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Generation of magnetic seed fields in protogalactic clouds due to plasma-neutral gas friction

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Abstract. We show that plasma - neutral gas friction in a weakly ionized rotating protogalactic system creates magnetic fields of seed field order. For these purposes we present results of 2-fluid simulations. Considering primordial gas clouds, this mechanism yields a magnetic field of the order of $10^{-15}G$ on time scales of $10^{6}yr$ and spatial scales of several hundred parsec. Such fields are a reasonable input for further amplification processes.

Key words: galaxies: magnetic fields – galaxies: formation – methods: numerical – cosmology: miscellaneous

1. Introduction

Observations show that large scale magnetic fields of the order of μG are a typical phenomenon in galaxies and galaxy clusters (e.g. Kronberg 1994; Beck et al. 1996; Lesch and Chiba 1997). The field strength exhibits almost no cosmological evolution. Magnetic fields in objects like damped Lyα at redshifts $z \approx 2-3$ and nearby spiral galaxies have similar field strengths of several μG (Kronberg et al. 1992; Wolfe et al. 1992). In principle, these fields can be understood as a consequence of some dynamo process together with field amplification due to the galalactic collapse (e.g. Lesch & Chiba 1995).

However, the concept of galactic dynamos demands for the existence of protogalactic magnetic seed fields. It is important that these seed fields are strong enough (i.e. $10^{-15} - 10^{-20}G$) for amplification up to μG on time scales of the order $10^{8}yr$. So far no self-consistent model is available for the first steps of the galactic magnetization. Whereas many simulations have been performed, where some initial magnetic flux was assumed (see Beck et al. 1996 and references therein), a self-consistent treatment of the seed field generation and subsequent amplification via magnetohydrodynamical mechanisms is still missing.

Several mechanisms for the generation of magnetic seed fields at different epochs of the universe are discussed in the literature. Here we concentrate on electrodynamic processes relevant after the period of recombination. During this era, seed fields may be created in the gravitational field of a rotating plasma due to different inertia of electrons and ions (Biermann 1950). A second battery process is based on the different interactions of electrons and ions in a rotating plasma with the intense cosmic background radiation (Mishustin & Ruzmaikin 1973).

More recently, a third elementary mechanism for the self-generation of magnetic fields in weakly ionized protogalactic plasmas was proposed by Lesch & Chiba (1995). Originally, this model was developed by Lesch et al. (1989) for active galactic central regions and later by Huba & Fedder (1993) in the context with the generation of magnetic fields in unmagnetized planetary ionospheres.

In this approach magnetic fields are created due to collisional interactions between plasma and neutral gas in protogalactic shear flows. From a kinematic point of view the resulting seed fields can be estimated to be of a reasonable order of magnitude (e.g. $\approx 10^{-20}G$, depending on the system under consideration). However, self-consistent dynamical studies of this process in protogalactic systems are still missing.

The generation of magnetic fields in protogalactic objects has to be investigated in the global scenario of galaxy formation. The origin of the damped Lyman α- systems (DLAS) is still under debate. They have often been interpreted as large (radii up 100 kpc) high-redshift progenitors of present-day spirals (e.g. Wolfe et al. 1995). Most recently Prochaska and Wolfe (1997) have investigated a variety of models for the spatial distribution and kinematics of the absorbing gas to test whether they could produce the observed absorption line profiles of low ionisation species. They concluded that only a rotating thick disk model can explain the large velocity spreads (up to 200 km s^{-1}) and the characteristic asymmetries of the observed absorption lines. They concluded that this result is inconsistent within a cold dark matter (CDM) structure scenario (e.g. Frenk et al. 1990).

Another approach was used by Haehnelt et al. (1997). They showed via hydrodynamic simulations of galaxy formation in a cosmological context, that the superposition of irregular protogalactic gas clumps can reproduce the observed velocity width distribution and asymmetries of the absorption profiles. Such clumps are an essential prediction of all CDM-models of hierachical structure formation (e.g. Kauffmann 1996). The importance of structure formation on spatial scales much smaller then the DLAS was also shown by several models of reheating of

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the intergalactic medium (Tegmark et al. 1994; Tegmark et al. 1997). It is shown that the first structures which become non-linear at redshift between $z \simeq 100 - 15$ no more massive than about $10^3 - 10^6 M_\odot$ may responsible for the reheating of the intergalactic medium, thereby fulfilling the constraints given by the COBE observations (Smoot et al. 1992). Such clouds would have a size of about 100-200 pc and they would present the basic building blocks for galaxies.

