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Letter to the Editor

Is γ-ray absorption by induced electric fields important in the pulsar magnetospheres?

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Abstract. Although the unified formula for γ-ray absorption process involving both the magnetic field and a perpendicular electric field derived by Daugherty & Lerche (1975) is correct, we argued in this paper that their conclusion that the induced electric fields are important in the pair formation process in the pulsar magnetospheres is wrong and misleading. The key point is that usually the direction of a γ photon at the emission point observed in the laboratory frame should be $(v/c, 0, [1-(v/c)^2]^{1/2})$ rather than $(0, 0, 1)$, where v is the corotating velocity. This emission direction is just the one which results in zero attenuation coefficient of the γ photon. Calculation shows that after the photon has moved a distance, its direction lead to the result that the induced electric field is also of minor importance. Thus only $\gamma - B$ process is the important mechanism for the pair production in the pulsar magnetospheres. The implications of the modification by ejecting the induced electric field are also discussed.

Key words: pulsars: general – pair production

1. Introduction

Pair production process plays an important role in pulsar physics. It is not only a necessary process for the multiplication of the particles to account for emissions of different bands from pulsars (e.g. Sturrock 1971; Ruderman & Sutherland 1975; Arons & Scharleman 1979; Arons 1983; Cheng, Ho, & Ruderman 1986), but also an important mechanism to absorb γ-rays produced in the pulsar magnetospheres, especially near the polar cap region (e.g. Hardee 1977; Harding, Tademaru, & Esposito 1978; Harding 1981; Daugherty & Harding 1982, 1996; Zhao et al. 1989; Lu & Shi 1990; Lu, Wei, & Song 1994; Dermer & Sturner 1994; Sturner, Dermer, & Michel 1995; Wei, Song, & Lu 1997). Furthermore, the way by which the γ-rays are absorbed is also the key factor to limit the parameters of the inner magnetospheric accelerators of pulsars (e.g. Ruderman & Sutherland 1975, hereafter RS75; Zhang & Qiao 1996; Qiao & Zhang 1996; Zhang et al. 1997a; Zhang, Qiao, & Han 1997b, hereafter ZQH97b).

Pair formation in intense magnetic fields ($\gamma - B$ process) has been studied explicitly by different authors (e.g. Erber 1966; Tsai & Erber 1974; Daugherty & Harding 1983, Rifert, Mészáros & Bagoly 1989), and its importance in pulsar physics was first pointed out by Sturrock (1971). Daugherty & Lerche (1975, hereafter DL75) first dealt with the case involving a relatively weaker electric field perpendicular to the magnetic field ($\mathbf{E}^2 - \mathbf{B}^2 \le 0, \mathbf{E}\cdot\mathbf{B} = 0$), and came to a unified formula of the attenuation coefficient of the γ photons. The more general case involving both the perpendicular and the parallel components of the electric field with respect to the magnetic field ($E_\perp$ and $E_\parallel$) was presented by Daugherty & Lerche (1976) and Urrutia (1978). In the specific case of pulsars, although $E_\parallel$ is usually sufficiently small so that its effect is negligible, $E_\perp$ induced by the fast spin of the neutron stars was demonstrated to be very important in pair formation process by DL75. This leads many authors to take this effect seriously into account in their studies (e.g. Hardee 1977; Harding, Tademaru, & Esposito 1978; Harding 1981; Daugherty & Harding 1982; Lu & Shi 1990; Lu, Wei, & Song 1994; Qiao & Zhang 1996).

In this paper, we'll argue that although the unified formula of DL75 is correct, their conclusion that the induced electric fields are important in the pair formation process in the pulsar magnetospheres is wrong and misleading. The detailed argument is presented in Sect. 2 and Sect. 3. Finally, we discuss the possible implications of this modification.

