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Structure formation in the one dimensional gravitational gas

A. Muriel[1], **A. Miciano-Cariño**[1], **and R. Cariño**[2]

[1] World Laboratory Research Centre for Fluid Dynamics, University of the Philippines at Los Baños, 4031 College, Laguna, Philippines
[2] Institute of Computer Science, University of the Philippines at Los Baños, 4031 College, Laguna, Philippines

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Abstract. We adopt a new result in non-equilibrium statistical mechanics to follow the time evolution of the density of an exploding one-dimensional gravitational gas. We show analytically that the resulting structure formation is a natural consequence of many-body physics.

Key words: instabilities – galaxies: formation – large-scale structure of Universe

1. Introduction

In two analytic studies of the one-dimensional gravitational gas, starting with the Vlassov-Poisson equation, it was shown that sub-structures could develop from some initial conditions. In one case (Minneau, et al. 1990), starting with an initial Dirac-delta distribution in space coupled with a Maxwellian distribution in momentum, it was shown that two symmetric depressions propagate away from the origin. These minute structure in the density, four orders of magnitude lower than the average density, are superimposed on a spreading Gaussian. We have called this caricature of an exploding gravitational gas the "petit-bang" model. In another case (Jirkovsky & Muriel 1993), we started with a step discontinuity in the density and a Maxwellian distribution of momentum. Again we find the development of equally tiny sub-structures, a propagating depression in the high-density side, and a pulse travelling in the low-density side. The depression and the pulse are anti-symmetric, conserving probability, and travel with the same speed away from the initial discontinuity. The initial discontinuity itself gradually smoothens. These two analytic studies distinguishing themselves from several numerical experiments of which Hohl and Feix (1967) are typical. Such numerical experiments have typically shown the formation of clusters, but it seems that analytic proofs of structure formation are difficult to establish, except for the above two cases.

In this work, we will show much more developed structure formation, displaying a hierarchy of structures, in a manner that exceeds our early studies. We generalize the "petit-bang" model by assuming an initial Maxwellian explosion out of a diffuse Gaussian spatial distribution at the origin. To arrive at our results, we use a new classical many-body approach that was developed in non-equilibrium statistical mechanics (Muriel & Dresden 1995), and applied it to the one-dimensional gravitational gas to give further proof of structure formation (Muriel & Esguerra 1996). This extends our early results which used conventional time-dependent perturbation theory (Minneau, et al. 1990; Jirkovsky & Muriel 1993). This recent theory is different from the usual BBGKY approach (Binney & Tremaine 1987), it uses an integral formulation instead of developing a hierarchy of equations. As this new approach is not yet used in stellar dynamics, we describe this approach briefly, referring quite heavily to the formal theory of Muriel and Dresden.

Send offprint requests to: A. Muriel

2. An integral formulation of many-body physics

We start with an exact, formal result derived by Muriel and Dresden for the one-particle distribution function, $f(r,p,t)$, given as Eq. (1), where, $L_0 = \frac{p}{m}\frac{\partial}{\partial r}$, and p, m, $V(r-r')$ are respectively, the momentum, mass of the identical particles, and the pair potential of two particles in the coordinates r, r'. The symbol $O(\frac{\partial^2}{\partial p^2}\cdots)+\ldots$ stands for an infinite number of terms each of which starts with $\frac{\partial^2}{\partial p^2}$. These omitted terms are given in detail in Muriel and Dresden, and it will become clear shortly why we do not even write out the terms. The symbol $f_3^1(r,r',r'',p,0)$ is a mixed distribution function representing the probability at time zero, that 3 particles are in their coordinates r, r', r'' and that in addition, the particle at r possesses the momentum p. The symbols f_2^1, f_1^1, f_2^2, are given analogous meaning. The occurrence of mixed distribution functions, instead of multi-particle distribution functions, distinguish our work from the conventional BBGKY hierarchy used also in stellar dynamics (Binney & Tremaine 1987). The 3-d version of Eq. (1) may be written properly, but we restrict this work to the 1-d version.

