On the dynamo driven accretion disks

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Abstract. We add the $\alpha$ effect in the dynamo driven accretion disk model proposed by Tout & Pringle (1992), i.e., a dynamo model depends on the physical processes such as Parker instability, B-H instability, magnetic field reconnection and $\alpha-\omega$ mean field dynamo as well. The $\alpha$ effect in the dynamo mechanism is determined by the strength of turbulence of the accretion flow. When the turbulent Mach number $M_t$ is less than 0.25, the solutions of the magnetic fields oscillate around their equilibrium values. Increasing the value of $M_t$ makes the amplitude of the oscillation smaller and the period longer, but does not affect the equilibrium values. The Shakura-Sunyaev viscosity parameter $\alpha_{SS}$ oscillates around the equilibrium value of 0.33. When the turbulent Mach number $M_t$ is larger than 0.25, the magnetic field components reach a stable state. In the non-linear dynamo region, the critical turbulent Mach number $M_t$ is 0.44 rather than 0.25. The oscillating magnetic fields and viscosity parameter can explain the basic properties of the dwarf nova eruptions and some properties of quiescent disks (Armitage et al. 1996).

Key words: magnetic fields – MHD – instabilities – accretion, accretion disks – turbulence

1. Introduction

There is observational evidence for the presence of magnetic fields in accretion disks, and they influence the dynamics of the system, such as angular momentum transport and poloidal outflow of the disk gas. The nature and magnitude of the viscosity is uncertain in the current theoretical descriptions of standard accretion disk, but almost all detailed modeling of the structure and evolution of accretion disks depends on the value of the viscosity (Duschl et al. 1997). In general, the molecular viscosity is inadequate and some kind of turbulent viscosity is required. Shakura & Sunyaev (1973) introduced a single dimensionless parameter which we denote here as $\alpha_{SS}$ to describe all the unknown physics about the viscosity, so that the kinematic viscosity $\nu$ is written as

$$\nu = \alpha_{SS} C_s^2 / \Omega,$$

where $C_s$ is the sound speed, $\Omega$ the angular velocity in the disk.

When a magnetized accretion disk is concerned, the dimensionless measure of the strength of the viscosity $\alpha_{SS}$, i.e., the Shakura-Sunyaev viscosity, is determined by the magnetic field in the form

$$\alpha_{SS} = B_r B_\phi / 4 \pi \rho C_s^2 = V_{Ar} V_{A\phi} / C_s^2,$$

where $B_r$ and $B_\phi$ are the radial and azimuthal components of the magnetic field, $\rho$ the density of the disk, and $V_{Ai}$ denotes the Alfvén speed corresponding to the $i$ component of the magnetic field. Most investigators think that the magnetic fields in the accretion disk are sustained by a magnetic dynamo. Tout & Pringle (1992) set forward a physical model simply considering the Balbus-Hawley (B-H) instability, Parker instability and Shear- ing rotation without $\alpha$ effect. They argued that the three well-known physical processes are all included in their model. This model was used to explain the basic properties of dwarf nova eruptions and some properties of quiescent disks (Armitage et al. 1996). But the $\alpha$ effect which arises from the correlation of small scale turbulent velocities and magnetic fields is important in maintaining the dynamo action by relating the mean electrical current arising in helical turbulence to the mean magnetic field. It plays the key role in the current studies of the astrophysical dynamo and many authors have studied it in detail (Krause & Rädler 1980, Stepinski & Levy 1994, 1991, Elstner et al. 1994). Furthermore, the nonlinear effect of the $\alpha-\omega$ dynamo models have been shown to be capable of producing a number of important features of different astrophysical objects. The presence of strong differential rotation plus a vertical density gradient will give rise to turbulence helicity. Considering that $\alpha$ effect arising from the helicity of turbulence is commonly accepted in Keplerian disks as well as stellar (Krause & Rädler 1980, Pudritz 1981a, b, Mangalam & Subramanian 1994, Vishniac & Brandenburg 1997, Reyes-Ruiz & Stepinski 1997), we add the $\alpha$ effect in their model and...
reconsider the behaviour of magnetic fields and the viscosity parameter in this paper. Instead of the B-H instability considered in this model, the magnetic buoyancy combined with the Coriolis twist was adopted to close the dynamo cycle by Rozyczka et al. (1995). They found that the magnetic fields reach a stable saturation state. We feel that their scenario is appropriate for a magnetized disk in which the B-H instability is restrained.

