Astron. Astrophys. 335, 647–653 (1998)

ASTRONOMY
AND
ASTROPHYSICS

Very low mass stars: non-linearity of the limb-darkening laws*

A. Claret

Instituto de Astrofísica de Andalucía, CSIC, Apartado 3004, E-18080 Granada, Spain

Received 5 February 1998 / Accepted 27 March 1998

Abstract. Linear and non-linear limb-darkening coefficients for the photometric bands u v b y U B V R I J H K are computed for very low effective temperatures stars. The atmosphere models used (PHOENIX-NextGen) do not include the effects of dust formation and dust opacities. These calculations are presented for the first time. The calculations extend the range of effective temperatures (2000 K - 50000 K) covered by our previous papers. These data are important in order to deal with stars in the lower part of Main-Sequence like M or brown dwarfs. The non-linear behavior of the limb-darkening laws, particularly in this effective temperature range, is emphasized and we urge users to take it into account.

Key words: stars: atmospheres – stars: binaries: eclipsing – stars: low mass, brown dwarfs

1. Introduction

The limb-darkening coefficients are an important tool in the study of light curves of eclipsing binaries, in measurements of stellar diameters, in the investigation of line profiles perturbed by rotation and more recently in the study of the effects of gravitational micro-lensing (Alcock et al. 1997)

In the last years several papers have been dedicated to compute and analyze the limb-darkening coefficients and their use in the light curve synthesis (see for example, Wade & Ruciński 1985, Claret & Giménez 1990, Van Hamme 1993, Díaz-Cordobés et al. 1995, Claret et al. 1995). However, the basic instruments used in these papers were stellar atmosphere codes designed for hot and moderate cold models.

This gap may be explained in part because the first code devoted especially to M dwarfs was only developed in 1976 by Mould. The situation has changed drastically since then. The investigations on lithium depletion in brown dwarfs and in Pre Main-Sequence stars, for example, opened an exciting field giving to the low effective temperature stars a leading role hard to imagine some years ago.

In the next sections we present calculations of limb-darkening for this missing range of effective temperatures. As input we have used the results of the PHOENIX code (Allard & Hauschildt 1995; Allard et al. 1997; Hauschildt et al. 1997a; Hauschildt et al. 1997b). In addition to fulfill the gap, we show once more that linear fitting is not adequate to describe the intensity variation over the stellar disk, particularly for low effective temperature stars.

In the second section we describe briefly the model atmosphere and the numerical method used to compute the limb-darkening coefficients. In section 3 we discuss the results and present the conclusions.

2. A brief description of the code PHOENIX and the numerical method

A complete description of the code PHOENIX can be found in Allard & Hauschildt 1995 and Hauschildt et al. 1994. The code can be used to solve radiative transfer equation in more complex situations, as for example, expanding envelopes in supernovae. Here we only recall some basic input physics. The adopted geometry is the plane-parallel. It was assumed total flux conservation, hydrostatic equilibrium and LTE. The models include several hundred molecules and about 2 million atomic lines. Dust formation and dust opacities are not included. For $T_{eff} \lesssim$ 3000 K such effects are important. In spite of these limitations I think that the general discussion presented here remains valid.

Under the physical point of view some phenomena may change the distribution of intensities over the stellar disk. As shown by Claret & Giménez 1992, the mutual irradiation of the components in an eclipsing binary results in that the irradiated stellar atmosphere presents a more uniform distribution of brightness (see also Baron et al. 1994). Here we shall only deal with standard models, that is, without any external radiation field.

The covered effective temperature is 2000 K $\leq \log T_{eff} \leq$ 4000 K though some additional models were considered in order to compare with calculations performed using the results from the ATLAS code (Kurucz 1993). The surface gravity range is characterized by $3.5 \leq \log g \leq 5.0$. Only models with solar metalicity were considered. The adopted mixing-length param-

---

Send offprint requests to: A. Claret, (claret@iaa.es)

[*] Tables 1-3 are available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/Abstract.html

eter is 1.0. The intensities are given for 5399 wavelengths (from 3000 Å up to 29995 Å) for 16 values of μ.

We have used the least-squares method to fit the intensities distribution. The methods often used do determine limb-darkening coefficients are a matter of discussion (see Díaz-Cordobés et al. 1995). The reasons why we selected the least-squares one are given in the quoted paper and they are not to be repeated here. The more simple fitting to the intensities is the linear:

$$I(\mu) = I(1)(1 - u(1 - \mu)) \tag{1}$$

while the quadratic one can be written as

$$I(\mu) = I(1)(1 - a(1 - \mu) - b(1 - \mu)^2) \tag{2}$$

and the root-square approximation is given by

$$I(\mu) = I(1)(1 - c(1 - \mu) - d(1 - \sqrt{\mu})) \tag{3}$$

where u is the linear limb-darkening coefficient, a and b the quadratic ones, c and d the root-square ones, μ = cosγ (γ is the angle between the line of sight and the emergent flux), and I(1) is the monochromatic specific intensity at the center of the disk.

