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An observational approach for the determination of gravity darkening in contact binaries of W UMa type

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Abstract. A new method for the determination of the gravity darkening exponents in close binaries of W UMa type is presented. The method is based on Kopal's method of Fourier analysis of the light changes of eclipsing variables in the Frequency Domain. In the analysis, the exponents for both components of a W UMa system are assumed to be the same. First, the method was successfully applied to two theoretical (test) light curves, belonging to two systems with radiative and convective envelopes, respectively. Then the method was applied to 36 W UMa systems for which geometric and photometric elements have been derived by the most powerful techniques. Systems showing large light curve anomalies were excluded from the analysis, since the effectiveness of the method depends strongly on the quality of the observations. Our results confirm the previous results with the assumed, same geometrical parameters.The derived values of the gravity darkening exponents are very close to the predicted ones by the existing theory of radiative transfer or convective equilibrium.

Key words: stars: atmospheres – stars: binaries: eclipsing – stars: fundamental parameters

1. Introduction

One of the most important observable phenomena which enable us to probe the stellar structure is the gravity darkening exhibited by the distorted stars in close binary systems. The general law of gravity darkening is written as

$$H \propto g^{\beta} \tag{1}$$

where H is the bolometric surface brightness at any arbitrary point on the stellar surface; g is the local gravity and β the exponent of gravity darkening.

According to von Zeipel(1924) for stars in hydrostatic and radiative equilibrium the value of $\beta = 1$, while Lucy (1967) calculated values for this exponent from data of convective stellar models and gave $\beta = 0.276 - 0.352$ with the representative value $\beta = 0.32$. Later, Anderson and Shu (1977) suggested that for convective envelopes, particularly for contact systems of W UMa-type, the bolometric flux depends only on the local variables and not on their gradients. They concluded that $\beta = 0.0$, which implies no gravity darkening.

A few years later, Sarna (1989) obtained a modified form of Lucy's gravity darkening law appropriate for a star with a convective envelope. He showed that for the secondary component in W UMa systems $\beta \approx 0.32$ and does not change significantly with mass-ratio and the mass of the secondary component. Very recently, Alencar and Vaz (1997) studied the gravity darkening exponent β in non-illuminated convective grey and non-grey atmospheres for $3700 < T_{eff} < 7000K$. Their results confirmed the value $\beta = 0.32$ only for $T_{eff} \approx 6500K$. They also found that β depends upon T_{eff}, being rather insensitive to variations of mixing length parameter, of the stellar mass and to the use of grey or non-grey atmospheres.

Several observational approaches for the gravity darkening determination have been made so far by Kitamura, Tanade and Nakamura (1957), Kopal (1968), Kopal and Kitamura (1968), Budding and Kopal (1970). From a quantitative analysis of the ellipticity effect in the light curves of W UMa systems Kopal (1968) found the observed values of the gravity darkening to be larger than the theoretical values at the respective wavelength of observation.

A different approach based on light curve synthesis has also been made by Rafert and Twigg (1980). From an analysis of a dozen contact binaries, the individual determinations for cooler, convective stars showed a scatter within a range of about $0.16 < \beta < 0.48$, on the average, confirming the prediction of Lucy. Eaton et al. (1980) used a quite different approach, specifically targeted at determining β for convective stars. Their method was based on a correlation of color indices formed from UV measurements with the expected variation of gravity over the contact configuration. From an analysis of the ANS satellite light curves of the prototype W UMa they obtained a well-defined value for convective stars $\beta = 0.12 \pm 0.04$.

From a study of four contact systems, Hilditch (1981) found very small values of β, and that imposing $\beta = 0.32$ yields a photometric mass ratio which is significantly different from the spectroscopic one, something that does not happen with $\beta = 0.0$. However, all the four systems, he studied, are spotted systems and, therefore, any attempt to determine temperature

Send offprint requests to: P.G. Niarchos variations over the surfaces (Rucinsci 1989) would be very difficult.

Later, in a series of papers Kitamura and Nakamura (1983, 1986, 1987a, 1987b, 1988a, 1988b, 1989), and Nakamura and Kitamura (1992) have deduced empirical values of the exponent of effective gravity darkening from an analysis of the photometric ellipticity effect in close binaries of various types. Their calculations give for W UMa binaries $\beta \simeq 0.56 - 1.84$ (Kitamura and Nakamura 1988a) which is much larger than previous values given by Rafert and Twigg (1980) and Eaton et al. (1980). As Rucinski (1989) pointed out, despite the efforts that have been made so far, our knowledge about gravity darkening is still incomplete. Therefore, we need new determinations of gravity darkening exponents, especially in the case of convective stars, both theoretical and observational.

The purpose of the present work is to propose a new method for the empirical determination of gravity darkening in close binary systems. The new approach is applied to several W UMa systems for which photometric and spectroscopic elements have been derived by the most powerful techniques.