In Sect. 2, we write down the full dynamo equations. The physical processes cover the Parker instability, the B-H instability, the $\alpha - \omega$ dynamo mechanism, and magnetic reconnection. The dynamo equations are solved in Sect. 3. Finally, we present conclusion and discussion in Sect. 4.

2. The model

2.1. Parker instability

The Parker instability is a kind of Rayleigh-Taylor instability, i.e., an interchange instability, introduced by magnetic buoyancy (Parker 1979). It was found that the gas disk supported in part by the magnetic field against vertical external gravity is unstable against a long wavelength disturbance along the field lines. The horizontal components of the magnetic field are denoted by $B_r$ and $B_\phi$, and the vertical component by $B_z$. The Parker instability leads to a loss of horizontal magnetic field and converts it to a vertical field. If the horizontal magnetic field is dominated by the azimuthal component $B_\phi$, we have

$$\frac{dB_z}{dt} \sim \frac{B_\phi}{\tau_P}, \quad \frac{dB_r}{dt} \sim \frac{B_r}{\tau_P}, \quad \frac{dB_\phi}{dt} \sim -\frac{B_\phi}{\tau_P} \tag{3}$$

The growth time of the Parker instability is

$$\tau_P = \frac{H}{V_{A\phi}}, \tag{4}$$

where $H$ is the half-thickness of the disk and $\eta \approx 3$ in a non-self-gravitating accretion disk according to the calculation given by Houruichi et al. (1988). The wavelength of the instability in the azimuthal direction is

$$\lambda_{P\phi} = \xi H, \tag{5}$$

where $\xi \approx 8$.

2.2. Balbus-Hawley instability

In recent years, Balbus & Hawley et al. published a series papers concerning the stability of weakly magnetized disks (Balbus & Hawley 1991a b, 1992). It was found that the shear instability is local and extremely powerful provided that the field energy density is much less than the thermal energy density. The maximal growth rate is given by the local Oort A-value of the disk, which is not only independent of the magnetic field strength but independent of the field geometry as well (Balbus & Hawley 1992). If a fluid element is outwardly displaced in a differentially rotating disk threaded by a vertical magnetic field, rigid rotation will be enforced because it is elastically tethered by a magnetic field, i.e., the field is trying to force the element to rotate too fast for its new radial location if $\frac{d\Omega}{dt} < 0$. The excess centrifugal force drives the element still farther outward. Therefore, the effect of the instability is to tap the energy present in the shear flow and to use it to generate a radial field from the initial vertical field, i.e., the vertical magnetic field $B_z$ results in the amplification of the radial component of the magnetic field $B_r$ because of the B-H instability

$$\frac{dB_r}{dt} \sim \gamma_{BH} B_z, \tag{6}$$

where $\gamma_{BH}$ is the growth rate of the instability,

$$\gamma_{BH} =$$

$$\begin{cases}
\gamma_{max}, & \frac{V_{A\perp}}{C_s} < \frac{\sqrt{2}}{\pi}, \\
\gamma_{max} \left[1 - \frac{(1 - \frac{V_{A\perp}}{C_s})^2}{(\sqrt{3} - 1)^2}\right]^{1/2}, & \frac{\sqrt{2}}{\pi} \leq \frac{V_{A\perp}}{C_s} < \frac{\sqrt{3}}{\pi}, \\
0, & \frac{V_{A\perp}}{C_s} \geq \frac{\sqrt{3}}{\pi},
\end{cases} \tag{7}$$

