectral lines with high ionization), followed in the next years investigating the possibility of a thin corona-like layer in OB stars. Lamers & Snow (1978) suggested $7\,10^4\ K \leq T \leq 4\,10^5\ K$ using UV satellite observations. Cassinelli, Olson & Stalio (1978) calculated the Hα profile for ζ *Ori* concluding that the possible hot corona must be very thin ($0.1R_*$). Olson (1978) used data for ζ *Pup* concluding that coronal temperatures in the range $(2-50)\ 10^5 K$ are possible and the combination of UV and Hα observations are necessary. However, Cassinelli & Olson (1979) found that the possible corona must be very thin. However, Hubeny et al. (1985) discussed the spectroscopic diagnostics of superionization in UV spectra of B stars with the use of CIV, SiIV, NV lines noting that absorption near 1550A is not CIV but a mix of FeIII lines. In this case there is not observational evidence for a hot corona in B stars. Furthermore, recent development of thermally driven stellar winds by Lima

& Priest (1993) extent Parker's model in 2 – D and also relax the isothermal assumption. In that work, an example of a thermally driven solution to B stars was given.

Radiatively driven winds from early type stars use two types of line-forces which coexist with the electron scattering force: the optically thick which employes the Sobolev mechanism and first used by Castor, Abbott & Klein (1975) (CAK model) and the optically thin (Cassinelli & Castor 1973, Marlborough & Zamir 1975). The thick-line force is supposed to drive winds with high terminal velocities ($\sim 10^3 km/sec$). The thin-line force is thought to drive winds with high mass loss rate and low ($\sim 10^2 km/sec$) terminal velocities. In order to model winds from Be stars, de Araujo (1995) studied the wind driving transition from the optically thick to the optically thin case.

In Paper II we incorporated the thin-line radiative force in the thermally driven solution of Paper I. In that work we showed that when the thermal mechanism excites the wind solely, the temperature at the stellar surface is about $10^6\ K$. By incorporating a significant radiative force close to the star the temperature at the stellar surface is reduced to $10^5\ K$. In all these cases the temperature drops with distance to $10 - 10^3\ K$ at 100 stellar radii. The 2-D solutions of Paper II can be addressed as follows (full mathematical analysis and expressions of all flow quantities can be found in Paper II):

Paper II solution

The outflow is steady state ($\partial/\partial t = 0$), axisymmetric to the rotational axis ($\partial/\partial\phi = 0$) and helicoidal ($V_\theta = 0$) (with (r, θ, ϕ) the usual spherical coordinates). The fluid is ideal, inviscid, non-magnetized and non-polytropic. In this case, the flow bulk velocity given by the expressions:

$$V_R(R,\theta) = \sqrt{f(R) - \frac{\xi(R) - \omega^2}{\mu R^2}\sin^{2\mu}\theta} \qquad (1)$$

$$V_\phi(R,\theta) = \frac{\omega \sin^\mu \theta}{R} \qquad (2)$$

satisfies the governing HD equations which conserve mass and momentum:

$$\nabla(\rho\mathbf{u}) = 0 \qquad (3)$$

$$\rho[\mathbf{u}\cdot\nabla]\mathbf{u} = -\nabla P + \mathbf{F}_{rad} - \rho\frac{(1-\Gamma)\mathrm{G}M}{r^2}\widehat{r} \qquad (4)$$

The symbols have the usual meaning: $R = r/R_*$ is the dimensionless radial distance (R_* is the stellar radius), $V = u/V_p$ is the dimensionless velocity ($V_p = \sqrt{2k_B T_p/m_p}$, k_B is Boltzmann's constant, m_p the proton mass, T_p the temperature parameter), $\omega = V_{rot}(\theta = \pi/2)/V_p$ is the dimensionless rotational velocity of the star at the equatorial plane, ρ is the fluid density. The fluid temperature is related to pressure and density by the usual equation of state:

$$T(R,\theta) = \frac{m_p}{2k_B}\frac{P(R,\theta)}{\rho(R,\theta)} \qquad (5)$$

Eq. (2) implies differential rotation for the fluid which is controlled by the parameter μ. In the force balance Eq. (4) P is the flow thermal pressure, $\mathbf{F}_{rad}$ is the line radiative force and the last right hand term is the effective gravity (gravitational force reduced by the Thomson electron scattering force) where G is the gravitational constant, M is the mass of the star and Γ is the ratio of the stellar luminosity L to Eddington luminosity L_E:

$$\Gamma = \frac{L}{L_E} = \frac{L\ \sigma_T}{4\pi \mathrm{G} M m_p c} \qquad (6)$$

where c is the speed of light and σ_T the Thomson scattering cross section ($\sigma_T = 6.7\ 10^{-25} cm^2$). In addition, we use the dimensionless parameter $\nu = V_{esc}/V_p$ which is the ratio of the escape velocity at the stellar surface to the velocity parameter.