2. Equations of the problem

The light changes of a close eclipsing system can be expressed (Kopal 1976, 1977, 1986) by a series of the form

$$L(\psi) \ = \ \frac{A_0}{2} + \sum_{j=1}^{n} A_j cos(j\psi) \qquad (2)$$

where $L(\psi)$ is the observed light of the system at any phase angle ψ; A_0, A_1, ..., A_n are properly determined coefficients and n denotes the order of the approximation considered. Moreover the variation of light between minima, where no eclipses occur and the light changes are due only to proximity effects, can be approximated (Kopal 1986) by a series of the form

$$L_{prox}(\psi) = \sum_{j=0}^{n} C_j cos^j \psi \qquad (3)$$

where C_j's constitute the so called **coefficients of the proximity effects** and n denotes the order of approximation ($n = 4$ for first order).

The coefficient C_2, which is the dominant among C_j's, can be considered (Kopal 1979) as a function of several parameters associated with the particular system, namely

$$C_2 = f(q, i, r_1, r_2, T_1, T_2, x_1, x_2, \beta_1, \beta_2, \lambda) \qquad (4)$$

where $q = m_2/m_1$ is the mass-ratio, where the subscripts 1 and 2 refer to the eclipsed star and the eclipsing one at primary minimum, respectively; i is the orbital inclination; r_1, r_2 are the fractional radii of the two components; T_1, T_2 are the effective temperatures; x_1, x_2 the linear coefficients of limb darkening; β_1, β_2 denote the exponents of the gravity darkening, and λ the respective wavelength of observations.

All the quantities (except the two exponents β_1, β_2) occurring in both sides of Eq. (4) can be determined from photometric and spectroscopic observations by using proper methods of analysis. Thus, the only unknown quantity in Eq. (4) are the two exponents β_1, β_2. Particularly for contact systems of W UMa type, with common convective envelopes, the two gravity exponents can be considered to be equal, so that in Eq. (4) there is only one unknown $\beta = \beta_1 = \beta_2$, for which it can be solved.

3. Determination of gravity darkening exponents

3.1. The light curve

In the previous section it was explained that Eq. (2) constitutes the expression for the whole light of the system at any phase angle ψ. A least square method was used to calculate the coefficients A_j in Eq. (2) by using the observational points (light intensities l_i and phase angles $\psi_i, i = 1 \ldots N1$, where N1 is the number of observations). The number n of terms used in Eq. (2) is defined for each case by the requirement that the coefficient A_j should not be smaller than the respective uncertainty. For all the cases we studied (except for some special cases) 6-7 terms were enough for the previous criterium to be fulfilled. For reasons of smoothing the data and saving computational time normal points were used.

It is well known that the light curve of a W UMa-type system is produced by pure eclipse effects and the so-called **proximity effects**. A necessary step in our method is to separate the eclipse from the proximity effects, e.g. to find the phase angle where the eclipses begin (or end). It has been shown by Kopal (1959) that the parts of the light curve around each quadrature (phase intervals $90^o \pm 32.5^o$ and $270^o \pm 32.5^o$) are free from eclipses in the case of contact binaries. But in most of the cases these intervals turn out to be larger and their exact determination is a crucial point in our method.

The methods proposed so far for the specification of the angle of the first contact assume spherical stars or Roche geometry and require the knowledge of the geometrical elements (r_1, r_2 and i) of the system. Such a determination is an approximate one and is based on geometry which does not describe precisely the real stars. We propose a new method for the determination of the angle of the first contact by simply studying the light curve, an approximation of which is given by Eq. (2). Assuming that the coefficients A_j are known, the derivatives of any order of Eq. (2) can be evaluated. In Figs. 1 and 2 the graphic representation of the light curves and of the first and second derivatives, derived from Eq. (2), are shown for the test systems AB And and V535 Ara, respectively. In both cases six terms in Eq. (2) were used.

In these figures it is clearly shown that a discontinuity exists in the values of the first derivative, which becomes more evident in the values of the second derivative. This is what should be expected, since the light changes in (the two parts of) the light curve are caused by two different physical phenomena, the eclipses and the proximity effects.

In the phase angle interval $0 - \pi/2$ we expect a diminishing rate of the light intensity increase at the point exactly where the eclipse ends. Therefore the first derivative assumes suddenly lower values, still positive. In fact, when the eclipse ends, the

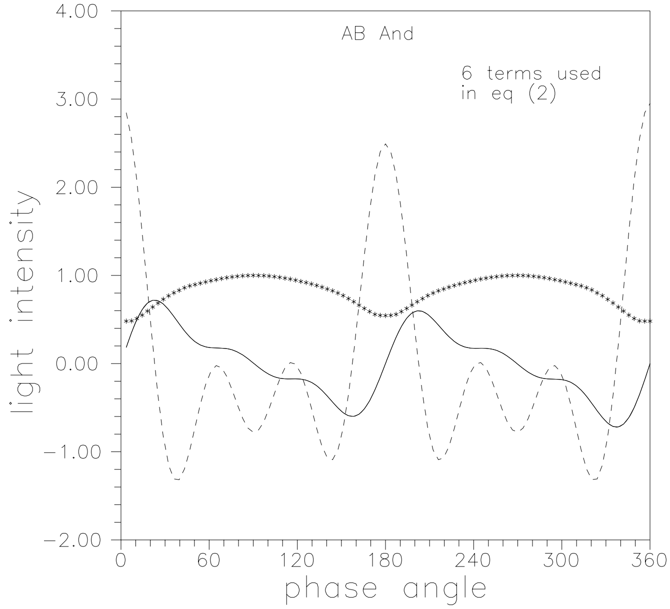

Fig. 1. Graphic representation of the light curve and of its first and second derivative of AB And. The asteriscs are theoretical V observations; the solid line represents the 1^{st} derivative and the dashed line the 2^{nd} derivative, derived from Eq. (2).