In this context we consider in our contribution the generation of magnetic fields in such clouds as a first step towards a more complete model for the large-scale magnetic fields in more evolved structures like DLAS. The primordial clouds are supposed to contain a mixture of neutral and ionized gas (Blanchard et al. 1992). Due to tidal interaction they can attain some angular momentum, which afterwards add to the global rotation of fully developed galaxies. We note that the tidal torque scenario is clearly supported by observational finding of intense merging of protogalactic clumps at high redshifts (e.g. Lavery et al. 1996). According to several observational programs the merging rate increases as $(1+z)^m$, all consistent with $m = 3.5 \pm 1$. Since the galaxy density at high redshifts was obviously drastically higher than today and since gravitational torques easily induce shear flows, the appearance of shear flow driven neutral gas-plasma interactions is quite natural.

In the present paper we discuss first results of self-consistent 2-fluid simulations considering the creation of magnetic seed fields due to plasma-neutral gas interaction. We show that these simulations yield both qualitative and quantitative very satisfying results. Assuming parameters typical for rotating protogalactic gas clumps in Lyα clouds we find magnetic fields of the order of $10^{-15} G$ generated on time scales of $10^6 yr$.

The paper is organized as follows. In the following section we discuss basic aspects of the creation of magnetic fields in a weakly ionized protogalactic plasma / neutral gas system with shear flow. A quantitative example of relevance with respect to rotating primordial clouds in DLAS is shown in Sect. 3. In the final section we summarize and discuss our results.

2. Consequences of protogalactic shear flow

To illustrate the consequences of shear flow in a weakly ionized protogalactic plasma with respect to the self-generation of magnetic seed fields, we start from the following set of balance equations for a 3-fluid system consisting of electrons, ions and neutrals (cf. Birk & Otto (1996) for a detailed derivation of the plasma - neutral gas balance equations)

$$\frac{\partial \rho_\alpha}{\partial t} = -\nabla \cdot (\rho_\alpha \mathbf{v}_\alpha) + Q_\alpha^C \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_\alpha \mathbf{v}_\alpha) = -\nabla \cdot (\rho_\alpha \mathbf{v}_\alpha \mathbf{v}_\alpha) - \nabla p_\alpha + \mathbf{F}_\alpha + \mathbf{v}_\alpha Q_\alpha^C + \mathbf{Q}_\alpha^P \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_\alpha \epsilon_\alpha) = -\nabla \cdot (\rho_\alpha \epsilon_\alpha \mathbf{v}_\alpha) - p_\alpha \nabla \mathbf{v}_\alpha + Q_\alpha^E. \tag{3}$$

Eq. (1) shows the balance equation for mass where ρ_α and $\mathbf{v}_\alpha$ denote mass densities and velocities and the index α denotes the species of particles, i.e. electrons, ions and neutrals, respectively. Q_α^C are source terms due to ionization and recombination processes. Eq. (2) describes the balance of momentum for each species of particles where p_α denotes the thermal pressures. The force term $\mathbf{F}_\alpha$ is given by

$$\mathbf{F}_\alpha = n_\alpha q_\alpha (\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{B}) + \rho_\alpha \mathbf{g} \tag{4}$$

where $n_\alpha, q_\alpha, \mathbf{E}, \mathbf{B}, c$ and $\mathbf{g}$ denote the number densities, charge, electric field, magnetic field, the speed of light and gravitational acceleration, respectively. The source terms $\mathbf{Q}_\alpha^P$ describe the effects of momentum transfer between electrons, ions and neutrals. The balance of the internal energy densities $(\rho_\alpha \epsilon_\alpha)$ is described by Eq. (3). The source terms Q_α^E consider energy changes due to thermalization, ionization, recombination and momentum transfer due to collisions.

