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Global reduction with proper motions for meridian observations

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Abstract. When we deal with astrometric observations spread over periods longer than a few years, it is of major importance to take into consideration the proper motions of all objects involved in the reduction process.

Not to consider such quantities, mainly when they are poorly known to the majority of the objects, may significantly degrade the final results of the reduction.

In this paper, we present a way to treat this problem. As an example, we reduced a set of over 170 000 observations made with the Bordeaux photoelectric automatic meridian circle, collected during almost ten years of work of the instrument.

In our reduction model, proper motions were included as unknowns, along with positions and instrumental parameters.

The method shown here, which we call *global technique*, is an extension of the overlap method introduced by Eichhorn (Eichhorn 1960), which was developed by many authors afterwards.

Key words: astrometry – methods: numerical

1. Introduction

Meridian observations are generally organized into series, which usually correspond to the data collected during a whole night of work.

When strictly differential astrometric observations are reduced in the traditional way, that is to say series-by-series reductions, it is necessary to consider two different classes of objects in each of these series (or nights): *reference* and *program* stars. Reference stars are those objects whose positions must be previoulsy known. Program stars are those whose positions will be derived from the reference ones.

The traditional or *classical* reduction consists basically of two steps. First, instrumental parameters are determined by a least squares solution involving reference stars only. It should be emphasized that, in this step, reference stars are regarded as having their positions free from errors. Next, positions to all program stars are calculated from these instrumental parameters.

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Although the classical approach has always provided good results, some weak points can be outlined from its technique: loss of information, since instrumental parameters are determined only from the reference objects; strong dependence on the reference catalogue, since reference objects are regarded as having their positions perfectly known; finally, information is not shared among series. For these reasons, considerable improvement, without further difficulties, can be achieved if the whole data is treated altogether.

In this way, an efficient solution to this problem is provided by the global reduction technique, which overlaps series through iterations of a unique system of equations, to all observations, in each coordinate. This is possible due to the existence of common stars among these series. Therefore, information is constantly shared as iterations run and each observation is a real contribution to the determination of stellar and instrumental parameters (Benevides-Soares & Teixeira 1992, Teixeira et al. 1992).

Consequently, the list of unknowns includes the corrections $\Delta\alpha\cos\delta$ and $\Delta\delta$ to the positions of <u>all</u> objects, putting them on the same footing. Furthermore, since observations are spread over a considerably long period of time, of about 10 years, and the proper motions of most of the objects were poorly known, we felt the need to include the corresponding corrections in our list of unknowns. We also included corrections to the proper motions of the fundamental stars, to provide a standard against which the results could be checked afterwards.

The rather strong linking provided by the overlapping condition does not preclude the need for fixing the zero points in right ascension and declination, as well as two additional parameters, by external constraints. These will be explicited later in the text.

Finally, it is worth remembering that the new CCD meridian circles observe a huge number of objects whose positions and proper motions are, in general, unknown, requiring reduction techniques that meet the considerations outlined here.

2. Reduction

The reduction model is written to all objects as follows:

$$m_k\cos\delta_{p_{k,i}} + n_k\sin\delta_{p_{k,i}} + c_k + \dot{c}_k\Delta t_{k,i} - (\Delta\alpha_p\cos\delta_{p_{k,i}} + \Delta\mu_{\alpha_p}\Delta T_{k,i}\cos\delta_{p_{k,i}}) = (\alpha_{p_{k,i}} - TS_{k,i})\cos\delta_{p_{k,i}} + r_{k,i} \quad (1)$$

and

$$R_k \tan z_{p_{k,i}} + f_k \sin z_{p_{k,i}} + P_k + \dot{P}_k \Delta t_{k,i} - (\Delta\delta_p + \Delta\mu_{\delta_p}\Delta T_{k,i}) = (z^*_{k,i} - z_{k,i}) + r'_{k,i}, \quad (2)$$

for right ascension and declination respectively. Since the global reduction technique considers a unique system containing all the observations, we identified in Eqs. (1) and (2) a series by the index *k*, a transit within series *k* by the index *i*, and a particular star by the index *p*. Least squares residuals are given by $r_{k,i}$ and $r'_{k,i}$ for right ascension and declination, respectively. The meaning of each unknown in Eqs. (1) and (2) is explained below:

$\underline{\alpha}_{p_{k,i}}$ and $\underline{\delta}_{p_{k,i}}$ are the approximately known right ascension and declination of the object, used to point the meridian instrument.