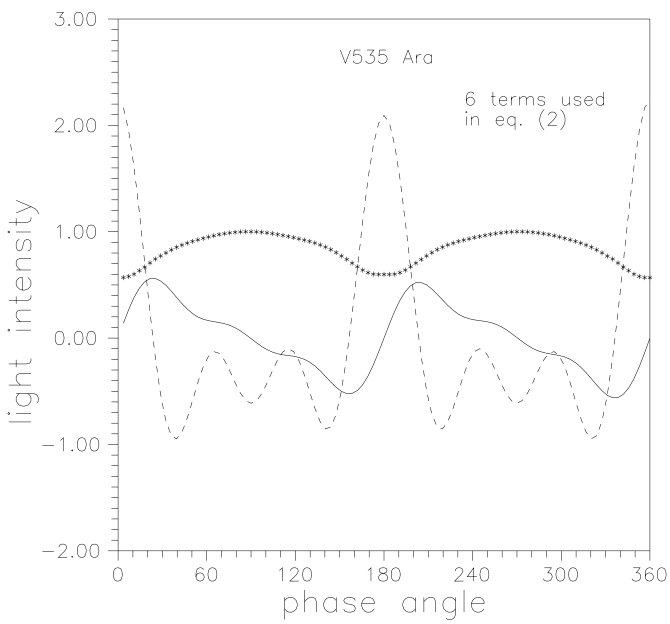

Fig. 2. Graphic representation of the light curve and of its first and second derivative of V535 Ara. The asteriscs are theoretical V observations; the solid line represents the 1^{st} derivative and the dashed line the 2^{nd} derivative, derived from Eq. (2).

light intensity continues increasing with lower rate than before, because all the points of the stellar surface are already visible from the observer and the light changes are due only to the varying projected surfaces of the two stars and to the distribution of brightness over the stellar surfaces. On the other hand, during the eclipse the light changes are due to the fact that areas of stellar surfaces, not visible to the observer, continuously reveal themselves producing a high rate of variation in the observed brightness.

It is expected that the light curve will exhibit a point of inflection in the range $0-\pi/2$, since at phase zero is concave upward and at phase $\pi/2$ is concave downward. In the region where the eclipse ends we expect the light curve to be less bended (more 'wide') than it was in the previous phase interval. This results in smaller absolute values of the second derivative. It means that, if the eclipse ends after the point of inflection of the light curve, where we have negative values of the second derivative, the diminution of the absolute value leads to increased algebraic value of the second derivative, as it is shown in Fig. 1. The opposite happens, if the eclipse ends before the point of inflection, when a diminution of the algebraic value of the second derivative occurs.

According to the above analysis, in the phase interval $0-\pi/2$ and at the point where the eclipse ends, we expect an abrupt increase of the second derivative and an abrupt diminution of the first derivative of the light curve. Similar conclusions can be drawn for the rest phase intervals: $\pi/2-\pi$, $\pi-3\pi/2$ and $3\pi/2-2\pi$. These sudden changes in the values of the derivatives can be used to locate the points where the eclipse begins or ends, and, consequently, to define the phase interval of the light curve which is free from eclipse effects. This method has the advantage that it is based on the observations alone and no previous knowledge of the system parameters is required.

3.2. Evaluation of the proximity effects

According to the previous analysis, the part of the light curve which is unaffected by eclipses can be specified from a study of the second derivative of the light curve. When the eclipse ends, the second derivative shows a sudden increase, while previously was being reduced. Therefore, the ends of the phase interval, where no eclipses occur, can be defined by the points where the third derivative becomes zero. These points could also be defined by other characteristic points of the derivative curves, e.g. the points where the second or the fourth derivative becomes zero. We experimented with all these cases, but the best results were obtained using the phase intervals defined by the points where the third derivative becomes zero. By best results we mean that we succeeded to calculate gravity darkening exponents almost identical to those used to produce theoretical light curves of the test systems AB And and V535 Ara (see Sect. 6).

It should be noticed that the pattern in the derivatives depends on the order of approximation that it is used. In Fig. 3 the light curve and its first and second derivative of AB And is shown, when 8 terms in Eq. (2), instead of 6 in Fig. 1, are used. It is obvious from Figs. 1 and 3 that the derivative patterns are slightly different in these figures, e.g. there is a small change in the phase interval of the light curve, which is free from eclipse effects. But we must repeat that the number of terms used in Eq. (2) is defined by the requirement that the coefficients should not be smaller than their respective uncertainties.