2. The role of the induced electric fields in pair formation at the emission point

The attenuation coefficient of converting a γ photon into electron-positron pairs in a pure strong magnetic field with a

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perpendicular component $B_\perp$ is expressed as

$$\zeta = 0.23c\frac{\alpha_0}{\lambda_e}\frac{2mc^2}{E_\gamma}\chi\exp\left(-\frac{4}{3\chi}\right), \quad (1)$$

with $\chi = \frac{E_\gamma}{2mc^2}\frac{B_\perp}{B_c}$ under the condition of $\chi \ll 1$ (Erber 1966). Here $\alpha_0 = e^2/\hbar c$ denotes the fine structure constant, $\lambda_e = \hbar/mc$ is the reduced Compton wavelength of the electron, $B_c = m^2c^3/e\hbar = 4.414 \times 10^{13}$G is the critical magnetic field, and E_γ is the energy of the γ photon.

If a relatively weaker electric field perpendicular to the magnetic field (i.e. $\mathbf{E}^2 - \mathbf{B}^2 \leq 0, \mathbf{E}\cdot\mathbf{B} = 0$) exists, the attenuation coefficient can be derived by performing a Lorentz transformation (DL75). With the positive y and z-axes being the direction of $\mathbf{E}$ and $\mathbf{B}$, respectively, this attenuation coefficient, which is a function of the direction cosines (η_x, η_y, η_z) of the photon, is expressed by DL75 (see their Eq. (9)). Note that the positive x-axis is just the moving direction of a frame with a "drifting" velocity of $\mathbf{v} = c\frac{\mathbf{E}\times\mathbf{B}}{B^2}$, in which the electric field vanishes completely. This velocity is also the "co-rotating velocity" of the particles in a pulsar's magnetosphere when the magnetic axis is aligned with the rotational axis.

DL75's derivation of the general formula (their Eq. (9)) is by all means correct. For $(\eta_x, \eta_y, \eta_z) = (E/B, 0, \pm[1 - E^2/B^2]^{1/2})$ (their *case (d)*), they derived $\zeta \equiv 0$, which means that in two special directions, the γ photons can traverse the electromagnetic field freely without being absorbed at all. Another special case is $(\eta_x, \eta_y, \eta_z) = (0, 0, 1)$ (their *case (e)*), i.e. the γ photon is moving along the direction of the magnetic field. And they obtained

$$\zeta = 0.23c\frac{\alpha_0}{\lambda_e}\frac{2mc^2}{E_\gamma}\chi\left(1 - E^2/B^2\right)\exp\left(-\frac{4}{3\chi}\right), \quad (2)$$

where $\chi = \frac{E_\gamma}{2mc^2}\frac{E}{B_c}$. It just like that E in Eq. (2) has replaced $B_\perp$ in Eq. (1) apart from a multiplicative factor $(1 - E^2/B^2)$.

In a pulsar magnetosphere, when we observe in the laboratory frame, a spin-induced electric field satisfying $\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} = 0$ should exist to fulfill the force-free condition (Goldreich & Julian 1969), where $\mathbf{v}$ is the co-rotating velocity at a given point in the magnetosphere. This electric field is perpendicular to B and is much smaller than B since $v \ll c$ in the pulsar polar cap region. So DL75's formula can be used to deal with this problem. DL75 argued that the photon's direction should be nearly parallel to B in the pulsar magnetosphere, so they came to the conclusion that their *case (e)* is proper and the induced electric field plays a considerable role in the pair production process, especially for those rapidly spinning pulsars.

But this argument is unfortunately wrong, since it results in very unnatural conclusions. First, consider a special magnetic field configuration (although it does not exist at all in nature) with straight field lines co-rotating with the magnetosphere. In the co-rotating frame where the electric field vanishes, a particle will move strictly along the field line and hence, emit a γ photon by a certain mechanism in the direction (0, 0, 1). The γ ray cannot be absorbed since there is no perpendicular magnetic field component at all. But if one observes the same process in the laboratory frame, according to DL75, Eq. (2) shows that the absorption is quite severe since there exists a very strong rotation-induced electric field. We see contradictory pictures in two different observer's frames. Secondly, in a curved magnetic field, a γ-ray can travel a distance until $B_\perp$ achieves a sufficient value to absorb it. But according to Eq. (2), a γ-ray can be directly absorbed severely at the very position it is produced. This means that in a rapidly rotating pulsar magnetosphere, the relatively high energy γ photons can hardly be formed at all. This is also quite unnatural. What is wrong?

