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Periods, period changes and the nature of the microvariations of Luminous Blue Variables

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Abstract. We present period determinations of the microvariability of the six luminous blue variables AG Car, HR Car, 164 G Sco, S Dor, R 127, and R 71. In total, we were able to determine 22 periods in these stars, ranging from 18 days up to 195 days. All stars have period changes by up to a factor 4 within time scales of a few hundred days. For all stars the amplitude of the pulsations in V increases with increasing periods. The slope of the correlation between the amplitude and the period decreases with increasing luminosity.

The values of the pulsation constant Q were determined. HR Car, 164 G Sco, R 71 and R 127 have Q-values in the range of 0.07 to 0.18 days. This is about a factor two larger than those of most other B-type supergiants, possibly because the LBVs have a higher L/M ratio as they have lost more mass. The most common value for the pulsational constant of LBVs is $Q = 0.07 \pm 0.01$ days, but Q can increase temporarily by as much as a factor four. This is not related to a particular phase in the light curve. The long periods might be due to a beat of two frequencies.

For the two stars R 71 and R 127, which showed significant changes in M_V, and hence in radius during the course of the observations, the pulsational period increased with increasing radius. The Q-values of R 71 and R 127 increase when the stars get brighter and their radii increase. This is probably due the changes in the density structure of the stars as their outer envelope expands.

We compare the observed variations with those predicted for strange modes by Kiriakidis et al. (1993). The periods of the observed microvariations are orders of magnitudes longer than predicted for strange modes. A comparison with the variations of slowly pulsating B-stars (SPBs) suggests that the microvarions of LBVs are due to g-mode pulsations. A first attempt for mode identification, based on a simple linear pulsation model by means of the multicolour Strömgren data, shows that none of the variations can be explained by means of a radial pulsation. The amplitude-wavelength relations suggest g-modes of low ℓ.

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1. Introduction

The pulsation of stars provides information about the structure and stability in the interior. The Luminous Blue Variables (LBVs) offer the unique opportunity to study the pulsation of stars when their radius changes by factors up to 4 or 8 in the course of years to decades. This paper deals with the study of the pulsational periods and the period changes during the variations in radius of six LBVs.

Luminous Blue Variables are the most unstable massive stars, apart from supernovae. They are variable on all timescales:
(a) small photometric variations (or microvariations) with $\Delta V \simeq 0.^m2$ on timescales of weeks to months,
(b) typical LBV variations (or moderate photometric variations) with $\Delta V \simeq 0.5$ to 2.0 mag. on timescales of years to a decade,
(c) giant eruptions on timescales of about 10^3 years.
In this paper we study the microvariations during several phases of the moderate variations.

Van Genderen et al. (1997a) have shown that there are two types of microvariations:
(1) near the visual minimum, when the star is hot and has a small radius, the star is bluer in the bright phase and redder in the faint phase of the microvariations.
(2) near the visual maximum, when the star is cooler and has a larger radius, the star is redder in the bright phase and bluer in the faint phase of the microvariations.
This points to different pulsational mechanisms.

Soukup et al. (1994) suggested that the microvariations of the LBVs could be due to non-radial g-modes of order between 10 and 20 with periods of about 10 days for $l = 1$. These modes could be due to the frozen-in convection in the region of the Fe opacity peak near 2×10^5 K. However, more recent calculations by Cox et al. (1995) suggest that the microvariations of LBVs are mostly or all radial pulsations due to strange modes. They argue that these occur when the stars have lost sufficient mass

Table 1. The number of observations for the program stars

identification		photometric system	
HD	name	Strömgren[a]	Walraven[b]
269006	R 71	240	162
90177	HR Car	118[c]	116
160529	164 G Sco	477[d]	-
35343	S Dor	208	17
269858	R 127	224	-
94910	AG Car	158[c]	153

a: Manfroid et al. (1991) and Sterken et al. (1993a,b); b: van Genderen (1979; 1982) and by van Genderen et al. (1985; 1988; 1990); c: Kilkenny et al. (1985); d: Sterken (1977; 1982).

to expose He, which could drive the pulsation in the helium ionization zone.

In this paper we study the pulsational characteristics of six LBVs, observed in the LTPV (Long Term Photometry of Variables) project of ESO (Sterken et al., 1993a,b). The purpose of this study is to understand the periods of the pulsations and period changes and to derive information on the nature of the instability. In Sect. 2 we describe the program stars and the observations. In Sect. 3 we describe the lightcurves and in Sect. 4 we determine the pulsational periods during various phases of the light curves. Section 5 deals with the pulsational constant Q and its changes when the radius changes. We also discuss the amplitude-period relations. In Sect. 6 we consider the possibility that the periods are due to strange modes and we compare the observed microvariations with those of Slowly Pulsating B-stars (SPBs). A very preliminary mode identification in terms of a linear theory is described in Sect. 7. Section 8 gives the discussion and conclusions.

The properties of the LBVs have been reviewed by Humphreys & Davidson (1994). The stars have been discussed extensively during the workshop *Luminous Blue Variables: stars in transition* in October 1996. The reader is referred to the proceedings of this workshop for reviews on the properties and evolutionary stage of LBVs (Nota & Lamers, 1997).

2. Photometry and program stars

2.1. The photometry

The photometric data used in this study are from Spoon et al. (1994). He collected Strömgren and Walraven photometry of the program stars, mainly from the Long-Term Photometry of Variables project organized by Sterken (1983) and published by Manfroid et al. (1991, 1994) and Sterken et al. (1993, 1995). The data are summarized in Table 1. The photometry other than from the LTPV project is indicated separately. Spoon et al. homogeneized the data and calculated the brightness of the stars in the Johnson V magnitude. We study the variations in the V magnitudes, which have a typical uncertainty of $0.^{m}01$, to search for periodicities. The colour variations of the microvariability will be used for the mode identification.

2.2. The luminosities of the program stars

The distance, bolometric magnitude and $E(B-V)$ of the program stars are listed in Table 2. We do not list the spectral type, temperature or radius because they vary, whereas the luminosity remains approximately constant during the moderate photometric variations upon which the microvariations are superimposed. We discuss the parameters listed in Table 2:

1. **R 71**: We show two sets of parameters for R 71. The first is from Wolf et al. (1981). They derive a luminosity of log $L_*/L_\odot$=5.3 by integrating the spectrum during minimum state. They adopt an LMC mean value of $E(B-V) = 0.05$. No correction for internal extinction within the LMC was made. The second set is from model calculations by Leitherer et al. (1989). They find that the model reproduces the observed energy distribution if $\log L/L_\odot = 5.71$ during maximum state and $E(B-V) = 0.15$. We use both sets throughout this paper. Note that a difference in $E(B-V)$ of 0.1 results in a difference in $M_{\rm bol}$ of 0.3 if $R_V \simeq 3$.
2. **164 G Sco**: The bolometric magnitude is from Humphreys & Davidson (1994) and was estimated from parameters derived by Sterken (1977).
3. **HR Car**: We calculated the bolometric magnitude with the parameters given by van Genderen et al. (1991) and the more accurate distance from Hutsemékers & van Drom (1991). This gives $M_{\rm bol}$=–9.05.
4. **S Dor**: From the maximum of $V = 9.0$ de Koter (1993) derived $M_{\rm bol} < -9.66$. Leitherer et al. (1985) derived a luminosity during maximum of 0.5 magnitude lower than the one derived by de Koter. Both assume an LMC mean value of $E(B-V) = 0.05$. De Koter finds that his value of $M_{\rm bol}$ is in good agreement with the observed and predicted photometry at visual minimum. Therefore we adopt the value of $M_{\rm bol} = -9.7$.
5. **R 127**: We show three sets of parameters of R 127. The first set of parameters is derived by assuming the LMC mean value $E(B-V) = 0.05$, which gives $M_{\rm bol}$= -10.14. The second set is from de Koter (1993) who compared the observed and predicted energy distributions and concluded that at minimum state the observations fit his models if $E(B-V) = 0.12 \pm 0.03$, which leads to $M_{\rm bol}$= -10.35. The third set is from Stahl et al. (1983) who derived $M_{\rm bol}$=–10.6 during maximum state with $E(B-V) = 0.20$. The extinction is obtained during minimum state by a colour comparison with an O9 supergiant (Schmidt-Kaler, 1982).
6. **AG Car**: The parameters have been determined by Humphreys et al. (1989) from UV and visual observations in maximum and minimum, based on a new distance determination of 6.2 kpc. The extinction is determined by comparison of the energy distribution with a set of standard Ia supergiants. This results in a bolometric magnitude of $M_{\rm bol}$= -10.8 ± 0.4.

Table 2. Parameters of the program stars

	distance [kpc]	$M_{\rm bol}$ [mag]	log L_* $L_\odot$	E(B–V) [mag]	M_* [$M_\odot$]
R 71	51.9 ± 3.1^a	-8.5^b	5.3	0.05^b	11^{+13}_{-2}
		-9.5^c	5.7	0.15^c	21^{+16}_{-2}
164 G Sco	1.98 ^{d}	-8.9^e	5.5	1.22^d	13^{+15}_{-2}
HR Car	5.4 ± 0.4^f	-9.1	5.5	0.9^g	15^{+15}_{-1}
S Dor	51.9 ± 3.1^a	-9.7^h	5.8	0.05^i	24^{+16}_{-2}
R 127	51.9 ± 3.1^a	-10.1	6.0	0.05	34^{+16}_{-3}
		-10.4^h	6.0	0.12^h	40^{+16}_{-3}
		-10.6^j	6.1	0.20^j	46^{+17}_{-3}
AG Car	6 ± 1^k	-10.8^k	6.2	0.63^k	53 ± 8

a: Panagia et al. 1991; b: Wolf et al. 1981; c: Leitherer et al. 1989; d: Sterken 1977; e: Humphreys & Davidson 1994; f: Hutsemékers & van Drom 1991; g: van Genderen et al. 1990; h: de Koter 1993; i: Stahl & Wolf 1982; j: Stahl et al. 1983; k: Humpreys et al. 1989;

2.3. The masses of the program stars

The masses of LBVs are not well known, because they cannot be derived from spectroscopic analysis in a reliable way. For instance, Pauldrach & Puls (1990) derived a spectroscopic mass for P Cyg of only 23 $M_\odot$. However, the mass cannot be smaller than about 30 $M_\odot$, otherwise the star would not have any hydrogen left at its surface. Therefore we derived the mass of the program stars from evolutionary tracks from Schaller et al. (1992) for Galactic stars (Z=0.02), and from Schaerer et al. (1993) for S Dor, R 71 and R 127 in the LMC (Z=0.008). We assume that a star reaches the LBV phase when its surface composition has reached a He/H ratio of 0.40 by number. This is the composition derived from spectroscopic studies of several LBVs (Najarro et al., 1997; Crowther, 1997). The ratio He/H=0.4 corresponds to the phase where X=0.377, Y=0.603 and Z=0.020 for Galactic stars and X=0.382, Y=0.610 and Z=0.008 for LMC stars.