The total radiative acceleration $\mathbf{F}^t_{rad}/\rho$ is in the radial direction, because photon scattering in the atmosphere is isotropic, and proportional to the incident Eddington radiation flux H^o_ν and absorption coefficients χ^o_ν (Mihalas 1978, p. 554): $F^t_{rad}/\rho = (4\pi/c)\int_0^\infty \chi^o_\nu H^o_\nu \mathrm{d}\nu$. We can evaluate the force due to Thomson electron scattering separately and this force (due to continuum absorption) is merged with gravity giving the effective value of it (Eq. 4). The remaining part of F^t_{rad} (i.e. F_{rad}) is due to the line contribution. Adopting the *optically thin atmosphere approximation* and according to CM F_{rad} can be expressed by line absorption opacities κ_i and subsequently by Thomson cross section σ_T as $k_i = \delta_i \sigma_T/m_p$. An average value of δ can be determined by the quantity $N\ <\delta> = \sum_{i=1}^N L_i\delta_i/L$, where $<\delta>$ is the average of the opacity of all thin lines, N is the number of thin lines and L_i is the local monochromatic luminosity in each line. By setting $W(R) = \Gamma N <\delta> /(1-\Gamma)$ the authors suggest that the number of lines and the average opacity scale with distance depending upon the local excitation and ionization equilibrium. In this way the thin line radiative acceleration is written as:

$$\frac{1}{\rho}F_{rad} = \frac{\sigma_T L}{4\pi m_p c r^2}N<\delta> = \frac{\mathrm{G}M}{R_*^2}\frac{(1-\Gamma)W(R)}{R^2} = \frac{\nu^2}{2}\frac{V_p^2}{R_*}\frac{(1-\Gamma)W(R)}{R^2} = \frac{\kappa\Gamma\nu^2}{2}\frac{V_p^2}{R_*}Q(R) \qquad (7)$$

where κ is a constant and $\kappa Q(R) = (1-\Gamma)W(R)/\Gamma R^2$.

By this formalism there is a correspondence between Paper II parameters and the works of CM and de Araujo (1995). CM introduce a power law $W(R) = \eta R^\varepsilon$. de Araujo considers a similar parameter κ which expresses the effect of all lines and $W = 1$. Obviously, there is a direct correspondence of the parameters of the three works:

$$\kappa = \left(\frac{1-\Gamma}{\Gamma}\right)\eta \qquad (8)$$

The parameter κ, or equivalently η, depends on the number of thin lines and the mean line opacity and increases with them. The characteristic value:

