Count laws and projection effects in clusters of galaxies[*]

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Abstract. We show that a 2D projection is representative of its corresponding 3D distribution at a confidence level of 90 % if it follows a King profile and if we consider the whole spatial distribution. The level is significantly lower and not decisive in the vicinity of the 2D cluster center. On the other hand, if we verify the reciprocal statement of the Mattig's distribution (1958) -i.e. a flux limited sample is represented by a 0.6 slope of its count law-, we point out that, due to the usual inaccuracy of the slope determination, a slope of 0.6 is not a sufficiently strict criterion for completeness and uniformity of a sample as often used in the literature.

1. Introduction

A large part of modern observational cosmology focuses on statistical studies of the galaxy population in the Universe. Two problems are commonly faced that concern the properties of galaxy samples:

First, the spatial distribution of galaxies belonging to clusters is basically unknown. The image of a cluster is a 2-dimensional projection of the true 3-dimensional distribution, and one is led to question the quality of this 2D projected distribution as estimator of the true 3D one. Let us recall that this point is important for understanding the true physics of the cluster, and that properties as the core radius (Lubin & Postman 1996) and morphological segregation (e.g. Whitmore & Gilmore 1991, Stein 1996) are derived from the 2D distributions only.

Second, one usually uses the slope of the count law of flux-limited extragalactic samples to estimate their completeness (e.g. Paturel et al. 1994) and uniformity. This test is based on Mattig's demonstration (1958) that, in an Euclidian Universe, the number N of objects brighter than an m magnitude is: $N(m) \propto 10^{0.6m}$. However, the validity of the reciprocal statement, that a count law with slope 0.6 in $(m, logN)$ co-

ordinates implies uniformity and completeness, remains to be checked carefully.

The present paper aims at clarifying these two questions by means of Monte-Carlo simulations based on input parameters consistent with observed ones. For this purpose, a series of test samples, containing either simulated and real data have been considered. For this purpose, series of test samples containing either simulated or real data, have been considered.

The samples are presented in Sect. 2. Sect. 3 describes the relationship between the projected 2D and real 3D distributions of galaxies in clusters. Sect. 4 discusses the slope of counts laws and completeness. Sect. 5 contains our conclusion and summary.

2. Description of the samples

2.1. Density laws

– To check projection effects in clusters of galaxies, two projected density laws $\rho(r)$ have been considered:
a uniform one with a constant density
a generalized 2D King law of β exponent with core radius r_c: $\rho(r) \propto [1/(1 + (\frac{r}{r_c})^2)]^\beta$
From these distributions, 3D spherical distributions are generated by affecting to each particle (galaxy) a radial coordinate z using a gaussian random generator. We recall that the core radius r_c is the same for a 2D or a 3D King profile (Girardi et al. 1995).

– To check the validity of the reverse count law, we generate 3 artificial spherical random distributions. These populations of galaxies are based on uniform space density with various clustering structure.

2.2. Cluster samples

We used also four samples taken from 52 clusters of galaxies:
The first one (Sample I) is built from Amram et al. (1992, 1994, 1995) samples containing 34 spiral galaxies taken from 8 clusters. To be more complete, other morphological type galaxies have been added in order to follow a King profile according to Whitmore & Gilmore (1991). The core radius of this synthetic cluster is 150 kpc.

The second (Sample II) is a sample of 43 Edinburgh-Durham Southern Galaxy Catalogue (hereafter EDSGC) and Eso Nearby Abell Cluster Survey (hereafter ENACS) rich Abell clusters (Table 1) kindly provided by ENACS and COSMOS team. These clusters are described at length in Mazure et al. (1996), in Katgert et al. (1997), and in Heydon-Dumbleton et al. (1989). EDSGC catalog gives the projected distribution of all galaxies. The EDSGC data are nearly complete for b_j=20. Core radii, exponent β and center for these 43 clusters are known (see Adami et al. 1997: AMK98 hereafter). We have selected here only nearly circular clusters to minimize ellipticity effect (Table 1). We take into account only galaxies in an homogeneous zone extending across 5 core radii. This corresponds to the main part of each cluster.

The third sample (Sample III) is taken from Virgo cluster. We have more than 550 galaxies in the whole available area (Binggeli et al. 1985). We do not have the density law information but this cluster will be used only for uniformity and completeness tests.