The quadratic law was investigated before by several authors (Manduca et al. 1977, Wade & Ruciński 1985, Claret & Giménez 1990). The root-square fitting was introduced by Díaz-Cordobés & Giménez 1992 and as an alternative, Kinglesmith & Sobieski 1970 proposed a logarithmic dependence.

In order to be useful to users we present our results in the most common photometric systems: Strömgren u v b y, Johnson UBV and the R I J H K bands. Before performing the fitting we integrate the intensities for a given band B following the equation

$$I_B(\mu) = \int_{\lambda_1}^{\lambda_2} I(\lambda, \mu) S(\lambda) d\,\lambda \tag{4}$$

where I$_B(\mu)$ is the specific intensity in the band B, I(λ,μ) is the monochromatic specific intensity. S(λ) is the response function that depends on the terrestrial atmospheric transmission, filter transmission curves, detector sensitivity and reflection from the aluminum coated mirror. Atmospheric transmission and reflection from aluminum mirror were taken from Allen 1976. For R and I, the transmission curves were taken from Bessel 1990 while the sensitivity of the CCD Tektronics was taken from Peletier 1994. For J, H, and K, the transmission curves were taken from Alonso et al. 1994 (the sensitivity of the In-Sb detector has been included). The resulting effective wavelengths are close to the values tabulated by the quoted papers with a slight dependence on the spectral type.

3. Discussion of the results and conclusions

To compare our present calculations with previous ones we have also computed the limb-darkening coefficients for hotter models (up to 10000 K). In this way the zone of overlapping between PHOENIX and ATLAS models is covered. Recall that the minimum effect ive temperature of ATLAS models is 3500 K. As reported in our papers on the subject (Claret & Giménez 1990, Díaz-Cordobés & Giménez 1992, Díaz-Cordobés et al. 1995, Claret et al. 1995) the linear fitting is not adequate to describe the distribution of intensities. In Fig. 1 we can see that the behavior of σ as a function of the effective temperature does not recommend it. In fact, in all the bands considered the linear fitting is not accurate and only for high effective temperature or large effective wavelength it is acceptable. The gap in Fig. 1 and in the subsequent figures for log T$_{eff}$ around 3.8 appears because the corresponding models were not available.

On the contrary, the quadratic (Fig. 2) and root-square (Fig. 3) fittings present a more acceptable behavior. For longer wavelengths (R I J H K) the superiority of root-square law over the quadratic ones is clear. For the other bands one fitting is superi or to other depending on the wavelength and/or effective temperature range. Generally speaking, we can say that the root-square law describes better the intensity distribution than the quadratic, and of course, than the linear.

The relative flux F$_B$/F$_{B,limb}$ in a band, i. e., the ratio of the flux calculated using quadrature and that using the limb-darkening laws, can be also used to test the goodness of the fitting. The latter can be written as follows:

$$F_{B,limb} = 2I_B(1) \int_0^1 I_B(\mu)/I_B(1)\mu d\,\mu \tag{5}$$

where the index B indicate a given band.

Examining Figs. 4, 5 and 6 we can conclude again that the linear fitting is not adequate. Moreover, it was detected a systematic dependence of F$_B$/F$_{B,limb}$ on the effective temperature and on the band in question.

In spite of the problems of the linear fitting described above, it can be used to compare the results derived from PHOENIX code with those calculated using the intensities generated by ATLAS. Fig. 7 illustrates this comparison. For higher effective temperatures - log T$_{eff}$ $\geq$ 3.8 - the PHOENIX limb-darkening coefficients compare well with the ATLAS results but with one peculiarity: the PHOENIX coefficients are systematically larger that the ATLAS ones for all photometric bands. Such a behavior may be caused by abundances and mixing-length differences used in both codes.

On the other hand, for cooler models this tendency is maintained with the exceptions of Strömgren u and v and Johnson U. For these filters the tendency is inverted and the linear coefficients computed following ATLAS code is larger reaching a difference of about 25 %. The worst situation is for the U band. In this range of effective temperatures the results using ATLAS are less accurate due to the missing of molecules and of molecular opacities.