$\underline{m}_k$, $\underline{n}_k$, $\underline{c}_k$ are Bessel parameters, where $\underline{m}_k$ also contains the clock correction.

$\underline{TS}_{k,i}$ is the observed sidereal time of the upper culmination.

$\underline{z}^*_{k,i}$ is the circle reading, corrected for division errors and added to the micrometer position. Also, a correction has been added due to refraction, which is calculated as a function of pressure, temperature and humidity measured on the ground during the observation.

$\underline{z}_{k,i}$ is the zenith distance, calculated from δ, that is, $z = \phi - \delta$, where ϕ is the mean latitude of the observatory.

$\underline{R}_k$ is the correction to the adopted refraction coefficient.

$\underline{f}_k$ is the amplitude of the instrument flexure.

$\underline{P}_k$ is the offset between the instrumental and the instantaneous pole.

$\underline{\dot{c}}_k$ and $\underline{\dot{P}}_k$ were introduced to take into account a possible time-dependent behaviour of *c* and *P*, respectively.

$\underline{\Delta T}_{k,i}$ is the difference (observation epoch - mean epoch of observations) for a particular object.

$\underline{\Delta t}_{k,i}$ is the difference between the sidereal time of the beginning of a series and that of the transit of its respective object.

$\underline{\Delta\alpha}_p$ and $\underline{\Delta\delta}_p$ are corrections to the right ascension and declination, respectively.

$\underline{\Delta\mu}_{\alpha_p}$ and $\underline{\Delta\mu}_{\delta_p}$ are corrections to the proper motions in right ascension and declination, respectively.

The least squares solution is accomplished by means of the Gauss-Seidel iteration. Instrumental parameters are considered constants for each series. Corrections $\Delta\alpha$, $\Delta\delta$, $\Delta\mu_\alpha$, and $\Delta\mu_\delta$ are constants throughout the reduction process (Benevides-Soares & Teixeira 1992, Teixeira et al. 1992).

In this way, the calculations made in the $j^{\underline{th}}$ step of the iterations can be described as follows, through matrix notation, taking as example the right ascension model given in Eq. (1):

$$m_k^{(j)}\cos\delta_{p_{k,i}} + n_k^{(j)}\sin\delta_{p_{k,i}} + c_k^{(j)} + \dot{c}_k^{(j)}\Delta t_{k,i} - (\Delta\alpha_p^{(j-1)}\cos\delta_{p_{k,i}} + \Delta\mu_{\alpha_p}^{(j-1)}\Delta T_{k,i}\cos\delta_{p_{k,i}}) = (\alpha_{p_{k,i}} - TS_{k,i})\cos\delta_{p_{k,i}} + r_{k,i}^{(j)}. \quad (3)$$

Now, we shall rewrite the system expressed by Eq. (3) as:

$$[M : A : B] \begin{bmatrix} X^{(j)} \\ Y^{(j-1)} \\ W^{(j-1)} \end{bmatrix} = Z + r^{(j)}, \quad (4)$$

where *M*, *A* and *B* are coefficient matrices for their respective unknowns, represented by the column vectors *X*, *Y* and *W*. A better insight into Eq. (4) can be obtained by a closer inspection of its components.

Matrix *M* is block-diagonal, where each diagonal block contains the values $\cos\delta_{p_{k,i}}$, $\sin\delta_{p_{k,i}}$, 1 and $\Delta t_{k,i}$ for a particular series *k*. Outside these blocks there are only zero values. Matrix *A* contains the coefficients for $\Delta\alpha_p\cos\delta_{p_{k,i}}$ for each observation, so that it has only 1's and 0's conveniently placed. Similarly, but now for the proper motions, *B* has only ΔT's and 0's.