$$\kappa_o = (1-\Gamma)/\Gamma \qquad (9)$$

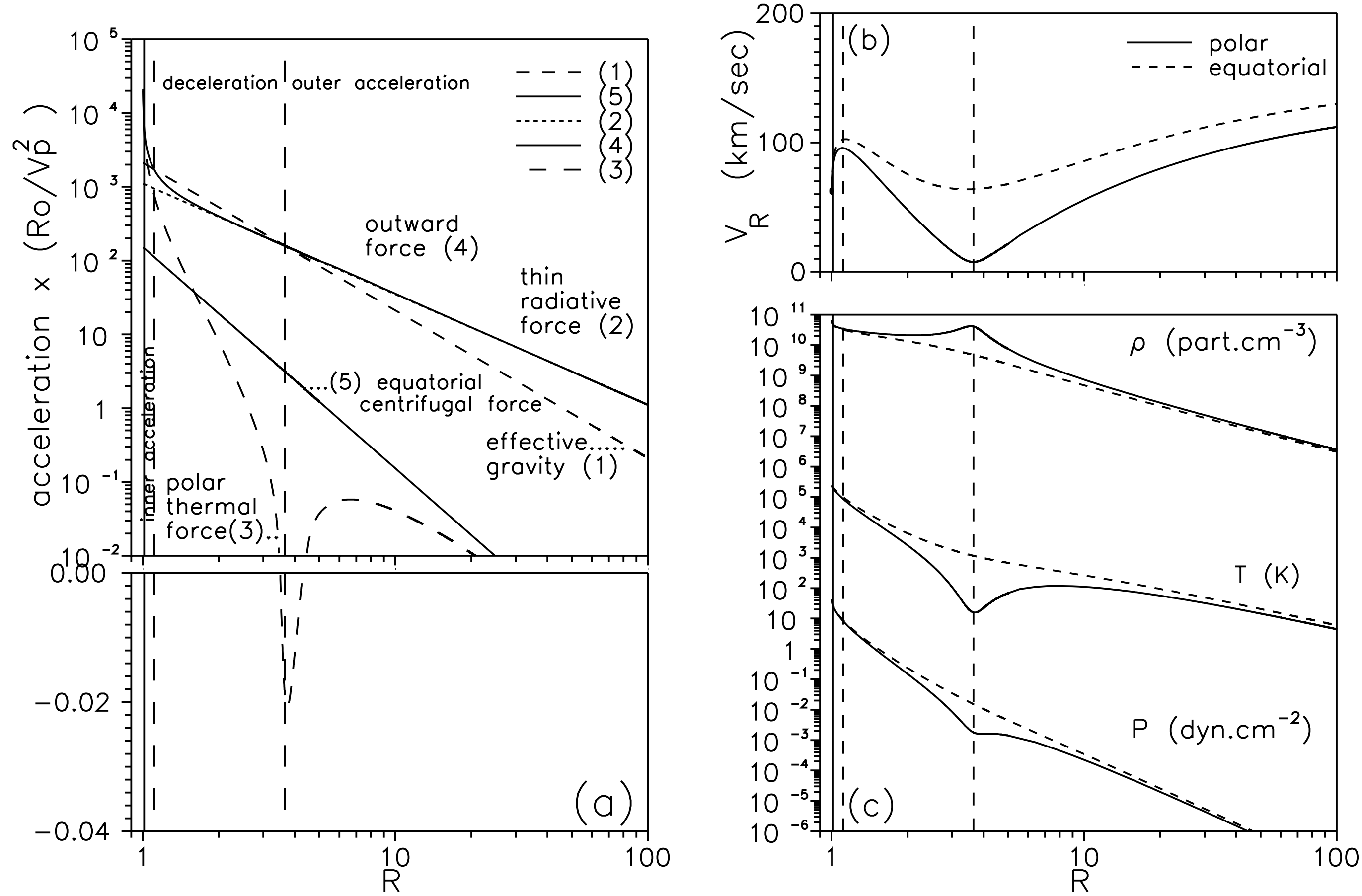

Fig. 2a–c. Steady shell formation for case 1 parameters of Table 1. **a** force balance **b** outflow radial velocity **c** outflow density, temperature and pressure. In plots **b** and **c** solid curves are polar and dashed are equatorial. In plot **a** five different forces are shown. The equatorial one is the centrifugal (5) (much smaller than the others). The thermal (3) and outward (4) are polar forces. The balance of the effective gravity (1) with the total outward force (4) (i.e. thin radiative (2) plus thermal (3)) separates the stellar envelope into inner and outer acceleration regions along the polar axis. Note that the outward force almost coincides with the thermal force at the stellar surface and with the thin radiative force after 2 stellar radii.

equals the radiative force and the effective gravity at the stellar surface. κ_o diverges when $\Gamma \to 0$ and vanishes when $\Gamma = 1$.

It was shown in Paper II that an analytical self-consistent 2-D solution is obtained for any function $Q(R)$ (Eq. 7) by solving Eqs. (3) - (4) and determining the functions f and $\xi(R;\mu;\omega)$. It is:

$$(\mathcal{C}R)^{2(\mu+3)} = \frac{|\xi - (\mu+4)\omega^2|^{(\mu+4)}}{|\xi - \omega^2|}$$

$$f(R) = f_o(R;\mu;\omega)\,[A + A_1 I_1(\xi;\mu;\omega) + A_2 I_2(\xi;\mu;\omega)] \quad (10)$$

where f_o is a function, $\mathcal{C}$, A, A_1, A_2 are constants and the I_1, I_2 integrals are evaluated numerically (Paper II). The integral I_2 involves the function Q.

$$A_1 = (1-\Gamma)\frac{\nu^2}{2}, \quad A_2 = \kappa\Gamma\frac{\nu^2}{2} \quad (11)$$

The solutions are of four types. The solution which describes subsonic outflow at the stellar surface which becomes transonic close to the star and terminates supersonic at infinity (named as Range I solution in Paper II) is applied in next section. The sonic surface is spherical and very close to the stellar surface ($R_s << R_*$). This is a solution with maximum radial velocity in the equatorial plane of the central object.

4. Steady shells modeled by the 2-D thermo-radiative solution

4.1. *Force balance*

As deduced in Paper II, when the thermal pressure force is employed to excite and drive a wind from typical early type stars, then, a sharp temperature gradient appears close to the star which leads to $\sim 10^6\ K$ surface temperature. Once a radiative force contributes in that region the temperature gradient smooths leading to $\sim 10^5\ K$. In isothermal models the thermal force is much less since no gradient of the sound speed exists. It was also seen that even if we adopt a strong thermal driving near the star the outflow cannot start from very low initial velocities because of the strong gravitational attraction after a certain distance. This means that the thermal force decays very rapidly with distance. By introducing a radiative force comparable to gravity (or greater than it) the previous decelerating effects and high initial velocities disappear. On the other hand, in optically thin stellar atmospheres, the radiative mechanism which is able to produce such a high force must be investigated. This is a well known subject in radiatively driven winds since the work of Lucy & Solomon (1970). Obviously, the main problem exists in astrophysical objects which posses a strong gravitational field and low luminosities without rotating close to the Keplerian-speed limit. The situation is still complicated when outflows from Be stars are studied, as well as from P Cygni type supergiants (Kuan & Kuhi 1975). Most of these stars show P Cygni profiles and there is also evidence for shell or blob ejections (Lamers 1994). These two observational features give some basis for a deceleration region in the atmosphere but the exact mechanism is not known (Kuan & Kuhi 1975, Hearn 1975, Nugis et al. 1979, Lamers et al. 1985).

In order to use the thin-line radiative force (described in the previous section) we have to choose the parameters κ and $Q(R)$ (Eq. (7)). According to CM the radial dependence of the thin radiative force is a power law:

$$Q(R) = \frac{1}{R^{2-\varepsilon}} \tag{12}$$

with ε constant. In order to create shells and not monotonously accelerating outflow solutions we have to choose $\kappa \leq \kappa_o$ (Eq. (9)). Choosing $\kappa \leq 1$ the line opacity is less than the continuum.

