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The space distribution of HIPPARCOS carbon stars*,**

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Abstract. More than 300 galactic carbon-rich stars have been observed by the HIPPARCOS satellite. Parallaxes and proper motions are given with an unprecedented accuracy. Carbon stars are however distant giants and there are only 4 stars with a parallax accuracy better than 20% (26 stars if 30%). Direct use of observed data is thus severely hampered by statistical biases. We propose here a model of the distribution of observed parallaxes as deduced from the cumulative distribution of proper motions.

A simple $P(\varpi) \propto \varpi^{-2.35}$ law for true parallaxes is found to give a good fit to observed parallaxes. The minimum true parallax of this magnitude-limited sample amounts to (0.3 ± 0.1) mas. True parallaxes are then generated for statistical purposes. They are found to agree with the relative photometric angular diameter of Knapik & Bergeat (1997) following the theoretical relation $\Phi/\Phi_0 \propto \varpi$. Diagrams such as Fig. 3b show no distortion even at small parallaxes which suggests they are free of systematic effects. Finally, the space distribution of the HIPPARCOS carbon stars is discussed. An evaluation of the Malmquist bias is obtained and the local (presumably unbiased) distribution of carbon stars is found to be $P(\varpi) \propto \varpi^{-3.02\pm0.09}$. A flat subsystem or slab is proposed $\left(P(\varpi) \propto \varpi^{-3}\right)$. It is shown that the HIPPARCOS sample is reasonably complete for $\varpi \geq 1\,\mathrm{mas}$ with the possible exception of the faint R0-R2 stars. The Malmquist bias appears also as negligible for the bright N-variables down to $\varpi \simeq 0.6\,\mathrm{mas}$. No clear correlation with the local arms structure and interstellar extinction could be seen in this magnitude-limited sample of the Sun vicinity. The space density of carbon stars is estimated to range from 40 to 70 stars kpc^{-3} which is in good agreement with previous estimates of Fuenmayor (1981).

Key words: stars: AGB and post-AGB – stars: carbon – stars: fundamental parameters – Galaxy: stellar content

1. Introduction

A very difficult problem is dealt with in the present paper. Biases need to be considered for relative errors larger than 0.2. The distribution of the true (not observed) parallaxes should be known to make a correct statistical use of observed parallaxes. This is however impossible at present since the true distribution of carbon stars is not known for certain. They tend to concentrate toward the galactic plane (e.g. Stephenson 1989). No clear correlation is observed between the distribution of the HIPPARCOS carbon stars and the local arms structure or intervening interstellar extinction (Sect. 2). Their scarcity among galactic red giants is an additional difficulty (about 300 stars with limiting $\mathrm{V}_1 \simeq 12.4$ in ESA 1997 and 5987 stars in Stephenson's catalogue). The bias studied by Lutz & Kelker (1973, LK) is twofold: the effect of the true spatial distribution of stars on measurements with a given standard deviation (LK considered stars uniformly distributed in space), and the non-gaussian distribution of distances and absolute magnitudes even for a gaussian distribution of parallaxes (e.g. Smith & Eichhorn 1996), with the additional effect of truncation (at least of negative parallaxes).

Specific modeling of samples is preferable to blind application of the LK correction (Brown et al. 1997). There is no need to deal with both biases simultaneously and we restrict ourselves to the former one here. A uniform spherical distribution in space was adopted in the LK-analysis. Their Z-variable, namely the ratio of true to observed parallax, admits of no minimum value unless truncation (of negative and very small positive parallaxes) is adopted. No truncation is applied in our method. Following a different approach, we estimate the distribution of true parallaxes from the cumulative distribution of proper motions (Sect. 3 and Fig. 1), with the assumption of a uniform velocity distribution (Hanson 1979). The calculated distributions of observed parallaxes for various ranges of the standard deviation are then compared to the observed distributions (Sect. 4 and Fig. 2). A good agreement is observed. True parallaxes are then generated for statistical purposes (Sect. 5) and screened with success against photometric data (Sect. 6). The completeness of the HIPPARCOS sample of carbon stars is discussed. An evaluation of the Malmquist bias is proposed (Sect. 7) to deduce the (true and presumably unbiased) distribution of local carbon stars from the HIPPARCOS one. Finally, the results are discussed (Sect. 8) in terms of a flat space distribution (slab), the results deviating significantly from a spherical distribution. Estimates of the number density of carbon stars are given.