The problem lies in the DL75's misusing (0, 0, 1) as the photon direction in the laboratory frame. In a strong magnetic field, the very short lifetime on the higher Landau energy levels makes an electron almost always stay in its ground state, so it is described that an electron moves along the field lines and emits photons at the tangent moving direction (e.g. the curvature radiation or the inverse Compton scattering). But this picture is strictly correct ONLY in the frame where there is no perpendicular electric field, or in the "co-rotating" frame for a aligned pulsar. If the observer moves with respect to the magnetic field (e.g. in the laboratory frame) so that an induced electric field exists, the directions of the field line, of the electron, and of the photon are all different from each other since they obey different transformation laws.

Suppose two inertia frames: frame S', the instantaneous co-rotating frame where the electric field vanishes so that the three directions are aligned; and frame S, the laboratory frame where the three directions may be misaligned (denoted by the angles between them and the positive x-axis, θ_B, θ_u and θ_γ). Frame S moves at a relative speed $-\mathbf{v}$ with respect to frame S', where $\mathbf{v}$ is the co-rotating velocity. The positive x-axis is defined as the direction of $\mathbf{v}$. The Lorentz transformation of the electromagnetic field then results in (in order to be simple, we assume the direction of $\mathbf{B}'$ is in $x' - z'$ plane)

$$\tan\theta_B = \gamma_r \tan\theta'_B, \quad (3)$$

where $\gamma_r = 1/\sqrt{1 - v^2/c^2}$ and the subscript "r" tells the Lorentz factor of the rotation from that of the particle. And the velocity Lorentz transformation comes to

$$\tan\theta_u = \frac{1}{\gamma_r}\frac{\sin\theta'_u}{\cos\theta'_u + \frac{v}{u'}}, \quad (4)$$

where u' is the velocity of the particle along the magnetic field line in the frame S'. The aberration effect makes

$$\tan\theta_\gamma = \frac{1}{\gamma_r}\frac{\sin\theta'_\gamma}{\cos\theta'_\gamma + \frac{v}{c}}. \quad (5)$$

Although $\theta'_B = \theta'_u = \theta'_\gamma$ in the frame S', usually $\theta_B \neq \theta_u \neq \theta_\gamma$ in the frame S. Physically, since an induced electric field $\mathbf{E}$ exists in the laboratory frame S, a "drifting" velocity component $\mathbf{v} = c\frac{\mathbf{E}\times\mathbf{B}}{B^2}$ will be added to the electrons besides the component along the magnetic field $\mathbf{B}$.

In the case of an "aligned rotator" whose magnetic and rotational axes are in the same direction, the relative velocity of the two frames is perpendicular to the magnetic field line in the

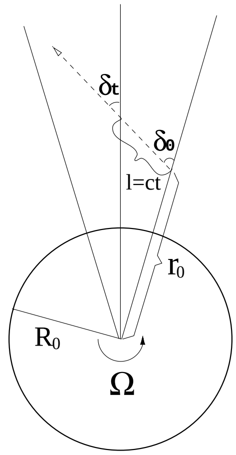

Fig. 1. The geometry of the photon moving direction with respect to the magnetic field configuration in an aligned rotator, viewed from the magnetic pole, i.e. the rotational pole.