For conservation of total mass, momentum and energy, the source terms have to meet the following constraints

$$\sum_\alpha Q_\alpha^C = 0 \tag{5}$$

$$\sum_\alpha (Q_\alpha^C \mathbf{v}_\alpha + \mathbf{Q}_\alpha^P) = 0 \tag{6}$$

$$\sum_\alpha (\frac{1}{2} Q_\alpha^C \mathbf{v}_\alpha^2 + Q_\alpha^P \mathbf{v}_\alpha + Q_\alpha^E) = 0. \tag{7}$$

Besides, we assume quasineutrality

$$n_e = n_i = n \tag{8}$$

and Maxwell's equations have to be considered

$$\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E} \tag{9}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} \tag{10}$$

In the following, we will neglect all effects of recombination and ionization for reasons of simplicity and clearness. A detailled discussion of the corresponding source terms can be found e.g. in Flower et al. (1985) and Draine (1986).

Combination of Eqs. (1) and (2) yields the equations of motion for electrons (11) and single charged ions (12) in the following form

$$0 = -\nabla p_e - ne(\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}) - \rho_e \nu_{ei}(\mathbf{v}_e - \mathbf{v}_i)$$
$$- \rho_e \nu_{en}(\mathbf{v}_e - \mathbf{v}_n) \tag{11}$$

$$\rho_i(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i) = -\nabla p_i + ne(\mathbf{E} + \frac{1}{c}\mathbf{v}_i \times \mathbf{B})$$
$$- \rho_i \nu_{ie}(\mathbf{v}_i - \mathbf{v}_e) - \rho_i \nu_{in}(\mathbf{v}_i - \mathbf{v}_n). \tag{12}$$

In Eq. (11) the electron inertia term has been neglected, but we consider the collisional momentum transfer between electrons, ions and neutrals due to shear flow. The terms $\nu_{\alpha\beta}$ denote the respective elastic collision frequencies. Writing $\mathbf{v}_i$ and $\mathbf{v}_e$ in the form

$$\mathbf{v}_i = \mathbf{v} + \frac{m_e}{e\rho}\mathbf{j} \tag{13}$$

$$\mathbf{v}_e = \mathbf{v} - \frac{m_i}{e\rho}\mathbf{j} \tag{14}$$

and using $\rho = n(m_e + m_i)$, the inertialess equation of motion of the electrons yields Ohm's law

$$\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} = \frac{1}{ne}\nabla p_e + \frac{1}{4\pi ne}\nabla\times\mathbf{B}\times\mathbf{B}$$
$$-\frac{1}{ne}(\rho_e\nu_{ei}(\mathbf{v}_e - \mathbf{v}_i) - \rho_e\nu_{en}(\mathbf{v}_e - \mathbf{v}_n)). \tag{15}$$

Finally, combination of (9) and (15) yields the induction equation that governs the temporal evolution of the magnetic field

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\mathbf{v}_i\times\mathbf{B} + \frac{c}{e}\nabla\times(\frac{\nabla p_e}{n}) - \frac{c^2}{4\pi}\nabla\times(\eta\nabla\times\mathbf{B})$$
$$- \frac{c}{4\pi e}\nabla\times(\frac{\nabla\times\mathbf{B}\times\mathbf{B}}{n})$$
$$+ \frac{cm_e}{e}\nabla\times\nu_{en}(\mathbf{v}_e - \mathbf{v}_n). \tag{16}$$

In this form of the law of induction we find an electron pressure term, a Hall term and the common resistive term with $\eta = \frac{m_e^2}{n_e e^2}\nu_{ei}$. Besides, however, there is a further non-ideal term resulting from the collisional momentum transfer between electrons and neutrals due to shear flow. This term is crucial for the generation of magnetic seed fields.

For electron-hydrogen scattering the collision frequency ν_{en} is given by $\nu_{eH} = n_H\sigma(kT_H/m_H)^{1/2}$ with a scattering cross section of $\sigma \simeq 4\cdot10^{-15}cm^{-2}$ and k, T_H denoting the Boltzmann constant and the hydrogen temperature, respectively. Thus, taking into account the term $\frac{cm_e}{e}\nabla\times\nu_{en}(\mathbf{v}_e - \mathbf{v}_n)$ only, the generation rate of magnetic field due to plasma - neutral gas friction can roughly be estimated as follows

$$\dot{B} \approx 10^{-16}\frac{n_H T_H^{1/2}}{L_{shear}}v_{shear}. \tag{17}$$

Assuming parameters which might be considered to be of a typical order of magnitude for primordial clumps in DLAS as first structures in the framework of CDM hierarchical clustering models, i.e. $T \approx 10^4K, v \approx 10km/sec, n_H \approx 1cm^{-3}$ and $L \approx 100pc$ (e.g. Khersonsky and Turnshek 1996; Rauch et al. 1997; Tegmark et al. 1997) we find $\dot{B}$ to be of the order of $\cdot 10^{-29}G/sec$. In this estimation the efficiency of the magnetic field generation is somewhat underestimated by assuming L_{shear} to be of the order of the extension of the system. The effective shear length will be smaller, however, it is unknown a priori.