where $\gamma_{max} = \frac{3}{4} \Omega$, which equals the Oort A-value of a Keplerian disk. The scale of the instability in the vertical direction, $\lambda_{BH}$, is given by

$$\frac{\lambda_{BH}}{2\pi} = \begin{cases}
1, & \frac{V_{A\perp}}{C_s} > \frac{\sqrt{2}}{\pi}, \\
\frac{\pi V_{A\perp}}{2\sqrt{2} C_s}, & \frac{\sqrt{2}}{\pi} \leq \frac{V_{A\perp}}{C_s} < \frac{\sqrt{3}}{\pi}.
\end{cases} \tag{8}$$

We can see that B-H instability is cut off if the magnetic field is sufficiently strong.

2.3. The $\alpha - \omega$ dynamo

The mean field dynamo theory has been successful in many kinds of astrophysical objects (Ma & Wang 1995, Stepinski & Levy 1991, Krause & Rädler 1980). The radial component of the magnetic field is amplified by means of the $\alpha$--effect following the equation

$$\frac{dB_r}{dt} \sim \frac{\alpha}{H} B_\phi, \tag{9}$$

The parameter $\alpha$ denotes the product of the mean helicity and the correlation time of the turbulent flows. In term of the disk’s angular velocity and turbulent Mach number, we take the form (Stepinski & Levy 1991)

$$\alpha = H M_t^2 \Omega, \tag{10}$$

where $M_t = V_t/C_s$ is the Mach number of the turbulence and $V_t$ the turbulent velocity. The recent numerical simulations indicated that the nonmagnetized astrophysical accretion disks are both linear and nonlinearly stable to shearing instabilities. Balbus et al. found no evidence for any hydrodynamical instabilities in local simulations of thin disks (Balbus et al. 1996). If the source of the turbulence is the B-H instability in a weakly magnetized disk, the turbulent Mach numbers fall in the region between 0.1 and 0.25 (Hawley et al. 1996). So, the turbulent
Mach number $M_i$ is adopted the order of 0.1 in our calculations.

At the same time, the amplification of azimuthal magnetic field results from the $\omega-$effect, i.e., the differential rotation, satisfying the equation

$$\frac{dB_\theta}{dt} \sim r \frac{\partial \Omega}{\partial r} B_r = \frac{3}{2} \Omega B_r,$$

(11)

Finally, we have closed the cycle for the dynamo to work.

2.4. Dissipation of the magnetic energy

The dominant flux loss mechanism is the reconnection of the vertical component $B_z$ in the radial direction. Consider two patches of $B_z$ of opposite sign coming together and reconnecting within distance of $\lambda_{rec}$. We have a term of dissipation in the vertical equation in the following form (Tout & Pringle 1992):

$$\frac{dB_z}{dt} = - \frac{B_z}{\tau_{rec}},$$

(12)

The magnetic flux is removed from the disk owing to reconnection at a rate

$$\frac{1}{\tau_{rec}} = \frac{1}{\tau_a^{rec}} + \frac{1}{\tau_{b^{rec}}},$$

(13)

where $\tau_a^{rec}$ is determined by the length scales of Parker instability, B-H instability and the shearing of the rotation,

$$\tau_a^{rec} = \frac{2}{3\sqrt{2}} \eta^{-1} V_{A\phi} \lambda_{rec\phi},$$

(14)

while $\tau_b^{rec}$ results from the shearing itself (Tout & Pringle 1992),

$$\tau_b^{rec} = \left( \frac{2\lambda_{rec\phi}}{3\Gamma \Omega V_{AZ}} \right).$$

(15)

Here $\Gamma^{-1} \sim \ln(\Re_m)$, where $\Re_m$ is the magnetic Reynolds number and $\Gamma$ is expected to be in the range 0.01 to 0.1 (Tout & Pringle 1992). According to the analysis made by Tout & Pringle, we adopt $\lambda_{rec\phi} = 0.5\lambda_{PH} \lambda_{BH}/H$.

