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LETTER

Letter to the Editor

On the spin-down of Be stars

John M. Porter

Astrophysics Research Institute, School of Engineering, Liverpool John Moores University, Byrom Street, Liverpool L3 3AF, UK

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Abstract. The spin-down of Be stars due to angular momentum transport from star to disc has been considered. This has been prompted by empirical studies of observed optical and IR line profile studies indicating that the disc is rotating in a Keplerian fashion. It is found that substantial spin-down may occur, especially for late B stars throughout their main-sequence lives for the "strongest" discs (most dense $\sim 10^{-11}$g cm^{-3} with high radial velocity ~ 1km s^{-1} at their inner edge and with large opening angle $\sim 15°$). This is in conflict with studies of rotational velocity distributions for different luminosity classes, which show no significant evolution. The implications of this for Be star discs are considered.

Key words: stars: emission-line, Be – stars: rotation – stars: evolution – circumstellar matter

1. Introduction

Be stars are now widely accepted to have two distinct regions of circumstellar matter: a diffuse polar stellar wind and a dense equatorial "disc" (Dachs 1987, Slettebak 1988). One of the major objectives of Be star research is to develop a theory which describes both of these components. The fast diffuse polar wind is well described by standard line-driven wind theory (Castor, Abbott & Klein 1975, Friend & Abbott 1986, Kudritzki et al. 1989). However, it has been much more difficult to describe the equatorial disc. Empirical models of the disc structure have been presented by e.g. Marlborough (1969), Waters (1986) and Hanuschik (1996), whilst theoretically driven models have been developed by Poe & Friend (1986), Chen & Marlborough (1992), Bjorkman & Cassinelli (1993), Willson (1986), Ando (1986) and Lee, Saio & Osaki (1991). These involve phenomena such as magnetic winds, latitudinal variation of driving lines, wind compression, stellar pulsation and viscous excretion. The most promising disc theory for several years, Bjorkman & Cassinelli's (1993) wind compressed disc model, has been shown to be incapable of reproducing observed discs by Owocki, Cranmer & Gayley (1996) and Porter (1997) via different routes.

One feature of empirical studies of line profiles in Be star discs is that they imply a rotationally supported disc, and that the rotation falls off in an Keplerian fashion (e.g. Dachs et al. 1986, Hanuschik 1989, 1996). The half-line width is typically larger than $v\sin i$ (Hanuschik 1996).

Also, the current model for V/R variations (described in e.g. Dachs 1987) assumes that $m = 1$ perturbations arise in a Keplerian disc (Papaloizou et al. 1992). Excretion disc models proposed by Lee, Saio & Osaki (1991) provide naturally this sort of rotationally supported disc – the disc material is rotating at its Keplerian speed, and drifts outward due to the effect of viscosity. These models require that angular momentum is supplied at the inner boundary of the disc. Given a prescription for the viscosity (e.g. 'α' from Shakura & Sunyaev 1973), then the surface disc density, and disc scale height may be integrated from the equations conserving angular momentum and mass (e.g. see Pringle 1981). A similar model has been used by Pringle (1991) applied to the cessation of accretion ("decretion") by a young stellar object.

In this investigation, assuming that the disc is indeed rotationally supported, the spin-down of the central star is calculated. In Sect. 2 the evolution of a star's rotational velocity is derived given that it is supplying angular momentum to the disc. Estimates of spin-down times are presented in Sect. 3 across the B star range of stellar parameters. This is discussed in Sect. 4 and conclusions are presented in Sect. 5.

2. Angular momentum transfer and spin down

It is here assumed that the disc material is moving round the star in approximately a Keplerian fashion. The star, however is not rotating at its Keplerian velocity, and so some angular momentum needs to be transferred to the disc. This may be achieved through non-radial pulsations (described by Osaki 1986) or through the action of magnetic fields. However, the exact mechanism is not important for the discussion here.

The specific angular momentum of the gas at the star's surface l_*, rotating at velocity $v_{\phi,*}$ is

$$l_* = v_{\phi,*} R_* = f \left(\frac{2GM_*}{3R_p} \right)^{\frac{1}{2}} R_*, \quad (1)$$

where R_* and R_p are the equatorial and polar radius of the star respectively ($R_* > R_p$), M_* is the mass of the star, and the star is rotating at a fraction f of its critical (or break-up) velocity (G is the gravitational constant).