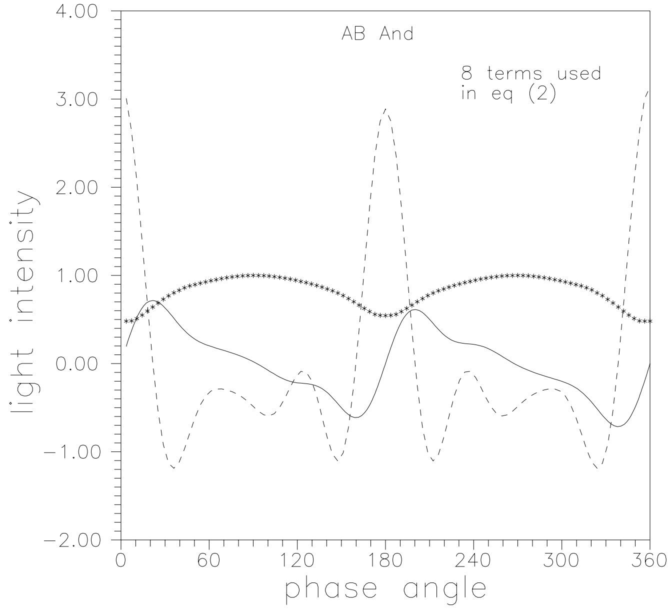

Fig. 3. Graphic representation of the light curve and of its first and second derivative of AB And. The asteriscs are theoretical V observations; the solid line represents the 1^{st} derivative and the dashed line the 2^{nd} derivative, derived from Eq. (2) with 8 terms.

An analytic expression for the approximation of the proximity effects curve should rely on the phase intervals which are free from eclipse effects. Such an expression is given by Eq. (3), which is a periodic function with period 2π. The coefficients $C_j, j = 0, 1, 2, 3, 4$ can be evaluated by using the observations of the part of the light curve which is not affected by eclipses. The coefficient C_0 can be found directly from Eqs. (2) and (3) and it is equal to $C_0 = L(\pi/2)$, while the coefficients $C_j, j = 1, 2, 3, 4$ follow from the relation

$$\sum_{i=1}^{N}(L_{prox}(\psi_i) - l_i)^2 = min \quad (5)$$

where l_i, ψ_i, are the N observational points (light intensity, phase angle) lying inside the phase interval where no eclipses occur.

In order for the proximity effects curve to be reliable in phase regions where eclipses occur, we must exploit most of the information included in the regions where no eclipse effects are present. Assuming that the functions $L(\psi)$ and $L_{prox}(\psi)$, defined by Eqs. (2) and (3), are known from observations, we can secure the identity of the two functions in the phase interval outside eclipses by minimizing, in addition to the quantity defined by Eq. (5), the quantity

$$\sum_{i=1}^{N}(L'_{prox}(\psi_i) - L'(\psi_i))^2 = min \quad (6)$$

defined in the phase interval outside eclipses. The $'$'s stand for the respective derivatives of 1^{st} order. A simultaneous solution of Eqs. (5) and (6) yield the coefficients C_j's. Similar systems of equations can be derived by taking derivatives of higher order. But, as it is shown below, derivatives of first order lead to

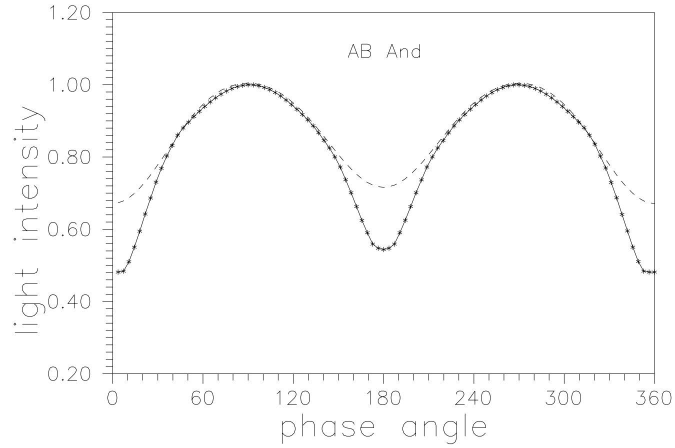

Fig. 4. Theoretical V light curve (solid line) and proximity effects curve (dashed line) of AB And.

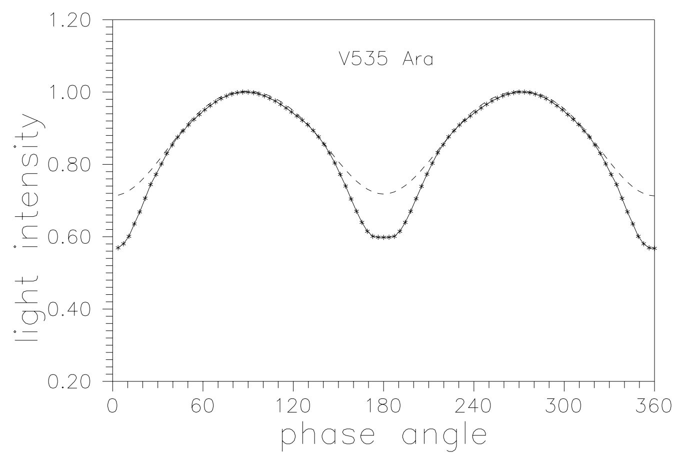

Fig. 5. Theoretical V light curve (solid line) and proximity effects curve (dashed line) of V535 Ara.

a satisfactory solution of the problem. The proximity effects curves together with the theoretical light curves for the two test systems are shown in Figs. 4 and 5, respectively.