We adopt conservative upper and lower limits of the mass. The LBV phase occurs after the main sequence phase and before the Wolf Rayet phase. So, the maximum mass of a star in the LBV phase is the mass at the end of the H-core burning phase. The minimum mass of an LBV is the mass at the beginning of the N rich WNL phase of Wolf Rayet stars. From the models of Schaller et al. (1992) and Schaerer et al. (1993) we derived the luminosities and masses at the three phases where (a) He/H=0.40 and (b) at the beginning of the WNL phase and (c) after the core contraction at the end of the main sequence phase, for stars with initial masses of 85, 60, 40 and 25 $M_\odot$. These M_*–L_* relations were interpolated logarithmically to derive the mass and its upper and lower limits for each LBV of a given luminosity. The estimated masses with their upper and lower limit are listed in Table 2. The relation between mass and luminosity of the individual stars can be fitted by the relation

$$\log{(L/L_\odot)} \simeq 3.99 + 1.29\ \log{(M/M_\odot)}. \qquad (1)$$

2.4. Estimates of $T_{\rm eff}$ and log g

For the purpose of this paper we need an estimate of $T_{\rm eff}$ and log g of the stars during the various phases of their variability. These quantities are not well known because they would require a detailed study of the energy distribution or the spectrum of each star during the various phases. This has not been done. Therefore we estimate $T_{\rm eff}$ and log g in a simpler, approximate way.

The variations in V are largely due to variations in the Bolometric Corrections (BC) of the stars, because LBVs vary at approximately constant bolometric magnitude (Wolf et al. (1981) for R71; Leitherer et al. (1985) for S Dor; Stahl & Wolf (1982) for R127; Lamers et al. (1989) for AG Car). This implies that the variations in V of the typical LBV variations (not of the micro variations!) can be related to variations in BC, which can be used to estimate the variations in $T_{\rm eff}$ and log g. We adopted the empirical relation between BC and $T_{\rm eff}$ for supergiants from Schmidt-Kaler (1982). In the temperature range of 12000 to 35000 K, the BC can be approximated quite accurately, i.e. within about $0.^m1$, as

$$\begin{aligned} BC &= +23.68 - 5.94 \times \log T_{\rm eff} \quad &\text{if } \log T_{\rm eff} > 4.182 \\ BC &= +15.40 - 3.96 \times \log T_{\rm eff} \quad &\text{if } \log T_{\rm eff} < 4.182 \end{aligned} \qquad (2)$$

with $BC < 0$. Using this expression we can estimate the values of $T_{\rm eff}$ during the various phases of the LBVs, by deriving BC from the difference between M_V and $M_{\rm bol}$. With $M_{\rm bol}$ and $T_{\rm eff}$ we can derive R_* in the usual way. The value of log g then follows from M_* and R_*. It is easy to show that for a constant value of $M_{\rm bol}$ and Eq. (2) the gravity varies as $g \sim BC^{-0.673}$. So if the star gets fainter in V by half a magnitude the gravity increases by a factor 1.6, if $M_{\rm bol}$ is constant.

For AG Car, which we study at the epoch of visual minimum $V \simeq 7.99$, we adopt a value of $T_{\rm eff} = 23\,000$ K. This value was derived from a detailed comparison between its energy distribution (visual and UV) at a phase when $V = 7.92$ and those of other supergiants by Lamers et al. (1989) (see also Humphreys et al. 1989).

There is some doubt about the constancy of the $M_{\rm bol}$ of LBVs during their variations, because the IUE data used for these studies have an accuracy of only about 25 percent. A study by Vennix (see Lamers, 1995) of the changes in the energy distribution of S Dor and a comparison with extended model atmospheres has suggested that $M_{\rm bol}$ of this star can vary by as much as $0.^m4$ in $M_{\rm bol}$ during a complete cycle of the typical LBV variations with $M_{\rm bol}$ being fainter during visual maximum. If this trend is confirmed for other stars as well, it implies that the changes in BC and the resulting changes in $T_{\rm eff}$ are larger than for constant $M_{\rm bol}$. However, the change in $M_{\rm bol}$ then counteracts this effect on the determination of log g. So, given the maximum changes in V of about $0.^m5$ in the time intervals studied here, we conclude that the assumption of constant $M_{\rm bol}$ results in sufficiently accurate estimates of the variations in log g for the purpose of this paper.

3. The light curves of the LBVs

Since the microvariations depend on the phase of the star during its moderate variations, we show the lightcurves of the program stars from Spoon et al. (1994) in Fig. 1. Below we briefly review the photometric variablity in V of the stars during the last decades, i.e. the time interval over which we study the microvariations.

The data consists of intervals with frequent measurements alternated with intervals of no available measurements. For each star we selected time intervals of the lightcurve that contained enough data points to allow a meaningful search for periodicities. These time intervals have a length of 50 to 500 days, except for R 127 which has an interval of 800 days. These intervals were chosen on visual inspection of the lightcurve. We also searched for periodicities in other intervals. However, we only found periodic variations in the intervals which were originally selected. This does *not* imply that the visual magnitude is not varying periodically outside the selected intervals. It means that there are insufficient data to find a periodicity with any degree of reliability. The selected regions in which we found a periodic behaviour are indicated in Fig. 1 by horizontal bars.

1. **R 71**: From 1975 until half 1979 the visual magnitude has dropped from about $V = 10$ to 10.8, where it stayed until 1995. We studied the microvariations halfway down the descending branch and at six intervals during the faint phase.
2. **HR Car**: The star has varied in visual magnitude between 7.4 and 8.4 mag. Between 1982 and 1985 and between 1985 and 1989 the visual magnitude went through a dip. Thereafter the star brightened, reaching V=7.4 mag in 1993. We studied the microvariations during 5 intervals at V between 8.0 and 8.4.
3. **164 G Sco**: The star showed a slow decrease in visual brightness from $V = 6.5$ to 6.8 between 1980 and 1985. The star remained faint at $V \simeq 6.8$ between 1985 and 1995. This star has been showing microvariations during the complete observational period from 1974 until half 1994 (van Genderen et al. 1997b). We studied the microvariations in two intervals at $V \simeq 6.8$.
4. **S Dor**: Intense observations started just before S Dor begun its moderate variation. Its visual brightness decreased from 1982 to 1985 from $V \simeq 9.4$ to 10.2, but it increased again from 10.2 to 9.0 between 1985 to 1989. After 1989 the magnitude decreased again, reaching a visual minimum in 1993 from which it is now recovering. We studied the microvariations in two intervals, close to the minimum.
5. **R 127**: The star has slowly been rising to visual maximum from $V \simeq 10.1$ in 1983 to 8.8 around 1990. From 1990 to 1994 it has slowly become fainter. We studied the microvariations in two intervals at $V \simeq 9.5$ and 8.9.
6. **AG Car**: The lightcurve of this star showed a huge dip during the time of observations. AG Car varied from $V \simeq 6.0$ to 8.0 between 1982 and 1985. Between 1985 and 1990, when the star was in this minimum, it showed clear microvariations. In late 1989 the star started to become brighter again. In early 1994 it had reached $V = 6.0$ again. We studied the microvariations in three intervals during visual minimum.

Table 3. Intervals of microvariations

Star	Nr	JD –24440000	V [mag]	N_{pts}
R 71	1	3200–3650	10.55	32
	2	6010–6155	10.81	62
	3	6380–6500	10.74	17
	4	6675–6860	10.80	40
	5	7025–7250	10.90	17
	6	7745–7850	10.95	15
	7	8100–8290	10.79	22
	8	8485–8660	10.78	17
HR Car	1	4595–4635	8.40	21
	2	6415–6525	8.15	29
	3	7135–7180	8.14	65
	4	7195–7260	8.19	36
	5	7305–7390	8.11	19
164 G Sco	1	7230–7775	6.83	83
	2	8060–8175	6.81	60
S Dor	1	5895–6155	10.12	52
	2	6305–6495	9.95	27
R 127	1	6305–6520	9.94	38
	2	7035–7850	8.86	51
AG Car	1	6420–6610	7.97	24
	2	6780–6870	7.99	39
	3	7135–7180	7.99	64

The intervals chosen for the study of the microvariations are listed in Table 3. Column 2 gives the interval-number that will be used later to indicate the interval. Column 3 gives the duration of the interval. The mean absolute visual magnitude in the interval is listed in Column 4. Column 5 gives the number of photometric observations in each interval.

4. Period determinations of the microvariations

4.1. The period-search methods

The search for periodicities was made with two methods: the Fourier method and the Phase Dispersion Minimization Method (PDM) from Stellingwerf (1978). The two methods differ in that the Fourier method deconvolves the lightcurve in a series of sinusoidal variations whereas the PDM method makes no apriori assumptions about the shape of the variations. The results of the two methods were compared to find information about the shape of the variations and about the reliability of the derived periodicities.

During 11 of the selected intervals the microvariations occurred while the visual magnitude was slowly changing. In these intervals we subtracted the slower variation.

When the Fourier method was applied, the HWHM of the main peak in the Fourier data spectrum was taken as the uncertainty of the frequency. The PDM method usually gives a slightly different uncertainty of the frequencies as the variation is not exactly sine-shaped. We fitted Gaussians through the minima of

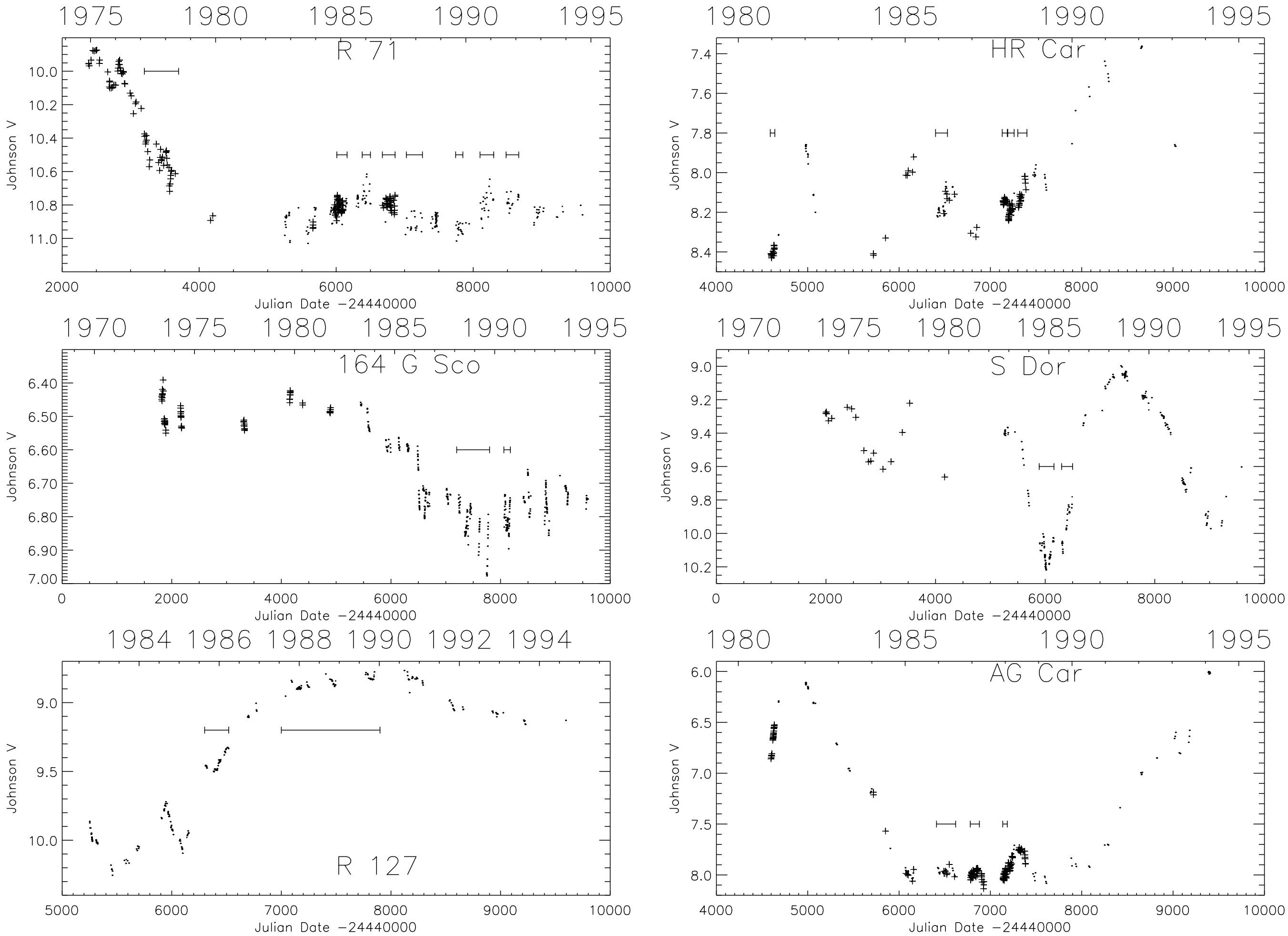

Fig. 1. The light curves of the program stars over periods of about 12 to 25 years (data collected by Spoon et al. 1994). Different symbols refer to data from different photometric systems (see Spoon et al.). The horizontal bars indicate the time intervals in which we found periodicity of the microvariations.