2.3. Artificial samples to test reciprocal Mattig's law

To check the validity of the reverse count law assumption, we generate in an Euclidean universe spatial distributions of objects with various uniformity and apparent magnitude completeness conditions. We assume that a sample is complete if it follows in absolute magnitude a Schechter luminosity function with de Lapparent parameters (1989). The range of generated absolute magnitude is [-11,-23].

The first one (Sample IV) has no structure in its space distribution, the position of each galaxy inside the volume is purely random. The uniform density is fixed to 0.06 galaxies Mpc^{-3} consistently with a projected surface density of $3\ 10^{-5}$ $gal\ arcsec^{-2}$ (Colless 1989). For computing limitation reasons, we populate only the 1/8 fraction of a spherical volume of 300 Mpc radius (H$_0$=100 km.s^{-1}Mpc^{-1} and q$_0$=0) with $6.55\ 10^6$ galaxies. The 1/8 fraction does not affect the validity of the results. The radius of the sphere corresponds to a redshift lower than 0.1 i.e. to a space volume within which the Universe is assumed to be quasi-euclidian. The galaxy absolute magnitude distribution is complete according to a Schechter luminosity function with α=1.1 and M_*=-19.2 (de Lapparent et al. 1989).

In the second sample (Sample V), the galaxies are grouped into 50 clusters of identical population. Each of these clusters is a sphere of 1 Mpc diameter. Two spheres are in average separated by 66 Mpc. The position of the cluster centers and the position of the galaxies inside the clusters are randomly generated. This mimics a distribution of galaxies in discrete groups bathing in large voids.

Sample VI combines a cluster distribution analog to Sample V case, encompassing 2/3 of the total galaxy population with a remaining 1/3 randomly distributed as isolated objects across the whole volume. The ratio 2/3 and 1/3 are given by Combes et al. (1991).

Table 1. The 43 clusters of galaxies with the correlation coefficient C, the representiveness confidence level of the mean value of C, and the slope of the count law before truncation (Slb) and after truncation (Sla).

name	C	P	Slb	Sla
A0087	0.68±0.08	95	0.48	0.48
A0118	0.70±0.08	≥ 99	0.62	0.43
A0119	0.68±0.07	≥ 99	0.45	0.39
A0151	0.67±0.09	≥ 99	0.54	0.45
A0168	0.67±0.05	≥ 99	0.47	0.43
A0367	0.62±0.07	72	0.58	0.55
A0524	0.67±0.05	82	0.58	0.50
A0978	0.72±0.11	≥ 99	0.64	0.56
A1069	0.67±0.04	97	0.44	0.40
A2353	0.65±0.08	86	0.69	0.64
A2362	0.63±0.07	77	0.31	0.29
A2383	0.71±0.11	≥ 99	0.57	0.47
A2426	0.70±0.08	≥ 99	0.63	0.53
A2480	0.68±0.07	81	0.51	0.46
A2644	0.68±0.09	83	0.43	0.30
A2715	0.71±0.09	≥ 99	0.61	0.57
A2717	0.71±0.03	≥ 99	0.52	0.47
A2721	0.68±0.04	≥ 99	0.79	0.67
A2734	0.66±0.06	≥ 99	0.46	0.35
A2755	0.59±0.10	54	0.64	0.54
A2764	0.65±0.07	93	0.55	0.51
A2765	0.69±0.13	96	0.62	0.50
A2778	0.67±0.08	98	0.62	0.52
A2871	0.60±0.07	61	0.43	0.28
A2911	0.70±0.06	≥ 99	0.29	0.29
A2923	0.59±0.09	52	0.45	0.34
A3093	0.59±0.12	53	0.65	0.53
A3094	0.68±0.05	≥ 99	0.66	0.55
A3111	0.65±0.06	90	0.33	0.30
A3122	0.68±0.04	≥ 99	0.66	0.53
A3128	0.67±0.02	90	0.60	0.51
A3151	0.66±0.09	88	0.54	0.47
A3158	0.68±0.07	≥ 99	0.57	0.49
A3194	0.71±0.06	≥ 99	0.64	0.55
A3341	0.66±0.09	≥ 99	0.51	0.43
A3528	0.62±0.04	82	0.40	0.35
A3744	0.69±0.06	≥ 99	0.54	0.49
A3781	0.65±0.09	84	0.45	0.43
A3809	0.68±0.03	≥ 99	0.68	0.58
A3822	0.69±0.03	≥ 99	0.77	0.63
A3825	0.68±0.06	≥ 99	0.60	0.54
A3827	0.70±0.06	≥ 99	0.69	0.55
A3897	0.66±0.10	90	0.59	0.55

