

# Modelling the disruption and reaccumulation of Miranda

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**Abstract.** The heavily cratered surfaces of the largest Uranian satellites and the unusual surface geology of Miranda suggest that Miranda could have been catastrophically disrupted by collision and then reaccumulated over solar system history (Smith et al., 1986; Mckinnon et al., 1991). Using the numerical model described by Marzari et al. (1995) we have simulated the breakup of Miranda by a high velocity impact and computed the size and orbital distributions of the collisional fragments. These distributions have been adopted as realistic initial conditions for the numerical algorithm of Spaute et al. (1991) with which we have simulated the reaccumulation of the satellite from the ring of debris.

Our results show that the reaccumulation of Miranda occurs on a short timescale ( $\sim 10^3$  years), in spite of the initial large dispersion of the ring debris and the presence of Ariel at the outer border of the ring. However the reaccumulation process depends strongly on the poorly known outcomes of collisions. If collisions dominately result in accretion, the reaccumulation of Miranda proceeds as an orderly growth with larger bodies accreting mass from the smaller ones. If cratering and fragmentation are included, the reaccumulation is characterized by two stages: an initial stage during which shattering dominates and all bodies smaller than few tens km are destroyed. In the second stage the large surviving fragments grow by accumulating the small comminuted fragments and finally, colliding with each other, re-build a new Miranda.

Different breakup reaccumulation scenarios have been analyzed to account for the variation of some physical parameters.

**Key words:** Miranda – satellites – solar system: formation – methods: numerical

## 1. Introduction

Miranda is the innermost and smallest of the five major Uranian satellites. Voyager imaging during the flyby of the Uranian system revealed that Miranda’s surface has a complex geologic structure with tectonic canyons and coronae (Smith et al. 1986). A possible explanation of how a small body, like Miranda, was heated enough to drive such an extensive geological activity,

is that it was catastrophically fragmented and then reaccumulated at least once over the Solar System history (Gore, 1986; Shoemaker, see Smith et al., 1986; Johnson et al., 1987). This hypothesis is consistent with the cratering record found on Miranda and the other Uranian satellites. Smith et al. (1986) have identified two different crater populations: large impact craters (Population I), similar to that of the lunar highlands and of the most ancient bodies in the solar system; smaller craters (Population II), resembling craters generated by ejecta from a primary impact and similar to the population of some of Saturn’s satellites. Smith et al. pointed out as the observed large number of Population I craters cannot be explained by bombardment at the estimated impact cratering rate over the last 3 to 4 billion years. The Population I craters have been probably produced early in the history of the solar system, either by Uranus and Neptune planetesimals or by short period comets of the Uranus family. Extrapolation of the Population I craters recorded on Oberon inward to the inner satellites implies a cratering over-saturation of inner satellite surfaces. This is true in particular for Miranda which is the innermost and more affected by the gravitational focusing of Uranus. The heavy impactor flux on Miranda could have been responsible for one or more catastrophic events of disruption and reaccumulation of the satellite. Also Umbriel, Ariel and perhaps Titania could have been disrupted and re-accreted at least once.

In this paper we explore by numerical modeling the possible fragmentation history of Miranda, from the breakup event through the subsequent reaccumulation process. The collisional breakup of the satellite generates a family of fragments with independent planetocentric orbits initially clustered around the orbit of the pre-impact body. The strong  $J_2$  of Uranus randomizes, after few revolutions, the values of the perihelia and nodes of the satellite fragments and the initial clustered orbital distribution of the debris relaxes in a ring. After repeated encounters and collisions in the ring, the fragments can reassemble into a large body of mass comparable to the initial satellite. According to Stevenson et al. (1986) and Burns et al. (1984), the reaccumulation of a satellite from a ring is a rapid process ( $\leq 10^4$  yr). The orbit of Miranda is located outside the classical Roche limit and tidal forces do not limit the re-accumulation process (Canup & Esposito, 1995).

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We have simulated the breakup of Miranda with a self-consistent numerical model developed to predict the outcomes of high-velocity impacts between asteroids or other small bodies of the solar system (Marzari et al., 1995). The model computes the size and orbital distribution of the fragments produced in the satellite breakup. The size and orbital data of the fragments are used as initial conditions for the multi-zone “planet building” code (Spaute et al., 1991; Weidenschilling et al., 1997) which models statistically the reaccumulation process via mutual collisions. The spatial resolution of the code allows us to use the information on the initial orbital distribution of the fragments, the outcome of the collisional code. Moreover, the planet building code is able to treat all the collisional outcomes between two fragments: accretion, inelastic rebound, cratering and fragmentation. In our simulations we include also the perturbations induced by Ariel which is located close to the outer boundary of the Miranda debris ring.

In Sect. 2 we describe in detail the numerical algorithms used to simulate the disruption and reaccumulation of Miranda. In Sect. 3 we describe the reaccumulation process either when only accretion is considered as outcome of a two-body collision between two satellite fragments, or when cratering and fragmentation are included. Sect. 4 is devoted to the conclusions.

## 2. Numerical models

The breakup of the proto-Miranda has been numerically simulated by using the model by Marzari et al. (1995). This model computes for a two-body high-velocity impact the list of fragments with their sizes and proper orbital elements. The algorithm is based on semiempirical laws inferred from the results of high velocity impact experiments (Fujiwara et al., 1989) and includes the possible gravitational reaccumulation of fragments with ejection velocity less than the escape velocity. The relevant input parameters are the strength  $S$  of the target body, defined as the minimum energy to cause catastrophic fragmentation of the body, and  $f_{KE}$  the fraction of impact velocity released to the escaping fragments. The orbital elements of the fragments are calculated assuming an isotropic distribution of the ejection velocities in three-dimensional space. The fragment speed is related to its size according to the Nakamura & Fujiwara (1991) and Nakamura et al. (1992) relationship derived from hypervelocity impacts in laboratory experiments. To simulate the dispersion in the velocity values for fragments of the same size, we have introduced a random component of Maxwellian form in the computation of the speed of each individual fragment. This algorithm has been already used to simulate the catastrophic breakup of a proto-Hyperion (Farinella et al., 1997) and to derive the initial orbital elements of the Hyperion fragments to test if they subsequently impact on Titan.