the θ–spectrum of the PDM method. The standard deviation of the Gaussian is taken as uncertainty in the frequency.

One example of a microvariation of R71 is given in Fig. 2. The top part shows the observed lightcurve during an interval with a clear periodicity; the middle part shows the comparison with a sine curve of the main frequency; the lower part shows the lightcurve folded with the sine wave.

4.2. Periods and amplitudes of microvariations

The results of the period search are listed in Table 4. For each star and each interval we give the stellar data and the data about the periodicity.

Columns 3 to 6 give information about the M_V, $T_{\rm eff}$, R_* and log g derived in the way described in Sect. 2.4. The values of $T_{\rm eff}$ were derived from the BC, using the relation between BC and $T_{\rm eff}$ from Schmidt-Kaler (1982) for supergiants. For R 71 these data refer to the stellar parameters with $E(B-V)=0.15$ and $M_{\rm bol}=-9.5$ and for R 127 they refer to $E(B-V)=0.12$ and $M_{\rm bol}=-10.35$.

Columns 7 and 9 give the dominant period of the microvariations and its half-amplitude. The period of the dominant variation is derived from the highest peak in the Fourier data spectrum, the frequency of the minimum value of θ and from the aliases and the subharmonics. The quoted uncertainty is the 1σ error, that was derived by fitting the dip in the θ-spectrum by a Gaussian profile and from the HWHM of the peak in the Fourier spectrum. In Column 10, we give the probability (in percentage) that the minimum in θ, corresponding to that period, is due to a random fluctuation. In all cases, except for R71, this probability is less than 20 percent. Column 11 gives the number of periods covered by the interval. If the photometry shows a gradual trend during the interval, we subtracted this trend from the data before determining the period. This correction is indicated in the last column, where "up" and "down" indicate a trend of increasing or decreasing visual brightness respectively.

The Q-values are listed in Column 8. They were derived from the mass listed in Table 2 and the radius. They will be discussed in Sect. 5.2.

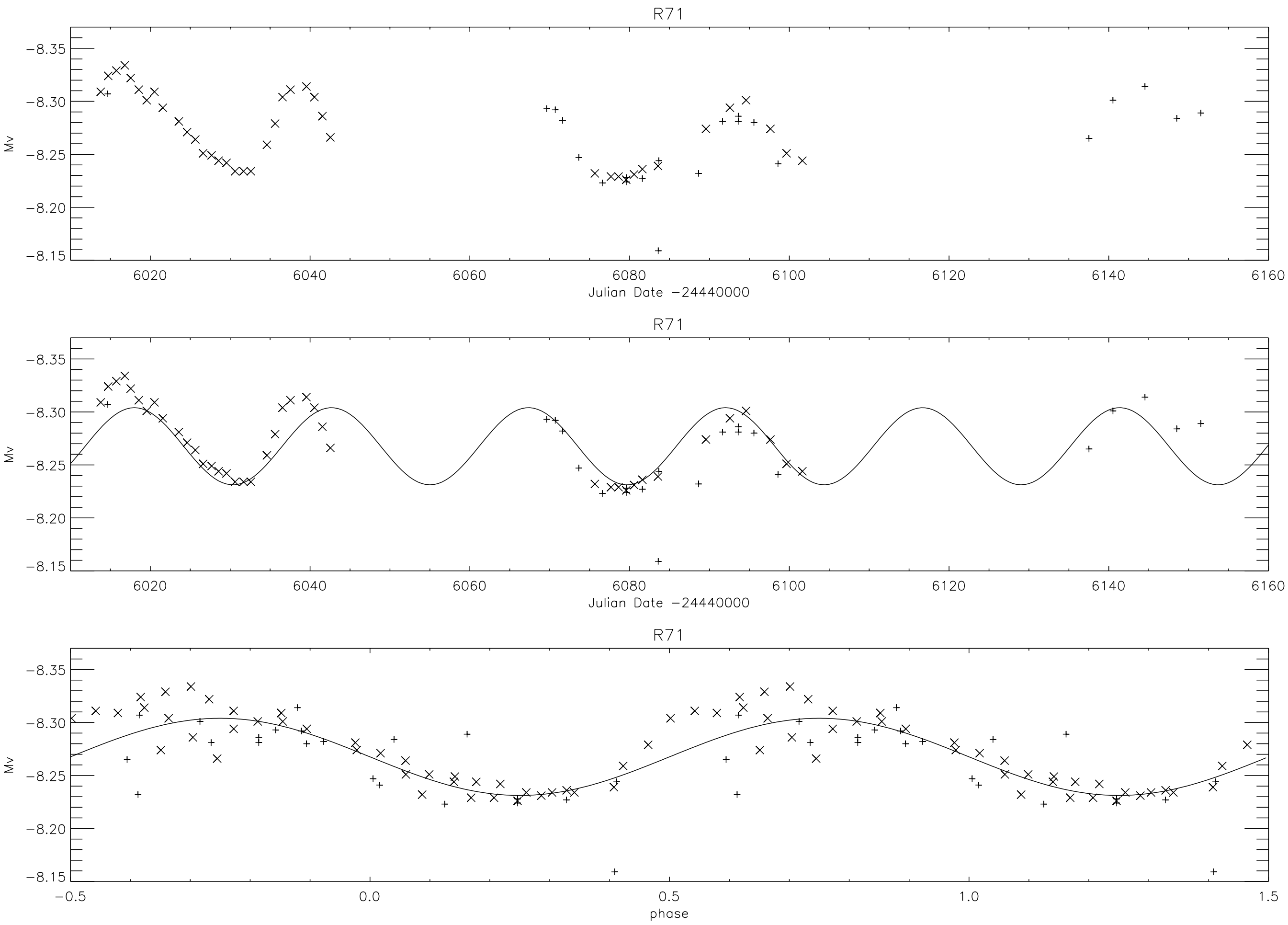

Fig. 2. a The photometric data of R 71 between Nov 1984 and April 1985; **b** The data with a sine wave of the calculated frequency; **c** The data folded with a sine wave. The lightcurve is given by two different symbols because the measurements were done with two different instruments (Spoon et al., 1994)

We briefly describe the results for each of the stars individually:

1. **R 71**: Eight intervals with a sufficient density of photometric data points were selected. The first one is during a phase of decreasing brightness. A rather long interval of 450 days was selected in order to have enough data points for a meaningful search for periodicities. We corrected for the linear decrease during this period before we searched for periodicity of the microvariations. The other intervals are during visual minimum. The periods of the microvariations varies between 76 days and 19 days, and the amplitude varies between 0.031 and 0.058 mag. The longest period occurred when the star was brightest. The periods are shorter during subsequent intervals, when the star has about a constant visual magnitude. All but two periods have a probability of less than 20 percent for being due to random fluctuations. The variation in the interval from JD +6010 to +6160 was chosen for the example in the previous section (Fig 2).
2. **HR Car**: Five intervals were selected. The first one was when the star was at a visual minimum of V=8.4 mag.; the other ones when V$\simeq$8.1 mag. The period of the microvariations ranges from 18 to 41 days, and the amplitudes vary between 0.010 and 0.059 mag.
3. **164 G Sco**: We selected two intervals: one of 545 days and a shorter one of 115 days. Both occurred when the star was in a relative minimum. During the first interval the star was slowly decreasing in visual brightness, whereas it was at about a constant magnitude during the second interval. The microvariations had periods of 55 and 45 days. The amplitudes are 0.043 and 0.037 mag. respectively. Sterken et al. (1991) determined a mean period of 57 days during a long time interval from about JD +5500 to +7500 during a period when V decreased from 6.5 to 6.9 mag.
4. **S Dor**: We found only two intervals of sufficient density of data points to search for periodicities of the microvariations. The first coincides with the visual minimum, the second one with the phase of increasing brightness. The microvariations during visual minimum have a period of 195 days. This is the longest period of microvariations found in our sample. The amplitude is 0.098 mag. During the second interval the

Table 4. Periods and amplitudes of microvariations

Star	Nr	M_V [mag]	log $T_{\rm eff}$	log R_* [$R_\odot$]	log g	Period [days]	Q [days]	Amplitude[1] [10^{-3} mag]	Prob [%]	$N_{\rm per}$	trend corr.
R 71[2]	1	–8.49	4.153	2.07	1.62	76 ± 4	0.270	51 ± 10	13	6.0	down
	2	–8.23	4.206	1.97	1.83	24.6 ± 0.9	0.126	36 ± 5	0.03	5.6	–
	3	–8.30	4.194	1.99	1.78	39 ± 4	0.187	58 ± 14	14	3.0	–
	4	–8.24	4.204	1.97	1.82	18.9 ± 0.3	0.096	31 ± 5	7.9	9.7	–
	5	–8.14	4.221	1.94	1.89	39.5 ± 2.1	0.225	55 ± 11	19	5.6	–
	6	–8.09	4.229	1.92	1.92	18.9 ± 0.5	0.114	31 ± 8	30	5.2	–
	7	–8.25	4.203	1.98	1.82	43.3 ± 1.3	0.217	49 ± 12	12	4.4	up
	8	–8.26	4.201	1.98	1.81	36.1 ± 2.0	0.179	31 ± 9	36	4.8	–
HR Car	1	–8.05	4.140	2.00	1.60	18.5 ± 1.3	0.070	19 ± 4	2.0	2.0	–
	2	–8.30	4.077	2.13	1.35	41 ± 4	0.102	59 ± 9	1.4	2.7	up
	3	–8.31	4.074	2.13	1.34	18.6 ± 2.4	0.045	9.8 ± 1.2	2.9	2.3	–
	4	–8.26	4.087	2.11	1.39	23.2 ± 1.8	0.061	30 ± 4	0.07	2.7	up
	5	–8.34	4.069	2.14	1.32	25.7 ± 0.8	0.060	35 ± 8	8.5	3.2	up
164 G Sco	1	–8.44	4.005	2.24	1.08	55.1 ± 0.9	0.087	43 ± 5	0.43	9.8	down
	2	–8.46	4.000	2.25	1.06	45.1 ± 2.8	0.069	37 ± 5	3.6	2.4	–
S Dor	1	–8.61	4.153	2.10	1.61	195 ± 6	0.672	98 ± 7	0.01	1.3	–
	2	–8.78	4.110	2.18	1.44	131 ± 17	0.335	48 ± 6	7.4	1.4	up
R 127[3]	1	–9.51	4.098	2.35	1.35	35.3 ± 2.5	0.067	8.5 ± 1.6	18	6.0	up[4]
	2	–10.09	3.954	2.63	0.77	111.4 ± 2.3	0.079	31 ± 3	0.34	7.3	up
AG Car	1	–7.87	4.362	1.90	2.36	40.6 ± 1.6	0.421	29 ± 6	1.9	4.6	–
	2	–7.85	4.362	1.90	2.36	18.1 ± 0.6	0.188	26 ± 4	4.0	4.6	up
	3	–7.84	4.362	1.90	2.36	10.9 ± 0.9	0.113	26 ± 4	7.9	3.9	up

(1): the quoted value is the half of the peak-to-peak amplitude
(2): values for $E(B-V) = 0.15$ and $M_{\rm bol}$=-9.54
(3): values for $E(B-V) = 0.12$ and $M_{\rm bol}$=-10.35
(4): sine of 296 days.

period is considerably shorter, 131 days, and the amplitude is also smaller.