3. 2D/3D representativeness

To check the consistency between projected 2D and spatial 3D distributions of galaxies in a cluster, we compute the Pearson correlation coefficient C between the projected radius r and the real radius $s = \sqrt{r^2 + z^2}$ for the whole sample. We have $C = \frac{cov(r,s)}{\sqrt{var(x)var(y)}}$.
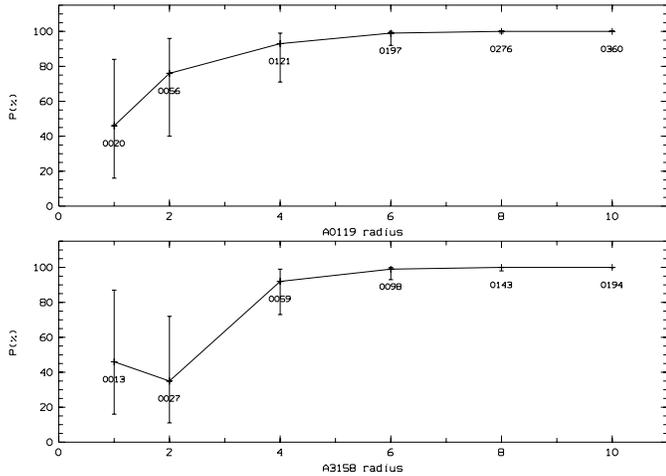
Fig. 1. Variation with the normalized radius D_i of the representativeness confidence level for A0119 and A3158. The number of galaxies within each radius is labeled.

In order to quantify the signification of C, we generate artificial 2D distributions and their corresponding 3D ones with exactly the same number of objects than the observed clusters, but without any 2D structures. We impose a low ellipticity of about 0.1 along the line of sight. This is the mean observed value of the projected ellipticities of our real clusters (AMK98). We have then for the artificial cases the lowest possible level of representation between a 2D and a 3D structure. For a given observed cluster, we proceed 10000 realizations of the artificial structure. By this way, we are able to calculate the percentage P of realizations (without structure) which give a lower value of C than the observed case (with structure). P is called here the confidence level of C.

For Sample I, we generate 10000 3D distributions according to a King density law with r_c = 150 kpc and β=1. We find C=0.70$\pm$0.15. The 2D distribution is representative of the 3D one with a confidence level of more than 99%. However, this sample contains less than 200 galaxies and is partially ideal. To deal with more realistic clusters, we use Sample II.

For Sample II, we generate 1000 3D distributions for each cluster according to a King density law with the parameters derived for each cluster by AMK98. The average C value is 0.67 for Sample II. The standard error on each real cluster is typically 0.07 and the standard error of mean C determination on all clusters is 0.03. We note that the synthetic Sample I and the real Sample II give very consistent values of C. We give in Table 1 the level P for each cluster. As we can see, only 10 clusters show a percentage lower than 90%. The 33 others are significantly representative of the 3D distribution at a level of more than 90%.

To check this representativeness variation as a function of the included area, we use 2 specific clusters, member of Sample II, that are very regular and contain more than 200 galaxies: A0119 and A3158. This ensures good statistics. Applying the same 3D generation method, we consider 5 values of the ratio $D_i = \dfrac{i^{th} \text{ limiting cluster radius}}{\text{core radius}}$. We impose D_i = 1, 2, 4, 6 and 8.

We calculate the correlation coefficient C and its representative confidence level P for each case (Fig. 1). P increases from about 50 % with large error bars to more than 99%. The central distributions are significantly less constrained than more extended ones and are only poorly representative of the corresponding 3D ones. We note here, that this tendency is certainly dependent on the actual cluster spherical degree: a strongly elongated cluster with a major axis along the line of sight will exhibit a lower P. Moreover, this representation is directly related to the morphological type of the galaxies: we know that the spirals are significantly less concentrated than the ellipticals (e.g. Adami et al. 1998 and included references: ABM98).