The multi-zone planet building code (Spaute et al., 1991; Weidenschilling et al., 1997) used to model the reaccumulation of Miranda, has been originally developed to study numerically the accretion into planetary embryos of a swarm of planetesimals. The initial population of bodies in Keplerian orbits around a central body is divided into logarithmic size bins and semima-

major axis zones. Each subgroup of bodies generated by this division is characterized by two indexes  $i, j$ : the index  $i$  determines the size bin, the index  $j$  the semimajor axis zone. An eccentricity and inclination, with the respective dispersions, are assigned to each  $i, j$  group: their values can change during the evolution of the population due to two-body collisions and mutual gravitational perturbations. Larger bodies in the size distribution are treated separately with their individual Keplerian orbits in order to avoid problems associated with fractional numbers in the largest size bins and to track where the first large bodies in the population will grow. The number of collisions between different  $i, j$  groups, or discrete bodies and  $i, j$  groups, are computed with a sophisticated particle-in-a-box algorithm which takes into account of the orbital distribution of the population. At the end of each timestep, the number of bodies in each group, the mass of discrete bodies, and their values of eccentricity and inclination are updated taking into account collisions and gravitational perturbations.

The spatial resolution, which characterizes the planet building code, makes it particularly suited for studying the reaccumulation of Miranda. The initial ring of fragments produced from the Miranda breakup is not distributed uniformly but has a dynamical structure which depends on the characteristics of the collisional event. The isotropic distribution of the fragment ejection velocities is in fact transformed into a complicated pattern in the orbital element space. Moreover, according to the mass-velocity distribution derived from Nakamura & Fujiwara (1991) and Nakamura et al. (1992), small fragments have on average higher velocities with respect to the larger ones. As a consequence at the wings of the Miranda debris ring we expect to find more small bodies with large eccentricities and inclinations, while in the middle of the ring the larger bodies are clustered with lower eccentricity and inclination.

The “collisional” structure of the Miranda debris ring is largely preserved when we create the input population for the planet building code from the list of fragments generated by the collisional model. Each fragment of the list produced by the collisional code is grouped in a  $i, j$  bin depending on its size and orbital semimajor axis. When all the fragments have been grouped, an average eccentricity and inclination value is computed for each  $i, j$  group. In this way the initial population for the planet building code is completely defined.

To represent the proto-Miranda fragment population we used 50 logarithmic size bins, spanning from 500 m to 1300 km in diameter, and 10 semimajor axis zones covering a region from  $10^5$  km to  $2 \times 10^5$  km from the center of Uranus. If, during the reaccumulation process, a two-body collision produces fragments smaller than 500 m, their masses are recycled in the smallest size-bin of each semimajor axis zone. In this way no mass is lost through the lower size end of the distribution. We also tested different values for the size of the smallest bin ( $d_o$ ) and no significant differences were observed in the reaccumulation process when  $d_o$  was lower than 2 km.

In the two models there are some common collisional parameters like the strength of the bodies, the coefficient  $f_{KE}$ , the bulk density  $\rho$ . For the bulk density we have assumed the

value of  $1.3 \text{ g/cm}^3$  derived by Voyager data (Smith et al., 1986). As  $f_{KE}$  we have assumed 0.05, smaller than the value usually used in asteroid families (0.2) but larger than the experimental values ( $\approx 0.01$ ). This choice is motivated by the composition of Miranda which is for more than 60 % water ice while asteroids are in large percentage rock.

To compute the strength  $S$  of the proto–Miranda parent body and its fragments we have used the following formula:

$$S(D) = S_0 + \frac{K\pi g\rho^2 D^2}{15}, \quad (1)$$

(Davis et al., 1985; Davis et al., 1994).  $S_0$  represents the material strength of the body deduced from laboratory experiments; the second term is the contribution to the impact strength of the gravitational self–compression, where  $g$  is the gravitational constant and  $D$  the diameter of the considered body. For the constant  $K$  we assume, as in Davis et al. (1994), a lower value of 1 and a maximum value of 10 suggested by the experiments of Housen et al. (1991). We do not consider strain–rate effects in our simulations because in the range where the material strength dominates, fragmentation is so intense that the weakening of the bodies predicted by the strain–rate scaling would not affect our results (see Sect. 3).

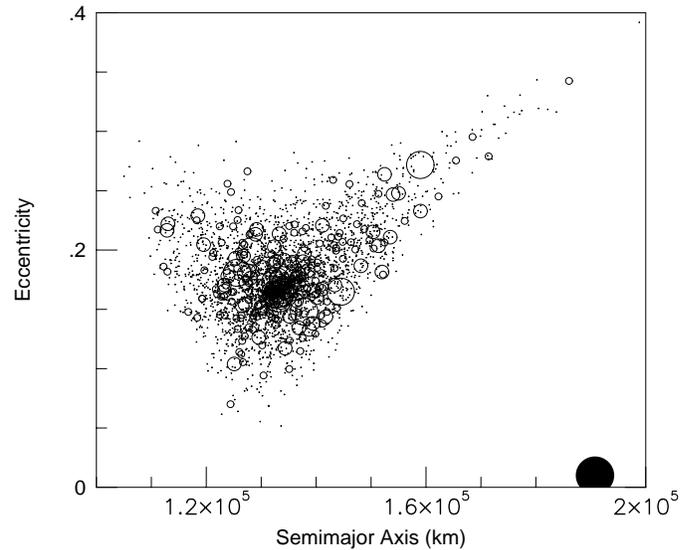
In the planet building code, the fragments produced in each collision are assumed to have a power–law distribution in velocity as in Davis et al. (1989):

$$f(> V) = (V/V_o)^{-\alpha} \text{ for } V > V_o, \quad \text{and} \\ f(> V) = 1 \text{ for } V < V_o,$$

where  $f(> V)$  is the fraction of collisional fragment mass ejected at a velocity larger than  $V$ ,  $V_o$  is the minimum ejecta velocity derived by conserving the collisional energy and  $\alpha$  is the velocity distribution index. The standard value of  $\alpha$ , derived from laboratory cratering experiments (Fujiwara & Tsukamoto, 1980), is about 9/4. When larger values of  $\alpha$  are assumed the computed average ejection velocity for the fragments of each shattering event is lower and, consequently, the fraction of escaping mass is reduced.