5. **R 127**: Two intervals were selected. The first interval of 215 days coincides with a phase of visual brightening. The microvariations during that time have a period of 35 days and an amplitude of 0.008 mag. The second interval occurs when the star is close to maximum visual brightness. The period of the microvariations has increased to 111 days and the amplitude has also increased to 0.031 mag.
6. **AG Car**: Three intervals were selected, all of them during the phase of visual minimum. The microvariability period varies from 41 to 11 days, but the amplitude remains approximately constant near 0.027 mag.

5. The Q-values and the relations between the stellar parameters and the periods

5.1. The Q-values of pulsating early-type stars

The periods of adiabatic pulsations are expected to depend on the mean density of the star via the well-known period mean-density relation

$$P = Q \left(\frac{\bar{\rho}}{\bar{\rho}_\odot} \right)^{-\frac{1}{2}}. \tag{3}$$

The parameter Q depends on the adiabatic index Γ_1, which determines the dynamical stability of the star, and on the density distribution within the star. The average Q-value for the radial fundamental mode of Cepheids is 0.040 days (e.g. Cox, 1980). The (non-)radial p-mode pulsations of β Cephei stars have an average Q-value of 0.03, while the non-radial g-mode pulsations that appear in Slowly Pulsating B-stars have 0.68 as average Q-value (Heynderickx et al., 1994). The Q-values of BA-type supergiants range from 0.032 up to 0.172 (Burki 1978), while O-type supergiants reach values up to 0.6 (van Genderen, 1985).

Lovy et al. (1984) have determined the period and the value of Q for radial modes (fundamental and first and second overtones) of supergiants, using the evolutionary tracks of Maeder (1981). They find that the observed periods in 40% of the supergiants are compatible with those predicted by their models. In the other cases, however, the periods are much longer than the ones predicted by their models and they conclude that these supergiants undergo non-radial g-mode pulsations or that they are in a post-red supergiant stage.

5.2. The Q-values of the microvariations of LBVs

We have determined the Q values for all the epochs in which periodicity was found. They are listed in Column 8 of Table 4. The Q-values are in the range of 0.058 to 0.909 days. We find that the Q-values differ significantly for different stars and moreover that they change considerably from one epoch to the other for some stars (e.g. AG Car and S Dor). In Table 5 we list the mean values for the stars. We also give an indication of typical LBV phase (visual minimum, visual maximum, or halfway) during the observations as shown by the lightcurves of Fig. 1. We give the data for the two values of $M_{\rm bol}$ for R 71 and for the three values of $M_{\rm bol}$ for R 127.

The values of Q are sensitive to the adopted stellar masses, since $Q \sim \sqrt{M_*}$. The conservative uncertainty of the masses are listed in Table 2. The resulting uncertainties in Q, expressed in terms of minimum and maximum corrections are also listed in Table 5.

The data in this table show that the mean value of Q is between 0.07 and 0.18 days for most of the stars (R 71, HR Car, 164 G Sco, and R 127). The two exceptions are S Dor with $Q = 0.34$ and 0.67, and AG Car with $Q = 0.42$, 0.19 and 0.11. The behaviour of AG Car shows that the period of the microvariations and Q can change by as much as a factor four, even when the visual magnitude of the star hardly changes. The data in Tables 4 and 5 suggest that the most common microvarations of LBVs have $Q \simeq 0.07 \pm 0.01$ days. However the stars can go through phases of slower pulsations when Q increases by a factor up to four. This does not seem to be related to a specific phase in the lightcurve. The two high values of Q for S Dor possibly represent temporary phases of slow pulsation.

The same behaviour might also be present in the microvariations of normal supergiants. Van Genderen (1985) lists three other supergiants that have abnormaly high Q-values. All other supergiants studied by him and by Burki (1978) have Q's that are at least a factor-of-two smaller than these three cases.

Lovy et al. (1984) predicted Q values for the fundamental and the first overtone of the radial pulsations of a grid of supergiant models. For the early type supergiant models in the range of $5.3 < \log\ L_* < 6.0$, with masses in the range of $25 < M_* < 47\ M_\odot$, the predicted Q-values for the fundamental radial mode are $0.037 < Q < 0.055$. Our mean values of Q for the four stars R 71, HR Car, 164 G Sco, and R 127 are larger than this by a factor 1.5 or 2. This might be due to the higher L_*/M_*- ratio of the LBVs compared to normal stars, because they may have lost more mass already. Post-main sequence stars which have lost mass will have a mass distribution that is more concentrated in the center than stars without mass loss, because the core is hardly affected by the mass loss, but the envelope mass has decreased. So we expect that LBVs, which have lost more mass than normal B supergiants, will have higher Q-values.

5.3. *The period-M_V relation*

When the visual brightness of an LBV increases, its effective temperature decreases. At the same time the radius of the star increases. This change in structure of the star may also affect the period of the microvariations. In this section we search for relations between the visual magnitude of the stars and the period of the microvariations.

The mean density can be expressed in terms of M_* and R_* while R_* can be expressed in $T_{\rm eff}$ and L_* or $M_{\rm bol}$. This results in a predicted relation for adiabatic pulsations:

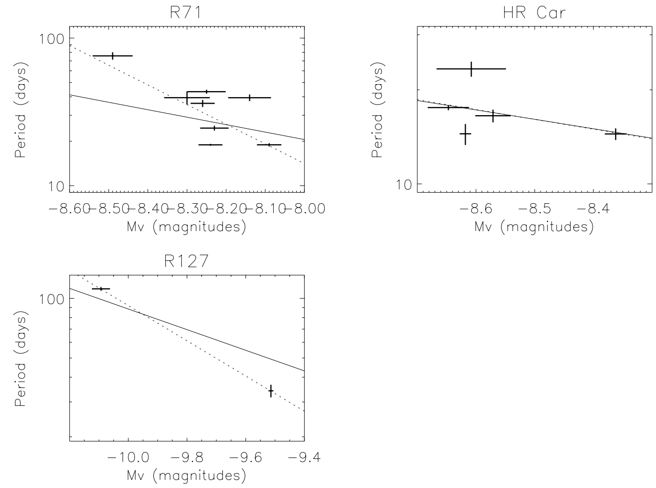

Fig. 3. The values of $\log\ P$ versus M_V for three stars. The dashed lines show the weighted least square fits. The full lines show fits with a fixed slope of 0.505, which is predicted for adiabatic radial pulsations for constant Q and $M_{\rm bol}$.

$$\log P \ = \ 12.71 + \log Q - 0.3 M_{\rm bol} - 0.5 \log \frac{M}{M_\odot} - 3 \log T_{\rm eff}. \quad (4)$$

The mass can be eliminated from this equation by means of the mass luminosity relation for LBVs, given in Eq. (1). The temperature can be expressed in terms of the bolometric correction, BC. The dependence of the bolometric correction on $T_{\rm eff}$ is taken from the empirical relation for normal supergiants, approximated by Eq. (2). This results in the following predictions for P:

$$\log P = 1.56 + \log Q + 0.36\ M_{\rm bol} - 0.505\ M_V \quad (5)$$

This equation is valid if Q and $M_{\rm bol}$ are both constant during the typical LBV variations. Equation (4) shows that the pulsational period of the LBVs with the same effective temperature are expected to vary with $M_{\rm bol}$. Equation (5) shows that for an LBV that varies with constant luminosity or $M_{\rm bol}$, the pulsational period is expected to vary as $10^{-0.505 M_V}$, if Q remains constant.

Fig. 3 shows the relation between the observed values of $\log\ P$ and M_V for the stars R 71, HR Car and R 127. We only show the relation for these three stars for which we either have several period determinations or where these determinations refer to significantly different values of M_V.

We know beforehand that we cannot expect an excellent empirical correlation. This is simply due to the fact that the period of the microvariations changes irregularly after a few pulsation cycles, even if the visual magnitude of the star remains almost constant. This is most obvious in the data of the star HR Car (Table 4) for which we derived four reliable periods at almost constant visual magnitude between M_V= -8.57 and -8.61. The period varies between 18.6 and 41 days, which is more than a factor 2.

Table 5. Empirical Q-values

Star	$M_{\rm bol}$	$< Q >$ days	$\frac{<Q>^a_{\rm min}}{<Q>}$	$\frac{<Q>^b_{\rm max}}{<Q>}$	phase^c
R 71	-8.53	0.082 ± 0.026	0.91	1.48	half/min
R 71	-9.54	0.177 ± 0.061	0.95	1.22	half/min
HR Car	-9.05	0.068 ± 0.021	0.92	1.47	min
164 G Sco	-8.90	0.078 ± 0.013	0.97	1.41	min
S Dor	-9.66	0.504 ± 0.239	0.96	1.29	min/half
R 127	-10.10	0.076 ± 0.008	0.95	1.21	half/max
R 127	-10.35	0.073 ± 0.008	0.95	1.18	half/max
R 127	-10.60	0.066 ± 0.007	0.97	1.17	half/max
AG Car	-10.80	0.241 ± 0.161	0.92	1.07	min

[a]: the minimum value of $< Q >$ corresponding to the minimum mass
[b]: the maximum value of $< Q >$ corresponding to the maximum mass
[c]: the phase of the visual lightcurve (see Fig. 1).