The role of the thermal force in the present work is to excite and drive the outflow in the inner acceleration region and to prevent gravitational attraction in the deceleration region up to the shell distance. An increasing with distance thin radiative force (which is less than the thermal force close to the stellar surface) contributes in the inner part and dominates in the outer. This force equals gravity at the shell distance. In this way, we fit a thermal force in order to obtain almost equilibrium conditions at the shell (across the rotational axis). The thermal driving in the inner acceleration region introduces a corona-like layer with temperature higher than the effective of the star. In present work, we restrict our analysis for early type supergiants to cases with $T_{\max} \leq 2\ 10^5\ K$ according to theoretical and observational aspects referred in Sect. 3.

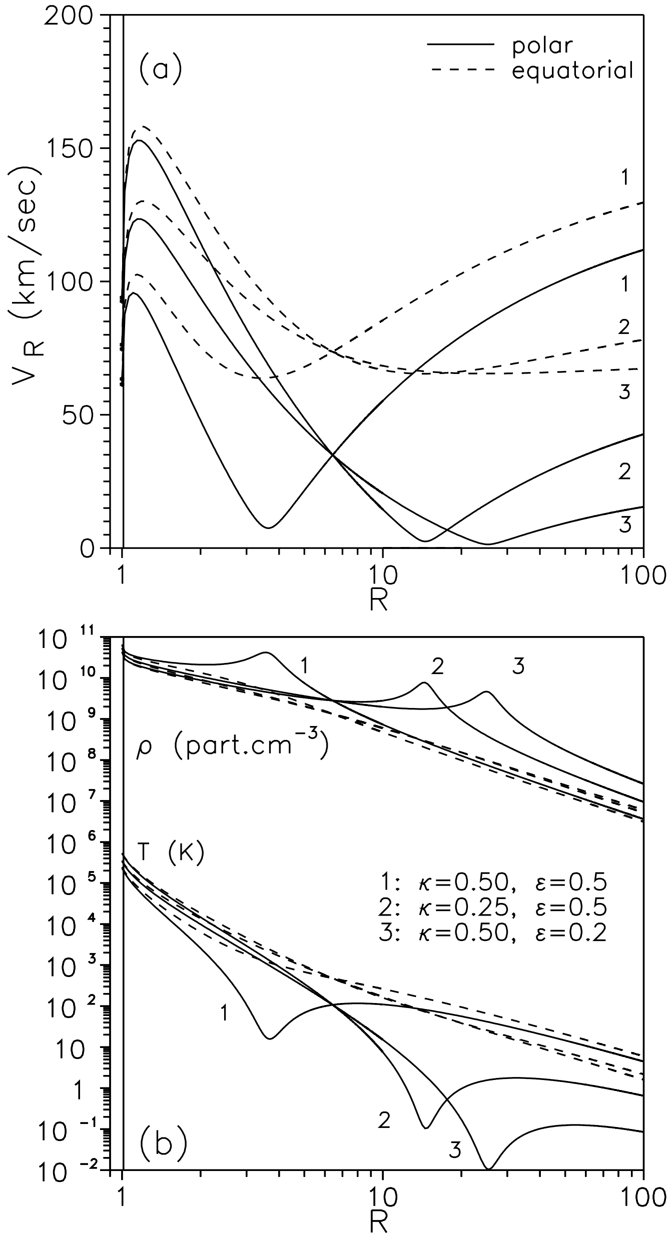

Fig. 3a and b. Dependence of the shell distance upon parameters κ and ε for values of Table 1. Solid curves are polar and dashed are equatorial.

4.2. Deduction of shell solutions

An approximate shell distance along the polar axis is easily evaluated neglecting the thermal and the centrifugal force. By equalizing the effective gravity and the thin radiative force we obtain:

$$R_{shell} \simeq \left(\frac{1-\Gamma}{\kappa\Gamma}\right)^{\frac{1}{\varepsilon}} R_* \tag{13}$$

Taking into account the centrifugal force at other co-latitudes θ, Eq. (13) becomes:

$$(1-\Gamma)\frac{R_{shell}}{R_*} - \kappa\Gamma\left(\frac{R_{shell}}{R_*}\right)^{1+\varepsilon} = \frac{2\omega^2}{\nu^2}\ \sin^{2\mu}\theta \tag{14}$$

Obviously, as θ increases the right hand term of (14) destroys the equilibrium at the shell distance and the shell disappears for $\theta > \theta_{max}$. With increasing μ, the rotating fluid concentrates close to the equator and consequently, the shell overlaps the polar regions reaching the equatorial area. Conclusively, it is seen that the shell distance depends on the stellar luminosity (Γ), the number and the opacity of the thin lines (ε, κ). The shape of the shell depends on the differential rotation of the fluid (μ) and the rotation of the star (ω). The exact distance and shape of the shell are found by taking into account the thermal force which is maximum at the equatorial plane.