Assuming further that v_{shear} will be of constant order of magnitude on a time scale of 10^5yr we can expect a self-generated magnetic field of the order of $10^{-16}G$. Thus, B will be small but of reasonable order of a seed field further amplified by some dynamo (Lesch & Chiba 1995).

Although this is only a very qualitative, rough estimate, we will see that it is in rather good quantitative agreement with results of self-consistent dynamical simulations.

3. Simulation of a rotating primordial cloud

We study the self-generation of magnetic seed fields in a weakly ionized protogalactic plasma by the help of 2-fluid plasma - neutral gas simulations. For these purposes we have to reduce the set of 3-fluid equations by a unified treatment of electrons and ions. The full set of plasma - neutral gas equations under consideration can be found in the Appendix. The simulations are 2-dimensional in the present case.

We want to illustrate the results of our simulations by the help of an example being both qualitatively and quantitatively of relevance with respect to a primordial cloud which can be interpreted as one protogalactic gas clump in a high-redshift ($z \approx 2-3$) DLAS consisting of many of these clumps. In this context, we follow the line of e.g. Haehnelt et al. (1996) or Rauch et al. (1997) discussing high redshift DLAS to be built up by numerous gas clumps in the framework of hierarchical structure formation. We consider a weakly ionized system with 1% plasma and neutral gas else. This ionization rate for $z \approx 2-3$ is in the range of predictions of models for the cosmological evolution of ionization taking into account re-ionization processes in the early Universe (e.g. Peebles, 1993). For simplicity we assume both components to be distributed homogeneously. The neutral gas is at rest at $t = 0$. Considering the initial plasma velocity we assume a rotation pattern as can qualitatively be seen in Fig. 1a. The initial maximum velocity in this eddy-like pattern is about $10km/sec$ which corresponds to the typical virial velocity of a cloud of mass $10^6M_\odot$ and size $100\,pc$. Shear flow between plasma and neutral gas can be interpreted as result of a gravitational collapse of both components in CDM potential well having different pressure or temperature, respectively. This flow is motivated by similar velocity pattern used in hydrodynamical simulations of rotating high redshift DLAS consisting of several protogalactic clouds, which yield observed asymmetric absorption line profiles (Haehnelt et al. 1997).

The initial configuration is completely unmagnetized. The system with its rotating shear dynamics has an extension of about $100-150pc$ (with length scales measured in units of $25pc$). We neglect ionization and recombination but consider collisional interactions between plasma and neutral gas with a collision frequency of about $1.7\cdot10^{-7}sec^{-1}$.

Fig. 1 shows snapshots of the flow pattern of the plasma. It can be seen, that the flow velocity of the plasma decreases more and more due to collisional interactions with the neutral gas. After $0.1t_0$, i.e. 10^5yr (Fig. 1a), the velocity is reduced by a factor of about 2 as compared with the initial flow pattern. At $t = 0.9$ the plasma flow is reduced by a factor of 5. However, there is still significant shear as the neutral gas is dragged to a velocity of about 1% of the plasma flow. Finally, at $t = 7$, we have saturation, i.e. both plasma and neutral gas rotate with a velocity of about 0.5% of the initial plasma velocity.

As mentioned above, we have no magnetic field at $t = 0$, but generation of a magnetic field of seed field order is predicted due to plasma - neutral gas interaction in the course of the dynamics. Fig. 2 shows snapshots of the magnetic field found in our simulation.