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** Based on data from the ESA HIPPARCOS astrometry satellite

2. The HIPPARCOS carbon stars

The HIPPARCOS sample of carbon stars includes more than 300 stars proposed for parallax and/or proper motion studies. The effective visual apparent magnitude reached is discussed in Sect. 7. Thanks to the authors of 1982 proposals, a good coverage of the whole sky was achieved, including the southern hemisphere. It is well known that carbon stars are concentrated toward the galactic plane, the effect being more pronounced for the N stars (Stephenson 1989, Fig. 2 p. 76) than it is for the R stars (ibid., Fig. 1 p. 75). This is also observed for the HIPPARCOS sample whose map is roughly similar to these two figures. The strong clumps observed in Stephenson's Fig. 2 around $l = 90^\circ$ $(60-120)$ and $l = 270^\circ$ $(240-300)$ are however absent. They were interpreted as structures correlated with the local spiral arms (Cygnus-Perseus and Sagittarius-Carinae respectively) which are deeply explored in those two directions. Presumably, they are located too far from the Sun and thus not included in the HIPPARCOS catalogue, which makes things simpler. Such features would have required the introduction of a function of the galactic coordinates for bar-like structures.

Deep near infrared surveys (Fuenmayor 1981) show less carbon stars in the galactic center direction $(l = 330^\circ \text{ to } 30^\circ)$ than observed in the anticenter direction $(l = 150^\circ \text{ to } 210^\circ)$. This is also illustrated by Stephenson's Fig. 2. This observation is understood in terms of metallicity decreasing with increasing galactocentric distance. The scheme also applies to the extragalactic sequence Milky Way-LMC-SMC where the carbon stars proportion among red giants increases with decreasing metallicity (e.g. Blanco & McCarthy 1983). The smaller the $[\mathrm{O}]\,/\,[\mathrm{H}]$ ratio in a star, the easier its transformation to a carbon star $([\mathrm{C}]\,/\,[\mathrm{O}] \geq 1)$ by enriching its atmosphere in carbon through convective drag or mass transfer in a close binary. No such effect of galactocentric gradient is observed in the HIPPARCOS sample. The main concentrations of Fuenmayor's stars are actually located at 5 kpc from the Sun in both directions. Large distances are required for the metallicity effect to be noticeable and the HIPPARCOS stars are located at shorter distances.

Although their distribution in galactic coordinates appears rather smooth, we searched for correlations with maps of interstellar extinction (see Knapik & Bergeat 1997 for a recent study and references therein). The effect of local clouds seems quite moderate and no void was detected. The extinction corrections of Knapik & Bergeat are applied hereafter but no attempt is made to take into account a selection effect due to the patchy distribution of interstellar reddening. Finally, the concentration of carbon stars to the galactic plane is the only feature obviously exhibited by the HIPPARCOS sample.

3. The cumulative distribution of proper motions

The accuracy on proper motions is far better than it is on parallaxes. The ratio typically ranges from 2 to 20 in our sample, with the few exceptions of perturbed (inaccurate) HIPPARCOS solutions. To estimate the spatial distribution of the HIPPARCOS carbon-rich stars, we have applied the method described by Hanson (1979). The velocity distribution being assumed not to vary appreciably on the volume of space studied, the parallax distribution can be obtained as:

$$P\,(\varpi)\;\propto\;\varpi^{-n} \tag{1}$$

if the cumulative distribution of proper motions follows the power law:

$$N\,(\mu)\;\propto\;\mu^{-n+1} \tag{2}$$

The latter diagram is shown in Fig. 1 for HIPPARCOS carbon stars. Three samples were considered according to selected ranges of the quoted standard deviation (in mas) on observed parallaxes. Proper motion intervals of 5 mas and 2.5 mas were considered for the first sample and the latter two respectively. It can be seen that a power law is appropriate over a large range of proper motions, with only slight differences from one sample to the other. We have adopted $n = 2.35 \pm 0.04$ from a mean weighted slope of -1.35 shown in Fig. 1. This is far lower than the value of 4 adopted by LK who assumed their stars to be uniformly distributed in space. A smaller effect is thus anticipated here. This result will be discussed in terms of space distribution in Sect. 8.

Table 1. The model parameters for the parallax distribution of carbon stars

σ_{ϖ_0}	N(ϖ_0)	ϵ	σ_ϖ	$\mathrm{N_S}$
0.5-1.0	74	0.95	1.10	16.7 ± 2.0
1.0-1.5	97	0.50	1.45	9.0 ± 1.4
1.5-2.5	88	0.25-0.33	1.95	3.1 ± 1.0

4. A model of the distribution of observed parallaxes

The number of HIPPARCOS carbon stars can be predicted as:

$$N\,(\varpi_0) = \sqrt{2\pi} N_{\mathrm{S}} \sigma^{-1} \int_{\epsilon}^{\infty} \varpi^{-2.35} \mathrm{e}^{-(\varpi-\varpi_0)^2/2\sigma^2} \mathrm{d}\varpi \tag{3}$$

where ϖ and ϖ_0 are the true and observed parallaxes respectively, $\sigma_\varpi = \sigma$ the standard deviation on true parallaxes relative to observed ones, ϵ the minimum true parallax in the sample (a function of σ and geometry for a given limiting magnitude V_{l} : see Subsect. 7.1.), and N_{S} a constant related to the space density.