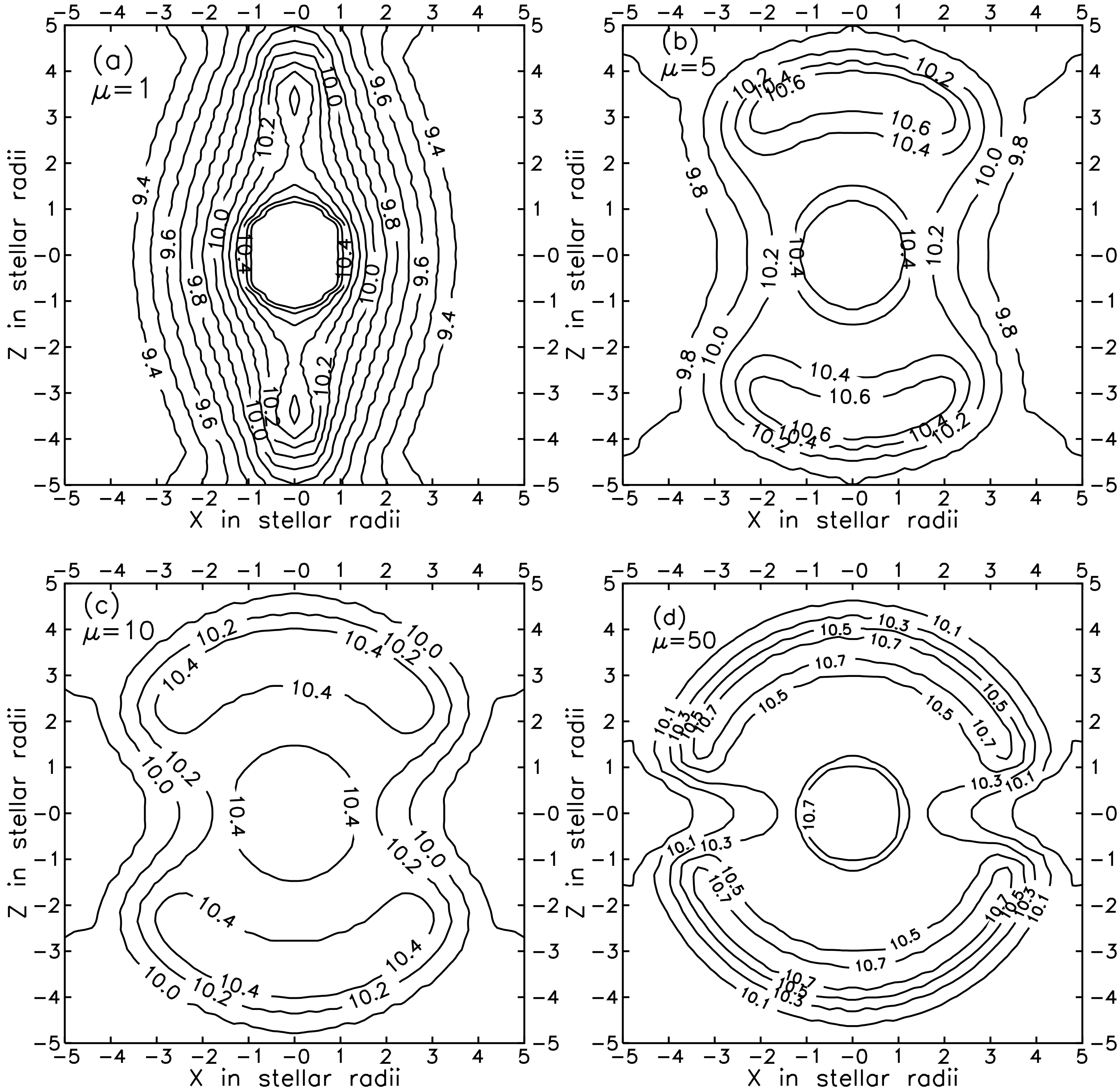

Fig. 4a–d. Dependence of shell shape upon differential rotation parameter μ for case 1 of Table 1 shown by selected logarithmic density ($\log \rho$) contours. **a** $\mu = 1$, **b** $\mu = 5$, **c** $\mu = 10$, **d** $\mu = 50$

In order to illustrate the previous results let us consider a hypothetical superluminous B1a+ type supergiant of:

$$M = 30\, M_{\odot},\; R_* = 80\, R_{\odot},\; V_{rot} = 50\, km/sec$$

$$L = 5\,10^5\, L_{\odot}, \Gamma = 0.51, T_{\rm eff} = 2\,10^4\, K$$

and adopt a density parameter $\rho_o = 10^{12}\, part/cm^3$ and a temperature parameter $T_p = 10^3\, K$ which correspond to a mass loss rate $\dot{M} = 4.2\, 10^{-6}\, M_{\odot}/yr$, $\omega \simeq 12.35$, $\nu \simeq 93$, $\kappa_o = 0.95$. We present three cases with the values of Table 1.