The column vector *X* contains the unknowns m_k, n_k, c_k and $\dot{c}_k$ for each series *k*. The same holds for *Y* and *W* when $\Delta\alpha_p\cos\delta_{p_{k,i}}$ and $\Delta\mu_{\alpha_p}\cos\delta_{p_{k,i}}$ are concerned.

The second member of Eq. (4) is, obviously, the observational data vector *Z* and the least squares residual fit one *r*. The $j^{\underline{th}}$ iteration step is given by

$$X^{(j)} = (M^T M)^{-1} M^T (Z - AY^{(j-1)} - BW^{(j-1)}) \quad (5)$$

$$Y^{(j)} = (A^T A)^{-1} A^T (Z - MX^{(j)} - BW^{(j-1)}) \quad (6)$$

$$W^{(j)} = (B^T B)^{-1} B^T (Z - MX^{(j)} - AY^{(j)}). \quad (7)$$

The matrices $(A^TA)^{-1}A^T$ and $(B^TB)^{-1}B^T$ act as an averaging operator over the residuals $(Z - MX^{(j)} - BW^{(j-1)})$ and $(Z - MX^{(j)} - AY^{(j)})$ respectively, so that each component of $Y^{(j)}$ and $W^{(j)}$ is explicitly given by

$$\Delta\alpha_p^{(j)}\cos\delta_{p_{k,i}} = \frac{\sum_{k=0}^{N} {}^*r_{k,i}^{(j)}}{N} \quad (8)$$

and

$$\Delta\mu_{\alpha_p}^{(j)}\cos\delta_{p_{k,i}} = \frac{\sum_{k=0}^{N} {}^{**}r_{k,i}^{(j)}\Delta T_{k,i}}{\sum_{k=0}^{N}(\Delta T_{k,i})^2} \quad (9)$$

where, to each object:

${}^*\underline{r}_{k,i}^{(j)}$ is a component of the residual $(Z - MX^{(j)})$ obtained in the $j^{\underline{th}}$ step, similarly to a classical reduction, subtracted from its temporal variation component contained in $BW^{(j-1)}$, calculated in the previous step.

${}^{**}\underline{r}_{k,i}^{(j)}$ is also a component of the residual $(Z - MX^{(j)})$ obtained in the $j^{\underline{th}}$ step as mentioned before, now subtracted from constant corrections $AY^{(j)}$ applied to the respective object.

It is clear that Eq. (9) is a least squares estimator for the temporal dependence of ${}^{**}r_{k,i}^{(j)}$. N is the number of observations of star p.

The $(j+1)^{\underline{th}}$ step is accounted by correcting the second member of Eq. (3) with the results from the $j^{\underline{th}}$ one and then restarting the calculations (Eqs. (5), (6) and (7) for $j = j+1$). The first step of the iterations may start by setting both corrections to positions and proper motions equal to zero. In order to obtain a faster convergence, we adopted as $\Delta\alpha_p^0$ the results from a classical reduction for these corrections. Corrections $\Delta\mu_{\alpha_p}^0$

Table 1. Objects and observations

Program	Objects	Observations
FK5	1057	56270
HIPPARCOS	7273	52052
Radio stars	242	8653
Asteroids	138	5015
Faint stars,		
Quasar 3C273,		
Stars for occultation, etc.	8564	56466
Total	17274	178456

were set equal to zero. It is important to notice that for a particular step j, these calculations represent a classical reduction. A detailed discussion on the relation between the classical and global reductions can be found in Benevides-Soares & Teixeira (1992).

Finally, since the system provided by Eq. (4) is rank deficient, the results from the reduction are just a particular solution of the problem. As a consequence, final results are given by:

$$m_k + u_1$$
$$n_k + u_2$$
$$c_k + u_3$$
$$\Delta\alpha_p \cos\delta_p + u_1 \cos\delta_p + u_2 \sin\delta_p + u_3$$

where the set of constants u_1, u_2, u_3 is obtained through constraints over the final (global) corrections of the FK5 stars, namely:

$$\sum \Delta\alpha = \sum \Delta\alpha \sin\delta = \sum \Delta\alpha \cos\delta = 0. \qquad (10)$$

The unknowns $\Delta\mu_{\alpha_p} \cos\delta_p$ and $\dot{c}_k$ remain unchanged since they are origin independent.