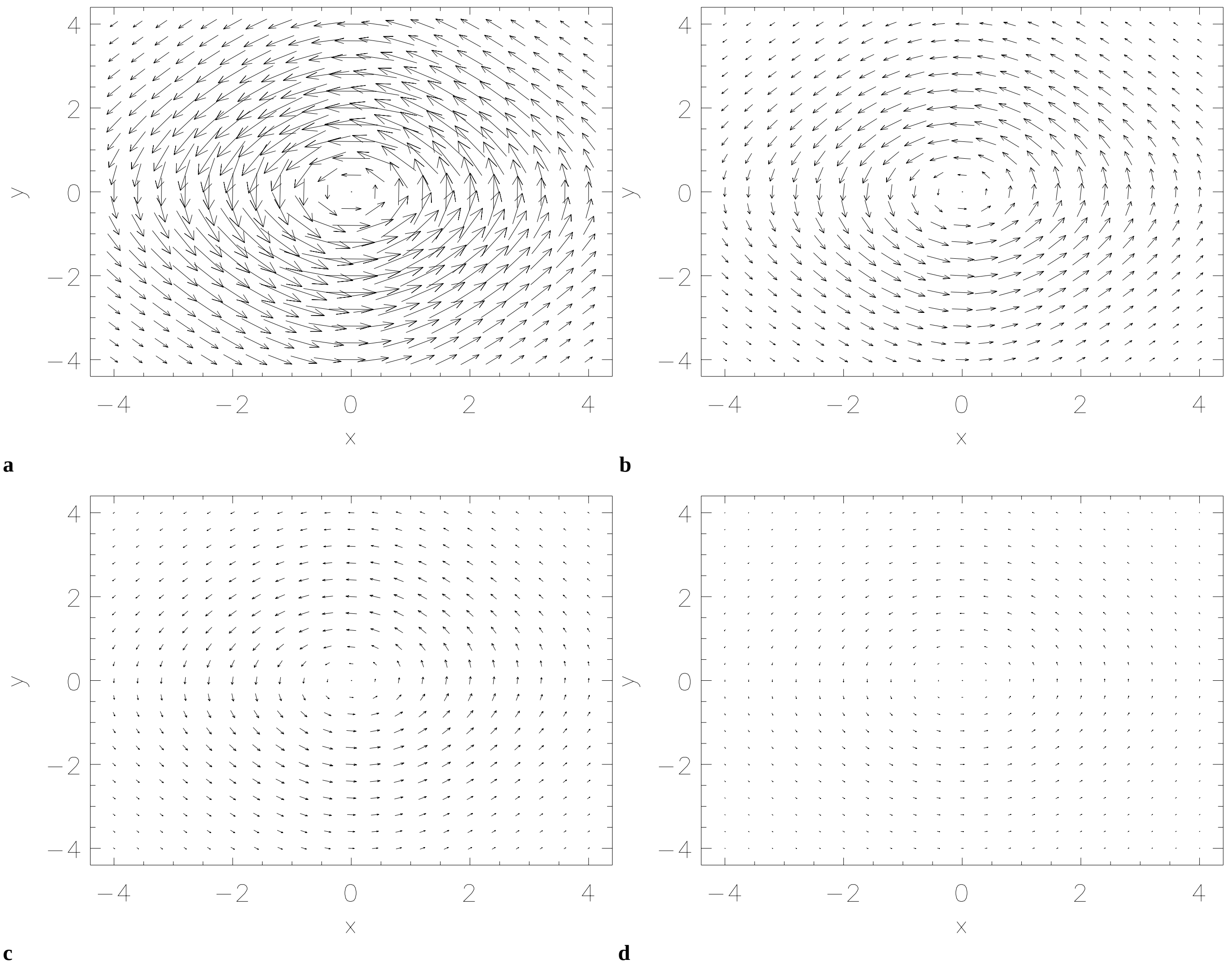

Fig. 1a–d. Snapshots of the plasma flow at **a** $t = 0.1$, **b** $t = 0.9$, **c** $t = 2$ and **d** $t = 7$ (measured in units of $10^6 yr$). The shear flow decreases due to collisions with neutral gas. At $t = 7$ plasma and neutral gas (being in rest at $t = 0$) have the same velocity.

After $0.1t_0$ (i. e. about $10^5 yr$, see Fig. 2a), we find a magnetic field of the order of $10^{-15}G$. This result is quantitatively in good agreement with what has been estimated in the previous section. The field increases up to one order of magnitude up to $t = 2$. Later, when the shear between plasma and neutral gas decreases more and more up to saturation the generated seed field remains more or less constant. The generated seed field is a large scale field, i.e. its scale lenght is of the order of the scale of the system.