Table 1. Parameters for the applications of Figs. 2, 3, 4

case	A	κ	ε	μ
1	1174.5	0.50	0.5	10
2	2668.5	0.25	0.5	10
3	1734.2	0.50	0.2	10

In Fig. 2a the force balance along the polar axis is shown for case 1. The outward acceleration is the sum of the thermal and radiative contributions and almost balances effective gravity at the shell distance. The equatorial centrifugal acceleration is also plotted (being negligible compared with radiative and effective gravity). As pointed out, the thermal force decays rapidly with distance. In Figs. 2b-2c the results for outflow radial velocity V_R, density ρ, temperature T and pressure P are shown. Solid curves are polar and dashed are equatorial. A mass concentration is formed at $\sim 3.5\ R_*$ which is the shell distance. The thermal pressure drops inside the shell (subpressure region) and as a result the fluid cools (Eq. 5). Note that the fluid temperature is about $2\ 10^5\ K$ at the stellar surface and reaches the effective value at $1.5\ R$. Thus, the corona-like region is $0.5\ R_*$.

In Fig. 3 the steady shell solutions are illustrated for cases 1 - 3. The shell distance depends on both κ and ε (Eq. (13)), so, the shell appears at $\sim 11\ R_*$ in case 2 and at $\sim 12.5\ R_*$ in case 3.

The change for the shell shape is shown in Fig. 4. Plot (c) corresponds to case 1 while in the other plots μ is different. For small μ (~ 1, plot (a)) the shell is restricted at polar regions and looks like a double blob. For large μ (~ 50, plot (d)) the shell reaches the equator.

4.3. *M supergiants*

Outflows from late type M5 supergiants have lower terminal velocities ($\sim 100 km/sec$) compared with early type stars. There is an uncertainty for their radii, but, let us consider the following values for a typical M5 supergiant:

$$M = 24\ M_\odot,\ R_* = 500\ R_\odot,\ V_{rot} = 25\ km/sec$$

$$L = 3\ 10^5\ L_\odot, \Gamma = 0.38$$

A steady shell at $\sim\ 10R_*$ appears (Fig. 5) using a density parameter $\rho_o\ =\ 10^{11}\ part/cm^3$ and a temperature parameter $T_p\ =\ 10^3\ K$ (which correspond to mass loss rate $\dot{M}\ =\ 1.65\ 10^{-5}\ M_\odot/yr$, $\omega \simeq 6.18$, $\nu \simeq 33$), $\kappa = 0.5$ (critical value $\kappa_o = 1.6$), $A = 378.3$, $\varepsilon = 0.5$, $\mu = 10$. There is observational evidence for chromospheric emission from late type supergiants seen in H, CaII and K spectral lines (Cassinelli 1979 and references therein). So, the temperature could reach values of the order $10^6\ K$ at the stellar surface (Fig. 5b).

4.4. *P Cygni*

The envelope of hypergiant *P Cygni* has been extensively studied with the use of observations. According to Lamers (1986) we adopt the parameters:

$$M = 30\ M_\odot,\ R_* = 76\ R_\odot,\ V_{rot} = 65\ km/sec$$

$$L = 7.24\ 10^5\ L_\odot, \Gamma = 0.74, T_{\rm eff} = 2\ 10^4\ K$$

In order to apply the solution we set the density parameter $\rho_o = 2\ 10^{12}\ part/cm^3$ and the temperature parameter $T_p = 10^3\ K$ (which correspond to a mass loss rate $\dot{M} = 7.6\ 10^{-6}\ M_\odot/yr$, $\omega \simeq 16$, $\nu \simeq 95.50$). The critical value of the line opacity is $\kappa_o = 0.35$ and we choose the value $\kappa = 0.25$. According to Lamers (1986) the total outward acceleration seems to scale as $R^{-1.8}$. In the analysis of Sect. 4.2 (Fig. 2a) it was shown that the total outward acceleration in the outer acceleration region is approximately equal with the thin radiative force. So, we set $\varepsilon = 0.2$. We present two cases: (1) where $A = 433$, $\mu = 10$, (2) where $A = 386.5$, $\mu = 50$. In both cases a steady shell is formed at $\sim 5R_*$ (Fig. 6). In Fig. 6c the meridional projection of the shell for case 2 is illustrated by contours of the density logarithm ($\log \rho$). The asymptotic outflow velocity at $100R_*$ is about $92\ km/sec$.

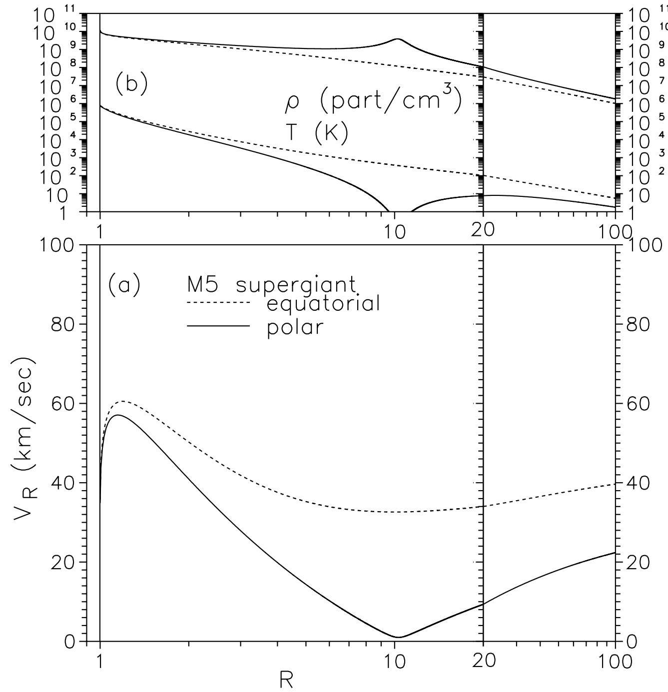

Fig. 5a and b. Steady shell solution for a hypothetical M5 supergiant **a** radial velocity **b** density and temperature. Solid curves are polar and dashed are equatorial.