2.5. The source of turbulence

Many numerical simulations have been done in accretion disks where the source of turbulence is the B-H instability (Hawley et al. 1996, Brandenburg et al. 1995). The turbulence based on these simulations is such that the turbulent Mach number is less than 0.3. These works find that kinematic dynamo theory does not appear to be an appropriate description.

There is no source known for the turbulence required for the $\alpha$-mechanism other than the one mentioned here already, the B-H instability. Therefore we need to ask what other sources of turbulence could be there in astrophysical disks. We cannot offer a final solution, but just probe several possibilities:

In entire galaxies that are modeled as optically thin large scale accretion disks, it is clear that the activity of supernovae is a persistent source of turbulence, almost certainly the dominant source. Supernovae and stellar winds inject magnetic fields into galaxies; these magnetic fields are very chaotic, but can be fairly strong at injection from the boundary shock of a stellar wind (Seemann & Biermann 1997).

In accretion disks around the black holes that are believed to be at the center of nearly any early Hubble type galaxy (Faber et al. 1997), there turbulence is injected by the passage of stars; whether this mechanism is sufficient to sustain an appreciable level of turbulence at a small distance to the central black hole is not clear. However, the collisions of stars with each other, clearly a less frequent process, already can produce considerable dissipation (Courvoisier et al. 1996), and so one may speculate that stellar passages can have a considerable influence on the accretion disk.

Again, in accretion disks around the black holes in centers of galaxies there is observed to be a appreciable hard X-ray emission, believed to arise from reprocessing in the accretion disk (e.g. George & Fabian 1991); since the power of this radiation is an appreciable fraction of the disk luminosity itself, one may well ask, whether the reprocessing induces turbulence in the disk.

And finally, in accretion disks around compact stars in binary systems, the very center of the disk emits X-rays, which impinge on the outer parts of the disk. In these outer parts thus disturbances can be induced, which may propagate into the inner part.

Thus, albeit no definitive answer to our initial question, there appear to be a sufficient number of open questions as regards to origin of turbulence in accretion disks, that we feel it is worthwhile to ask, what happens, if such sources do exist.

2.6. The full dynamo equations

Concluding all the physical processes stated above, and defining a dimensionless time $\tau = \sqrt{2\eta}/\Omega$, $\dot{\gamma} = \gamma/\Omega$ and a dimensionless velocity $w_i = V_{Ai}/C_s (V_{Ai} = B_i/\sqrt{4\pi B}, i = r, \phi, z)$, we get the full dynamo equations in the spatially local approximation

$$\frac{dw_r}{d\tau} = \begin{cases} \hat{\gamma}_{max} \sqrt{2\eta} w_z - w_r w_\phi + \sqrt{2\eta} \lambda_{BH}^2 w_\phi, & w_z < \sqrt{2}; \\ \hat{\gamma}_{max} - \left( \frac{\sqrt{2\eta} w_z^2 / \sqrt{2}}{\sqrt{2}^2} \right)^{1/2} w_z, & w_z \leq \sqrt{2}; \\ -w_r w_\phi + \sqrt{2\eta} \lambda_{BH}^2 w_\phi, & w_z > \sqrt{2}, \end{cases}$$

(16)

$$\frac{dw_\phi}{d\tau} = \frac{3\sqrt{2}}{2} \eta w_r - w_\phi^2,$$

(17)

$$\frac{dw_z}{d\tau} = \begin{cases} w_\phi^2 - 6\sqrt{2}\eta^2 \lambda_{BH} \frac{w_z^2}{w_\phi}, & w_z \leq \sqrt{2}; \\ -2^{3/4}\sqrt{3}\eta^2 \lambda_{BH} \frac{1/2}{1/2} \xi^{1/2} - 1/2 w_z^{3/2}, & w_z > \sqrt{2}. \end{cases}$$

(18)
We note that all the spatial effects are ignored in these equations. As compared with Tout & Pringle’s results, the terms including \( M_t \) present the \( \alpha \) effect in Eq. (16), which are not invoked in their model. In the next section, we will analyze the solutions of the full dynamo equations.