$$f(r,p,t) = f(r-pt/m,p,0) \; + \int_0^t ds_1 \exp(-s_1 L_0) \int dr' \frac{\partial V(r-r')}{\partial r}\frac{\partial}{\partial p} f_2^1(r,r',p,0) \; + \int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \frac{\partial V(r-r')}{\partial r} \times \frac{\partial}{\partial p}\left(\frac{p}{m}\frac{\partial}{\partial r} f_2^1(r,r',p,0)\right) \; -$$

$$\int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dp' \int dr' \frac{\partial V(r-r')}{\partial r} \times \frac{\partial}{\partial p}\left(\frac{p'}{m}\frac{\partial}{\partial r} f_2^2(r,r',p,p',0)\right) + \frac{1}{2}\int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \int dr'' \frac{\partial V(r-r')}{\partial r} \times \frac{\partial V(r-r'')}{\partial r}\frac{\partial^2}{\partial p^2} f_3^1(r,r',r'',p,0) + \frac{1}{2}\int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \left(\frac{\partial V(r-r')}{\partial r}\right)^2 \times \frac{\partial^2}{\partial p^2} f_2^1(r,r',p,0) + O(\frac{\partial^2}{\partial p^2}\cdots) + \ldots \quad (1)$$

Integrating Eq. (1) over momentum from $-\infty$ to $+\infty$, all terms represented in $O(\frac{\partial^2}{\partial p^2}\cdots)$ integrate out to zero by partial integration, leaving a truncated expression for the density $n(r,t)$.

$$\begin{aligned} n(r,t) &= \int dp f(r-pt/m,p,0) + \\ &\int dp \int_0^t ds_1 \exp(-s_1 L_0) \int dr' \tfrac{\partial V(r-r')}{\partial r}\tfrac{\partial}{\partial p} f_2^1(r,r',p,0) + \\ &\int dp \int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \tfrac{\partial V(r-r')}{\partial r} \times \tfrac{\partial}{\partial p}\left(\tfrac{p}{m}\tfrac{\partial}{\partial r} f_2^1(r,r',p,0)\right) - \\ &\int dp \int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dp' \int dr' \tfrac{\partial V(r-r')}{\partial r} \times \tfrac{\partial}{\partial p}\left(\tfrac{p'}{m}\tfrac{\partial}{\partial r} f_2^2(r,r',p,p',0)\right) + \\ &\tfrac{1}{2}\int dp \int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \int dr'' \tfrac{\partial V(r-r')}{\partial r} \times \tfrac{\partial V(r-r'')}{\partial r}\tfrac{\partial^2}{\partial p^2} f_3^1(r,r',r'',p,0) + \\ &\tfrac{1}{2}\int dp \int_0^t ds_1 \int_0^{s_1} ds_2 \exp(-s_2 L_0) \int dr' \left(\tfrac{\partial V(r-r')}{\partial r}\right)^2 \times \tfrac{\partial^2}{\partial p^2} f_2^1(r,r',p,0) \end{aligned} \quad (2)$$

For a system that obeys the symmetry of particle exchanges and the use of "reasonable" momentum distributions which vanish at the limits of phase space, Eq. (2) is properly truncated from its infinite series form. This truncation of the infinite series only works when the momentum integration is done for "reasonable" momentum distributions.

3. Time evolution of the density of the one-dimensional gravitational gas

For the one-dimensional gravitational gas, change r to x and use the following:

$$\frac{\partial V(x-x')}{\partial x} = \varepsilon(x-x'), \qquad \varepsilon(x) = \begin{cases} 1 \text{if } x>0, \\ 0 \text{if } x=0, \\ -1 \text{if } x<0, \end{cases} \quad (3)$$

$$\begin{aligned} f_1^1(0) &= \psi(x)\varphi(p), \\ f_2^1(0) &= \psi(x)\psi(x')\varphi(p), \\ f_2^2(0) &= \psi(x)\psi(x')\varphi(p)\varphi(p'), \\ f_3^1(0) &= \psi(x)\psi(x')\psi(x'')\varphi(p), \end{aligned} \quad (4)$$