Fig. 3 also shows the slope of the expected relation (Eq. 5) for constant Q and $M_{\rm bol}$. The figure shows that on the average the periods of the microvariations increase when the star gets visually brighter. This is to be expected because the radius increases. Although the data are scarce, the figure suggests that the observed relation between P and M_V is steeper than predicted for constant Q and $M_{\rm bol}$. This could be due to at least two effects:
(a) $M_{\rm bol}$ varies during a typical LBV varation in the sense that the luminosity is smaller at visual maximum (Lamers, 1995). If that is the case, then we have overestimated the BC and the temperature and underestimated the radius when the star gets visually brighter. So we have overestimated the mean density and overestimated the value of Q, derived from P and $\bar{\rho}$ when the star is optically bright. In that case we expect that Q gets smaller when the star gets visually brighter, which would result in a *flatter* $P - M_V$ relation than predicted for constant Q and $M_{\rm bol}$. This is contrary to the observations in Fig. 3
(b) As M_V gets brighter, the star expands considerably but only a small fraction of the stellar mass takes part in the expansion. Lamers (1995) and Maeder (1995) have suggested that less than 1 percent of the stellar mass takes part in the expansion. This means that the star will have a stronger density concentration and hence a higher value of Q when it is optically brighter. It would result in a *steeper* $P - M_V$ relation than predicted for constant Q. This is qualitatively in agreement with the steep empirical relations in Fig. 3.

We conclude that the observations show the expected trend of increasing period with increasing visual brightness, but that the observed relation is steeper than predicted for constant Q and $M_{\rm bol}$. The difference can be due to changes in Q due to changes in the density distribution as the outer envelope of the star expands.

5.4. The period-amplitude relation

Fig. 4 shows the amplitude of the variations versus the period. The stars are arranged in order of increasing luminosity. We find that larger amplitudes are seen for variations with longer periods. The lines in the six graphs are simple linear fits going through zero: $A \propto P$. Except for AG Car (and possibly R 71) these linear fits agree with the data. The slope of the amplitude-period relations are listed in Table 6. The data in Fig. 4 and in Table 6 show that, apart from AG Car, there is a trend of decreasing slope with increasing luminosity. Van Genderen (1985) already noticed this trend. AG Car behaves differently in the sense that the linear fit through the origin has a much steeper slope than suggested by its high luminosity. However, if we would relax the condition that the relation should go through the origin, the slope would be very flat (dashed line).

Table 6. The Amplitude-Period relation

Star	$M_{\rm bol}$	dA/dP [10^{-3} mag/day]
R 71	-8.5 – -9.5	1.13 ± 0.14
164 G Sco	-8.9	0.79 ± 0.07
HR Car	-9.1	0.98 ± 0.19
S Dor	-9.8	0.43 ± 0.06
R 127	-10.1 – 10.6	0.27 ± 0.03
AG Car	-10.8	1.12 ± 0.40
		0.09 ± 0.24 [1]

(1): Relation not through the zero point.

We conclude that the (A, P)-relation of the microvariations of LBVs has a slope that decreases with increasing luminosity of the star.

5.5. Conclusion about the photometric "microvariations"

- All our six LBV program stars show photometric variations with semi-amplitudes up to $0.^m1$. The periods range from 11 days (AG Car) to 195 days (S Dor). The variations are larger during visual minimum than during visual maximum.
- For each LBV the period varies significantly, up to about a factor four.
- The three LBVs that vary significantly in M_V during the phases that we studied, R 71, R 127 and HR Car, all show a period that increases with increasing visual brightness, i.e. with increasing radius. The relation is steeper than expected

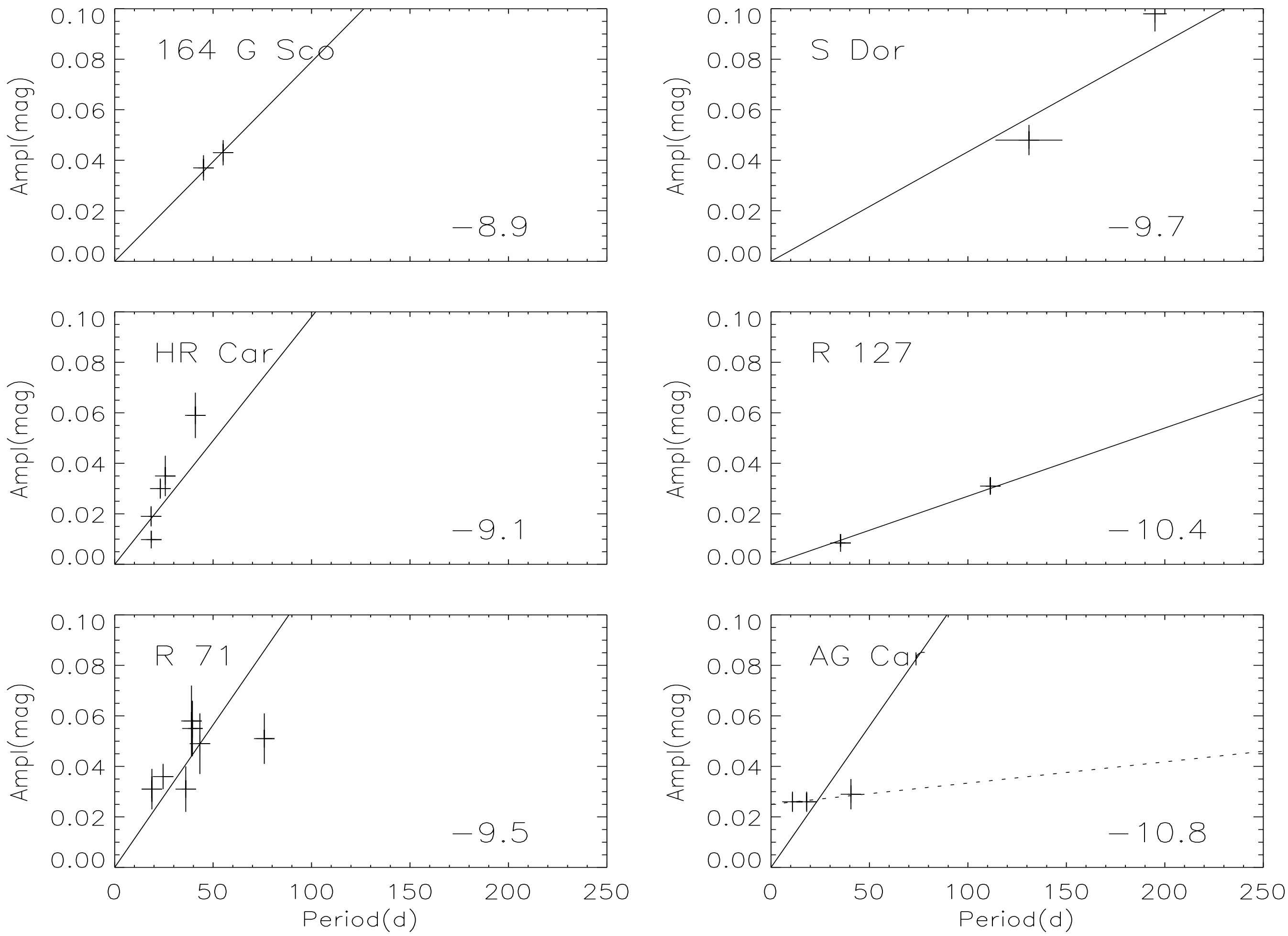

Fig. 4. The amplitude versus period of the microvariations. The stars are arranged in order of increasing luminosity, with the value of $M_{\rm bol}$ indicated in the lower right corner. The lines are linear fits going through zero with a slope given in Table 6. For AG Car we also show the slope of the relation that does not go through the zero point.

for a constant Q-value. This qualitatively agrees with the expected relation for stars of which only the envelope has expanded at visual maximum.

- The Q-values of the four LBVs (R 71, HR Car, 164 G Sco, and R 127) are in the range of 0.08 to 0.18 days. This is about twice as long as for normal B supergiants. This might be due to a higher L_*/M_* ratio of the LBVs.
- The Q-value of S Dor is much higher. This might be a temporary effect.

6. The nature of the pulsations

6.1. Strange-mode instabilities?

Kiriakidis et al. (1993) have suggested that the variability of LBVs could be due to strange-mode instabilities. They have calculated the location in the HR-diagram where this instability occurs using the OPAL opacities for various metallicities: $Z = 0.004$, 0.02 and 0.03. The location depends very strongly on the metallicity. This is shown in Fig. 5, where we plot the regions where the strange-mode instability occurs for two values of the metallicity: Z=0.004 and 0.03. The location of our program stars is also indicated. The LMC stars are compared with the prediction for $Z = 0.004$ (although this metallicity is lower than the LMC value of $Z \simeq 0.008$) and the Galactic stars are compared with the $Z = 0.03$ predictions. The values of $T_{\rm eff}$ of LBVs is varying. In this figure we used the mean value of log $T_{\rm eff}$ at the epoch where we measured the microvariations.

The temperature range in the HRD where the strange-mode instability occurs is very sensitive to the metallicity. The LMC stars are outside the instability region for $Z = 0.004$. However, the instability region for $Z = 0.008$ (not predicted) might cover the location of the LBVs of the LMC. Of the Galactic stars, AG Car is in the predicted instability region, but the other two program stars are outside the region. They are too faint by about 0.2 dex. However, given the uncertainty in the stellar parameters and in the location of the edges of the strange mode instability region, the disagreement between predictions and observations might not be significant.

It is *very unlikely* that the strange-mode instability is responsible for the microvariations of most of the LBVs. The reason

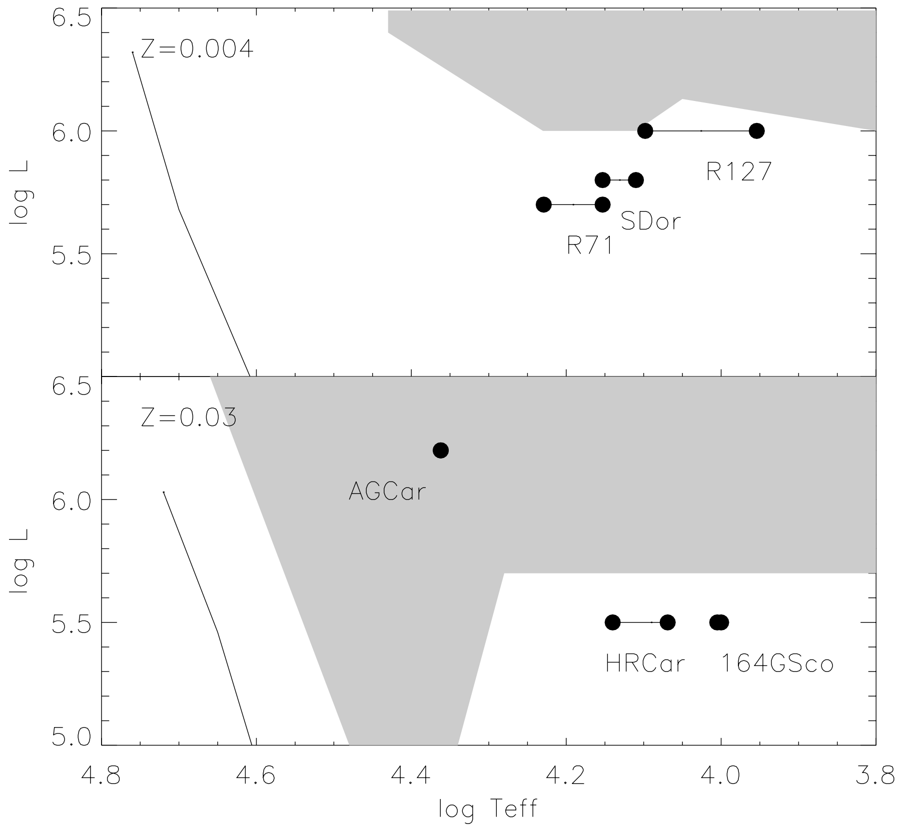

Fig. 5. The location of the strange-mode instabilities in the HR-diagram (shaded area) for $Z = 0.004$ and $Z = 0.03$ (from Kiriakidis et al. 1993). The location of our LMC program stars at the time of the observed variability are compared with the predictions of $Z = 0.004$ and the Galactic stars are compared with $Z = 0.03$. The left full line indicates the location of the main sequence.

is simple: the microvariations of LBVs are very similar to those of normal supergiants (where it is sometimes called α Cyg variability). The only difference is that the amplitudes and periods of the LBVs are larger than for normal supergiants by about a factor two. All early type supergiants, from types O through A, show these microvariations. This is not in agreement with the predicted limited location of the strange-mode instability in the HRD.