4. The gravity darkening effect and its relation to the wavelength λ

It has already been stated that the gravity darkening exponent β in Eq. (1) refers to the bolometric light of the star. Since the photometric observations are made in a specific wavelength λ, it is necessary to relate the exponent β with the gravity darkening effect determined from observations at any wavelength λ. From the well known relations

$$\frac{H}{H_0} = \left(\frac{T}{T_0}\right)^4 \quad (7)$$

and

$$\frac{H}{H_0} = \left(\frac{g}{g_0}\right)^\beta \quad (8)$$

Table 1. Results for the test systems AB And and V535 Ara

Coefficient	AB And	V535 Ara
A_0	0.81375 ± 0.00500	0.83822 ± 0.00500
A_1	-0.02431 ± 0.00005	-0.00269 ± 0.00005
A_2	-0.22405 ± 0.00005	-0.19586 ± 0.00005
A_3	-0.01208 ± 0.00005	-0.00455 ± 0.00005
A_4	-0.06070 ± 0.00005	-0.04854 ± 0.00005
A_5	-0.00382 ± 0.00005	-0.00122 ± 0.00005
A_6	-0.02371 ± 0.00005	-0.01685 ± 0.00005
C_2	-0.19810 ± 0.00005	-0.17751 ± 0.00005
τ	0.39 ± 0.02	0.86 ± 0.08
β	0.33 ± 0.01	1.08 ± 0.06
β_{th}	0.32^*	1.00^*

*theoretical value

where T_0 is the mean effective temperature of the star, g_0 is the mean gravity on the star's surface, and H_0 is the bolometric surface brightness at the point where $g = g_0$, we get

$$T = \left(\frac{g}{g_0}\right)^{\beta/4} \cdot T_0 \tag{9}$$

and from Planck's law we have

$$\left(\frac{H}{H_0}\right)_\lambda = \frac{e^{\frac{hc}{\lambda k T_0}} - 1}{e^{\frac{hc}{\lambda k T}} - 1} \tag{10}$$

where the constants h, c and k have the usual meaning. If we insert Eq. (9) in Eq. (10) and expand the latter one in a Taylor series in terms of $(g - g_0)/g_0$ in the neighborhood of $g = g_0$, we finally get

$$\left(\frac{H}{H_0}\right)_\lambda = 1 + \tau\frac{g - g_0}{g_0} + \cdots \tag{11}$$

where τ is now called **coefficient of gravity darkening**. The coefficient τ depends on the wavelength of observations and it is found that it is connected with the bolometric exponent β through the relation

$$\beta = 4\lambda k T_0 \tau \left(1 - e^{\frac{-hc}{\lambda k T_0}}\right)/hc \tag{12}$$

The exponent β follows now from Eq. (12), provided that the coefficient τ is derived from observations. It should be pointed out that the coefficient τ depends on the wavelength of radiation λ, while the exponent β is independent of λ and refers to the bolometric radiation.

5. Evaluation of the gravity darkening exponent

The coefficient C_2 in Eq. (4) is given (Kopal 1975) by

$$C_2 = C_2^* + \gamma_2 \tag{13}$$

where

$$C_2^* = \left(\frac{3}{2}sin^2 i L_1 q(15 + x_1)(1 + \tau)/(15 - 5x_1)\right) r_1^3 +$$

$$\left(\frac{3}{2}sin^2 i L_1 q 45(1 - x_1)(1 + \tau/3)/(24 - 8x_1)\right) r_1^5 +$$

$$\left(\frac{3}{2}sin^2 i L_2(15 + x_1)(1 + \tau)/(q(15 - 5x_2))\right) r_2^3 +$$

$$\left(\frac{3}{2}sin^2 i L_2 45(1 - x_1)(1 + \tau/3)/(q(24 - 8x_2))\right) r_2^5 \tag{14}$$

where $L_{1,2}$ are the fractional luminosities of the two components in the respective wavelength of observations; $r_{1,2}$ the fractional radii of the two stars in terms of their separation taken as unit of length; q is the mass-ratio; i is the inclination of the orbit with respect to the normal of the line of observation; $x_{1,2}$ are the coefficients of limb darkening, and

$$\gamma_2 = L_2\gamma_{2,1} + L_1\gamma_{2,2} \tag{15}$$

where

$$\gamma_{2,k} = \left(\frac{r_k^2}{3\pi} + \frac{3r_k^3}{8}\right) f_k sin^2 i, \qquad k = 1, 2 \tag{16}$$

and

$$f_k = (T_{3-k}/T_k)^4 (L_k/L_{3-k})(r_{3-k}/r_k)^2 \tag{17}$$

In the above Eq. (14) all the quantities occurring on the right-hand side, except for the coefficient τ, can be found from observations or (like $x_{1,2}$) from the theory. On the other hand, ${C_2}^*$, on the left-hand side, can be evaluated from Eq. (13), since C_2 and γ_2 can also be determined from observations through the Eqs. (5)-(6) and (15)-(17), respectively. Thus, Eq. (14) can be solved for the only unknown τ, and then from Eq. (12) we finally get the value of the gravity darkening exponent β.