where $\psi(x) = \sqrt{\frac{\alpha}{\pi}} e^{-\alpha x^2}$. We only require that $\varphi(p)$ vanish at the boundaries of phase space. Our model is similar to what we have called elsewhere "petit-bang" (Minneau, op. cit.), representing a one-dimensional Newtonian cosmology of expansion from a smooth central distribution at time zero. From Eq. (4), we see that we have used complete factorization or no correlation for the initial conditions. Substituting Eqs. (3) and (4) into Eq. (2), we get

$$\begin{aligned} n(x,t) &= \sqrt{\tfrac{\alpha}{\pi}} \int dp e^{-\alpha(x-\frac{pt}{m})^2}\varphi(p) \\ &+ \tfrac{\gamma m}{4}\int dp \left[(erf[\sqrt{\alpha}x])^2 - (erf[\sqrt{\alpha}(x-\tfrac{pt}{m})])^2\right]\tfrac{1}{p}\tfrac{\partial\varphi(p)}{\partial p} \\ &+ \gamma\int dp \begin{bmatrix} \frac{m^2}{4p^2}\left\{(erf[\sqrt{\alpha}(x-\frac{pt}{m})])^2 - (erf[\sqrt{\alpha}x])^2\right\} + \\ \frac{m^2}{2p^2}\sqrt{\frac{2\alpha}{\pi}}\left\{(x-\frac{pt}{m})(erf[\sqrt{2\alpha}x] - erf[\sqrt{2\alpha}(x-\frac{pt}{m})])\right\} + \\ \frac{m^2}{2\pi p^2}(e^{-\alpha x^2} - e^{-\alpha(x-\frac{pt}{m})^2}) + \\ \frac{mt}{p}\sqrt{\frac{\alpha}{\pi}} e^{-\alpha x^2} erf[\sqrt{\alpha}x] \end{bmatrix} \tfrac{\partial}{\partial p}\left(\tfrac{p}{m}\varphi(p)\right) \\ &+ \tfrac{\gamma^2 m}{12}\int \tfrac{dp}{p}\int_0^t ds \left[(erf[\sqrt{\alpha}x])^3 - (erf[\sqrt{\alpha}(x-\tfrac{ps}{m})])^3\right]\tfrac{\partial^2\varphi(p)}{\partial p^2} \\ &+ \tfrac{\gamma^2 m^2}{4}\int \tfrac{dp}{p^2}\begin{bmatrix} \frac{1}{\sqrt{\pi\alpha}}\{e^{-\alpha(x-\frac{pt}{m})^2} - e^{-\alpha x^2}\} + (x-\frac{pt}{m}) \\ \cdot\{erf[\sqrt{\alpha}(x-\frac{pt}{m})] - erf[\sqrt{\alpha}x]\} \end{bmatrix}\tfrac{\partial^2\varphi(p)}{\partial p^2} \end{aligned} \quad (5)$$

The first two terms are well behaved and bounded. The remaining terms have integrable singularities at $p = 0$, and could diverge for large x and t about which we make a comment now.

The original derivation of Eq. (1) assumes that the system is finite. Although a spatial Gaussian can be considered compact, strictly speaking, it has infinite extent, a violation of the initial assumption which results in the limited applicability of Eq. (5). We do not regard this feature of our theory as fatal, for even in the case of Poincare's time-dependent perturbation, secular divergences also show up. Elsewhere, we will discuss and illustrate all the formal requirements for the Muriel and Dresden formalism to be valid for all times, clarifying the limitation that we meet in this current investigation.

4. Graphical results

Let us put $\varphi(p) = \sqrt{\frac{\beta}{\pi}} e^{-\beta p^2}$ in Eq (5). All integrations over odd functions of momentum disappear. We then plot in Fig. 1 the time evolution of the density for the following parameters: $\alpha = \beta = m = 1, \gamma = 0.01$. To be sure that we do not go beyond the validity of the theory, we also plot in Fig. 2 the total mass and check that it is constant and finite. When the total mass, as calculated from integration over all space, becomes divergent, we stop the numerical plot.