4. Slope of count laws and completeness

4.1. Non uniform samples

For each of the Samples IV, V and VI, 100 Monte-Carlo runs are done. The observed counting laws, after projection into apparent magnitudes, are the following:

– Sample IV, as expected, gives a slope 0.59 ± 0.02, consistent with Mattig's (1958) law.

– Sample V gives a slope 0.89 $\pm$0.18, showing that extreme non-uniformity translates into a much steeper relation than Mattig's one and a non-reproducibility of the slope value, as evidenced by the statistical dispersion.

– Sample VI, supposed to be a more realistic picture of the nearby Universe than the purely uniform case, yields a slope 0.64$\pm$0.04, only slightly departing from Mattig's value. Therefore, a substantial degree of inhomogeneity in the distribution of galaxies should be quite difficult to detect from observation of the counting law slope only.

We use our available Abell clusters (Sample II) as the "real" sample. We have calculated the slope of the count law in the range [minimal cluster magnitude, minimal cluster magnitude + 3] for the largest available area of each cluster. We find a slope equal to 0.56$\pm$0.11. We note that the error on each slope is negligible compared to the global dispersion of the individual values. This mean value is consistent with the uniform case: 14 of the clusters show a slope greater than 0.56. We remark that one of the most non uniform clusters, like A0151, provides a slope of 0.54 close to the "uniform reference case" value of 0.59.

We do not have *very* different values of the slope between purely uniform samples and samples with a uniform part or real samples. The only significant discrepancy occurs when considering a totally non uniform sample, which is not observed, as soon as a survey encompasses a sufficient solid angle.

4.2. Effect of incompleteness

First, the slope of the count law of the Virgo sample (Sample III) is analyzed in various apparent magnitude ranges. It is found

to be equal to 0.59±0.09 in [9.0;12.2]. Hence, completeness is taken as working hypothesis across this range. Further, we cut randomly with a uniform law of the magnitude about 50 % of the objects and we recalculate the slope of the count law. We find 0.58±0.10, not different from the complete case.

We use now the structureless Sample IV and we cut it in a more realistic way according to the exponential law of the magnitude m: 1-e$^{(m-(min+2))/(m-max)}$. The variable min is the minimum observed magnitude in the sample and max is the theoretical limiting magnitude of the EDSGC survey. We deal now with appromatively $1.07\ 10^6$ galaxies out of the original $6.55\ 10^6$ ones. We find a slope of 0.53±0.02 to be compared with 0.59 ±0.02 (see previous section). The difference is significant, but we must take into account the very small errors due to the very large number of considered galaxies.

Third, we use the Abell clusters sample (Sample II). We cut their galaxy apparent magnitude distribution with the same exponential law .When m grows, the percentage of removed objects grows also. We reject by this way about 70% of the galaxies. We compare the calculated slopes before and after truncation (Table 1). The mean truncated value is 0.48±0.09 and the mean difference between the two slopes is 0.07 (15% of the slope), lower than 1 standard error. More than 30% of the clusters exhibit a difference smaller than 10%. This difference represents an error lower than the usual accuracy of the slope determination (when the counting errors are estimated with the usual square root of the count). We note however that the truncated values are systematically lower than the non truncated ones.

5. Conclusion

We have shown that a 2D projection is representative at a level of 90% of the corresponding 3D sample if it obeys a King profile and if we consider the whole cluster. The confidence level is significantly lower in the vicinity of the cluster 2D center (typically inside 4 core radii) and is not decisive there.

We have also shown that it is somewhat unsafe to use the slope of the count law of a sample to test its completeness level and its uniformity. Indeed, for the uniformity of the sample, we have almost no observable difference between a realistic artificial sample (with 2/3 of its population in clusters), and a completely uniform artificial sample. For the completeness level, the difference is about 15% between a complete sample and an incomplete one. This difference represents an error lower than the usual accuracy of the slope determination. We conclude that the slope of the count law, in itself, is not a decisive factor to assess uncompleteness and homogeneity of a sample.

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