The declination reduction is very similar to the right ascension one, and a full description of the whole process is found in Teixeira (1990), Teixeira et al. (1992) and Camargo (1997).

3. Observational data

In this work, we used observations made with the Bordeaux automatic meridian circle (Réquième & Mazurier 1986) from 1^{st} December 1983 to 29^{th} May 1992. Many observational programs were developed during those years, of which the most important was the construction of the input catalogue for the HIPPARCOS mission (Perryman et al. 1989). Table 1 summarizes these programs, as well as the effort devoted to each one. It is interesting to notice that the FK5 stars were the most observed objects, even though they were not the main target.

In this work was considered any object, except solar system ones, with 3 or more observations whose residuals were within an interval of 3σ, which was obtained from a classical reduction. As we said before, such reduction preceeded the global one, where the derived corrections to positions were used to allow a faster convergence of the iterations.

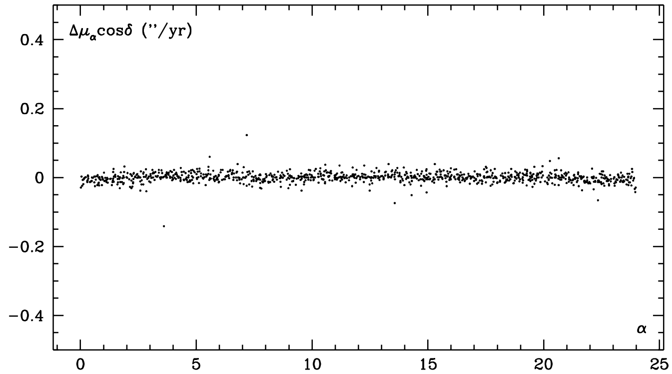

Fig. 1. Proper motion differences in the sense Bordeaux - FK5

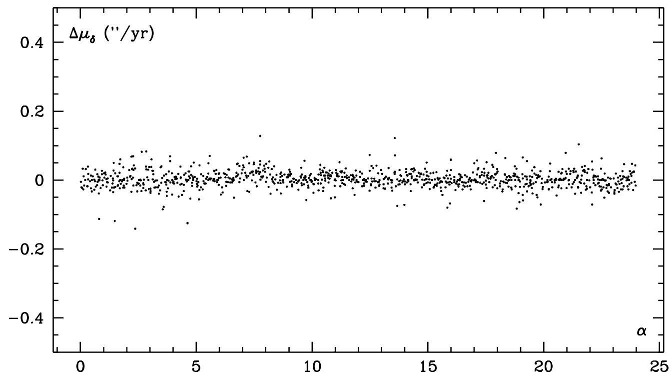

Fig. 2. Proper motion differences in the sense Bordeaux - FK5

Table 2. Precions for a single observation

Program	$\sigma_\alpha \cos\delta('')$	$\sigma_\delta('')$
FK5	0.09_4	0.15_4
HIPPARCOS	0.10_1	0.15_7
Radio stars	0.09_8	0.15_4
General	0.10_0	0.15_5

Corrections $\Delta\mu_\alpha \cos\delta$ and $\Delta\mu_\delta$ to proper motions were obtained to every object satisfying a minimum of eight observations spread over a time lag of at least two years. About one third of our data set was under these requisites, where the majority were non-FK5 stars and over 1500 objects had no previous determinations for their proper motions.

4. Discussion

Here, we are aiming to test the consistency and coherence of our results in proper motions, as well as the applicability of this technique.

The positional results did not differ much from those presented in Teixeira (1990) and Teixeira et al. (1992). The precision for a single observation is shown in Table 2.