This should be compared to the specific Keplerian angular momentum $l_k = v_{\phi,k} r = (GM_* r)^{\frac{1}{2}}$. It is now assumed that the extra angular momentum is added to the disc at or close to the star's surface – this is a conservative estimate leading to a lower bound on the angular momentum deficit. If the disc has a half-opening angle θ, and θ is small, then the rate of angular momentum supplied to the disc is

$$\begin{aligned} \frac{dL}{dt} &= (l_k - l_*) 4\pi R_*^2 \theta \mathcal{F} \\ &= 4\pi R_*^2 \theta \left(GM_* R_*\right)^{\frac{1}{2}} \left\{ 1 - f \left(\frac{2R_*}{3R_p} \right)^{\frac{1}{2}} \right\}, \end{aligned} \quad (2)$$

where $\mathcal{F}$ is the mass flux through the disc in g cm^{-2}s^{-1}.

The rate of angular momentum lost by the star is then simply $dL_*/dt = -dL/dt$. If the star rotates as a solid body at angular velocity Ω_*, and is regarded as a polytrope of index 3/2, then the total angular momentum of the star is

$$L_* = \epsilon M_* R_p^2\, \Omega_* = \epsilon f M_*^{\frac{3}{2}} \left(\frac{2GR_p}{3} \right)^{\frac{1}{2}} \quad (3)$$

where $\epsilon = 0.2046$ (this is a slight underestimate as the star's mean radius has been assumed to be R_p). This leads to the rate of angular momentum loss by the star:

$$\frac{dL_*}{dt} = \epsilon M_*^{\frac{3}{2}} \left(\frac{2GR_p}{3} \right)^{\frac{1}{2}} \frac{df}{dt} \quad (4)$$

which may be combined with Eq. 2 to finally give

$$\frac{df}{dt} = -\left(\frac{4\pi\theta}{\epsilon}\right)\left(\frac{\mathcal{F} R_*^2}{M_*}\right)\left(\frac{3R_*}{2R_p}\right)^{\frac{1}{2}}\left\{1 - f\left(\frac{2R_*}{3R_p}\right)^{\frac{1}{2}}\right\} \quad (5)$$

2.1. Disc parameters

To calculate the spin-down, the parameters of the disc need to be fixed. Many of the models for Be star discs may be described via power law expressions of density, disc height and velocity. For example, both Lee et al.'s (1991) and Okazaki's (1997) excretion disc models may be represented by power law density distributions.

The discs must also be normalized in some way such that they provide sufficient IR excess as the various disc models outlined above do not yield absolute values of densities, mass-loss rates etc. Here the density at the inner edge of the disc is used from Waters, Coté & Lamers' (1987) fitting to IR continuum excess.

The mass-flux in Waters et al.'s model is $\mathcal{F} = 7.7 \times 10^{-6} v_0 \rho_{-11}$ where the density ρ_{-11} is measured in 10^{-11}g cm^{-3}, the velocity at the inner edge of the disc v_0 in km s^{-1} and $\mathcal{F}$ in g cm^{-2}s^{-1}. Waters et al. calculate densities of $\rho_{-11} \approx 0.05$–2 (their Table 4) across a range of B spectral sub-types. The velocity of the flow assumed by Waters et al. was 5km s^{-1} (yielding a high mass flux) is almost certainly too high (Marlborough et al. 1997). Exactly what value of the initial velocity should be used depends upon which model is implemented, although v_0 has been calculated to be subsonic from both empirical and theoretical models. Poeckert et al. (1982), Marlborough & Cowley (1974) & Marlborough et al. (1997) find initial velocities of $\lesssim 1$km s^{-1} using empirical models, whilst for excretion disc models it can be shown that the radial velocity of an "alpha" accretion (or excretion) disc is $v_r \sim \alpha c_s^2/v_\phi$, where c_s is the sound speed (e.g. Pringle 1981) which becomes

$$v_0 \approx 0.3\alpha T_4 \left(\frac{R_*}{M_*}\right)^{\frac{1}{2}} \text{ km s}^{-1} \quad (6)$$

where T_4 is the disc temperature in 10^4K, and M_* and R_* are measured in Solar units. This leads to values for the velocity of $v_0 = 0.2\alpha$–0.6αkm s^{-1} across the range of stellar parameters for stars (see below). The final unknown in Eq. 5 is the disc half-opening angle θ. This have been calculated statistically to be $\theta \sim 5°$ (Porter 1996) to $\theta \sim 13°$ (Hanuschik 1996) for Be star samples, whilst Waters et al. (1987) use an opening angle of $\theta = 15°$.