We now investigate if the microvariations of the two LBVs for wich we found a much larger Q-value, i.e. S Dor and AG Car, could be due to strange modes.

AG Car

The luminosity of AG Car suggests an initial mass of about 70 $M_\odot$. The calculations of Kiriakidis et al. (1993) show the results for a model with an initial mass of 60 $M_\odot$ and a metallicity of Z=0.02. The temperature of AG Car was about $T_{\rm eff} \simeq 23\ 000$ K during the epoch for which we derived the period. For this temperature range the predictions show one unstable model with a period of $P = \sigma_r^{-1}\sqrt{{R_*}^3/3GM_*}$ with $\sigma_r = 1.2$, which corresponds to a period of only 0.3 days. This is much shorter than the observed periods by about a factor 10^2.

S Dor

Kiriakidis et al. (1993) did not publish the results for $Z \simeq 0.008$ which would be applicable to the pulsations of S Dor in the LMC. However they show the results for $Z = 0.004$. The luminosity of S Dor suggests that the initial mass was about 50 $M_\odot$. The nearest model in Kiriakidis et al. is for $M_{in} = 60\ M_\odot$. The predicted models of this star do not show unstable modes in the temperature region of $T_{\rm eff} \simeq 14\ 000$ K, which is the temperature of S Dor at the time of the measured microvariations. However, since the location of the instability is very strongly dependent of Z, let us assume that a model with $Z = 0.008$ is unstable. In that case we can use the eigen frequency calculations of Kiriakidis et al. to find the expected period. For a model with $M_{in} = 60\ M_\odot$ and $T_{\rm eff} \simeq 14\ 000$ K the predicted period of the lowest order strange-mode instability is 4.4 days. This is a factor 30 to 50 shorter than the observed periods.

We conclude that the microvariations of the LBVs cannot be explained by strange-mode instabilities.

6.2. *The similarity to Slowly Pulsating B-stars: non-radial g-modes?*

We compare the microvariations of the LBVs with those of the Slowly Pulsating B-stars (hereafter: SPBs, Waelkens 1991). Pamyatnykh (1998) recently predicted a continuous extension of gravity mode instability due to the κ -mechanism from the SPB-domain in the HR-diagram towards large stellar masses. The observational findings of Waelkens et al. (1998) with respect to the position in the HR diagram of variable supergiants discovered by Hipparcos are fully compatible with Pamyatnykh's theoretical prediction. The main period of the variation of the supergiants derived from the Hipparcos photometry also suggests an extension of the SPB instability domain. This can be seen on Fig. 3 of Waelkens et al. (1998): the position of all the supergiants suggests that they exhibit g-modes similar to those excited in SPBs.

Since the microvariations of the LBVs are very similar to those of the normal B-type supergiants (with the exception of AG Car and S Dor, see Sect. 5.2) this suggests that the microvariations of the LBVs are also due to pulsation in g-modes.

7. A first attempt for mode identification from the colour variations

If we accept that the pulsational behaviour of the LBVs is similar to the one of SPBs, we can attempt to identify the modes from multicolour photometry in the same way as it is achieved for these well-known non-radial g-mode pulsators. From a comparison of the photometric amplitudes in different filters of a photometric system, it is in principle possible to derive information on the degree of the pulsation mode in the context of linear pulsation theory. This method of mode identification is referred to as "the photometric amplitudes method" and was introduced by Dziembowski (1977), refined by Watson (1988), and by Heynderickx et al. (1994).

To achieve identification, a prediction for the photometric amplitude of a non-rotating pulsating star is derived as a function of the wavelength and of the degree ℓ of the pulsation mode (see Heynderickx et al. 1994). The method is, up to now, mostly used for identification in β Cep stars and also in some SPBs. In the derivation of the relation between colour and amplitude, one assumes a phase difference between the temperature variation and the radial displacement of 180°, while non-adiabatic effects are allowed for by means of a free parameter $S \in [0, 1]$ ($S = 1$: adiabatic pulsation). These approximations are good for β Cep stars and SPBs. In view of the fact that LBVs seem to be an extension of the instability domains of these two groups of non-radial pulsators towards massive stars (Waelkens et al., 1998), the use of the same approximations seems justified as a first attempt to identify modes in LBVs.

By calculating the relation between the photometric amplitude and wavelength for different values of the degree of the pulsation mode ℓ ($\ell = 0$ for a radial pulsation) and S, and by comparing the results with the observed photometric amplitudes as a function of λ, one determines the value of ℓ that best fits the observations. This is achieved by considering *ratios* of amplitudes, such that wavelength independent constants are eliminated. We have implemented Heynderickx et al.'s (1994) method for the Strömgren photometric system.

We considered only those epochs for which an accurate period was determined and for which at the same time sufficient multicolour data were available in the Strömgren system. The amplitudes of the variations were determined by subtracting the slower variation and fitting a sine to the data with the periods and the trend described in Table 4. The results are listed in Table 7, together with the fraction of the variability that is explained by the model. It can be seen that the fits are rather poor in the case of R 71. We therefore did not attempt a mode identification for this star.

We show the variations in the four filters of HR Car and 164 G Sco in the upper panels of Fig. 6, and of R 127 and S Dor in the upper panels of Fig. 7. The amplitudes were then normalised with respect to the u-filter. For R 127 and 164 G Sco the relations between amplitude and wavelength change from one epoch to another (see the lower panels of Figs. 6 and 7). This points towards a switching of modes of different character. In the case of S Dor and HR Car this is not clear.

The observed normalised amplitudes were subsequently compared with theoretical normalised amplitudes as a function of ℓ and S. The logarithmic derivatives that appear in the theoretical expressions for the amplitude (Heynderickx et al., 1994) were determined by interpolation in the grid of atmosphere models by Kurucz (1979) with $\log\ T_{\rm eff}$ and $\log\ g$ as close as possible to the estimated values listed in Table 4. Accurate stellar models are not available for LBVs, and we had to consider $\log\ g = 2$ for all stars except AG Car.

For the comparison between the predicted and observed relative amplitude versus wavelength relations, we use the square root of the sum of squares of the differences between the observed and theoretical amplitudes divided by two as an indicator of the agreement. This quantity is defined as the "amplitude difference" and was determined for $S = 0.1, \ldots, 1.0$ in steps of 0.05 and for $\ell = 0, \ldots, 6$. The most likely mode is the one with the smallest amplitude difference. The results of our analysis are shown in the middle panels of Fig. 6 for HR Car and 164 G Sco, and Fig. 7 for R 127, and S Dor. The mode identification of AG Car was considered as unreliable since the results were rather dependent on the gravity of the adopted Kurucz models. For the other four LBVs, we found the same results for $\log\ g = 2$ as for higher $\log\ g$-values. We now discuss the results for four LBVs.

HR Car (Fig 6, left)

We find that the best fit occurs for an $\ell = 2, S = 1$ mode for the first epoch (Nr 2 with P=41 days) while an $\ell = 2, S < 0.5$ is obtained for the second epoch (Nr 3 with P=19 days). The large difference in the periods indicates that the pulsations have a different radial order. Our identification thus points towards a pulsation with the same degree $\ell = 2$, but another overtone at the two epochs. In the lower panel of Fig. 6 we compare the theoretical amplitudes with the observed ones and it can be seen that the observations can be accurately described by the pulsation model with the parameters given above, especially for the second epoch.

164 G Sco (Fig 6, right)

For interval Nr 1 (P=55 days) we find an $\ell = 1$ solution, independent of the value of the non-adiabaticity parameter S. For interval Nr 2 (P=45 days) we find an $\ell = 3$ solution to give the best representation of the data. In both cases our simple linear non-radial pulsation model explains the observations very

