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The fractal octahedron network of the large scale structure

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Abstract. In a previous article, we have proposed that the large scale structure network generated by large scale magnetic fields could consist of a network of octahedra only contacting at their vertexes. Assuming such a network could arise at different scales producing a fractal geometry, we study here its properties, and in particular how a sub-octahedron network can be inserted within an octahedron of the large network. We deduce that the scale of the fractal structure would range from ≈100 Mpc, i.e. the scale of the deepest surveys, down to about 10 Mpc, as other smaller scale magnetic fields were probably destroyed in the radiation dominated Universe.

Key words: cosmology: large-scale structure of Universe

A fractal nature for the large-scale structure of the Universe has often been suggested (early references in Mandelbrot, 1984; Coleman & Pietronero, 1992; Guzzo, 1997; Sylos Labini et al. 1998a,b; Coles, 1998; de Vega et al., 1998 and others). On the other hand, it has also been suggested (Battaner et al., 1997a,b, and Florido & Battaner, 1997) that a network built up of octahedra only contacting at their vertexes (like an egg carton) is the simplest network comprising filaments produced by early large-scale magnetic fields. The purpose of this paper is to show how both suggestions may be compatible, i.e. how to produce a fractal based on filament octahedron elements, in other words, how to insert the sub-octahedron network inside an octahedron belonging to the larger scale network.

The size of the large octahedron can be 2, 3, 4, . . . times the size of the octahedra inside it. But let us exclude from this series even ratios 2, 4, 6 . . . because it is impossible to insert octahedra contacting at their vertexes in such a way as to have a sub-octahedron at each corner of the octahedron. The ratio of the sizes of octahedron and suboctahedron is therefore assumed to be 3, 5, 7 . . .. The simplest corresponds to a ratio 3, which is depicted in fig. 1. In this case, 7 sub-octahedra are contained in the octahedron.

For a ratio of 5, there are 25 sub-octahedra. For a ratio of 7, there are 63, and so on. Table 1 gives the series of sub-octahedra that are contained in the large octahedron, N.

This series can be represented by the equation

$$N(n) = \frac{4}{3}n^3 - 2n^2 + \frac{8}{3}n - 1 \tag{1}$$

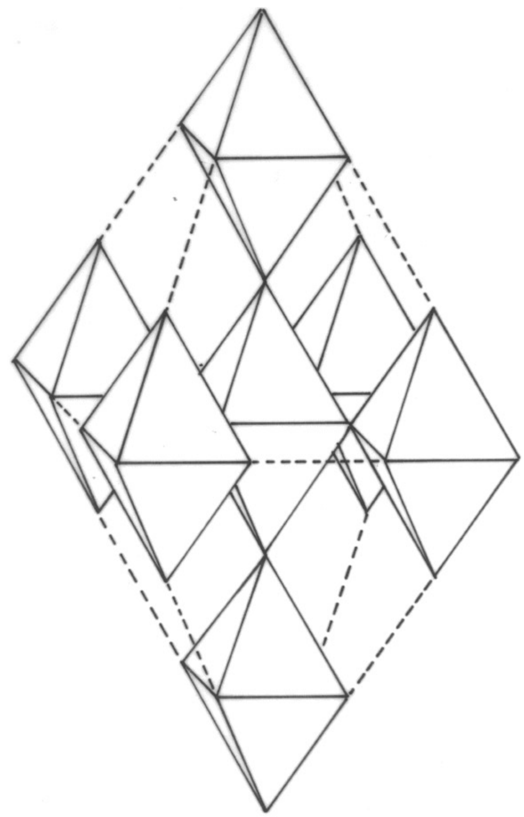

Fig. 1. 7 small octahedra inside a large octahedron when the size ratio is 3

The fractal dimension would be 1.77, 2, 2.13, 2.21, . . . (→3 for n→ ∞) not considering mass distributions, but rather lengths of filaments (smaller filaments contain smaller mass objects, Lindner et al., 1996). The values of the fractal dimension are similar to those obtained by other authors (see Einasto et al. 1997 and references therein).

For n=3, we obtain a fractal dimension of 2, which is precisely the value suggested by observations (Pietronero et al., 1996; Sylos Labini et al., 1998) but values given in the above list are all compatible with our network.

Lindner et al (1996) considered the range over which filamentary structures exist, from the observational point of view. Following the magnetic interpretation assumed here, the lower

Table 1.

Order number n	Ratio of sizes 2n-1	Number of sub-octahedra N
1	1	1
2	3	7
3	5	25
4	7	63
5	9	129
6	11	231
7	13	377

limit corresponds to the smaller scale below which primordial magnetic fields were destroyed in the radiation-dominated era by different mechanisms, mainly Silk damping (Silk, 1968; Jedamzik, Katalinic and Olinto, 1996) and magnetic diffusion (Lesch and Birk, 1998). Super horizon scale magnetic fields during this epoch, today of the order of 10 Mpc, were unaffected by these processes and this horizon scale should be identified with the lower limit, below which the fractal geometry is lost. The upper limit should be greater than the typical distance of the deepest surveys, about 100 Mpc. Therefore the distance range is relatively short, 10-100 Mpc. In the best case with a size ratio equal to 3, there would only be octahedra, sub-octahedra and sub-sub-octahedra, nothing more. Even the lowest size octahedra would probably be unrecognizable. We also conclude that $n \leq 2$. This situation is similar to that depicted by Lindner et al (1996).

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References

Battaner, E., Florido, E., Garcia-Ruiz, J.M. 1997a, A&A, 327, 8

Battaner, E., Florido, E., Jimenez-Vicente, J. 1997b, A&A, 326, 13

Coleman, P.H. & Pietronero, L. 1992, Physics Reports, 213, 311

oles, P. 1998, Nature, 391, 120

de Vega, H.J., Sánchez, N., Combes, F. 1998. To appear in "Proc. of the 6th Erice Chalonge School on Astrofundamental Physics". d. by N. Sánchez & A. Zichichi. Kluwer. Astro-ph 9801115

Einasto, M., Tago, E., Jaaniste, J., Einasto, J. & Andernach, H. 1997, A&AS, 123, 129

Florido, E., Battaner, E. 1997, A&A, 327, 1

Guzzo, L. 1997, astro-ph/9711206. New Astronomy, 2, 517

Jedamzik, K., Katalinic, V. & Olinto, A. 1996, astro-ph/9606080

Lindner, U., Einasto, J., Einasto, M., Frendling, W., Fricke, K., Lipovetsky, V., Pustilnik, S., Izotov, Y. & Richter, G. 1996, A&A, 314, 1

Lindner, U., Einasto, J., Einasto, M. & Fricke, K.J. 1997, astro-ph/9711046. Proc. IAU Symp. 183 "Cosmological parameters and the evolution of the Universe", Kyoto, Japan

Lesch, H. & Birk, G. 1998, to be published

Mandelbrot, B.B. 1984, Les objects fractals: forme, hasard et dimension. Flammarion. Paris

Pietronero, L., Montuori, M., Sylos Labini, F. 1996, "Critical Dialogues in Cosmology". Princeton

Silk, J. 1968, ApJ, 151, 459

Sylos Labini, F., Montuori, M., Pietronero, L. 1998a, Phys. Rep. (in press)

Sylos Labini, F., Montuori, M., Pietronero, L. 1998b, astro-ph 9801151