4.1. FK5 stars

Proper motion deviations relative to the FK5 stars were regarded as a diagnostic to our reduction method coherence and consis-

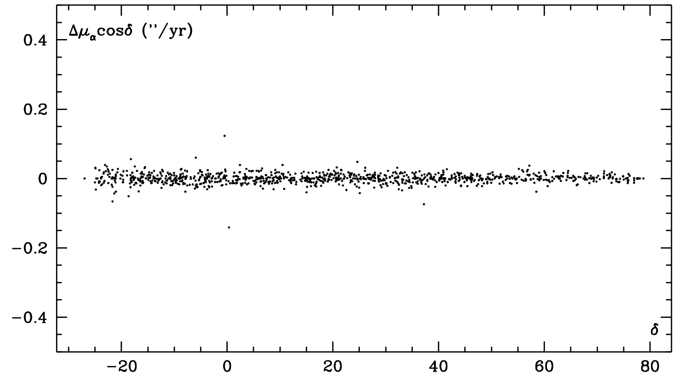

Fig. 3. Proper motion differences in the sense Bordeaux - FK5

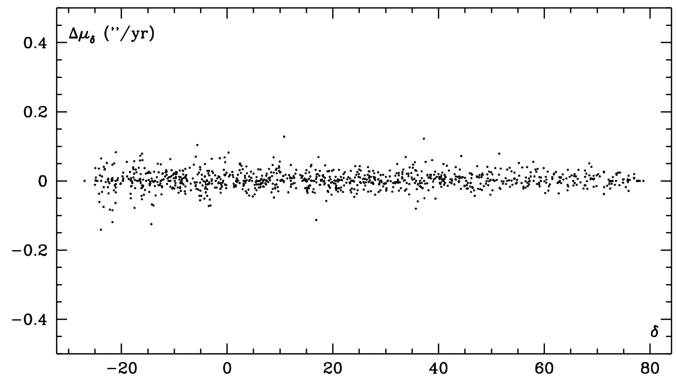

Fig. 4. Proper motion differences in the sense Bordeaux - FK5

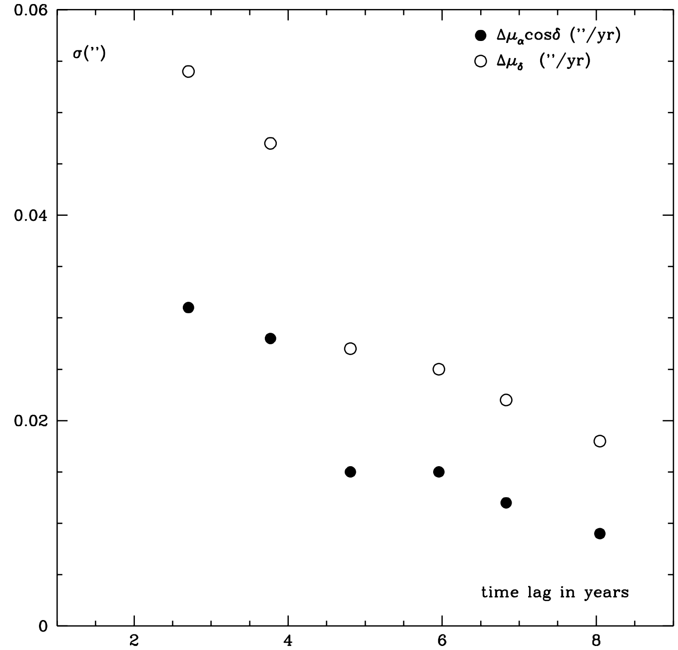

Fig. 5. Estimate for the precision of the proper motions as a function of the time lag – FK5 stars

tency, due to the good qualities of the proper motions contained in this catalogue.

In Figs. 1 to 4, each dot stands for as a deviation $\Delta\mu_\alpha\cos\delta$ and $\Delta\mu_\delta$ to the FK5 stars. One can notice that the differences are close to and well distributed around the zero value. In addition, no relevant systematic trend is depicted when such differences are given as a function of the equatorial coordinates α and δ.

The few points that tend to depart from the general trend deserved a closer investigation, and in all cases at least one out of two characteristics could be outlined: either a poor temporal distribution or a small number of observations when compared to other FK5 stars.

In Figs. 3 and 4, a larger dispersion is noticed for the low declination results. This fact is closely related to the large zenith distances that such observations were made, where atmospheric effects prevent precision from quality. It is worth remembering that Bordeaux observatory is close to latitude 44^oN.