### 3. Simulations and results

Miranda is an almost spherical body with an average diameter of 470 km, a mass of  $7.1 \times 10^{22} \text{ g}$  for a density of about  $1.3 \text{ g/cm}^3$ . In modelling the catastrophic disruption of the proto–Miranda, we have considered a parent body slightly larger ( $D_{PB} = 475 \text{ km}$ ) than the present Miranda in order to account for eventual loss of mass during the reaccumulation process (ejection from the ring, capture by Ariel). The projectile is a body of 230 km in diameter, possibly coming from Uranus family comets or Kuiper Belt comets (Smith et al., 1986; McKinnon et al., 1991), impacting on the surface of Miranda at a relative velocity of 10 km/s. The density of both target and projectile is assumed to be  $1.3 \text{ g/cm}^3$ , typical of bodies with a high percentage of volatiles. In this model of the proto–Miranda breakup the largest fragment has a diameter of 250 km while the ratio between its mass ( $M_{LF}$ ) and the parent body mass ( $M_{PB}$ ) is about 15%.



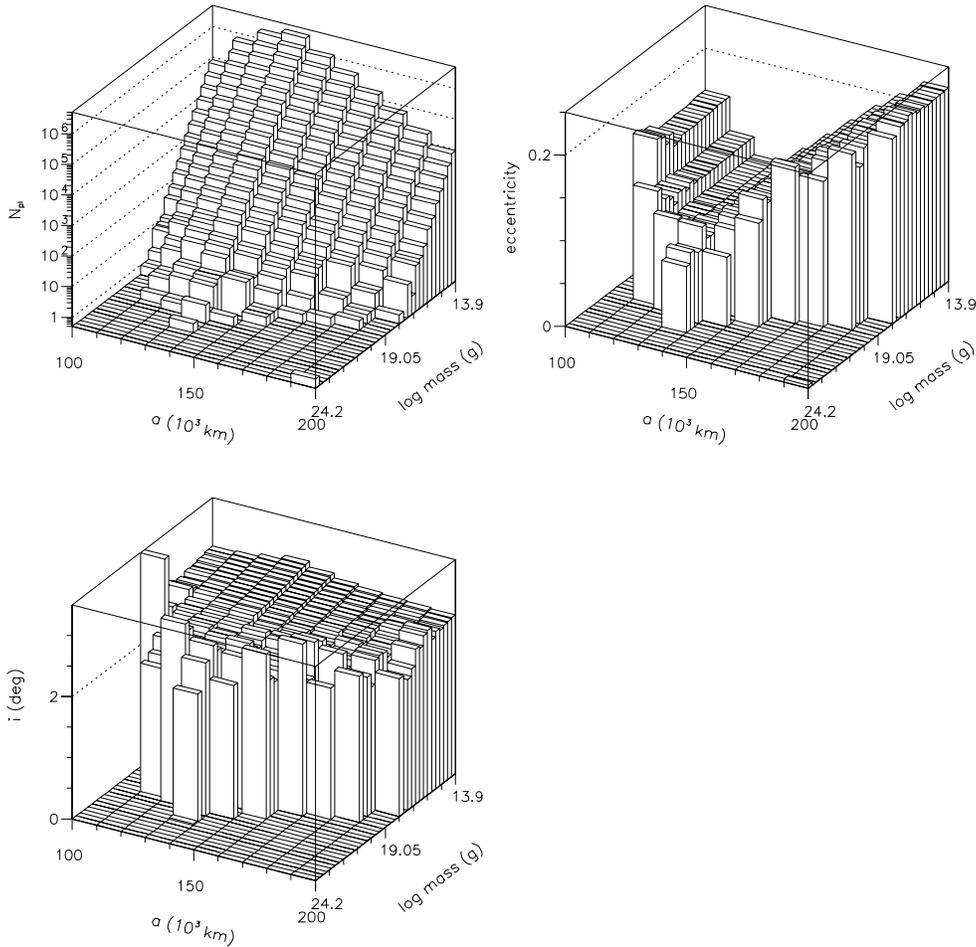
**Fig. 1.** Orbital distribution of the proto–Miranda fragments on the  $a$ – $e$  plane. The filled circle represents Ariel.

**Table 1.** Miranda breakup event.  $D_{PB}$  and  $M_{PB}$  are the diameter and mass of the proto–Miranda parent body, respectively.  $D_p$  is the projectile diameter and  $V_{imp}$  its impact velocity.  $f_{KE}$  is the fraction of the impact kinetic energy released into the fragments and  $\rho$  is the parent body and projectile bulk mass density.  $M_{LF}/M_{PB}$  is the ratio between the largest fragment and the parent body masses.

$D_{PB}$ km	$M_{PB}$ g	$D_p$ km	$V_{imp}$ (km/s)	$f_{KE}$	$\rho$ $\text{g/cm}^3$	$M_{LF}/M_{PB}$
475	$7.3 \times 10^{22}$	230	10	0.05	1.3	15%
600	$1.5 \times 10^{23}$	330	10	0.05	1.3	11%

In order to simulate a catastrophic event for the proto–Miranda breakup where the largest fragment is only a small fraction of the original parent body, we had to assume a projectile size larger than the value reported in McKinnon et al. (1991) (Table 1). They give the minimum impactor size for fragmentation and they do not take into account that a consistent fraction of the impact energy is required to disperse the fragments. Instead of considering a breakup event close to the disruption threshold, we simulate a case where most of the original proto–Miranda mass is in the small fragments and a significant reaccumulation is necessary to rebuild Miranda. Less catastrophic events, with largest mass ratio  $M_{LF}/M_{PB}$ , would result in a faster reaccumulation process.