The shell mass for case 2 (Fig. 6c) is evaluated numerically by considering an inner R_1 and an outer R_2 radius for the shell. In this case $R_1\ =\ 3.5\ R_*$ and $R_2\ =\ 5.5\ R_*$, so, the thickness of the shell is $H\ \simeq\ 2\ R_*$. The shell exists for colatitudes $\theta < 72^o$. The density in that region is bounded by $4.4\ 10^{10} < \rho < 1.3\ 10^{11} part/cm^3$. An approximate numerical integration gives:

$$M_{shell} \simeq 2.6\ 10^{24}\ kgr \simeq 4.37\ 10^{-8}\ M_*$$

It is also found that across the rotational axis the fluid spends $\sim 2\ yr$ to travel up to $10R_*$ and passes $\sim 0.8\ yr$ at the shell region.

The ejection of shells from the hypergiant *P Cygni* has been extensively investigated by Lamers et al. (1985) who found stable and variable shell components in the observed spectrum. Lamers (1986) also studied the acceleration in the envelope of this star. He suggested that the radiative force is mainly based on the large number of thin lines in the spectrum and the total outward force barely exceeds gravity, scaling as $R^{-1.8}$. The

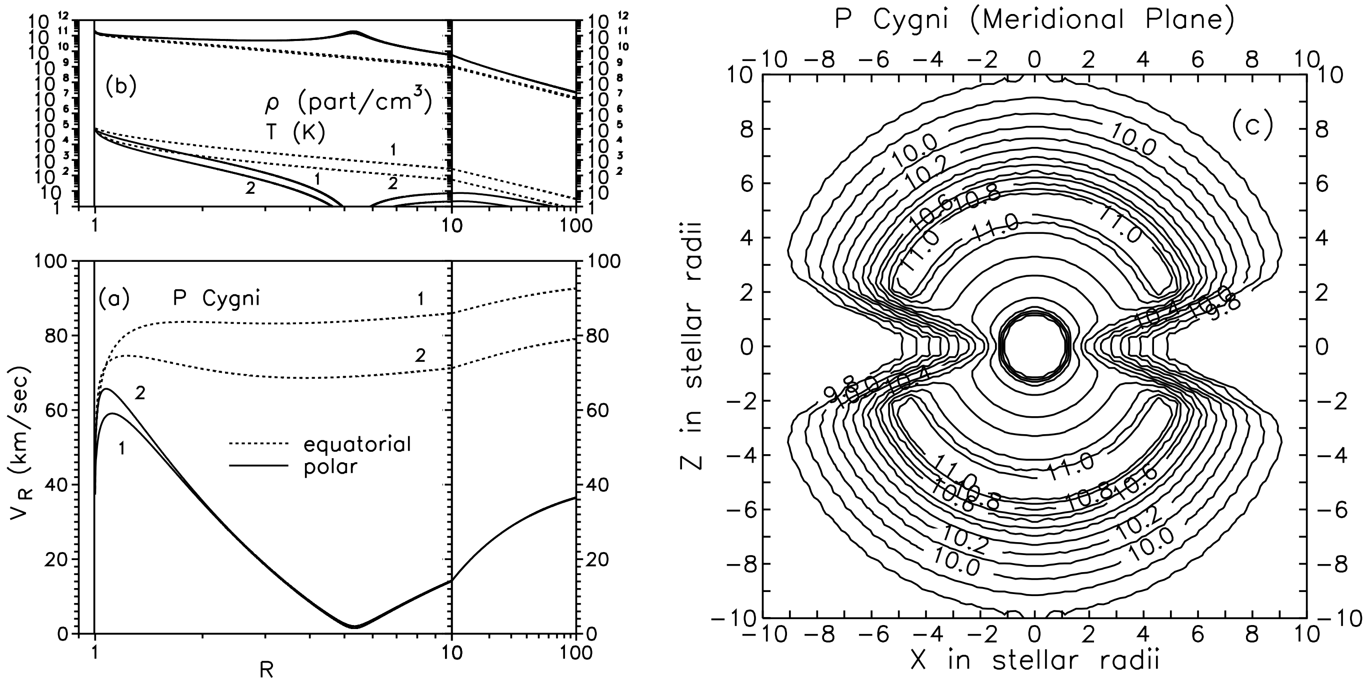

Fig. 6a–c. Steady shell solution for hypergiant *P Cygni* (Sect. 4.4). **a** radial velocity **b** density and temperature **c** selected logarithmic density ($\log \rho$) contours for case 2. A steady shell is formed at $\sim 5R_*$ when $\kappa = 0.25$ and the radiative force scales as $R^{-1.8}$.

mechanism described in Sect. 2 has also similarities with the work of Nugis et al. (1979) who proposed a *three zone* envelope in order to explain observations of *P Cygni*. In this way they introduced ADA - models because of acceleration - deceleration - acceleration of the flow in each zone respectively. They suggest radiative acceleration and gravitational deceleration for the outer zones but they correlate the inner acceleration and excitation of the flow with possible stellar pulsations. For *P Cygni* they found that inner acceleration occurs up to 2 R_* and then the flow is decelerated up to 5 R_*. Lamers et al. (1985) argue that this model explains their observations because it implies high velocities ($\sim 10^3\ km/sec$) not observed.