The mean square sum of the deviations $\Delta\mu_\alpha\cos\delta$ and $\Delta\mu_\delta$ relative to the FK5 stars provide an estimate for the precision obtained from our reduction method for these parameters:

$$< (\Delta\mu_\alpha\cos\delta)^2 >^{1/2} = 0".01_5/\mathrm{yr}$$
$$< (\Delta\mu_\delta)^2 >^{1/2} = 0".02_6/\mathrm{yr}.$$

The above numbers should not be understood as representative to all objects, for the FK5 stars were the most intensively observed ones (about 30 observations per object), having a mean time lag of about six years.

Fig. 5 shows how the mean square sum of these deviations varies as a function of the time lag for the FK5 stars. It is no surprise that the results become better as the time lag increases. Fig. 5 also shows that if larger lags (more than ten years) were available, one could infer that precisions of the order of those contained in catalogues like FK5, PPM and TYCHO, for example, would be possible mainly in right ascension. The difference between both curves reflects a better quality for right ascension observations, rather than those in declination. Such feature is particular to the Bordeaux observatory.

4.2. HIPPARCOS stars

Another way to verify the quality of our results was achieved by the direct comparison, as shown in Figs. 6 and 7 of the calculated proper motions of about one thousand and five hundred non-FK5 stars from our data set common to the HIPPARCOS main catalogue (ESA 1997). Although, in this case, the average time lag (3.4 years) as well as the average number of observations (11 per object) are not as favourable as they were for the FK5 stars, there is still a quite good agreement between the data, which clearly reinforces the quality of the global technique results. The mean square sum of the differences Bordeaux-HIPPARCOS, for proper motions in both coordinates, is given by

$$< (\Delta\mu_\alpha\cos\delta)^2 >^{1/2} = 0".03_7/\mathrm{yr}$$
$$< (\Delta\mu_\delta)^2 >^{1/2} = 0".05_9/\mathrm{yr}.$$