6. Application of the method

According to the previous analysis, the method for the determination of the gravity darkening exponent can be fully automatized. The main steps to be followed are: (i) Read in the photometric observations (magnitudes or intensities and phase angles) and the geometric and photometric elements (taken from a synthetic light curve analysis, where theoretical gravity darkening exponents are assumed); (ii) Formation of normal points and evaluation of the coefficients A_j of Eq. (2); (iii) Definition of phase intervals around each quadrature, which are free from eclipse effects; (iv) Evaluation of the proximity effects by using the observations of the above phase intervals. (v) Evaluation of the gravity darkening coefficient τ from the coefficient C_2 and then β from τ through Eqs. (13)-(17) and (12), respectively.

6.1. Application to theoretical (test) light curves

The above method was first applied to the theoretical light curves of the W UMa systems AB And and V535 Ara. These light curves were produced by the *Binary Maker 2.0* programme (Bradstreet 1993). As input parameters were used those derived by an analysis of the real light curves by using the Wilson &

Table 2. Results for W UMa systems

n	system	r_1	r_2	q (m_2/m_1)	i (degrees)	T_1 (K)	T_2 (K)	β	Ref.
1	AB And	O.320	0.451	2.03	86.80	5821	5450	0.29 ± 0.01	14
2	OO Aql	0.412	0.380	0.84	90.00	5700	5635	0.18 ± 0.04	4,15
3	VW Boo	0.470	0.322	0.43	75.64	5700	5190	0.27 ± 0.01	5,29
4	TY Boo	0.309	0.457	2.14	77.50	5864	5469	0.38 ± 0.02	21
5	TU Boo	0.471	0.342	0.50	87.50	5800	5787	0.32 ± 0.02	27,21
6	V523 Cas	0.332	0.436	1.75	83.70	4407	4200	0.53 ± 0.02	31
7	CW Cas	0.326	0.438	1.84	73.40	5440	5090	0.32 ± 0.01	1
8	AD Cnc	0.341	0.428	1.60	64.90	5164	4825	0.69 ± 0.01	31
9	CC Com	0.326	0.447	1.93	90.00	4300	4140	0.26 ± 0.01	8
10	FS Cra	0.360	0.411	1.32	86.50	4700	4567	0.39 ± 0.05	8
11	V677 Cen	0.580	0.243	0.15	89.50	5700	5700	0.33 ± 0.03	17
12	V700 Cyg	0.335	0.452	1.88	81.60	6890	6334	0.67 ± 0.02	28
13	BV Dra	0.318	0.472	2.43	76.28	6345	6245	0.40 ± 0.01	30,16
14	BW Dra	0.281	0.499	3.57	74.40	6164	5980	0.33 ± 0.01	30,16
15	YY Eri	0.303	0.470	2.49	82.50	5600	5362	0.30 ± 0.01	2,22
16	AK Her	0.517	0.276	0.26	80.47	6400	6033	0.33 ± 0.01	37
17	DF Hya	0.306	0.463	2.36	84.30	6000	5851	0.22 ± 0.01	24
18	ST Ind	0.438	0.350	0.60	71.30	6430	6414	0.47 ± 0.02	39,34
19	XY Leo	0.317	0.444	2.00	65.80	4850	4575	0.53 ± 0.02	13
20	RT LMi	0.306	0.481	2.59	83.38	6000	5800	0.29 ± 0.02	26
21	V502 Oph	0.299	0.477	2.65	71.67	6200	5954	0.49 ± 0.04	35,20
22	V508 Oph	0.441	0.324	0.53	86.13	6000	5830	0.12 ± 0.01	19
23	V566 Oph	0.534	0.271	0.34	79.80	6700	6618	0.35 ± 0.02	7
24	BX Peg	0.300	0.479	2.72	87.50	5528	5300	0.25 ± 0.03	32
25	AE Phe	0.305	0.480	2.22	86.00	6100	5809	0.12 ± 0.01	23,11
26	FG Sct	0.359	0.403	1.27	89.90	4800	4662	0.37 ± 0.01	8
27	V781 Tau	0.310	0.476	2.47	65.36	5950	5861	0.50 ± 0.03	9
28	W UMa	0.480	0.325	0.49	86.00	6500	6400	0.16 ± 0.01	3,38
29	BI Vul	0.363	0.431	1.44	78.80	4600	4598	0.19 ± 0.01	8
30	V535 Ara	0.432	0.287	0.36	82.14	8750	8572	1.04 ± 0.08	33
31	XY Boo	0.558	0.253	0.18	68.38	7200	7100	0.81 ± 0.08	36
32	RR Cen	0.555	0.247	0.18	78.70	7250	7188	0.87 ± 0.01	7,10
33	XZ Leo	0.412	0.353	0.73	72.03	7850	7044	1.27 ± 0.04	25
34	MW Pav	0.561	0.256	0.18	85.07	7620	7570	0.97 ± 0.01	18
35	TY Pup	0.559	0.257	0.18	81.80	7800	7567	1.36 ± 0.02	12
36	RZ Tau	0.502	0.320	0.37	82.88	7200	7146	0.86 ± 0.04	2