The occurrence of a sub-structure is apparent, with hints of micro-structure. In fact the micro-structure is hardly visible with the scale used in Fig. 1. To show the micro-structure, we go back to Eq. (1) and plot the phase space behavior of $f(x,p,t)$ in Fig. 3. In plotting $f(x,p,t)$, it must be remembered that Eq. (1) is an infinite series, and that one should include all the terms, but for the value of γ used, we may neglect the rest of the infinite series for short times. Adopting all parameters used in Fig. 1, we plot $f(x,p,t)$ in Fig. 3, which shows the rotation in phase space of the time-dependent distribution function, as well as the formation of sub-structure, for times up to $t = 6$. This rotation has

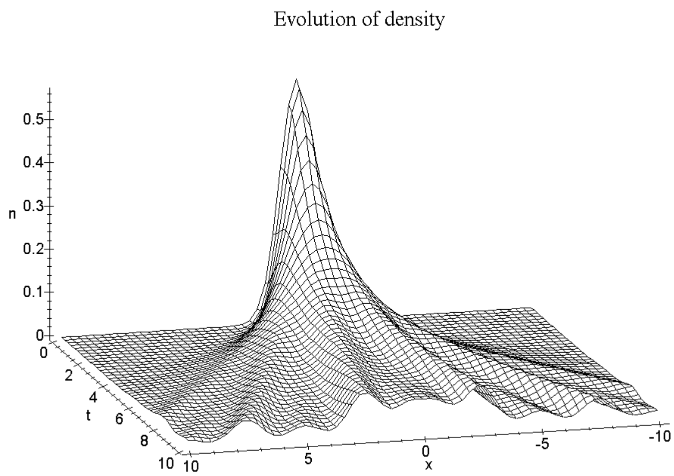

Fig. 1. Time evolution of the density up to $t = 10$ for the parameters $\alpha = \beta = m = 1, \gamma = 0.01$.

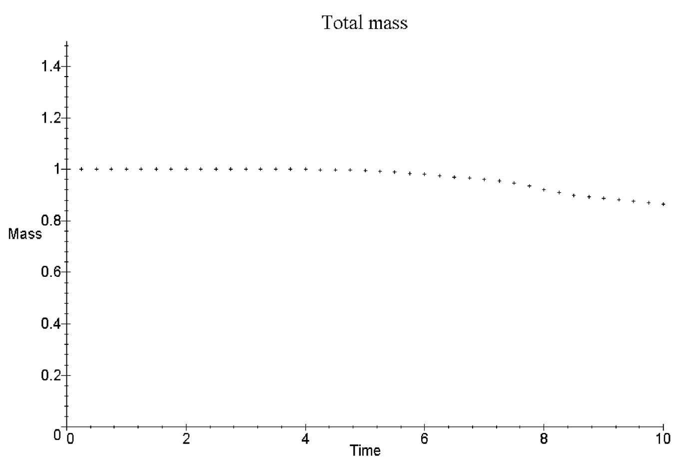

Fig. 2. Time dependence of total mass according to Eq. (5)