Considering only carbon stars with standard deviation $\sigma_{\varpi_0} = \sigma_0 \leq 2.5\,\mathrm{mas}$, we have plotted the number of stars against the observed parallax ϖ_0 for three samples of various ranges in σ_0 (Fig. 2). The observations (symbols) are confronted to Eq. (3) predictions (continuous curves). The agreement is found to be rather good for the set of parameters in Table 1. The total number density from Table 1 has to be increased of $15.8\,\%$ corresponding to stars with poor data $(\sigma_0 \geq 2.5\,\mathrm{mas})$. A total $N_{\mathrm{S}} = 33.3\,\mathrm{stars\,kpc^{-1.35}}$ is then deduced. As expected, widened gaussian curves are obtained whose width consistently increases with the central value of the σ_0 range. They also shift toward smaller ϖ_0 since more distant stars are included when allowing for lower accuracy. It is found with few exceptions

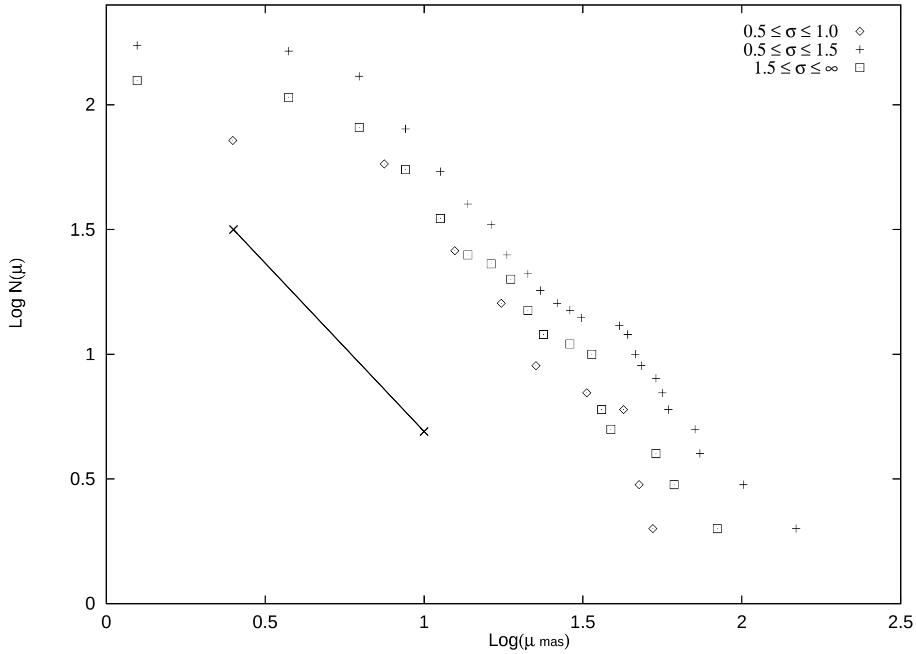

Fig. 1. The cumulative distribution of carbon stars proper motions from the HIPPARCOS catalogue (ESA, 1997). Three samples according to accuracies on parallaxes are considered and the adopted mean -1.35 slope is also shown.