With the above mass flux normalized to a base density, this should ensure that the disc should be able to produce enough free-free emission in the IR to account for the observed excesses. However, this is still subject to the functional form of the density distribution and thickness of the disc model. The above normalization is therefore a necessary, but not sufficient procedure for all disc models. The study by Marlborough et al. (1997) illustrates the effect of differing power law density distributions on the optical and IR line profiles.

3. Results

Combining the estimate for mass flux with Eq. 5 gives

$$\begin{aligned} \frac{df}{dt} &= -1.4 \times 10^{-15} \left(\frac{R_p^2}{M_*}\right) (v_0 \theta \rho_{-11}) \times \\ & \qquad \left(\frac{R_*}{R_p}\right)^{5/2} \left\{1 - f\left(\frac{2R_*}{3R_p}\right)^{\frac{1}{2}}\right\} \end{aligned} \quad (7)$$

where R_p is measured in Solar units. This function may now be integrated to obtain the evolution of the rotation of a Be star. Three model stars (with stellar parameters in Table 1) spanning the B spectral type are used.

Eq. 7 breaks into three separate parts: the stellar parameters, the disc parameters, and a function of rotation. For a given star the models may be characterised by the parameter $P = v_0 \theta \rho_{-11}$. With $\theta = 5$–$10°$, $\rho_{-11} = 0.05 - 2.0$, and $v_0 = 0.1$–1.0km s^{-1}, the disc parameter ranges between $P = (0.5$–$350) \times 10^{-3}$. Three values are used; $P = 5 \times 10^{-3}$, 10^{-2}, and 10^{-1} corresponding to "weak", "medium" and "strong" discs.

Fig. 1 displays the spin-down due to each of these discs with the top, middle and bottom panels corresponding to the strong, medium and weak discs respectively. The initial value of f was set to 0.81, and the calculation for each star was stopped at a time equal to the main sequence lifetime. This was taken to be

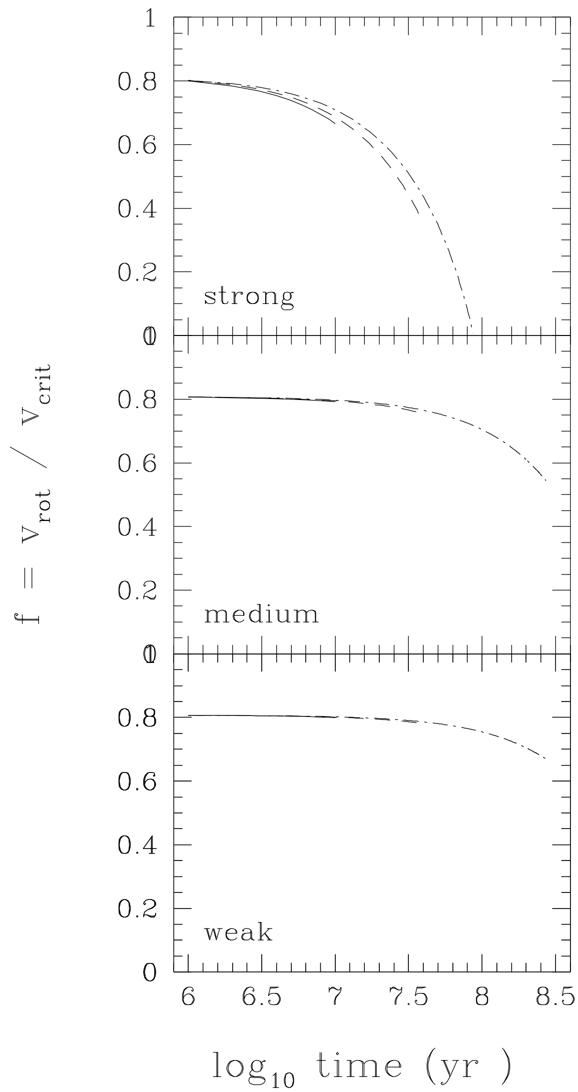

Fig. 1. The function $f(t)$ given by Eq. 7 for different disc models and different stellar parameters. For each panel the solid, dashed and dash-dotted lines correspond to the $17.5 M_\odot$, $7.7 M_\odot$ and $3.4 M_\odot$ models. An initial value of $f = 0.81$ at $t = 0$ is assumed.