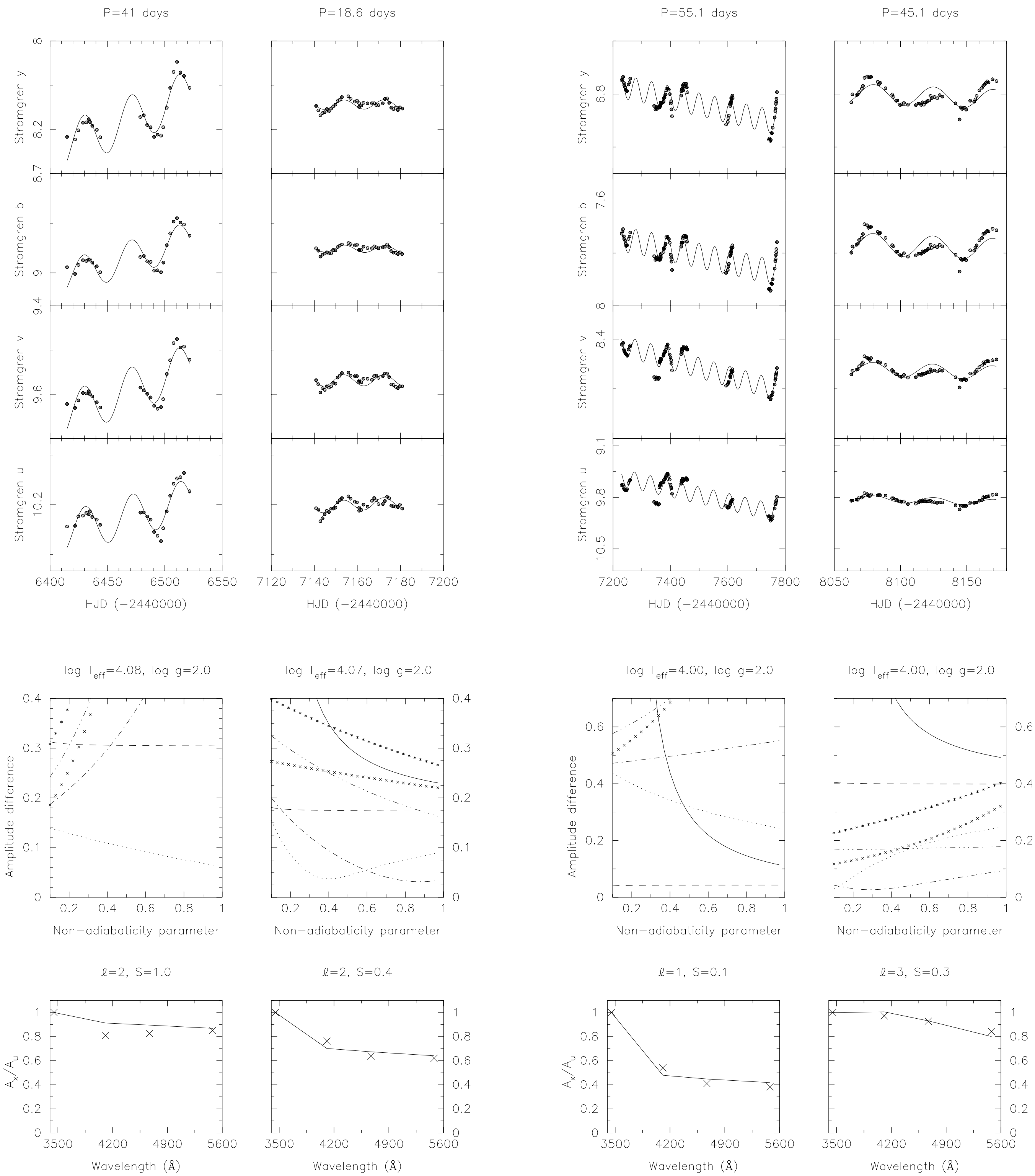

Fig. 6. Top four panels: multicolour data for the four passbands of the Strömgren system of HR Car (two left panels) for the period 2 and 3 (see Table 4) and for 164 G Sco (two right panels) for the period 1 and 2. The dots are the observations while the full lines are fits with the pulsation periods listed in Table 4 and given on top of each panel. Middle panels: the amplitude differences (in magnitudes) as a function of the non-adiabaticity parameter (see text for an explanation) for the most likely degrees of the pulsations. Different symbols denote different degrees: full line: $\ell = 0$, dashed line: $\ell = 1$, dotted line: $\ell = 2$, dashed-dotted line: $\ell = 3$, dashed-dot-dot-dot line: $\ell = 4$, $\star : \ell = 5$, $\times : \ell = 6$. Lower panels: the observed amplitude ratios A_x/A_u for $x = u, v, b, y$ (crosses) are compared with those of the best theoretical model (connected by the full line) for the four filters of the Strömgren system

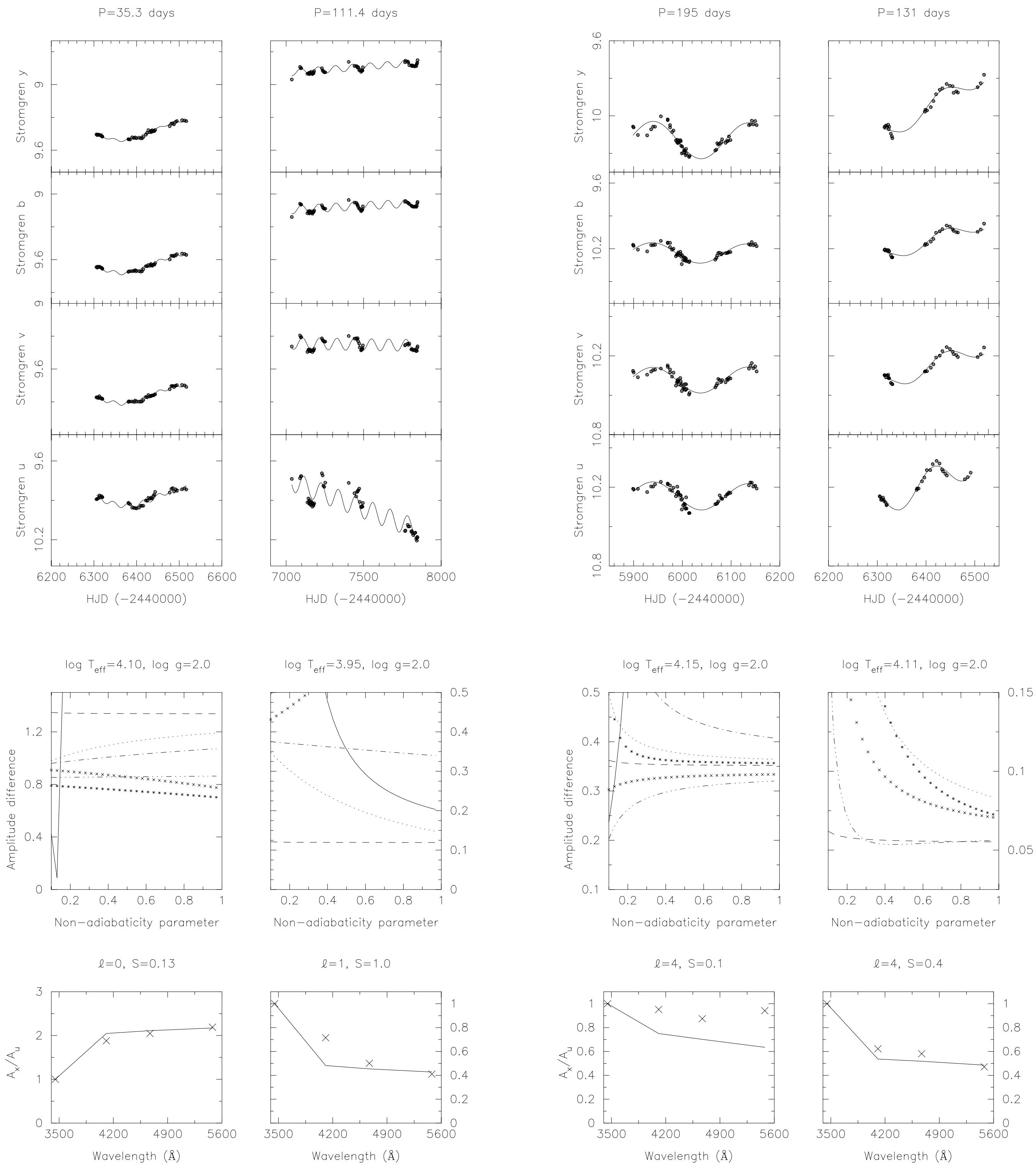

Fig. 7. Top four panels: multicolour data for the four passbands of the Strömgren system of R 127 (two left panels) for the period 1 and 2 (see Table 4) and for S Dor (two right panels) for the period 1 and 2. The dots are the observations while the full lines are fits with the pulsation periods listed in Table 4 and given on top of each panel. Middle panels: The amplitude differences (in magnitudes) as a function of the non-adiabaticity parameter (see text for an explanation) for the most likely degrees of the pulsations. Different symbols denote different degrees: full line: $\ell = 0$, dashed line: $\ell = 1$, dotted line: $\ell = 2$, dashed-dotted line: $\ell = 3$, dashed-dot-dot-dot line: $\ell = 4$, $\star$: $\ell = 5$, $\times$: $\ell = 6$. Lower panels: the observed amplitude ratios A_x/A_u for $x = u, v, b, y$ (crosses) are compared with those of the best theoretical model (connected by the full line) for the four filters of the Strömgren system

Table 7. Amplitudes A_u, A_v, A_b, A_y (in mag) of the multicolour data we considered. The Nr. listed corresponds to the epoch given in Tables 3 and 4. We fitted a sine function as well as a linear correction to the data according to the periods and the trend listed in Table 4. The fraction of the variance explained by the fit with the considered period in each of the filters is indicated as well

Star	Nr	A_y	Frac	A_b	Frac	A_v	Frac	A_u	Frac.
R 71	2	0.0351 ± 0.0081	54%	0.0368 ± 0.0066	63%	0.0317 ± 0.0060	62%	0.0412 ± 0.0064	71%
	3	0.0611 ± 0.0124	66%	0.0580 ± 0.0134	60%	0.0574 ± 0.0125	63%	0.0763 ± 0.0154	67%
	5	0.0553 ± 0.0110	66%	0.0363 ± 0.0140	34%	0.0288 ± 0.0143	24%	0.0302 ± 0.0248	10%
	6	0.0313 ± 0.0987	54%	0.0268 ± 0.0095	43%	0.0219 ± 0.0124	24%	0.0299 ± 0.0150	28%
	7	0.0485 ± 0.0154	52%	0.0469 ± 0.0147	52%	0.0453 ± 0.0149	50%	0.0578 ± 0.0182	45%
	8	0.0309 ± 0.0125	42%	0.0296 ± 0.0124	39%	0.0296 ± 0.0119	40%	0.0348 ± 0.0139	38%
HR Car	2	0.0539 ± 0.0061	88%	0.0523 ± 0.0066	85%	0.0513 ± 0.0062	86%	0.0633 ± 0.0075	82%
	3	0.0104 ± 0.0021	48%	0.0107 ± 0.0022	45%	0.0128 ± 0.0023	53%	0.0168 ± 0.0029	62%
164 G Sco	1	0.0443 ± 0.0049	71%	0.0474 ± 0.0045	73%	0.0625 ± 0.0080	74%	0.1155 ± 0.0168	67%
	2	0.0370 ± 0.0051	50%	0.0407 ± 0.0051	55%	0.0427 ± 0.0051	58%	0.0439 ± 0.0064	53%
S Dor	1	0.0977 ± 0.0072	82%	0.0907 ± 0.0074	79%	0.0987 ± 0.0077	81%	0.1037 ± 0.0079	78%
	2	0.0501 ± 0.0079	96%	0.0618 ± 0.0088	95%	0.0662 ± 0.0088	94%	0.1064 ± 0.0069	97%
R 127	1	0.0092 ± 0.0020	98%	0.0086 ± 0.0018	98%	0.0079 ± 0.0019	98%	0.0042 ± 0.0051	85%
	2	0.0308 ± 0.0031	89%	0.0376 ± 0.0035	82%	0.0537 ± 0.0060	64%	0.0751 ± 0.0137	80%
AG Car	1	0.0271 ± 0.0049	73%	0.0264 ± 0.0050	74%	0.0255 ± 0.0054	71%	0.0339 ± 0.0064	77%
	3	0.0261 ± 0.0056	65%	0.0260 ± 0.0057	63%	0.0263 ± 0.0057	64%	0.0297 ± 0.0069	60%

well. This star thus seems to switch between modes of different degree.

8. R 127 (Fig 7, left)

For interval Nr 1 (P=35 days) we find a strange behaviour of the radial mode ($\ell = 0$) below $S = 0.2$. We do not know if this is a mathematical artefact of the model or if it is real. Although this radial solution is able to explain the observed amplitudes with high accuracy, we have to treat this identification with caution because of the large errors on the amplitudes. The latter are due to the fact that we had to correct for the long-term variations by means of a sine with a long period. For interval Nr 2 (P=111 days) we find that an $\ell = 1$ mode gives the best fit with the observations. Our preliminary suggestion then is that there is a switching of modes, similar to for 164 G Sco.

S Dor (Fig 7, right)

The pulsational behaviour during interval Nr 1 (P=195 days) is difficult to describe with our model (see the lower panel of Fig. 7). The $\ell = 4$ mode gives the smallest amplitude difference. For interval Nr 2 (P=131 days), we find a good description by means of an $\ell = 4$ or $\ell = 1$ mode with intermediate S. It thus seems that we have a different radial order here, but not necessarily another type of mode.

We conclude that, besides the ambiguous result for the first epoch of R 127, the observed relations between amplitude and wavelength give no evidence for a radial pulsation in the considered LBVs. Instead we found evidence for g-modes of low ℓ. S Dor (high Q) and HR Car (low Q) seem to switch from one overtone to another, while R 127 and 164 G Sco pulsate in modes of different degree. We found that the modes which fit the multicolour data of the LBVs have a degree between 1 and 4 in terms of linear pulsation theory.