We have modelled also a second breakup event for Miranda, in which the parent body is twice as massive as the present Miranda, with a diameter of 600 km and a mass of  $1.5 \times 10^{23} \text{ g}$ . The largest fragment produced in this simulated impact event is 11% in mass of the assumed parent body. This second breakup model is adopted when in the reaccumulation process the capture of ring mass by Ariel becomes significant and less mass is available for the reaccumulation of the disrupted satellite. The parameters used in simulating the two breakup events are summarized in Table 1.



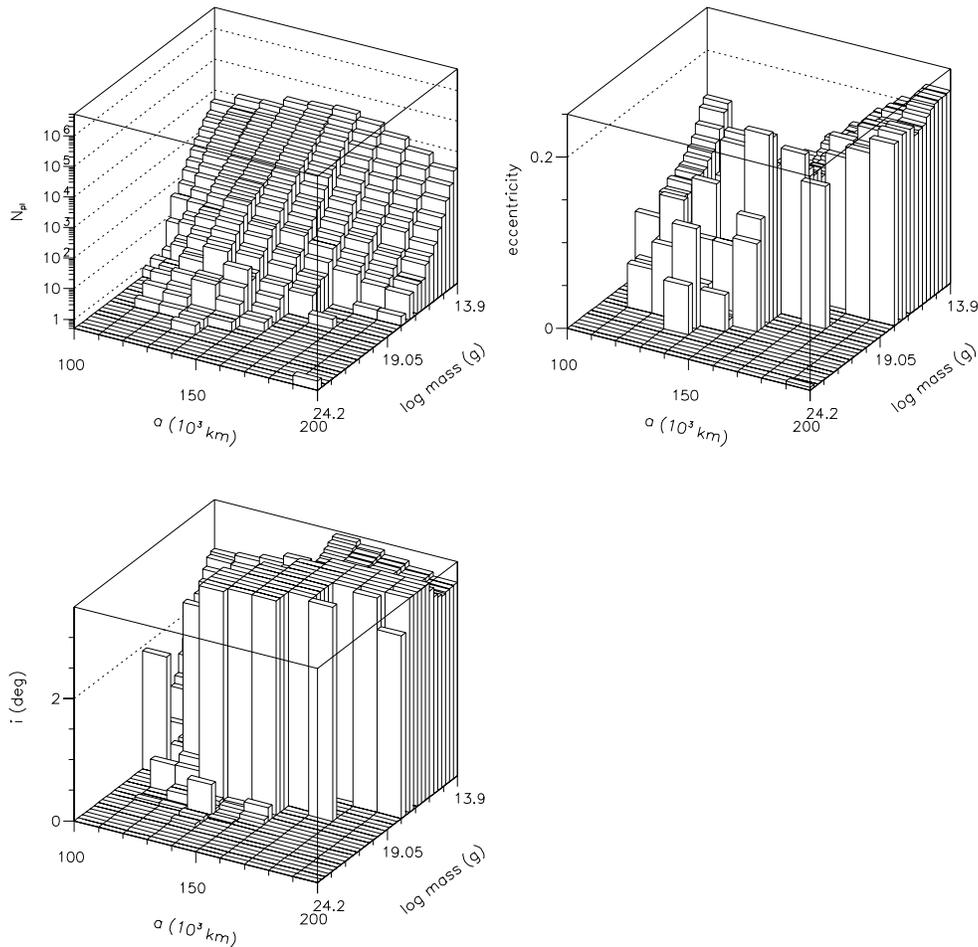
**Fig. 2.** Initial distribution of the Miranda breakup fragments used in the reaccumulation simulations. The three histograms show the number of planetesimals ( $N_{pl}$ ), the average eccentricity and inclination in each  $i, j$  bin, as a function of the Uranocentric distance in units of  $10^3$  km and logarithmic intervals of mass (each size bin spans a factor 2 in mass). The zones of semimajor axis in which the fragment population is divided have equal width ( $1 \times 10^4$  km) and range from  $1 \times 10^5$  to  $2 \times 10^5$  km.

The Marzari et al. (1995) collisional code allows us to derive the fragment size distribution complete down to 250 m of radius. For each fragment we compute an ejection speed normalized in such a way that 5 % of the impact energy is partitioned into ejecta kinetic energy. The orbital elements of the fragments are computed by adding to the fragment ejection speed the orbital velocity of the proto-Miranda at the moment of the breakup. The orbital elements of the proto-Miranda have been chosen in order to place the largest fragment after the breakup in the present Miranda location. The distribution of the satellite fragments with radius larger than 10 km in the  $a-e$  and  $a-i$  planes is shown in Fig. 1. Smaller fragments have higher eccentricities and inclinations, as expected when the Nakamura & Fujiwara (1991) mass-velocity distributions is used, while the largest remnants of the proto-Miranda breakup are clustered around the present Miranda location.

The output of the collisional code has been adapted to be used as input by the planet building code. The about one billion fragments are grouped in 28 size bins spanning a mass range from  $8 \times 10^{13}$  g to  $1 \times 10^{22}$  g and 10 zones covering a distance from Uranus between  $1 \times 10^5$  and  $2 \times 10^5$  km. In Fig. 2 we show in the form of 3-dimensional histograms the initial size and orbital distribution of the proto-Miranda fragments.

In simulating the reaccumulation of the satellite, we first consider only accumulation as outcome of a collision between two bodies (simulation ACC). This is because previous simulations of satellite accumulation and estimates of the reaccumulation timescale are based on this assumption. After  $\sim 20$  years the proto-Miranda fragments have grown in an orderly manner (runaway growth does not occur) to larger bodies (Fig. 3). The proto-Miranda core starts to accrete mass more rapidly than the neighboring large fragments and, as shown in Fig. 4, after 2000 years it has already reaccumulated almost all the mass of the original proto-Miranda and created a gap around its orbit. Only a small fraction of the original proto-Miranda is captured by Ariel (2%).