In this work, just using the thin radiative force as described by CM scaling as $R^{-1.8}$ (in accordance with Lamers (1986) for the stellar post - shell envelope) we find a three - zone envelope similar to Nugis et al. (1979). The inner acceleration is up to $\sim 0.5\ R_*$, and the deceleration is up to the shell distance $\sim 5\ R_*$. Afterwards the flow is accelerated by the thin radiative force. The present velocities are less than $100\ km/sec$ (Fig. 6). The main difference with Nugis et al. (1979) is that the inner acceleration is thermo - radiative and steady (not related with possible stellar pulsations and time dependent HD). We evaluate a shell mass of $M_{shell} \sim 1.31\ 10^{-6} M_*$ very close to the estimated by Lamers et al. (1986) ($M_{shell} \geq 6\ 10^{-5} M_{\odot}$).

We must note that the main purpose of this article is to study the mechanism of steady shell creation. So, the application to *P Cygni* shown in Fig. 6 is just indicative for shell formation in its envelope. It is seen that the mass loss rate and the shell mass are underestimated (observed $\dot{M} \simeq 1.5\ 10^{-5} M_{\odot}/yr$). This divergence can be removed by redefining the parameter ρ_o. Furthermore, the shell distance, shape and the terminal velocity can change as described in Sect. 4.2.

5. Discussion

In this work we presented a *thermo - radiative* mechanism which is able to create steady state shells in optically thin stellar atmospheres. This mechanism is based on the Paper II analytical 2 – D solution and the analysis of CM for the thin radiative force. The shell is formed in the supersonic region of the outflow. The results show that early and late type superluminous supergiants are expected to create massive shells in their envelopes even if the opacity in thin lines is smaller than the continuum opacity. This happens because these stars are close to their Eddington limit. Stars which are closer to the main sequence can possess shells with the same mechanism but with larger line opacity.

This mechanism is insensitive to stellar rotation. So, the stars do not have to rotate close to the Keplerian-speed limit. The centrifugal force is negligible in most applications, so, the mechanism applies similarly to fast and slowly rotating objects.

The thermal driving used for early type stars does not produce enormous temperature close to the stellar surface. The corona-like region is very thin ($0.5\ R_*$ for the example of 4.2, $0.1\ R_*$ for the P Cyg example of 4.4) and the maximum temperatures are bounded by the value $2\ 10^5\ K$. This is in accordance with Hearn (1975a,b), Lamers & Snow (1978), Cassinelli, Olson & Stalio (1978), Olson (1978), Cassinelli & Olson (1979) (Sect. 3). We note that possible incorporation of non-thermal driving mechanisms in the inner acceleration region will decrease the temperature values, and this will be a subject for future work.

Concerning the observed shell variabilities we have to note that they cannot be modeled by the present steady state solutions. On the other hand, the present shells depend upon the balance of the thin radiative force with gravity as well as the differential rotation of the fluid. If the radiative parameters change during the life of the star the shells may appear or disappear at several distances. The transition between these situations needs time - dependent HD, but, if the time periods between them are large enough the final stages of shell creation can be described by the present model as steady state. As seen in Sect. 4.2. this mechanism can either produce shells or double blobs beyond the poles. We note that the spectroscopic observations depend on the relative observer's direction. So, variabilities with periodicities correlated with the star's rotation may be appropriate to the present solutions. We also note that very massive shells with large opacity may cause deviations in the observed stellar mass and radius.

The adopted radiative force in this work decays monotonously with distance. If this force exhibit local maxima and minima the creation of multiple shells is also possible.

In the present work we study the dynamic nature of shells. The energetics of the outflow must also be studied. Just applying the 1^{st} Law of Thermodynamics we find that the outflow must be heated very close to the stellar surface and it is cooled at all other distances (Paper II). The cooling is possibly radiative, so, emission lines are expected in the spectrum. The flow energetics as well as the spectrum formation, the thermodynamic conditions in the shell and ionization stages will be the subject of a forthcoming work. Another point for future work is the incorporation of the thick radiative force, using the Sobolev mechanism, in the inner acceleration region where the velocity gradients are significant. The inclusion of other driving mechanisms in the inner part of the envelope, such as magnetic driving, could also be considered.

Conclusively, steady shells are consistent with single fluid HD when the thin radiative force is important. This result concerns early and late type supergiants which usually exhibit shell characteristics and emission lines in their spectrum.

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