9. Summary and conclusions

We have studied the periods and the period changes of the microvariations of 6 LBVs observed with the LTPV project in Strömgren photometry. We adopted two period search methods: the Fourier method and the Phase Dispersion Minimization Method (PDM). The Fourier method assumes that the pulsations are sinusoidal, whereas the PDM method makes no apriori assumption about the shape of the lightcurves. Both methods give very similar results. The variations are approximately sinusoidal but do not show a strict periodicity, even within one interval. It seems that the stars pulsate with one period only a few cycles before changing the period. This hampers an accurate study of the variability, because it would require long continuous series of observing times.

We have determined the periods and amplitudes in selected intervals ranging from 50 to 800 days, depending on the number of photometric observations available. The six LBVs show microvariability with half-amplitudes of about 0.01 to 0.1 magnitude in V and periods between 11 and 195 days. Whenever the variations were studied during more than one epoch, the periods and the amplitudes change. The changes in period are up to about a factor 3 or 4. The changes in amplitude are up to a factor 6.

The structure of the LBVs changes slowly due to the moderate variations, on a timescale of years to decades. In three LBVs (R 71, HR Car and R 127) the epochs of the microvariations occur when the star changes M_V by more than 0.3 mag. due to the slow moderate variations. These changes in M_V correspond to changes in the stellar radius of about 40 percent for R 71 and HR Car, and a factor two for R 127. This enables us to compare the pulsation periods when the star changes its radius. The data show a trend of increasing period with increasing optical brightness, i.e. with increasing radius (Fig. 3). If the pulsational parameter Q was constant and if $M_{\rm bol}$ was constant, we would expect log P to vary as $-0.5 M_V$. The observed trend is steeper. This steep trend indicates that Q is not constant when the star changes its radius, but that Q increases as the radius increases. This is qualitatively in agreement with the steepening of the density profile as the star expands, because only less than one percent of the mass of the star takes part in the expansion.

For each LBV the amplitudes of the microvariations increase as the period increases (Fig. 4). The slope of the period-amplitude relation depends on $M_{\rm bol}$, in the sense that the relation is steeper for the stars with lower luminosity than for stars with higher luminosity. The star AG Car may be an exception to this rule, but for this star the slope of its period-amplitude relation is not well defined.

The most common microvariations of LBVs have $Q \simeq 0.07 \pm 0.01$ days. However the stars go through phases of slower pulsations when Q can increase by as much as a factor four. This is not related to a particular phase in the lightcurve. The largest Q values are found for S Dor ($Q = 0.67$ and 0.34 days) and for one epoch of AG Car ($Q = 0.42$ days). The high Q-values of S Dor are probably only temporary, because S Dor does not differ significantly in its characteristics (luminosity, radius, mass, circumstellar nebula) from the other LBVs. Telting (private communication) has suggested that the slow phases might be due to a beat of two frequencies. Unfortunately the photometric data of LBVs are too sparse and the observing runs are too short (one observing season is about five periods of microvariations) to detect multiple periodicities.

The minimum value of the pulsational constant of the LBVs, $Q \simeq 0.07$ days, is about 1.5 to 2 times as large as that for normal supergiants (Burki 1978). This is possibly due to the fact that LBVs may have a higher L_*/M_*-ratio than normal supergiants, because they have lost more mass.

Kiriakidis et al.(1993) argued that the variations of LBVs might be due to strange-mode instabilities. To test this suggestion, we compared the observed periods of the LBVs with those predicted for the strange-mode instability in Sect. 6.1. The predicted periods for stars with parameters close to those of the LBVs are in the range of only a fraction of a day to a few days. The observed periods are much longer and on the order of tens to hundreds of days. So the microvariations of LBVs are probably not due to strange-mode instabilities. Another argument against the strange-mode interpretation of the microvariations is the fact that the strange-modes are expected to occur in a restricted region of the HR diagram (Fig. 5), whereas the microvariations of LBVs and the very similar α Cygni variations of normal supergiants occur in a much wider region of the HRD than predicted.

A more promising explanation of the microvariations is suggested by the comparison of LBVs with the β Cephei stars and the Slowly Pulsating B-stars (SPBs). Waelkens et al. (1998) have shown that the variations of the normal supergiants are a logical extension of those found in SPBs and β Cepheids. This is supported by calculations by Pamyatnykh (1997). Since the SPBs and β Cepheids are known to pulsate in g-modes, the microvariations of the LBVs are probably also due to g-mode pulsation.

We have attempted to identify the pulsation modes of the LBVs by studying the relation between amplitude and wavelength, in a similar way as has been applied to β Cepheids by e.g. Heynderickx et al. (1994). This implies the calculation of the variations of the energy distributions over the stellar surface due to modes of different ℓ. This is done using Kurucz model atmospheres. The results, shown in Figs 6 and 7., suggest that the stars are pulsating in g-modes of low order, typically $1 < \ell < 4$. We did not find evidence for radial ($\ell = 0$) modes.

During the period of observations used in this paper, the LBVs did not show the reversal of the colours-magnitude variation of the microvariations that was observed for visual maximum by van Genderen et al. (1997a), (see Sect. 1.). Therefore we have no information on this possible change of the nature of the pulsations during the maximum of the light curve.

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References

Burki G., 1978, A&A 65, 357

Cox J.P., 1980 in *Theory of stellar pulsations*, Princeton Press, p10

Cox J.P., Guzik A.N., Soukup M.S. 1995, in *Astrophysical Applications of Stellar Pulsation*, eds R.S. Stobie & P.A. Whitelock, ASP Conf. Series 83, 192

Crowther P.A. 1997 in *Luminous Blue Variables: Stars in Transition*, ASP Conf. Series 120, eds A. Nota & H.J.G.L.M. Lamers, 51

de Koter A., 1993, *Studies of the variability of Luminous Blue Variables*, Ph.D. thesis, University of Utrecht

Dziembowski W.A., 1977, Acta Astr. 27, 203

Heynderickx D., Waelkens C., Smeyers P., 1994, A&AS 105, 447

Humphreys R.M., Davidson K., 1994, PASP, 106, 1025

Humphreys R.M., Lamers H.J.G.L.M., Hoekzema N., Cassatella A. 1989, A&A 218, L17

Hutsemékers D., van Drom E., 1991, A&A 248, 141

Kilkenny D., Whittet D.C.B., Davies J.K. et al. 1985, South-African Astron. Obs. Circ., 9, 55

Kiriakidis M., Fricke K.J., Glatzel W. 1993, MNRAS 264, 50

Kurucz R.L., 1979, ApJS 40, 1

Lamers H.J.G.L.M. 1995 in *Astrophysical Applications of Stellar Pulsations*, ASP Conf. Ser. 83, eds R.S. Stobie and P.A. Whitelock, p176

Lamers H.J.G.L.M., Hoekzema N., Trams N.R. et al. 1989 in *Physics of Luminous Blue Variables*, IAU Col. 113, eds K. Davidson, A. Moffat, H.J.G.L.M. Lamers, 135

Leitherer C., Appenzeller I., Klare G. et al. 1985 A&A 153, 168

Leitherer C., Schmutz W., Abbott D.C. et al. 1989, ApJ, 346, 919 ApJ 428, 292

Lovy D., Maeder A., Noëls A., Gabriel M., 1984, A&A 133, 307

Maeder A. 1981 A&A 99, 97

Maeder A. 1995 in *Astrophysical Applications of Stellar Pulsations*, ASP Conf. Ser. 83, eds R.S. Stobie & P.A. Whitelock, p191

Manfroid J., Sterken C., Bruch A., et al., 1991, First Catalogue of Stars Measured in the LTPV Project (1982–1986), ESO Scientific Report no. 8, Garching.

Manfroid J., Sterken C., Conuw B. et al., 1994, Third Catalogue of Stars Measured in the LTPV Project (1990–1992), A&AS 109, 329

Najarro F., Kudritzki R.-P., Hillier D.J. et al. 1997 in *Luminous Blue Variables: Stars in Transition*, ASP Conf. Series 120, eds A. Nota & H.J.G.L.M. Lamers, p105
Nota A., Lamers H.J.G.L.M. (eds) 1997 *Luminous Blue Variables: Stars in Transition*, ASP Conf. Series 120
Pamyatnykh A.A. 1998, in *A Half Century of Stellar Pulsation Interpretations*, PASP Conf. Series 135, eds P.A. Bradley & J.A. Guzik, p 268
Panagia N., Gilmozzi R., Macchetto F. et al. 1991, ApJ 380, L23
Pauldrach A.W.A. & Puls J. 1990, A&A237, 409
Schaerer D., Meynet G., Maeder A., Schaller G., 1993, A&AS, 98, 523
Schaller G., Schaerer D., Meynet G., Maeder A., 1992, A&AS, 96, 269
Schmidt–Kaler Th., 1982, In: Landolt–Börnstein (Group 6, Volume 2, Subvolume 2b), Springer Verlag, Berlin
Soukup M.S., Cox A.N., Guzik J.A., Morgan S.M. 1994, BAAS 26, 907
Spoon H.W.W., De Koter A., Sterken C. et al. 1994, A&AS, 106, 141
Stahl O., Wolf B. 1982 A&A 112, 111
Stahl O., Wolf B., Zickgraf F.J. et al. 1983 A&A 120, 287
Stellingwerf R.F., 1978, ApJ 224, 953
Sterken C., 1977, A&A, 57, 361
Sterken C., 1982, In: *ESO Workshop: The Most Massive Stars*, Eds. D'Odorico, S., Baade, D., Kjär, K., 147
Sterken C. 1983, The Messenger 33, 10
Sterken C., Gosset E., Juttner A. et al. 1991, A&A 247, 383
Sterken C., Manfroid J., Anton K. et al. 1993a, Second Cat. of Stars Measured in the LTPV Project (1986-1990), A&AS 102, 79
Sterken C., Manfroid J., Anton K. et al. 1993b, Second Cat. of Stars Measured in the LTPV Project (1986–1990), ESO Scientific Report no. 12, Garching
Sterken C., Manfroid J., Beele D. et al., 1995, Fourth Cat. of Stars Measured in the LTPV Project (1992-1994), A&AS 113, 31
van Genderen A.M., 1979, A&AS 38, 249
van Genderen A.M., 1982, A&A 112, 61
van Genderen A.M., 1985, A&A 151, 349
van Genderen A.M., Steemers W.J.G., Feldbrugge P.T.M. et al. 1985, A&A, 153, 163
van Genderen A.M., Thé P.S., Augusteijn Th. et al. 1988, A&AS, 74, 453
van Genderen A.M., Thé P.S., Heemskerk M. et al. 1990, A&AS, 82, 189
van Genderen A.M., Robijn F.H.A., van Esch B.P.M., Lamers H.J.G.L.M. 1991, A&A 246, 407
van Genderen A.M., Sterken C., de Groot M. 1997a, A&A 318, 81
van Genderen A.M., de Groot M., Sterken C. 1997b, A&AS 124, 517
Waelkens C. 1991, A&A 246, 468
Waelkens C., Aerts C., Kestens E. et al. 1998 A&A 330, 215
Watson R.D., 1988, Ap&SS 140, 255
Wolf B., Appenzeller I., Stahl O. 1981 A&A 103, 94