When in the numerical simulation we consider all the possible collisional outcomes (accretion, inelastic rebound, cratering and fragmentation), the evolution of the size distribution is significantly different from the case ACC. Collisional fragmentation dominates the initial stage of the Miranda fragments evolution; due to the consistent initial eccentricities and inclinations of the fragment orbits (see Fig. 1 and 2), most collisions are characterized by high impact velocities. Shattering of both projectile and target body occurs and the fragment size distribution is depleted of bodies smaller than few tens km, depending on the adopted impact strength and the evolution of the relative



**Fig. 3.** Size and orbital distribution of the Miranda fragments after 20 yr from the breakup of the parent body in simulation ACC (only accumulation is considered as outcome of a two-body collision).

velocities. Larger bodies are preserved due to their higher impact strength induced by the gravitational self-compression. We have performed different simulations assuming extreme values for the impact strength of the bodies: in simulations FS1 and FS2 we assume that the fragments are weak bodies, with strength  $S_0 = 1 \times 10^6$  erg/cm<sup>3</sup> and coefficient for the self-compression  $K = 1$ . In simulation FS3, we consider strong bodies with  $S_0 = 1 \times 10^7$  erg/cm<sup>3</sup> and  $K = 10$ . Simulation FS1 has initial conditions derived from the breakup of a 475 km parent body.

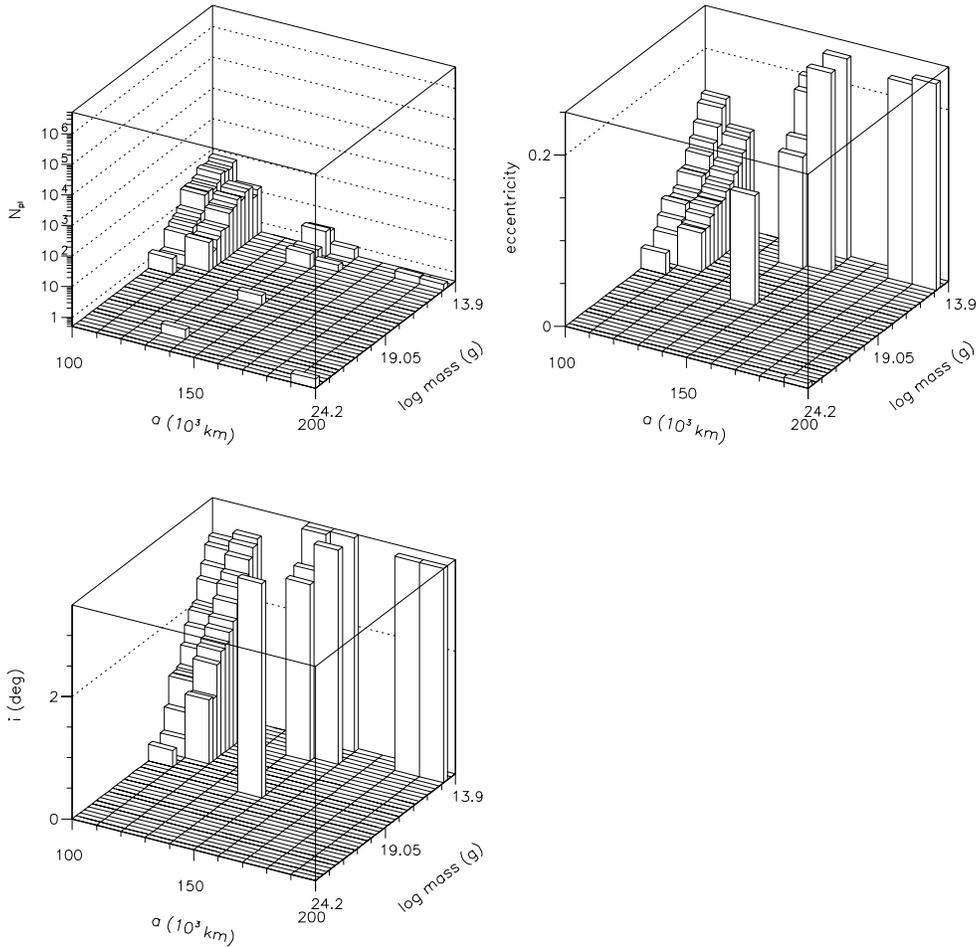
In Fig. 5 we show the size and orbital distribution of the proto-Miranda fragments after 20 yr from the collisional event. An initial intense shattering phase disrupts all bodies smaller than few tens km in diameter. A few large bodies survive due to their higher impact strength induced by the gravitational self-compression. At this stage most of the mass is in small fragments which populate the smallest size bins.

The subsequent evolution of the proto-Miranda swarm is characterized by the accretion of the small fragments by the large survivors of the intense shattering phase. These bodies, as they grow, collide each with another and generate at the end a single large body, the present Miranda, after about 2500 years from the breakup of the parent body (Fig. 6). In this simulation Ariel is particularly effective in subtracting mass from the reaccumulating Miranda by capturing about 44% of the total

mass of the proto-Miranda fragments. The reassembled body is in fact smaller in mass by almost a factor 2 with respect to the present satellite. As a consequence, to re-build Miranda as it is now when low values for the impact strength are adopted, it is necessary to assume that the proto-Miranda was at least twice as massive as the present satellite.

We performed a second simulation (FS2) with parameters given in Table 2. The size and orbital distributions are substantially similar to those shown Fig. 1 and 2 for FS1. In this case the reaccumulation of the satellite is faster (only 400 yr), due to the large amount of mass distributed into the fragments. Ariel traps 33% of the initial mass but at the end, when the ring of fragments is cleared, we are left with a re-assembled Miranda with mass equal to the present mass of Miranda.

In simulation FS3, when the bodies are more resistant to impact erosion, the reaccumulation process is substantially similar to that of simulation FS1 (we assume as in FS1 that the parent body was 475 km in diameter). In Fig. 7 we show the size and orbital distribution of the fragments after 20 years for FS3. The higher strength of the large bodies, survivors of the initial shattering phase, induces a faster growth of these bodies, with respect to FS1, and in particular of the largest fragment produced in the initial breakup. This body competes already in the initial stages with Ariel in capturing mass from the ring and grows



**Fig. 4.** Size and orbital distribution of the Miranda fragments after 2000 yr in simulation ACC. The new Miranda (mass equal to  $7 \times 10^{22}$ ) has reaccumulated at  $a = 135800\text{km}$ .

faster than in FS1. At the end of the reaccumulation process, the mass captured by Ariel is only 18% of the initial mass in the proto-Miranda swarm. In Fig. 8 we compare the growth of the largest fragment in the two cases FS1 and FS3. In FS3, after 1500 yr, the reaccumulated Miranda has reached about 86% of the present Miranda mass.