Table 1. Stellar parameters for the stars considered from Schmidt-Kaler (1982). The radii quoted are the polar radii. $'\tau$ is an approximation to the main sequence lifetime taken from the hydrogen burning times from Maeder & Meynet (1989)

Spectral type	M_* ($M_\odot$)	R_* ($R_\odot$)	τ (yr)
B0	17.5	7.7	9.8×10^6
B3	7.7	4.7	3.9×10^7
B9	3.4	2.8	2.7×10^8

the time at which hydrogen burning ceases in the models of Maeder & Meynet (1989). These times are given in column 4 of Table 1.

The results displayed in Fig. 1 clearly illustrate three things: (i) the amount of spin-down is strongly dependent on the disc parameter P, (ii) the amount of spin-down for a star is not strongly dependent on its stellar parameters, except through its lifetime, and (iii) for some combinations of disc and star, substantial spin-down will occur.

For early B stars, there is only significant spin-down during the main sequence for strong discs. Both medium and weak discs cause little evolution in the rotation. However, for late B stars unless $P < 5 \times 10^{-3}$ significant spin-down should occur. If there is a threshold rotational velocity below which no disc should form (previous studies have found this to be $f \sim 0.2 - 0.3$; Porter 1996), then late B stars may even spin-down so much that they cease to support a disc at all.

4. Discussion

It has been demonstrated above that there is a possibility of some Be stars significantly spinning down. However, in observational studies, Fukuda (1982), Slettebak (1982) and more recently Zorec & Briot (1997) have found that across the B star range, there is no significant change in the rotational velocity distributions between different luminosity classes. This, contrasted with the above numerical results makes it very difficult to see how, (for late B stars), even medium discs may be generated, as these stars should have spun-down significantly during their main sequence lives, or with strong discs they should have spun-down to small rotational velocities and may cease to support a disc. As the disc structure has little or no dependance on the spectral type of the star (van Kerkwijk et al. 1995) then, at a first glance, it is perplexing that observational studies indicate that the discs around Be stars rotate at Keplerian velocities.

It is possible that all the actual Be star discs are what have been termed weak here. Some small fraction of stars may indeed have strong discs and even though they may spin-down significantly during their lifetime they may not skew the rotational distributions enough to detect. The conclusion of this is that the disc population is dominated by weak discs. The least well known parameter in the disc is the initial radial velocity v_0. Assuming maximal values of $\rho_{-11} \approx 2$ and $\theta = 15°$ for the disc, then a star will not spin down significantly if the initial radial velocity is $v_0 \lesssim 0.01 \mathrm{km\,s^{-1}}$ ($P \lesssim 5 \times 10^{-3}$).

The estimates of v_0 stem from optical and IR line modelling of discs although these only provide upper limits of $\sim 1 \mathrm{km\,s^{-1}}$. Excretion disc models (e.g. Lee et al. 1991) do predict values of v_0, although at the expense of introducing the viscosity parameter α (Eq. 6). With $\alpha \lesssim 0.01$, values of v_0 applicable to weak discs are obtained. Imposition of no evolution in the rotational distribution therefore may be seen as a tighter constraint on v_0 (or on α).

In the discussion so far, the disc has been assumed to exist around the star at all times in the main-sequence lifetime of the star. Be stars are known to undergo phases when the disc is apparently lost (see e.g. Hirata 1995 and references therein). Depending on the fraction of their lifetime spent without discs (or with very weak discs), Be stars could easily support strong discs and still not evolve significantly in rotational velocity.

It is timely to discuss whether the disc material is genuinely being lost. Strong discs could exist around B stars if all, or a fraction of them are re-accreted at some phase and hence spinning-up the B star. Indeed, a similar process of viscous re-accretion of disc material has been suggested by Hanuschik et al. (1993) in their study of μ Cen. However, at low radial velocities such as these it is impossible to detect whether material is inflowing or outflowing as the rotation dominates the kinematics in the disc (also see Hanuschik et al. 1993). This renders the line profiles generated in the disc impotent as radial velocity detectors in the presence of the Keplerian rotation. Consequently it seems impossible to obtain an explicit observational measurement of v_0.

5. Conclusion

Under the assumption that Be stars maintain discs throughout their main-sequence lives, spin-down is found to occur for certain parameters of the circumstellar disc if the disc rotates in a Keplerian fashion. This is in conflict with observational studies of the distribution of rotational velocity for Be stars of different luminosity classes. The simplest resolution to this paradox is that the majority of Be star discs must occupy parameter space in which $P = v_0\theta\rho_{-11} \lesssim 5\times10^{-3}$. For the range of opening angles from observational studies (e.g. Porter 1996), and densities (e.g. Waters et al. 1987) this implies very low radial velocities of $v_0 \lesssim 0.01 \mathrm{km\,s^{-1}}$.

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