4. Summary and discussion

We have presented results of self-consistent two-fluid simulations showing that in a weakly ionized protogalactic system magnetic seed fields can be generated due to plasma - neutral gas interaction. These simulations confirm previous kinematic studies of this process (Lesch & Chiba 1995).

Quantitatively, the simulations presented in this paper are of relevance with respect to high redshift rotating $Ly\alpha$ clouds. In the frame of hierarchical structure formation such systems are the result of tidally interacting primordial gas clouds of size $100 - 150pc$ with temperature of $10^4 K$ and a typical velocity of about $10^6 cm/sec$. The velocity profile we assume in our simulations is similar to that used in hydrodynamical simulations by Haehnelt et al. (1997)in context with observed asymmetric absorption line profiles. The collisional interaction between plasma and neutral gas due to shear flow between both components yields magnetic fiels of the order of $10^{-15} - 10^{-14}G$ on a time scale of $10^5 - 10^6 yr$. Taking the observational evidence for intensive merging at high redshifts shear flows are a natural ingredient of protogalactic objects. The merging of different constitutents will lead to an inhomogeneous mixture of different fluids and depending on the degree of anisotropy one can expect to receive completely different effiencies of the magnetic field generation process in protogalactic complexes.

Concerning the magnetic field strengths on protogalactic scales we consider a region which starts to decouple from the overall expansion and which is filled with primordial clouds all