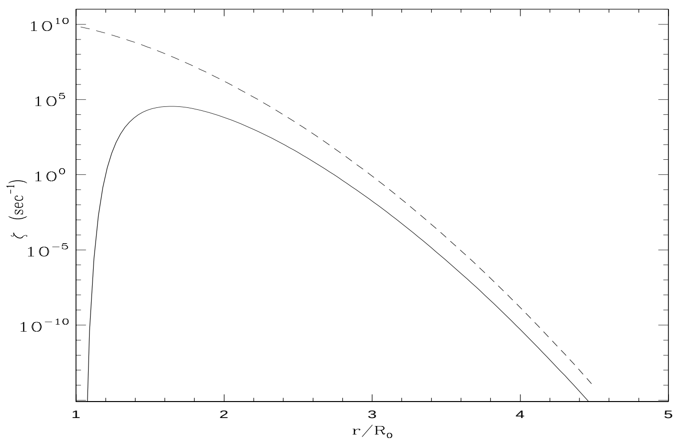

Fig. 2. The attenuation coefficient curve (the solid line) of a γ photon as a function of the distance it travels (see detailed description in the text). The curve of case $(0, 0, 1)$ is also plotted by the dashed line. Parameters adopted: pulsar period $P = 0.1$s, surface magnetic field $B = 10^{12}$G, height of emission point $r_0 = 1.01R_0$, energy of the photon $E_\gamma \sim 10^4$Mev.

co-moving frame, i.e. $\theta'_B = \theta'_u = \theta'_\gamma = \pi/2$. Using Eqs. (3,5), we can obtain $\theta_B = \pi/2$ and the photon's direction in frame S as $(\eta_x, \eta_y, \eta_z) = (\sin\delta, 0, \cos\delta) = (v/c, 0, [1-(v/c)^2]^{1/2})$ rather than $(0, 0, 1)$, where δ is the angle between the direction of γ-ray and that of the field line in frame S (i.e. $\theta_B - \theta_\gamma$). Note this direction is just the direction $(E/B, 0, [1-(E/B)^2]^{1/2})$ (for an aligned rotator $\mathbf{v}$, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other). This is just the *case (d)* rather than the *case (e)* in DL75, so that $\zeta = 0$ is satisfied, i.e. the induced electric field plays no role in the γ-ray absorption process at the emission point since the emission is produced at a preferred direction.

The two "unnatural conclusions" get away now. In the "straight field" configuration, the γ photon will not be absorbed in the laboratory frame, either, since it is just emitted in the absorption-free direction. So no contradiction lies in the two frames. Furthermore, no energy restriction on pair production is performed anywhere in the pulsar magnetosphere because a γ-ray will never be severely absorbed before it moves a distance for $B_\perp$ to achieve a sufficient value to absorb it.

In the polar cap region of a pulsar, the co-rotating velocity is much smaller than the speed of light, so that $\delta = \sin^{-1}(v/c)$ is very small and usually we can still regard the three directions described above to be the same. But in discussing the pair formation process, the attenuation coefficient ζ is very sensitive to the incident angle. It is just this very small deviation of the incident angle that really counts. Now another question arises: Propagation makes a γ photon encounter other bunches of field lines and changes the incident angle. Will the induced electric field be important at this time?

3. The role of the induced electric fields in pair formation after the photon moves a distance

According to the above discussions, a γ photon will be initially emitted at an incident angle $\delta_0 = \sin^{-1}(v/c)$ in the laboratory frame for an aligned rotator, where v is the co-rotating velocity at the emission point. Neglecting the general relativity effect, the photon will move along a straight line and intersect other bunches of the magnetic field lines. The incident angle of this γ photon gets smaller and smaller, and asymptotically merges to zero at infinity where the direction $(0, 0, 1)$ is usable, if we again assume the magnetic field lines are straight and are perpendicular to the rotating axis (i.e. there is no field-line-curvature-caused $B_\perp$ at all after the photon moves a distance, see Fig. 1). The magnetic field is assumed to be also declined as r^{-3}. Our adopting this unrealistic but simple magnetic field configuration is just for the sake of examining the importance of the propagation-caused absorption effect.