3. Solutions of the dynamo equations

3.1. The equilibrium solutions

When the turbulence is weak, the process is dominated by B-H instability and Parker instability, so the equilibrium analysis should be the same as Tout & Pringle’s results. On the other hand, if the turbulence is so strong that \( M_t > 0.25 \), we will see from the numerical calculation that the \( \alpha - \) effect takes over, and the equilibrium solutions read as

\[
\frac{V_{eq}^{Ar}}{C_s} = \sqrt{2} \eta M_t^2, \\
\frac{V_{eq}^{A\phi}}{C_s} = \sqrt{3} \eta M_t, \\
\frac{V_{eq}^{Az}}{C_s} = 3^{1/4} 2^{-3/4} \eta^{1/2} \Gamma^{-1/2} \xi^{1/2} M_t^{3/2}.
\]

In this regime, we estimate the viscosity according Eq. (2)

\[
\alpha_{SS} = \sqrt{6} \eta^2 M_t^3.
\]

3.2. The numerical solution

Based on the statements in Sec.2, we take the parameters \( \Gamma = 0.1, \eta = 3, \xi = 8, \gamma_{max} = 0.75 \) and the initial condition \( w_r = w_\phi = w_z = 0.01 \) at \( \tau = 0 \) in our calculations. When the turbulent Mach numbers are set to be 0.1, 0.2, 0.24 and 0.30, the time dependence of the magnetic field components are shown in Fig. 1 - Fig. 4 respectively. We can see from the figures that the dynamos are dominated by B-H instability, Parker instability and the shearing motion of the disk if the turbulent Mach numbers \( M_t \) are less than 0.25, and the magnetic fields oscillate around their equilibrium values. The Mach number of the turbulence does not affect the equilibrium value of the amplified magnetic fields so much, but the amplitudes of the oscillation of the amplified magnetic field decreases meanwhile the periods increase as the Mach number \( M_t \) increases. If the turbulent Mach number is larger than 0.25, the \( \alpha - \omega \) dynamo takes over and the oscillation of the amplified magnetic field disappears, and the equilibrium values depend on \( M_t \) as Eq. (21) — (23). The corresponding time-dependent behaviors of the viscosity parameter \( \alpha_{SS} \) are shown in Fig. 5, where the turbulent Mach numbers are set to be 0.1, 0.2 and 0.25 respectively. We can see that the time averaged value of \( \alpha_{SS} \) is about 0.33, which does not depend on the turbulent Mach number \( M_t \).

3.3. The non-linear dynamo

When the back-reaction of the amplified magnetic field on the turbulent motion of the fluids is considered, the mean helicity of the turbulent flow becomes

\[
\alpha' = \alpha \Psi(B^2),
\]

where the function \( \Psi(B^2) = (1 + V_A^2/C_s^2)^{-1} \) is quadratic in the magnetic field, representing the so called \( \alpha - \) quenching (Schultz et al. 1994, Rüdiger et al. 1997, Covas et al. 1997), and \( \alpha \) is determined by Eq. (10). The critical turbulent Mach number increases to \( M_t \simeq 0.44 \), as compared with the linear model, it is 0.25.

4. Conclusion and discussion

Tout & Pringle (1992) put forward a physical model including three processes: the Parker instability, the B-H instability and
magnetic field reconnection, to study the magnetic field configurations and viscosity parameter. The magnetic dynamo origin for the viscosity based on this model was used to explain the eruptions of accretion disks and dwarf nova (Armitage et al. 1996). Considering that the $\alpha$ effect has been well studied too, we added the $\alpha$ effect in their dynamo model and obtain the results as follows.