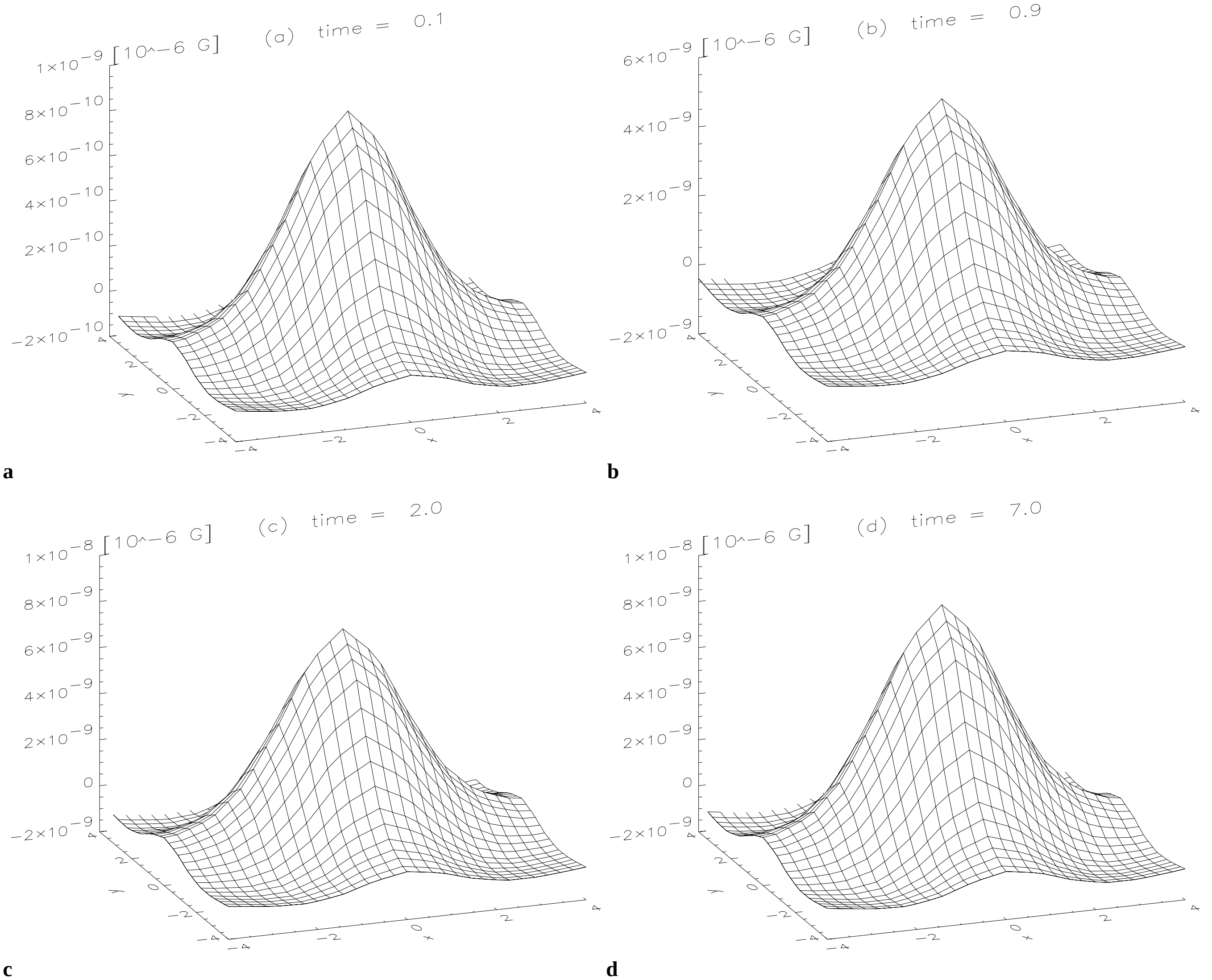

Fig. 2a–d. Snapshots of the generated magnetic seed field (measured in units of $10^{-6}G$) at $t = 0.1, 0.9, 2.0$ and 7.0 measured in units of $10^6 yr$.

generating their own magnetic field via plasma-neutral gas interaction. They are not causally connected with their neighbours. The clouds have magnetic dipol fields, i.e. magnetic bubbles with scales of 100pc. For simplicity we suppose that in each magnetic bubble, the field lines form loops that are confined to the bubble walls. When bubble walls eventually collide, the fields from each bubble are "stitched" to those of its neighbours by local magnetic reconnection (e.g. Lesch & Bender (1990) for an application of magnetic reconnection in galactic context). Because the magnetic generation is in the first place uncorrelated on scales larger than the bubble radius l_0, the loop structure in different bubbles is uncorrelated and an individual field, instead of turning back in a closed loop on the scale l_0, soon loses any memory of where it started and executes an infinite Brownian walk in space with step length $l_{\rm b}$. These lines traverse regions of space even on scales where no causal connection has yet occured. We define $B_{\rm l}$ as the flux density of B field lines remaining after tangles below a scale l are smoothed out by viewing the system with spatial resolution l. Thus, $B_{\rm l}^2$ is the kinetic energy density available from straightening tangles on scale l. To estimate the flux $B_{\rm l}$ we need to calculate the rms (root mean square) net flux through the "smeared" surfaces of radius l and effective half widths of order l. Since the orientation of the lines is random, the rms net magnetic flux through these fuzzy surfaces is $B \sim \sqrt{\sigma}/l^2$, where σ is the total number of lines which traverse the surface in any direction. If the lines were smooth on scale l, or if the surfaces were sharp, we would have $\sigma \sim l^2$, however for Brownian walks the total effective length of each line decreases like l^{-1}. This model consequently leads to a spectrum $B_{\rm l} \propto l^{-3/2}$. This argumentation was first introduced by Hogan (1983) for magnetic fields in the early universe.

It is worth noting that this acausal propagation is capable of producing large-scale magnetic fields that are just as strong as the causal superposition of dipole fields in a vacuum. A simple way to derive this scaling is to imagine space filled with randomly placed, randomly oriented current loops of radius $l_{\rm b}$

and local field strength B_{l_b}; The magnetic dipole moment of these loops will add linearly, and so the mean field strength at a distance l due to $N = (l/l_b))^3$ randomly oriented loops is about

$$B_l \simeq \sqrt{N} B_{l_b} \left(\frac{l_b}{l}\right)^3 \simeq B_{l_b} \left(\frac{l}{l_b}\right)^{-3/2}, \tag{18}$$

where $B_{l_b}\left(\frac{l_b}{l}\right)^3$ denotes the field strength contributed by each loop at a distance l. This example demonstrates that is not necessary to generate currents with correlations extending to infinity in order to create a field of infinite random walks.

In our scenario l_b is about 100pc, the clouds radius. The magnetic field strength B_{l_b} is about 10^{-15}G. If we use Eq.(27) we would reach about $5 \cdot 10^{-20}$G at a length scale of 100 kpc. As was shown by Lesch and Chiba (1995) this field can be amplified by subsequent global collapse and dynamo action up to a few μG at redshift 2-3.