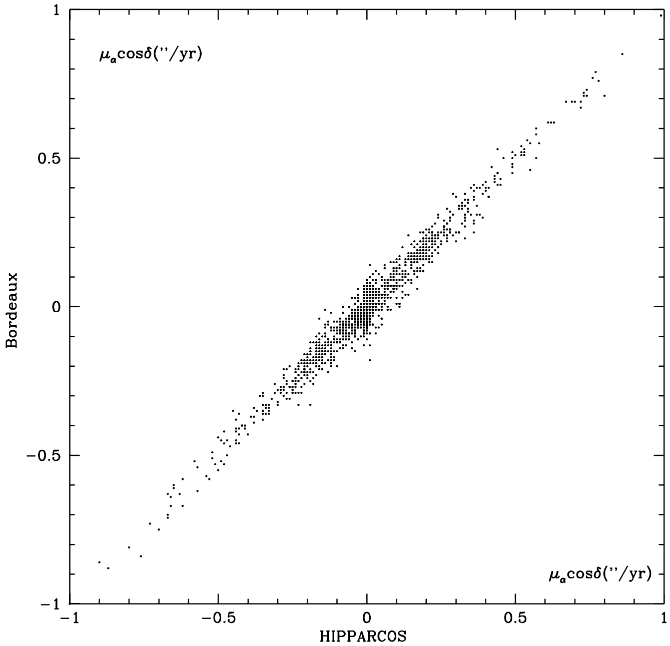

Fig. 6. HIPPARCOS vs Bordeaux proper motions in right ascension

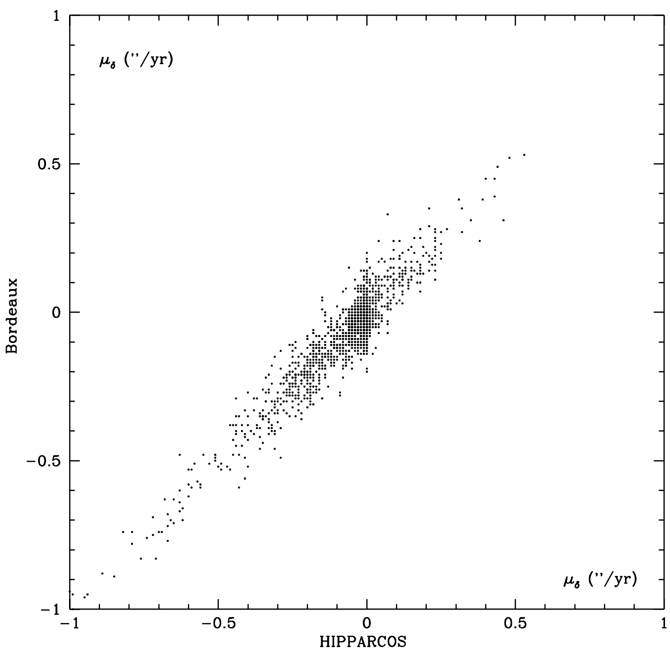

Fig. 7. HIPPARCOS vs Bordeaux proper motions in declination

Again, as in the case of the FK5 stars, Fig. 8 shows that results become better as time lag increases.

5. Conclusions

Our results show the reliability and feasibility of the global reduction method, besides providing a technique where new proper motions can be easily derived.

The quadratic average of the derived proper motions, for the objects with no previous estimates, is 0".05/yr, for $\mu_\alpha \cos\delta$, and 0".07/yr, for μ_δ. With these figures in mind, it becomes clear that neglecting the proper motion contribution will imply errors larger than the average observation accuracy, as soon as the interval is longer than two years.

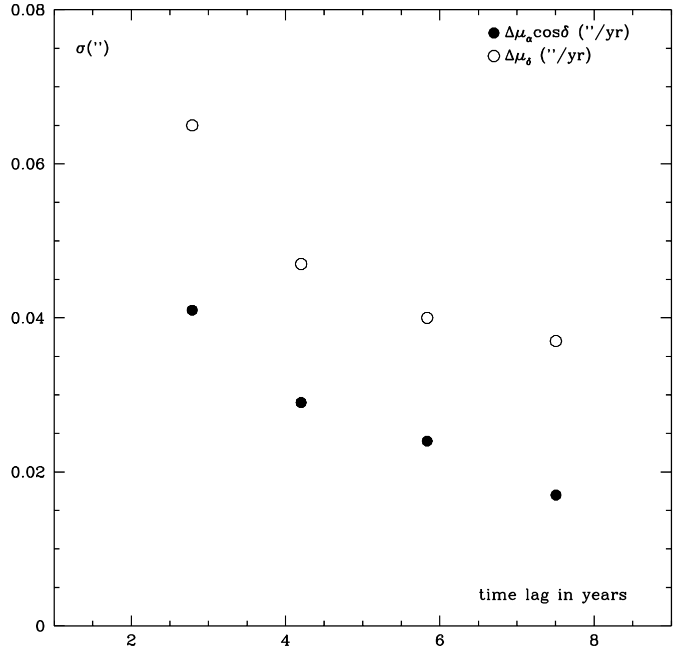

Fig. 8. Estimate for the precision of the proper motions as a function of the time lag – HIPPARCOS stars

Obviously, better values for the proper motions could have been derived, in terms of precision, if Bordeaux observational programs had been oriented to the determination of such quantities, providing larger time lags and more uniform temporal distribution for the observations.

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References

Benevides-Soares, P., Teixeira, R., 1992, A&A, 253, 307.
Camargo, J. I. B., 1997, "Análise das Observações Meridianas Fotoelétricas do Observatório de Bordeaux", MSc dissertation.
Eichhorn, H., 1960, Astron. Nachr. 285, 233.
ESA, 1997, The HIPPARCOS and TYCHO Catalogues, ESA SP-1200.
Perryman, M. A. C. et al., 1989, ESA SP - 1111, vol. 1,2 and 3.
Réquième, Y., Mazurier, J. M., 1986, IAU *Symp.* 109 "Astrometric Techniques", 435.
Teixeira, R. 1990, "Traitement Global des Observations Méridiennes de l'Observatoire de Bordeaux", PhD thesis.
Teixeira, R., Réquième, Y., Benevides-Soares, P., Rapaport, M., 1992, A&A, 264, 307.