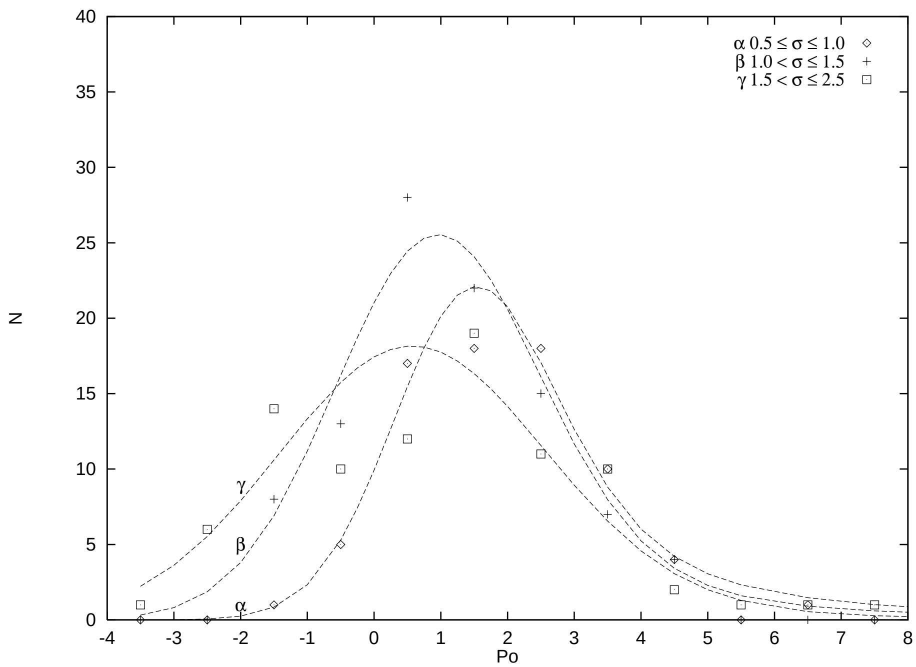

Fig. 2. The distribution of the number density of HIPPARCOS carbon stars as a function of the observed parallax in mas (ESA,1997) for three samples, according to accuracy ranges. The observations are shown as symbols and the results of calculations as continuous curves.

that our adjusted σ_ϖ is typically 0.15 to 0.35 mas larger than the mean values from the HIPPARCOS catalogue, except beyond 2.1 mas where the difference becomes negligible. Standard errors of the HIPPARCOS astrometric parameters might be slightly underestimated and the chromaticity effect results in larger systematic errors for the reddest stars, which is the case of carbon stars (ESA 1997, Vol. 3 pp 438-441). A 3-4 kpc estimate for maximum distances is deduced from the smallest values of ϵ in Table 1. This is consistent with the absence of structures correlated with the spiral arms or metallicity effects as discussed in Sect. 2. There are very few HIPPARCOS stars with distances from Sun larger than $1.6 - 2\,\mathrm{kpc}$. It will be shown in Sect. 7 that the Malmquist bias in a magnitude-limited sample is responsible for this situation. The tails for negative ϖ_0 are notably affected by the minimum true parallax in the sample. This is the way ϵ was estimated, independently of any specific geometry. The ϵ parameter should be considered as a mean quantity affected by interstellar extinction and averaged on many lines of sight. Such anisotropies have to be small (see Sect. 2).

5. Estimated values of true parallaxes

As a further step, we calculate the expectation of the difference $\Delta\varpi = \varpi - \varpi_0$ between the true and observed parallaxes on the basis of the model of Sect. 3. This quantity is introduced for statistical purposes only. It is required when some kind of a mean is to be computed. It is not intended to replace the catalogue (observed) value which should be used every time a star is individually considered. Here, every star is considered as a member of a parent population whose σ and ϵ are adapted from our model, with σ_{ϖ_0} from the catalogue. Then, the mathematical expectation is:

$$E\,(\varpi_0) = C \int_{\epsilon}^{\infty} (\varpi\prime - \varpi_0)\, \varpi\prime^{-2.35} \mathrm{e}^{-(\varpi_0 - \varpi\prime)^2/2\sigma^2} \mathrm{d}\varpi\prime \quad (4)$$

where $\varpi\prime$ is a parallax integration variable and $C = \sqrt{2\pi} N_S\,/\sigma N(\varpi_0)$. Except for a few nearby stars observed with reasonable accuracy or fortuitously, the calculated (positive or negative) difference can be substantial. We emphasize its statistical meaning : individual values may occasionally be wrong and a few cases were detected from data at hand such as galactic latitude, proper motion or even computed space velocity relative to Sun. In such a case, the catalogue value can be kept if positive or the star is abandoned. Parallaxes can also be estimated from companion data in binary systems.

6. Relative angular diameters and true parallaxes

Before making use of the calculated true parallaxes with full confidence, we must validate them against independent, i.e. non-astrometric, data. Knapik & Bergeat (1997) have published an analysis of the spectral energy distributions (SEDs) of about 200 carbon variables, a work now extended to about 550 carbon stars. For every studied star, their dedicated pair method gives the interstellar extinction on its line of sight and the classification of its dereddened SED in a photometric group (namely CV1 to CV6, with new extensions HC0 to HC5 for hot stars and CV7 for very cool variables). The extinctions thus obtained agree fairly well with the values from other sources in the literature. It is believed that the groups and effective temperatures are correlated. Assuming that the same brightness distribution prevails on the disks of the stars in a given group (eventually the same limb darkening law), Knapik & Bergeat also obtained the value of:

$$< k >^{1/2} = \Phi/\Phi_0 \quad (5)$$

where Φ is the angular diameter of the star and Φ_0 the angular diameter a star with magnitude $[1.08] = 0$ would have, on the average, in this group. Denoting by R_{p} the photospheric radius, the theoretical linear relation:

$$< k >^{1/2} = 2\, R_{\mathrm{p}}\, \varpi/\Phi_0 \, = \, a\, \varpi \quad (6)$$

is readily derived between this purely photometric quantity and the true parallax ϖ which is purely astrometric. The former quantity is shown, as a function of the observed parallaxes (Fig. 3a) and of the true parallaxes as estimated above (Fig. 3b), for the 35 CV2-stars ($T_{\mathrm{eff}} \simeq 3000 - 3150\,\mathrm{K}$) observed by HIPPARCOS. It can be seen that Eq. (6) is closely satisfied provided the effective range in slope is approximately:

$$0.08 \, < \, a \, < \, 0.21 \quad (7)$$

as shown by the two freehand straight lines drawn in Fig. 3b. The intrinsic range in a is of course broadened by the errors on both abscissae and ordinates. These stars have received uniform treatment: no outlier was found and no “wrong case” (see the end of Sect. 5) was detected in this CV2-group. The interesting point is that no systematic distortion can be seen in Fig. 3b, for instance of distant stars when compared to nearby ones. Diagrams similar to Fig. 3 were obtained for the other CV-groups with similar ranges in slope. A few outliers were eventually observed. Clearly, Fig. 3a is a good illustration of what observed parallaxes we get: the range in ϖ_0 increases with decreasing $< k >^{1/2}$ i.e. with increasing distance, for a given radius range. Finally, we consider the estimated true parallaxes of Sect. 5 as corrected statistically from the space distribution (LK) bias. We shall see in the next section that the Malmquist bias remains for $\varpi \, \leq \, 1.0\,\mathrm{mas}$. At increasingly larger distances, the HIPPARCOS sample is increasingly deficient in intrinsically faint stars.

7. The evaluation of the Malmquist bias

7.1. The magnitude-limited sample

It is well known that samples selected on the basis of a limiting magnitude are subject to the Malmquist bias (Malmquist 1924, 1936). The distant stars selected in such a biased distribution are increasingly brighter when distance increases. This effect was not investigated in Sects. 2 to 5 since we used the distribution of the HIPPARCOS carbon stars as it is. The magnitude of the bias has to be evaluated before discussing what the true space distribution of the local carbon stars may be. We start from:

$$P\,(\varpi) \,\propto\, \varpi^{-2.35} \quad (8)$$

which is the distribution adopted in Sect. 3 for the HIPPARCOS carbon stars.

The HIPPARCOS stars were actually selected on the grounds of a limiting magnitude which finally amounts to $V_{\mathrm{l}} \simeq 12.4$ and completeness is probably achieved up to $V \simeq 7.3$ to 9.0 (ESA 1997 Vol 1, XV). Many carbon stars are fainter than the latter values and were included from specific proposals. The former limit is certainly far from being reached since the authors of 1982 proposals used provisional values like $B_{\mathrm{l}} \simeq 13$. The carbon stars are specially faint in the blue and the unreddened B-V ranges typically from 1 to 5. The effective cut ranges from 9.8 to 10.7 with only three values larger than 11, when dealing with dereddened V magnitudes. The value seems to be somewhat depending on the colour index. Finally, we adopt $V_{\mathrm{l}}' \simeq 10.0 \pm 0.50$. The limiting absolute magnitude is then:

$$M_{\mathrm{l}} = V_{\mathrm{l}}' - 10 \, + \, 5 \, \log\,(\varpi) \simeq 5 \, \log\,(\varpi) \quad (9)$$

Now we attempt two differing approaches respectively based on a gaussian fit of our $N\,(M_{\mathrm{V}})$ (Subsect. 7.2.) and a rectangular distribution with data from other authors (Subsect. 7.3.).