Suppose the γ photon with $E_\gamma \sim 10^4$Mev is emitted at a height of $(r_0 - R_0)$ from the neutron star surface (R_0 is the radius of the neutron star). After it travels a distance $l = ct$ (t is the travel time), the incident angle changes to δ_t. Submitting $(\eta_x, \eta_y, \eta_z) = (\sin\delta_t, 0, \cos\delta_t)$ into DL75's Eq. (9), we can get the attenuation coefficient of the photon as a function of the travel distance (see Fig. 2). The extreme case of $(\eta_x, \eta_y, \eta_z) = (0, 0, 1)$ is also marked. From Fig. 2 we can see that the induced electric field is also of minor importance, since the attenuation coefficients are always at least two order-of-magnitudes smaller than that of the case (0, 0, 1).

4. Conclusion and discussions

Although the unified formula for the pair production process involving both the magnetic field and the perpendicular electric field derived by DL75 is correct, we have argued in this paper that their conclusion that the induced electric fields are important in the pair formation process in the pulsar magnetospheres is wrong and misleading at least for the "aligned rotator" case. At the emission point, the photon emitted by a certain mechanism (e.g. the curvature radiation or the inverse Compton scattering) just moves along the very direction in which the attenuation coefficient is zero. Considering the propagation of the photon, LETTER

we found that this rotation-induced electric field still plays a minor role in the γ-ray absorption process in the polar cap region of a pulsar.

For the general case of an "oblique rotator" in which the magnetic and the rotational axes are misaligned, the co-rotating velocity is no more perpendicular to the magnetic field so that θ'_B (also θ'_u and θ'_γ) is not $\pi/2$. The photon direction consequently deviate from $(v/c, 0, [1-(v/c)^2]^{1/2})$ slightly. From Fig. 2, we see that the attenuation coefficient is also very small around the direction $(v/c, 0, [1-(v/c)^2]^{1/2})$, so that the conclusion that the induced electric field plays a minor role in γ-ray absorption still holds for the oblique rotator case. Actually, DL75's result can only be applied to the aligned case strictly, since generally the co-rotating velocity $\mathbf{v}_r$ is not equal to $\mathbf{v}_{drift} = c(\mathbf{E}\times\mathbf{B})/B^2$, with which one can define a frame where the electric field vanishes completely. In the co-rotating frame of an oblique rotator, an electric field component parallel to the magnetic field will still remain so that DL75's application condition fails.

Our results in this paper may have some implications for some previous studies which regard the electric field as the important effect of γ-ray absorption (e.g. Hardee 1977; Harding, Tademaru, & Esposito 1978; Harding 1981; Daugherty & Harding 1982; Zhao et al. 1989; Lu & Shi 1990; Lu, Wei, & Song 1994; Qiao & Zhang 1996).

Although the polar cap models of the γ-ray pulsars are by all means sound in principle, their concrete details will alter much by ejecting the induced electric field. Specifically, based on DL75's result, Hardee (1977) got an absolute upper limit to the photon energies

$$E_\gamma < 9.6\times 10^9 B_{12}^{-1} r_6^2 P \mathrm{eV}, \quad (6)$$

(his Eq. (38)) at which escape of the γ-rays from the magnetosphere is possible. It should be replaced by a threshold in the pure magnetic field absorption scheme

$$E_\gamma < 2.6\times 10^9 B_{12}^{-1} P^{1/2} \mathrm{eV} \quad (7)$$

(Wei, Song, & Lu 1997, their Eq. (10), $r_6 = 1$ and the last open field line is assumed). Thus the generation order parameters proposed by Lu, Wei, & Song (1994) should take the form in Wei, Song, & Lu (1997, their Eq. (17)).

The three boundary lines (birth line, death line and appearance line) in the $\dot{P} - P$ diagram of pulsars derived by Qiao & Zhang (1996) are also based on the electric field absorption. The details will also be changed by ejecting the electric field, but the picture still remains and may give a hint to us about the magnetic field configuration in the neutron star vicinity (Qiao & Zhang, discussions).

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