References:1) Barone et al. 1988, 2) Binnendijk 1963, 3) Binnendijk 1966, 4) Binnendijk 1968, 5) Binnendijk 1973, 6) Bookmyer 1968, 7) Bookmyer 1969, 8) Bradstreet 1985, 9) Cereda et al. 1988, 10) Chambliss 1971, 11) Gronbech 1976, 12) Gu et al. 1992, 13) Hrivnak 1985, 14) Hrivnak 1988, 15) Hrivnak 1989, 16) Kaluzny & Rucinski 1986, 17) Kilmartin et al. 1987, 18) Lapasset 1976, 19) Lapasset & Gomez 1990, 20) Maceroni et al. 1982, 21) Milone et al. 1991, 22) Nesci et al. 1986, 23) Niarchos & Duerbeck 1991, 24) Niarchos et al. 1992, 25) Niarchos et al. 1994a, 26) Niarchos et al. 1994b, 27) Niarchos et al. 1996, 28) Niarchos et al. 1997, 29) Rainger et al. 1990, 30) Rucinski & Kaluzny 1982, 31) Samec et al. 1989, 32) Samec & Hube 1991, 33) Schoffel 1979, 34) Walter et al. 1989, 35) Wilson 1967, 36) Winkler 1977, 37) Woodward & Wilson 1977, 38) Zhai et al. 1984, 39) Zola et al. 1997.

Devinney code, where the gravity darkening exponents were assigned theoretical values (0.32 and 1.00, respectively). The input parameters were taken from the *Pictorial Atlas of Binary Stars* prepared by Terrel et al. (1992). The first system is a typical example of a system with convective envelopes (cool system), while the second one is a system with radiative envelopes (hot system). The results of our method are given in Table 1.

Figs. 4 and 5 show the normal points and the corresponding theoretical light curves, obtained by our method (Eqs. (2) and (3)), of the systems AB And and V535 Ara. It is obvious that the fitting is extremely good, while the evaluated gravity darkening exponents are almost identical, within the limits of their errors, to the theoretical values 0.32 and 1.00.

7. Application to W UMa systems

Having shown that our new method works very well, we applied it to a large enough sample of W UMa systems. The systems were carefully selected to have very accurately determined parameters. The majority of the systems were chosen to possess common convective envelopes, since, according to Rucinski (1989), the problem of the gravity darkening is still open, mainly for convective stars. A dividing line is drawn to the temperature 7200 K and all systems with $T < 7200K$ are considered to have convective envelopes, while those with $T > 7200K$ are supposed to possess radiative envelopes. The results are given in Table 2 together with the values of some basic parameters of the systems. The necessary input parameters and observations were taken from the literature listed in the last column of Table 2. The given values of β are mean values obtained from the analysis of light curves in different colours. In the case of systems with unequal maxima (O'Connell effect) the β values are calculated from the observations around the higher maximum (assumed as unspotted part of the light curve).

8. Conclusions

It has been pointed out by several investigators (e.g. Rucinski 1989) that the problem of gravity darkening in close binary systems is still open, mainly for systems possessing common convective envelopes. The existing theories propose different values for the gravity darkening exponents.

The crucial point in our method is to define the phase interval of the light curve which is free from eclipse effects. After several numerical experiments we found that the interval giving the best results is that defined by the points where the third derivative of the equation of the light curve becomes zero. A successful test of the method using theoretical light curves was made, before we analyse the light curves of W UMa systems.

As far as the limitations of our method are concerned, it should be noted: (i) the effectiveness of the method depends strongly on the quality of the observations, (ii) the method at its present form can be used only to W UMa systems, where we assume $\beta_1 = \beta_2$ for the two components, as a result of the common envelope of the system, and (iii) in case of systems with unequal maxima in their light curves (O'Connell effect), the part of the light curve around the higher maximum (unspotted part) should be used for the determination of gravity darkening exponent.

The results of our method confirm the previous results for β for W UMa systems, where the same geometrical parameters are used. These results show that for radiative (hot) systems the computed values of β are close to the theoretical value $\beta = 1$, as proposed by von Zeipel's theory, while for systems with convective envelopes (cool systems) the computed values of β cluster around the theoretical value $\beta = 0.32$ proposed by Lucy (1967). In conclusion, the importance of the results of the present work lies on that: (a) they confirm to a great degree the validity of the existing theories of von Zeipel and Lucy, and (b) the computed values of the gravity darkening exponents can be used as input parameters in the light curve synthesis programs for a more precise solution of the observed light curves.