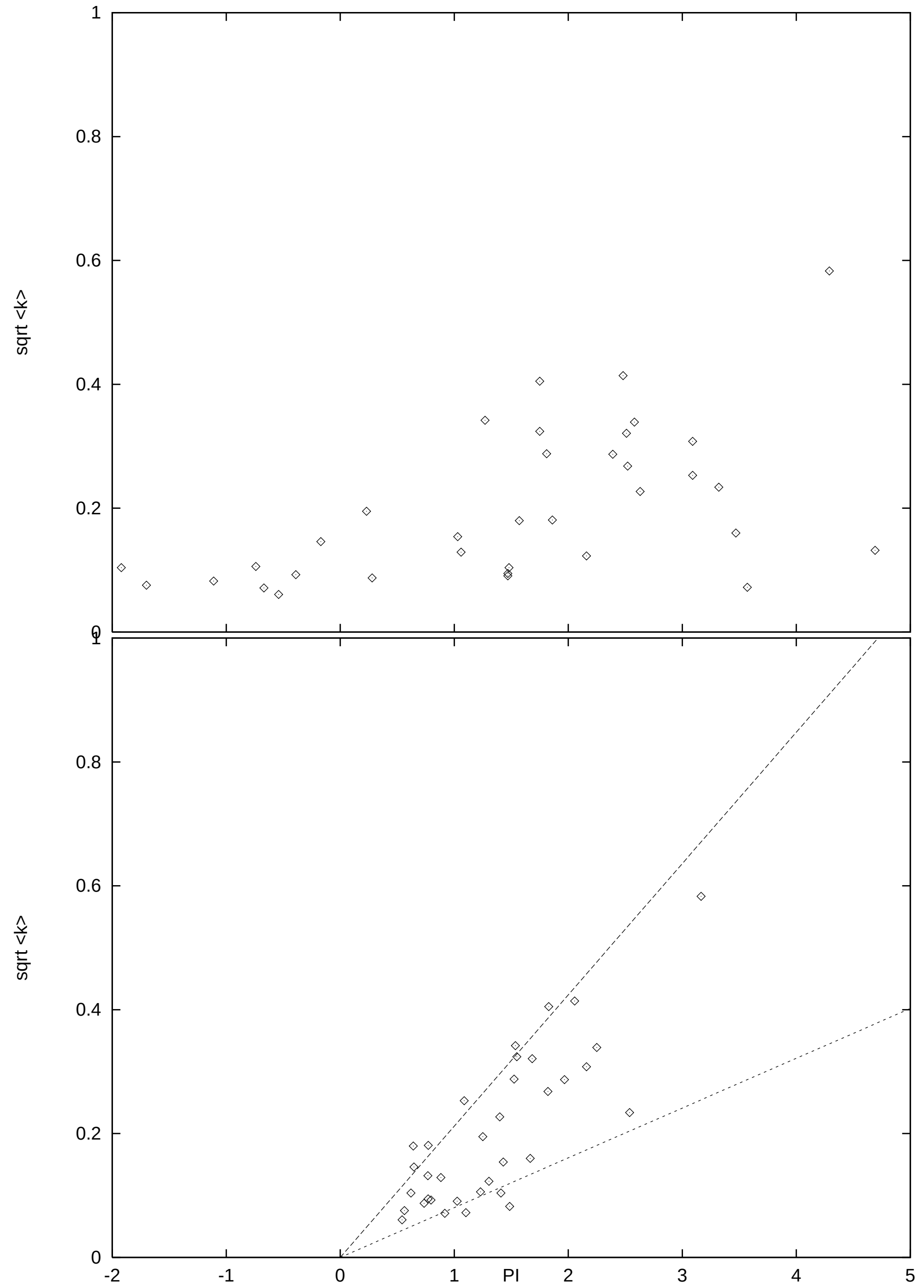

Fig. 3. The relative angular diameters of CV2-carbon stars from photometry as a function of their observed (ESA, 1997; Fig. 3a) and true (estimated in Sect. 5; Fig. 3b) parallaxes

7.2. Evaluation from a Gaussian $N\,(M_{\rm V})$

Making use of our evaluated true parallaxes ϖ (see Sect. 5), we have computed the absolute visual magnitudes $M_{\rm V}$. The used apparent magnitudes were taken from our data base. For variable stars they are usually close to maximum light values (Knapik & Bergeat 1997). For 262 documented stars, a good gaussian fit was obtained which is centered at $M_{\rm V} \simeq -1.5$ with dispersion $\sigma \simeq 1.06$, viz.

$$N\,(M_{\rm V}) \simeq 50.7 \exp\left[-(M_{\rm V}+1.5)^2/2\,(1.06)^2\right] \tag{10}$$

Then, the correction factor $(f \geq 1)$ on the number density is:

$$f\,(\varpi) = \left[\int_{-\infty}^{x_1} (2\pi)^{-1/2}\,\exp\left(-x^2/2\right)\,{\rm dx}\right]^{-1} \tag{11}$$

where:

$$x_1 = (M_1 + 1.5)\,/\,2^{1/2}\,(1.06) \simeq 1 + 3.34 \log \varpi \tag{12}$$

if $V_1' \simeq 10.0$ is adopted. The resulting $f\,(\varpi)$ can be closely approximated by the power law:

$$f\,(\varpi) = f\,(1)\,\varpi^{-m} \simeq 1.22\,\varpi^{-m} \tag{13}$$

with $m \simeq 0.67 \pm 0.08$ on the $0.5 \leq \varpi \leq 1.4\,{\rm mas}$ interval where $74.3\,\%$ of our stars are located. For stars with $\varpi \geq 1.4$ $(22.6\,\%)$, we obtained $f \leq 1.06$, a correction which can be neglected. For a remaining $3\,\%$ of stars at $\varpi \leq 0.5$, the f-factor is underestimated of typically $15\,\%$ when calculated from Eq. (13).