The reason why Ariel accretes a significant portion of the proto-Miranda mass, when fragmentation is considered, is due to the continuous transport of collisional fragments at larger Uranocentric-distances caused by the high rate of shattering events. If the collisional spreading of the fragments in semimajor axis is reduced, for example assuming a larger value for the coefficient  $\alpha$ , then the accretion by Ariel is less efficient. For larger values of  $\alpha$ , with respect to the standard  $9/4$ , the fragments produced in each shattering event are ejected with lower velocities and, as a consequence, the swarm of fragments is less dispersed in the orbital element space and the flux of mass towards Ariel is reduced. Miranda has then a larger mass supply and can grow faster. In simulation FS4 and FS5, we adopted the same parameters as in FS1 and FS3, respectively, but we increased  $\alpha$  to 4. With this value, the ejecta velocity are reduced by about a factor 2.8 respect to the standard case with  $\alpha = 9/4$ . By comparing simulation FS4 with FS1 (Table 2), a consistent reduction (more than a factor 2) of the mass fraction accreted by

Ariel is observed. A smaller but still significant decrease in the Ariel mass accretion rate is observed also when the parameter  $K$  is equal to 10 (compare FS5 with FS3 in Table 2).

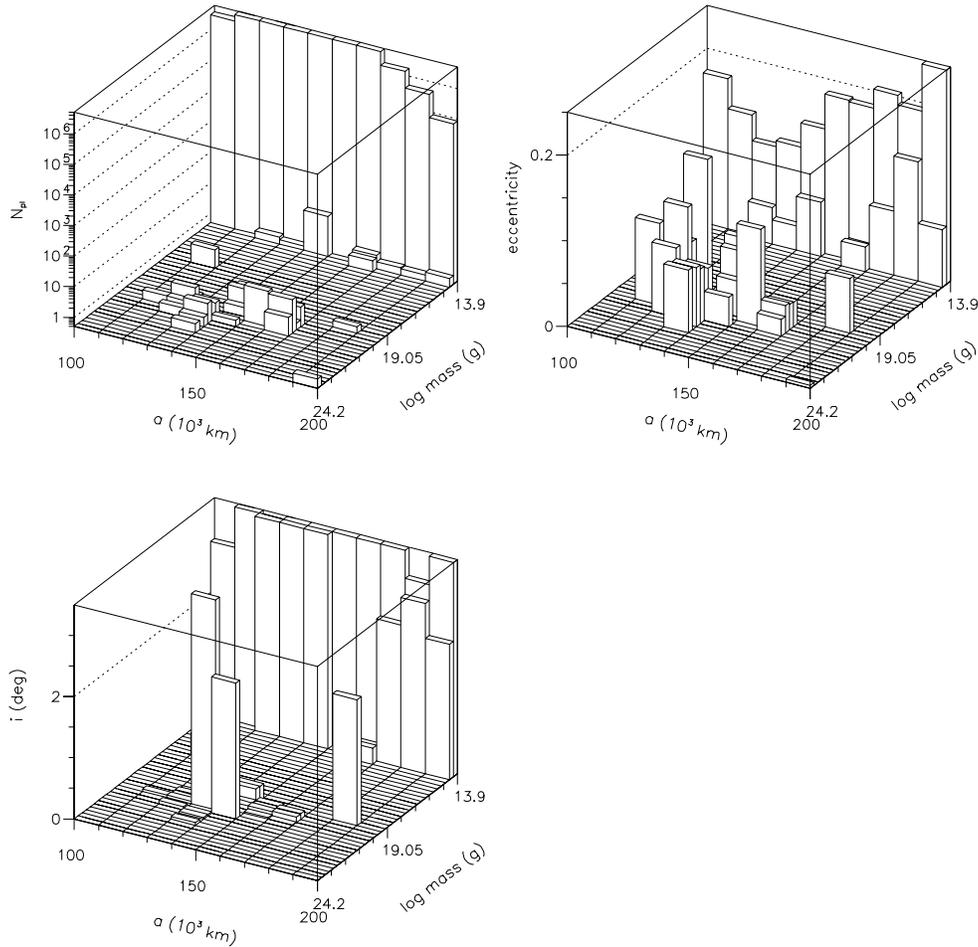
Simulations FS3, FS4 and FS5 are characterized by a shorter timescale compared to the ACC (only accumulation) simulation. In effect, the orderly growth of Miranda in ACC is slower than the runaway growth which distinguishes the FS simulations.

In all the models the percentage of mass loss through the borders of the initial collisional ring is a negligible fraction (maximum value 0.02%) of the total mass. Most of the outgoing flux of collisional fragments is in fact captured by Ariel before escaping out of the ring.

#### 4. Discussion and conclusions

Our numerical simulations confirm that the reaccumulation of a collisionally fragmented satellite is a fast process. By applying state of art algorithms to simulate the fragmentation and subsequent reaccumulation of Miranda, satellite of Uranus, we show how the re-assembling of the satellite occurs on a timescale of the order of  $10^3$  years.

We performed several numerical simulations to test the relevance of the assumed collisional model in the reaccumulation process. When only accumulation is considered as outcome of a two body collision, an orderly growth of the initial proto-



**Fig. 5.** Size and orbital distribution of the Miranda fragments after 20 yr for the FS1 simulation. Only few large fragments a few tens of km survive the initial intense shattering phase.