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References

Alencar S.H.P., Vaz L.P.R., 1997, A&A 326, 257
Anderson L., Shu F.H., 1977, ApJ 214, 798
Barone F., Maceroni C., Milano L., Russo G., 1988, A&A 197, 347
Binnendijk L., 1963, AJ 68, 22
Binnendijk L., 1966, AJ 71, 340
Binnendijk L., 1968, AJ 73, 32
Binnendijk L., 1973, AJ 78, 103
Bookmyer B., 1968, AJ 73, 26
Bookmyer B., 1969, AJ 74, 1197
Bradstreet D.H., 1985, ApJS 58, 413
Bradstreet D.H., 1993, Binary Maker 2.0 User Manual
Budding E., Kopal Z., 1970, ApSS 9, 343
Cereda L., Misto A., Poretti E., Niarchos P.G., 1988, A&AS 76, 255
Chambliss C.R., 1971, AJ 76, 64
Eaton J.A., Wu C.-C., Rucinski S.M., 1980, ApJ 239, 919
Gronbech B., 1976, A&A 24, 339
Gu S., Yong Y., Liu Q., Zhang Z., 1992, ApSS 203, 161
Hilditch R.W., 1981, MNRAS 196, 305
Hrivnak B.J., 1985, ApJ 290, 696
Hrivnak B.J., 1988, ApJ 335, 319
Hrivnak B.J., 1989, ApJ 340, 458
Kaluzny J., Rucinski S.M., 1986, AJ 92, 666
Kilmartin P.M., Bradstreet D.H., Koch R.H., 1987, ApJ 319, 334
Kitamura M., Tanabe H., Nakamura T., 1957, Publ. Astron. Soc. Japan, 9, 120
Kitamura M., Nakamura Y., 1983, Ann. Tokyo Astron. Obs., 2nd ser. 19, 413
Kitamura M., Nakamura Y., 1986, Ann. Tokyo Astron. Obs., 2nd ser. 21, 229
Kitamura M., Nakamura Y., 1987a, Ann. Tokyo Astron. Obs., 2nd ser. 21, 311
Kitamura M., Nakamura Y., 1987b, Ann. Tokyo Astron. Obs., 2nd ser. 21, 387
Kitamura M., Nakamura Y., 1988a, ApSS 145, 117
Kitamura M., Nakamura Y., 1988b, Ann. Tokyo Astron. Obs., 2nd ser. 22, 31
Kitamura M., Nakamura Y., 1989, Publ. Nat. Astron. Obs. Japan, 1, 43
Kopal Z., 1959, *Close Binary Systems*, Chapman-Hall and John Wiley, London and New York.
Kopal Z., 1968, ApSS 2, 23
Kopal Z., 1975, ApSS 38, 191
Kopal Z., 1976, ApSS 45, 269
Kopal Z., 1977, ApSS 51, 439
Kopal Z., 1979, *Language of the Stars*, D. Reidel Publ. Co., Dordrecht and Boston
Kopal Z., 1986, *Vistas in Astronomy*, Vol.29, pp. 295
Kopal Z., Kitamura M., 1968, Adv. in Astron. Astrophys. (Academic Press, New York), 6, 125
Lapasset E., 1976, ApSS 46, 155
Lapasset E., Gómez M., 1990, A&A 231, 365
Lucy L.B., 1967, Z. Astrophys. 65, 89
Maceroni C., Milano L., Russo G., 1982, A&AS 49, 123
Milone E.F., Groisman G., Fry D.J.I., Bradstreet D.H., 1991, ApJ 370, 677
Nakamura Y., Kitamura M., 1992, Ap&SS 191, 267
Niarchos P.G., Duerbeck H.W., 1991, A&A 247, 399
Niarchos P.G., Hoffmann M., Duerbeck H.W., 1992, A&A 258, 323
Niarchos P.G., Hoffmann M., Duerbeck H.W., 1994a, A&A 292, 494
Niarchos P.G., Hoffmann M., Duerbeck H.W., 1994b, A&AS 103, 39
Niarchos P.G., Hoffmann M., Duerbeck H.W., 1996, A&AS 117, 105

Niarchos P.G., Hoffmann M., Duerbeck H.W., 1997, A&AS 124, 291
Rafert J.B., Twigg L.W., 1980, MNRAS 193, 79
Rainger P.P., Bell S.A., Hilditch R.W., 1990, MNRAS 246, 47
Rucinski S.M., Kaluzny J., 1982, ApSS 88, 433
Rucinski S.M., 1989, Comm. on Astrophys. 14, 79
Samec R.G., Van Hamme W., Bookmyer B.B., 1989, AJ 98, 2287
Samec R.G., Hube D.P., 1991, AJ 102, 1991
Schoffel E., 1979, A&AS 36, 287
Terrell D., Mukherjee J., Wilson R.E., 1992, *BINARY STARS, A Pictorial Atlas*, Krieger Publ. Co., Florida.
Walter K., Niarchos P.G., Duerbeck H.W., 1989, ApSS 161, 1
Wilson R.E., 1967, AJ 72, 1028
Winkler L., 1977, AJ 82, 648
Woodward E.J., Wilson R.E., 1977, ApSS 52, 387
Zeipel H.V., 1924, MNRAS 84, 702
Zhai D.S., Leung K.C., Zhang R.X., 1984, A&AS 57, 487
Zola S., Kolonko M., Szczech M., 1997, A&A 324, 1010