7.3. Evaluation from a rectangular distribution

In order to test the influence of the adopted $N(M_{\rm V})$ law on the m-exponent, we adopt here a rectangular distribution with a magnitude range estimated from various references. Mikami (1986) applied the maximum-likelihood method and other statistical methods to the homogeneous kinematical data available. He obtained values ranging from -0.6 ± 1.5 for hot carbon stars to -1.6 ± 1.3 for cooler ones. Vandervort (1958) quoted +0.44 for the hottest (R0-R2) stars and -1.09 for R4-R8 stars. From various methods Gordon (1968) obtained +0.4 for the R-stars (i.e. like for the oxygen-rich K-giants) and -1.5 to -3.5 for the N-stars. From our best parallaxes, values as bright as -3.2 were deduced for N-stars. We tentatively adopt $M_{\rm v,1} \simeq -3.0$, $M_{\rm v,2} \simeq +0.5$ and thus $\Delta M = M_{\rm v,2} - M_{\rm v,1} \simeq 3.5$ for the ends and amplitude of our sample. For a simple rectangular distribution the correction factor $(f \geq 1)$:

$$f = \Delta M / \left(M_{\rm l} - M_{\rm v,1}\right) \tag{14}$$

is obtained where $M_{\rm l}$ as deduced from Eq. (9) is the absolute visual magnitude of the cut at the distance corresponding to the parallax ϖ. Of course the factor is to be applied only for $M_{\rm v,1} \leq M_{\rm l} \leq M_{\rm v,2}$. We have calculated the values of the function $f(\varpi)$ in the range $0.3 \leq \varpi \leq 5.0$ where it can be closely approximated by a power law:

$$f(\varpi) \propto \varpi^{-m} \tag{15}$$

with a mean $m \simeq 0.63$. Here the correction applies for $\varpi \leq 1.26\,{\rm mas}$. Adopting $M_{\rm v,2} = 0\,{\rm or} + 1.0$ would yield $\varpi \leq 1\,{\rm or}\,1.59\,{\rm mas}$ respectively. Varying $V_{\rm l}'$, $M_{\rm v,1}$ and thus ΔM within reasonable ranges results in a slightly affected exponent $(0.53 \leq m \leq 0.75)$.

7.4. Conclusion

Both approaches yield power laws with very similar exponents in the 0.6-0.7 range. The adopted $N(M_{\rm V})$ law has little influence on the result. We finally adopt $m \simeq 0.67 \pm 0.08$ from Subsect. 7.2.. It is remarkable that only the stars fainter than $M_{\rm V} = -1.1$ are affected at $\varpi = 0.6$. We conclude that the subset of the bright N-variables is little affected by the Malmquist bias while the opposite situation prevails for the faint hotter carbon stars.

Finally, the true distribution estimated from the HIPPARCOS one would be:

$$N(\varpi)\,{\rm d}\varpi \propto \varpi^{-{\rm q}}{\rm d}\varpi \tag{16}$$

obtained by correcting uniformly for the Malmquist bias. For the gaussian $N(M_{\rm V})$ law of Subsect. 7.2., the exponent found is $q = m + n \simeq 3.02 \pm 0.09$. This rough argument has to be refined since the Malmquist bias does not apply uniformly. Analysing the 296 estimated $\varpi'{\rm s}$ of Sect. 5, a least square fit of the number densities yields:

$$N(\varpi)\,{\rm d}\varpi \simeq 61.3\varpi^{-(2.90\pm0.27)}{\rm d}\varpi \tag{17}$$

for $\varpi \geq 1{\rm mas}$. This result is consistent with the exponent amounting to about 3 with a negligible Malmquist bias. We assume that these results are representative of the space distribution of carbon stars in the Sun vicinity. For $\varpi \leq 1{\rm mas}$. an increasing depression is observed for decreasing ϖ. Our conclusion is that the HIPPARCOS sample is reasonably complete for distances to Sun less than $0.9\,{\rm kpc}$.

8. Discussion and conclusion

First of all we discuss the results of Sects. 5-6. The expectations on true parallaxes derived from Eq. (4) were successfully checked. Relative angular diameters from photometry were used for this purpose. They can be calibrated with diameters from lunar occultations or interferometry with a 1 to 1 slope (Bergeat & Knapik, 1997), but there are unfortunately few stars in common. The main feature of Fig. 3b is that no distortion is observed, specially for distant stars. The range in the slope a proves well-defined in Fig. 3b but it is difficult to decide what the contribution of the errors is. The ratio of the maximum to minimum in a is 2.6 for the CV2-group. If it could entirely be attributed to an intrinsic range in photospheric radii, the corresponding range in bolometric magnitude would be nearly 2.1, assuming the same effective temperature prevails for every star in the CV2-group. This is only an upper limit since the errors contribute substantially.