**Table 2.** Reaccumulation of Miranda.

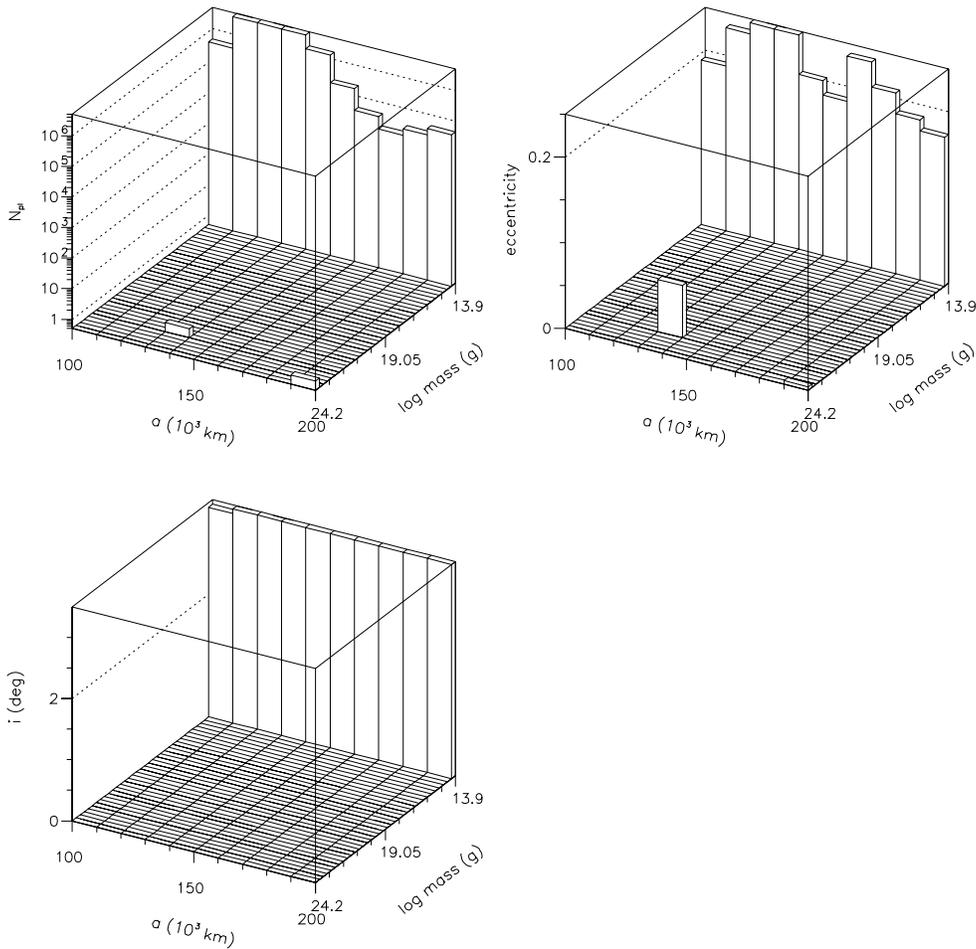
Simulation	Init. mass (g)	Time (yr)	Final mass of Miranda	% of mass on Ariel	$S_0$ erg/cm <sup>3</sup>	$K$	$\alpha$
ACC	$7.3 \times 10^{22}$	2000	$7.0 \times 10^{22}$	2%	-	-	-
FS1	$7.3 \times 10^{22}$	2500	$4.1 \times 10^{22}$	44%	$1.0 \times 10^6$	1	2.25
FS2	$1.5 \times 10^{23}$	400	$7.4 \times 10^{22}$	33%	$1.0 \times 10^6$	1	2.25
FS3	$7.3 \times 10^{22}$	1500	$6.1 \times 10^{22}$	18%	$1.0 \times 10^7$	10	2.25
FS4	$7.3 \times 10^{22}$	1800	$5.8 \times 10^{22}$	20%	$1.0 \times 10^6$	1	4
FS5	$7.3 \times 10^{22}$	1000	$6.3 \times 10^{22}$	14%	$1.0 \times 10^7$	10	4

Miranda fragments is observed. After about 2000 years from the breakup event, only the re-assembled satellite survives and it has a mass equal to the proto-Miranda mass. Ariel, which was included in the simulations, does not significantly influence the whole process.

When all the possible outcomes of a collision are considered, i.e. accumulation, cratering, shattering and inelastic rebound, the reaccumulation process is more complex. It is characterized by an initial stage (lasting few 10 years) dominated by fragmentation: all bodies smaller than few tens km are destroyed and a ring of small fragments is generated. Larger bodies survive this initial fragmentation phase because of their higher gravitational self-compression impact strength and grow by accreting mass from the fragment ring. In the final stage they impact onto

each other and a single new Miranda is re-built. In this second scenario, Ariel plays an important role by accreting significant portions of mass from the ring of small fragments. The fraction of mass subtracted by Ariel to the Miranda embryo depends strongly on the collisional parameters. It ranges from 18%, for high impact strength bodies, to about 44% for weak bodies and assuming the standard value of 9/4 for the velocity distribution index  $\alpha$ .

If we reduce the ejecta velocities, by increasing the parameter  $\alpha$  in the collisional model, then the mass accreted by Ariel is significantly less and ranges from 14 % to 20 %. In all these cases the pre-breakup Miranda had to be larger than the present Miranda by a fraction proportional to the mass accumulated by



**Fig. 6.** Size and orbital distribution of the Miranda fragments after 2500 yr (simulation FS1). The re-assembled Miranda has accumulated most of the mass from the initial ring of fragments.