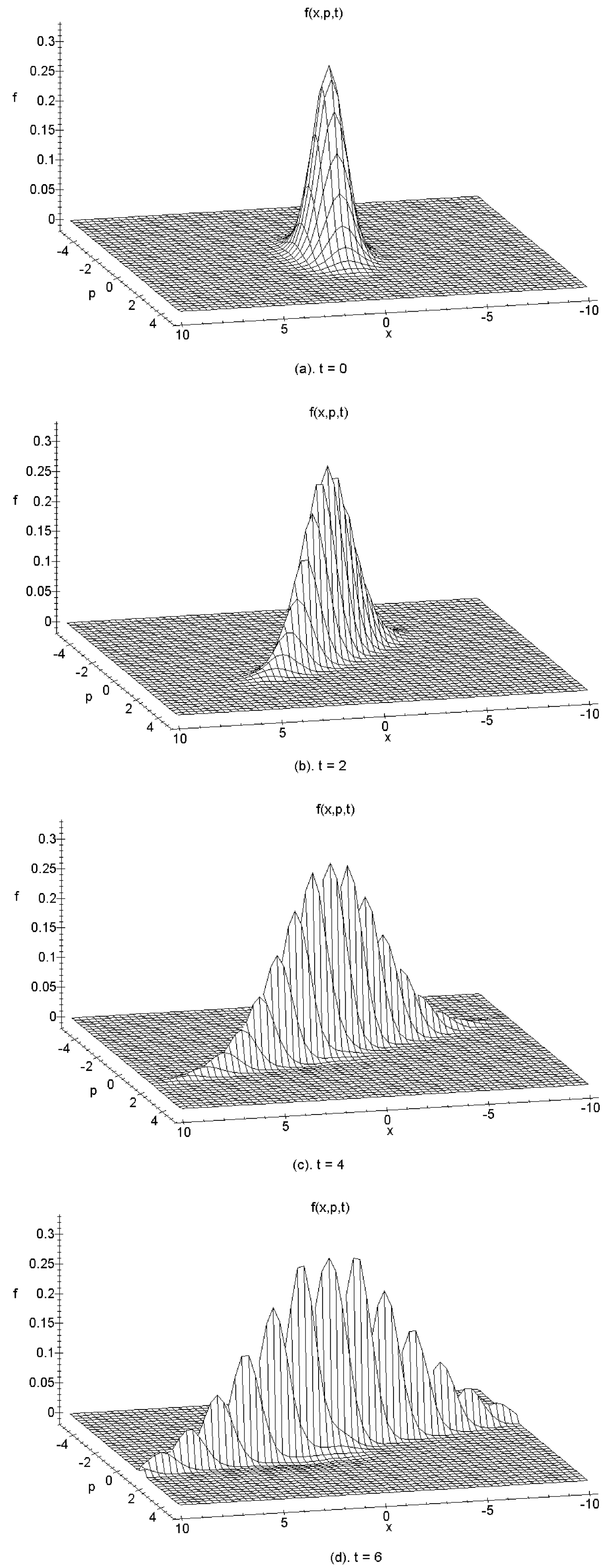

Fig. 3a–d. One particle distribution function at **a** $t = 0$, **b** $t = 2$, **c** $t = 4$, and **d** $t = 6$ for the same parameters as in Fig. 1

already been seen numerically by Hohl and Feix (1967). The use of only the first six terms in Eq. (1) for plotting Fig. 3 has limited validity with time. However, the justification for this work remains, a completely new qualitative feature of structure formation observed for "short" times, of the order of $1/\gamma$, obtained analytically, and only hinted at before by perturbation calculation. Furthermore, from a physical standpoint, calculating only $f(r,p,t)$, the single particle distribution function, cannot adequately describe the system. It would be interesting to calculate as well the two-particle distribution function $f(r,r',p,p',t)$ in the future. This would be the best way to show the development of binaries, or gravitational molecules.

To summarize, we observe the following: the main structure is determined by the zeroth term in γ, or the ballistic term; the sub-structure is proportional to γ; and another layer of structure, or micro-structure, is proportional to γ^2. For $\gamma << 1$, the amplitudes of the high-order terms diminish with powers of γ.

Finally, we discuss the relevance of this work to the existing literature on structure formation. From a numerical point of view, although not very much remarked upon, the development of clusters or sub-clusters in one dimension has been apparent for a long time, see for example Severne, et al. (1984) and Hohl and Feix (1967). In three dimensions, the formation of a hierarchy of structures has been physically justified in a qualitative, physical approach (Padmanabhan 1993; Peebles 1980). For example, using a continuum approach in 3-d, versus our own many-body approach in one dimension, Zeldovich (1970) described the growth of density perturbations using linear perturbation theory. In an improvement of this linear calculation, in what is called the adhesive model, Shandarin and Zeldovich (1990) confirmed the growth of density perturbations. Neither the first or second analytic attack described the growth of internal structures (Padmanabhan 1993). Apart from the mathematical difference between our Liouvillian approach and the Zeldovich continuum approach, our calculations show the development of gross structure, sub-structure and even microstructure, which has not been shown by any of the two models introduced by Zeldovich and Shandarin. So what this work contributes is an independent, analytic confirmation of structure and sub-structure formation in one dimension, in conformity with past numeric and qualitative studies.

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