Now we turn to the results of Sects. 2-4 & 7 in terms of space distribution of carbon stars. We stress that this is the HIPPARCOS sample of carbon stars which is simulated in Sects. 5-6. We have proposed in Sect. 7 a correction of the Malmquist bias from which the space distribution of carbon stars was recovered. Varying the parameters around our best fit results in more or less strong effects. For instance, it is possible to obtain reasonable fits in Fig. 2 if the n-exponant of 2.35 adopted from Fig. 1 is changed by no more than ∓ 0.6 apart. This is the distribution of HIPPARCOS carbon stars, i.e. of a subset of the complete sample we now try to describe. Assuming the stars to be uniformly distributed in a spherical volume, LK used a theoretical ${\rm n} = 4$ since:

$$N(\varpi)\,{\rm d}\varpi \propto 4\pi\,\varpi^{-4}{\rm d}\varpi \tag{18}$$

Such an assumption seems to be ruled out by the present evidence. As suggested by the concentration of carbon stars toward the galactic plane (e.g. Stephenson 1989) and the smaller $q \simeq 3$ value obtained in Sect. 7, we consider the case of:

$$N(\varpi)\,{\rm d}\varpi \propto 2\pi\,{\rm h}\,\varpi^{-3}{\rm d}\varpi \tag{19}$$

for the intrinsic distribution. It corresponds to an uniformly populated slab of height h, provided the explored length always exceeds $\mp\,{\rm h}/2$ along the third axis which is assumed to be perpendicular to the galactic plane. The interstellar extinction is not considered in this simple model where no variation with galactic longitude is assumed. Ideally, the same measurements in a complete sample (i.e. free of the Malmquist bias) would have given instead of Eq. (3):

$$N(\varpi_0) = \sqrt{\pi/2}Nh\sigma^{-1}\int_{\epsilon}^{\infty}\varpi^{-3}{\rm e}^{-(\varpi-\varpi_0)^2/2\sigma^2}{\rm d}\varpi \tag{20}$$

where $N_S = Nh$ is the surface density in stars per square kpc. The minimum parallax obtained in Sect. 4 ranged from 0.33 to 0.25 mas which yields a (biased) radius estimate of $R = 3 \text{ to } 4\,\text{kpc}$. We favor the lower 3kpc estimate. This is the radius of the part of the slab which is explored by HIPPARCOS, namely a (R, h) cylinder.

The used data was limited to $\sigma_{\varpi_0} \leq 2.5\,\text{mas}$ since increasing contamination by (eventually undetected) binarity is expected for larger values. We have no evaluation of the cylinder height but a minimum value of 0.6 to 0.8 kpc seems reasonable for the N stars, while 1.1 to 1.3 kpc might apply to R stars. The theoretical distribution corresponding to Eq. (17) is:

$$N(\varpi)\,\mathrm{d}\varpi = 2\pi \mathrm{hD}\varpi^{-3}\mathrm{d}\varpi \tag{21}$$

we integrate for a calibration of the number density D, viz.

$$\int_1^\infty N(\varpi)\,\mathrm{d}\varpi = \pi\mathrm{hD} = 152 \tag{22}$$

where 152 is the corresponding number of stars in the restricted interval. Adopting $h = 0.7\,\text{kpc}$ or $h = 1.2\,\text{kpc}$ yields $D = 69$ and 40 stars per cubic kpc respectively. The former estimate is probably better since the hot R stars are only 26% of the sample. Fuenmayor (1981) derived space densities from 15 to 50 stars kpc^{-3}. Both results seem in good agreement. From our values, one would then obtain about 1370 carbon stars for $R = 3.0\,\text{kpc}$ in both cases, i.e. 23% of the Stephenson's catalogue (1989) content, but its infrared carbon stars (IRAS C) fall beyond the HIPPARCOS limiting magnitude. The Malmquist bias in a magnitude-limited sample is responsible for observing only about 300 stars with HIPPARCOS. A substantial fraction of those missing stars is certainly made of hot and faint carbon stars (spectral types R0-R5 or C-classified stars with a first index lying between 0 and 3). Finally, they might be as numerous as their bright and cool counterparts (late R and N or C-classified with a first index larger than 3). The latter stars outnumber the former ones in Stephenson's catalogue because they are observed at larger distances. The above numbers are of course rough estimates like the values of R and h which were used. Some moderate population mixing is probably present which contributes in confusing the picture.

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