If the turbulent Mach number $M_t < 0.25$ in the linear dynamo model, dynamos are dominated by the B-H instability and the Parker instability. When the dynamo works in this region, we have the time-averaged viscosity $\alpha_{SS} \sim 0.33$. The turbulent Mach number associated with the $\alpha - \omega$ dynamo only affects the amplitude and the period of the oscillation of the amplified magnetic fields. The kinematic viscosity obtained from this simple model can be used to explain the outburst and quiescent phases of the disk around the Dwarf Nova (Tout & Pringle 1995, Armitage et al. 1996). The disk remains in a quiescent state with strong fields and low viscosity until sufficient mass has been added to restart the B-H instability. On the other hand, Gammie and Menou (1997) proposed an other scenario for the origin of episodic accretion in dwarf novae, in which the outburst cycle purely results from the global hydrodynamic instability and depends on the magnetic Reynolds number. This model is totally different from the standard disk instability model.

But if the turbulent Mach number $M_t \geq 0.25$, the B-H instability is restrained and the oscillations of the magnetic field components disappear. That means that the equilibrium magnetic fields are determined by $M_t$ alone. The corresponding viscosity is $\alpha_{SS} = \sqrt{6\eta^2 M_t^3}$. In this case, the disks should keep in a quiescent phase all the time, i.e., there would be no eruptions any more when the turbulence is so strong that the Mach number is larger than 0.25. But in the absence of the B-H instability, there would be no turbulence in the disk at all if no other source of turbulence is included. So we think that the accretion disks cannot work at this stage. In fact, the turbulent Mach number also depends on the temperature of the accretion flow, which means that a combination of the disk model and dynamo model is necessary to describe the behavior of the accretion disk more clearly. We note that the standard mean field dynamo theory does not cover the B-H instability.

The physical nature of turbulence in astrophysical objects is not clear, although most people believe that the turbulent Mach number in accretion disks or galaxies is of the order of 0.1 (Ruzmaikin et al. 1980, Stepinski et al. 1991, Moss et al. 1996). The three dimensional simulations of an accretion disk indicate that the values of the turbulent Mach number fall in the region between 0.1 and 0.25 (Table 4 of Hawley et al. 1996). Brandenburg et al. (1995) suggested that supersonic flows, initially generated by the Balbus-Hawley magnetic shear instability, regenerate a turbulent magnetic field, which, in turn, can reinforce the turbulence. The final turbulence is not so strong based on their simulations either. So it seems that the turbulent dynamo is not so effective to suppress the B-H instability completely. In fact, when a nonlinear dynamo is concerned, the critical turbulent Mach number increases to be 0.44 rather than 0.25 as compared to the linear model. We conclude that the $\alpha -$ effect does not
change the basic configuration of magnetic fields described by Tout & Pringle (1992).

There are two facts in the model that may need modification for further studies. The first is that the dissipation processes of magnetic fields in accretion disks is not clear to us, which are simplified to some degree in this model. The second is that the spatial behavior of a full magnetic dynamo model should be taken into account.

We note that the vertical component of the magnetic field is important for the outflow of jets in AGN (Yoshizawa & Yokoi 1993). Tout & Pringle (1996) demonstrated that a dynamo-generated field coupled with an inverse cascade process is able to produce sufficient field strength on large enough scales to drive a large-scale outflow. The similar inverse cascade process of the magnetic fields from small scale to large scale was recently proposed by Kulsrud et al. (1997) to explain the origin for cosmic magnetic fields.

The rotating magnetic fields in the accretion disk would introduce an electric field which can accelerate the charged particles. Some of the electrons will become run-away particles in the electronic field. The high energy electrons may emit X-rays or $\gamma$-rays from the disk and excite plasma instabilities which might be the source of turbulence (Pustil'nik & Ikhsanov 1994).

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