We note that this estimate is only a lower limit for the large-scale field strength, since we assumed that the clouds move completely uncorrelated. Defacto in a collapsing protogalactic system, all clouds are subject to the gravity of the dark matter in the system. They react to the gravitational potential of the dark matter. Their velocity field becomes more correlated, i.e. they rotate differentially and interact via tidal forces in a collapsing volume. We therefore expect that several clouds will generate magnetic fields in the intercloud medium, which will enhance the large-scale field strengths. Numerical simulations of several clouds in a dark matter halo are underway.

We believe that the generation of magnetic fields due to plasma - neutral gas interaction has to be considered as a fundamental process with respect to magnetic seed fields in weakly ionized protogalactic systems as it will work for a great variety of dynamical scenarios (in particular in context with the collapse dynamics of dense cores in molecular clouds) as far as there is shear flow between plasma and neutral gas. Studies considering systems with a collapse dynamics due to external gravitation and self-gravitation, as well, will be subject of future work.

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Appendix

In our simulations we use the following set of normalized plasma - neutral gas equations including collisional interactions due to shear flow (cf. Birk & Otto 1996)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\mathbf{v}\rho) \tag{19}$$

$$\frac{\partial \rho_n}{\partial t} = -\nabla \cdot (\mathbf{v}_n\rho_n) \tag{20}$$

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) = -\nabla\cdot(\rho\mathbf{v}\mathbf{v}) - \nabla p + (\nabla\times\mathbf{B})\times\mathbf{B} - \rho\nu_{pn}(\mathbf{v}-\mathbf{v}_n) \tag{21}$$

$$\frac{\partial}{\partial t}(\rho_n\mathbf{v}_n) = -\nabla\cdot(\rho_n\mathbf{v}_n\mathbf{v}_n) - \nabla p_n - \rho_n\nu_{np}(\mathbf{v}_n - \mathbf{v}_)) \tag{22}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times(\eta\nabla\times\mathbf{B}) - \nabla\times\nu_{pn}(\mathbf{v}_n - \mathbf{v}) \tag{23}$$

$$\frac{\partial p}{\partial t} = -\mathbf{v}\cdot\nabla p - \gamma p \nabla\cdot\mathbf{v} + (\gamma - 1)$$
$$\left(2\eta\left(\nabla\times\mathbf{B}^2\right) - 3\nu_{pn}\left(p - \frac{\rho}{\rho_n}p_n\right) + \rho\nu_{pn}\left(\mathbf{v}-\mathbf{v}_n\right)^2\right) \tag{24}$$

$$\frac{\partial p_n}{\partial t} = -\mathbf{v}_n\cdot\nabla p_n - \gamma_n p_n \nabla\cdot\mathbf{v}_n + (\gamma_n - 1)$$
$$\left(3\nu_{pn}\left(p_n - \frac{\rho_n}{\rho}p\right) + \rho_n\nu_{pn}\left(\mathbf{v}_n - \mathbf{v}\right)^2\right) \tag{25}$$

These 2-fluid equations are result of reduction of the set of 3-fluid Eqs. (1) - (7) by the help of Eqs. (13) and (14) together with the relations $\rho = n(m_e + m_i)$, $\rho\mathbf{v} = \rho_e\mathbf{v}_e + \rho_i\mathbf{v}_i$, and $p = p_e + p_i$.

For simplicity the Hall term and gradient pressure term are neglected in the induction equation. Consequences of the plasma neutral gas friction are found in the law of induction and in the equations of energy balance. The plasma - neutral gas collision frequencies ν_{pn} and ν_{np} meet the constraint of momentum conservation, i.e. they are given by

$$\nu_{pn} = \frac{m_i\nu_{in} + m_e\nu_{en}}{m_e + m_i} \tag{26}$$

$$\nu_{pn}\rho = \nu_{np}\rho_n \tag{27}$$

Eqs. (19) - (25) are dimensionless. In our simulations length scales, magnetic fields and number densities are measured in units of $25pc$, $10^{-6}G$, and $10^{-2}cm^{-3}$, respectively. This choice yields typical velocities of $2.1\cdot10^6 cm/sec$ and a time scale of $10^6 yr$. Other quantities follow in a generic way.

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