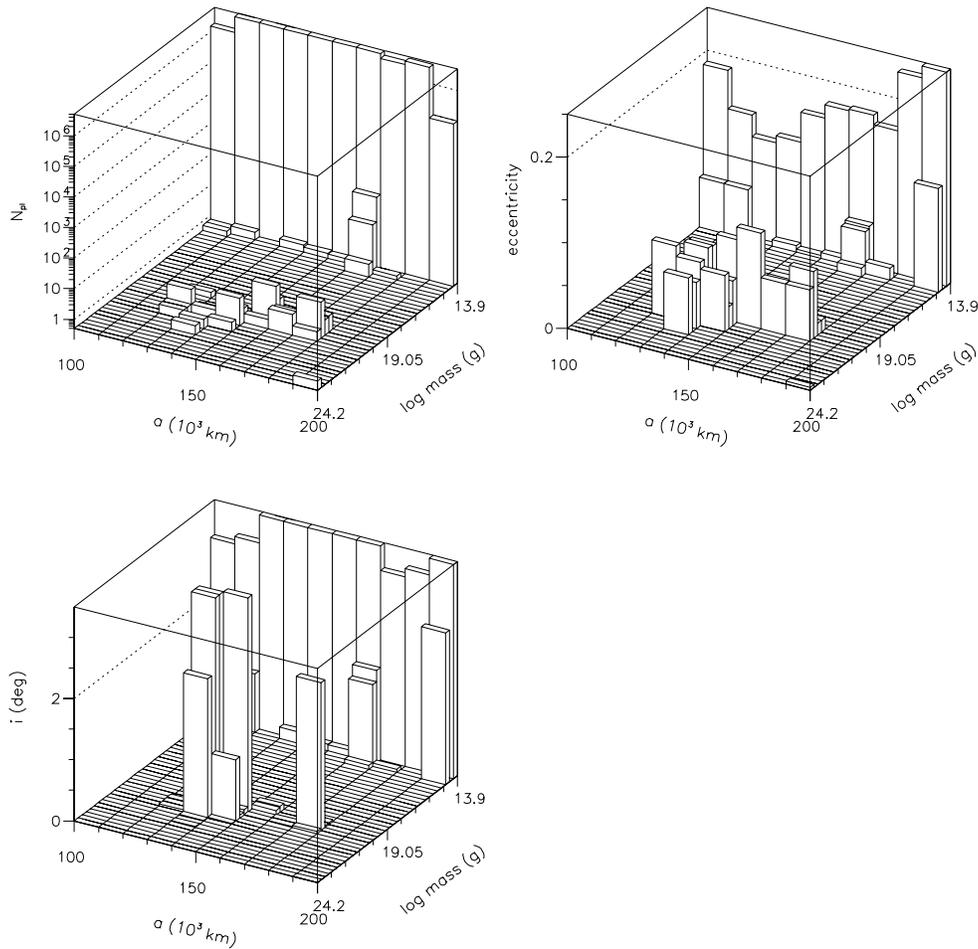
Ariel. Only a small amount of the initial mass is lost through the outer border of the fragment ring.

A full discussion of the implications of our results for the geology of the Uranian satellites is beyond the scope of this paper. As mentioned in the introduction, it has been claimed that there are two populations of craters on the satellites. Population I, with a shallow size distribution, is believed to be due to planetesimals and/or comets in heliocentric orbits, while Population II is deficient in large craters, and has been attributed to bodies in planetocentric orbits (Smith et al 1986; Strom 1987). Scaling Oberon's Population I crater density to the higher flux due to focusing by Uranus' gravity shows that Miranda's disruption was probable. It is tempting to ascribe the Population II craters to debris from that event. However, evidence for two distinct populations is uncertain. Crater counts by different researchers disagree, possibly due to limitations in coverage and resolution of Voyager images (McKinnon et al. 1991).

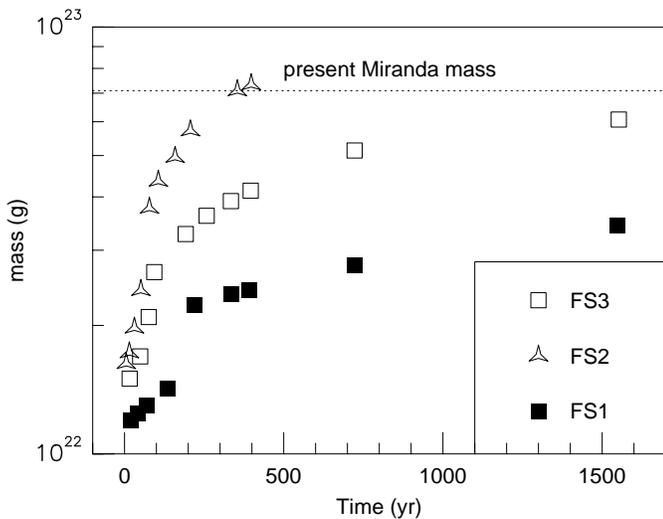
The timescale for reaccumulation of Miranda's debris is short. If Population II craters are due to disruption of Miranda, then they all would have essentially the same age (unless Miranda was disrupted more than once), and varied crater densities on terrains on a given satellite would be due to selective removal by endogenic processes, rather than different ages of surface units. While this scenario may not be ruled out (most

of the Uranian satellites show evidence for tectonic and/or cryovolcanic activity), it seems unlikely that resurfacing operated in such similar fashion on different satellites. Miranda itself shows a wide variety of crater densities on different surface units, and endogenic features that may be due to processes associated with reaccumulation (accretional heating, changes in rotational state, viscous relaxation of nonhydrostatic stresses, etc). If some of Miranda's craters are Population I, they might be explained by continued bombardment by heliocentric projectiles after its reaccretion. In that case, however, it is difficult to explain why there are relatively few Population I craters on Ariel.

Croft & Soderblom (1991) ascribe the lack of large craters on Ariel to a global resurfacing that mantled and buried pre-existing craters. In some of our simulations, the mass accreted from the disrupted Miranda by Ariel is a significant fraction (tens of percent) of the total, and would have covered its surface to a depth of several kilometers. It may be significant that smaller fragments from the disruption event tend to have higher speeds; the material impacting Ariel would have a different size distribution than that which reaccreted as Miranda. It is possible that Ariel was blanketed with fine material, while Miranda's "Population I" craters were due to large impactors, although both had the same source. The later (Population II) craters on



**Fig. 7.** Size and orbital distribution of the Miranda fragments after 20 yr for the FS3 simulation.



**Fig. 8.** Evolution of the mass of the Miranda embryo as a function of time for simulations FS1, FS2 and FS3. In FS3 the higher impact strength of the large bodies reduce the collisional erosion and allows a faster growth respect to FS1. The dotted line represents the mass of the present Miranda.

Ariel would then be due to cometary bodies in heliocentric orbits, possibly with some planetocentric component from sub-catastrophic basin-forming impacts on one or more satellites. The latter cannot be ruled out (only half of each satellite was imaged by Voyager), and would allow a range of ages for Population II craters. Full imaging of all the Uranian satellites at high resolution will be needed to decipher their geological histories.

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