The linear fitting is not adequate for monochromatic limb-darkening coefficients. Also in this case the root-square fitting produces the best results. However, as commented before, the linear coefficients are useful to compare different techniques and/or atmosphere models. Fig. 8 shows how monochromatic coefficients depends on wavelength and effective temperature. The bands u v b y and R I J H K are indicated for orientation. In the case of the hotter model the values of the monochromatic

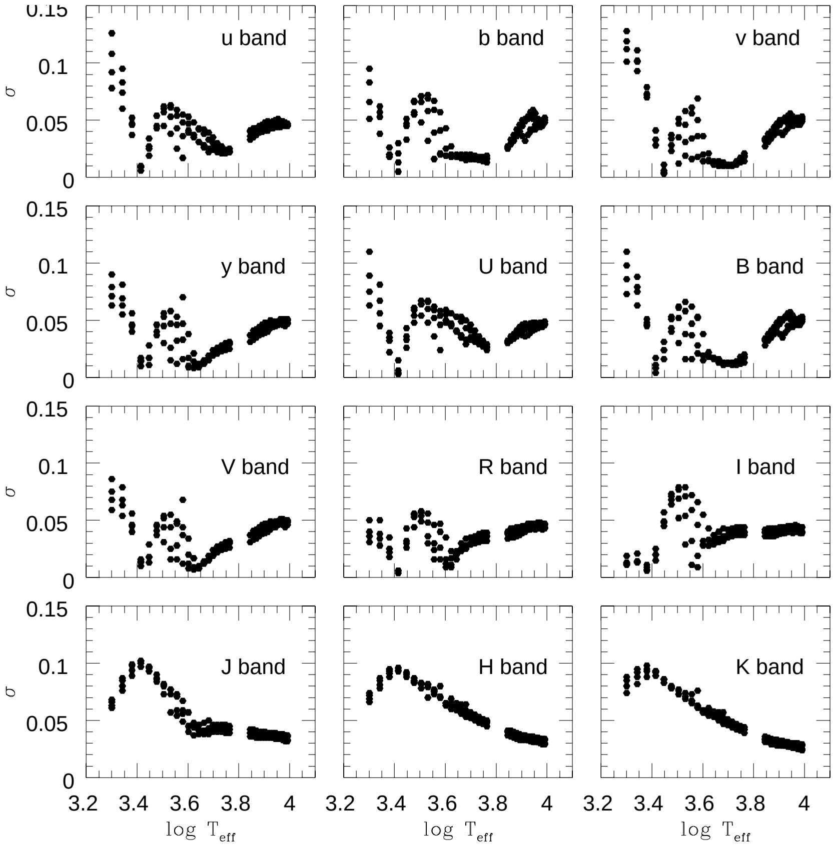

Fig. 1. σ of linear fitting as a function of effective temperatures for the 12 photometric bands.

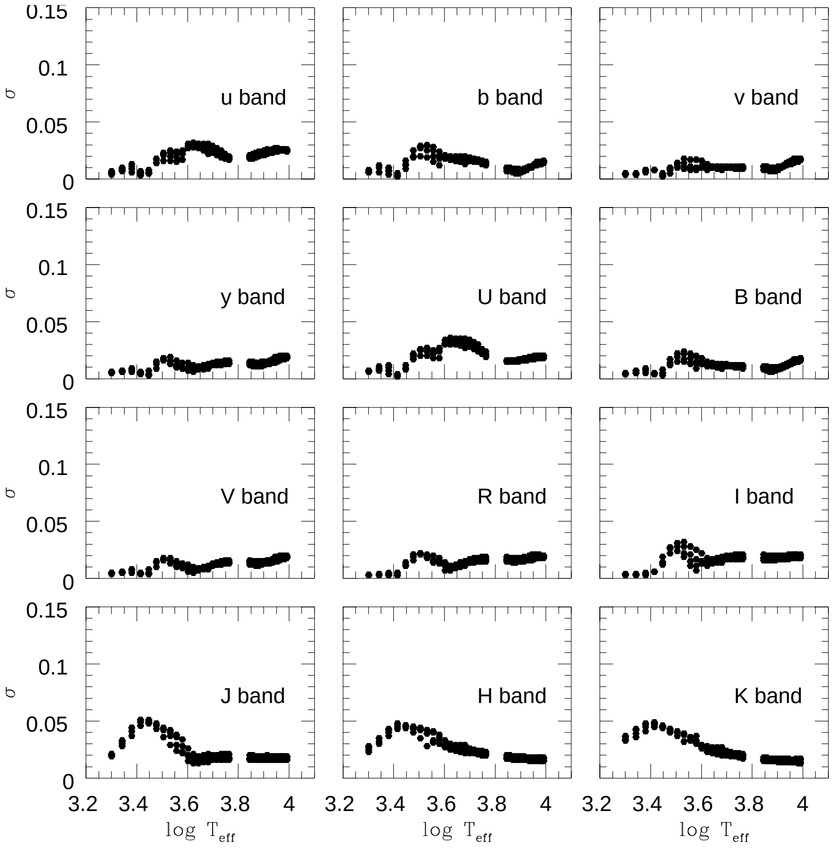

Fig. 2. σ of quadratic fitting as a function of effective temperatures for the 12 photometric bands.

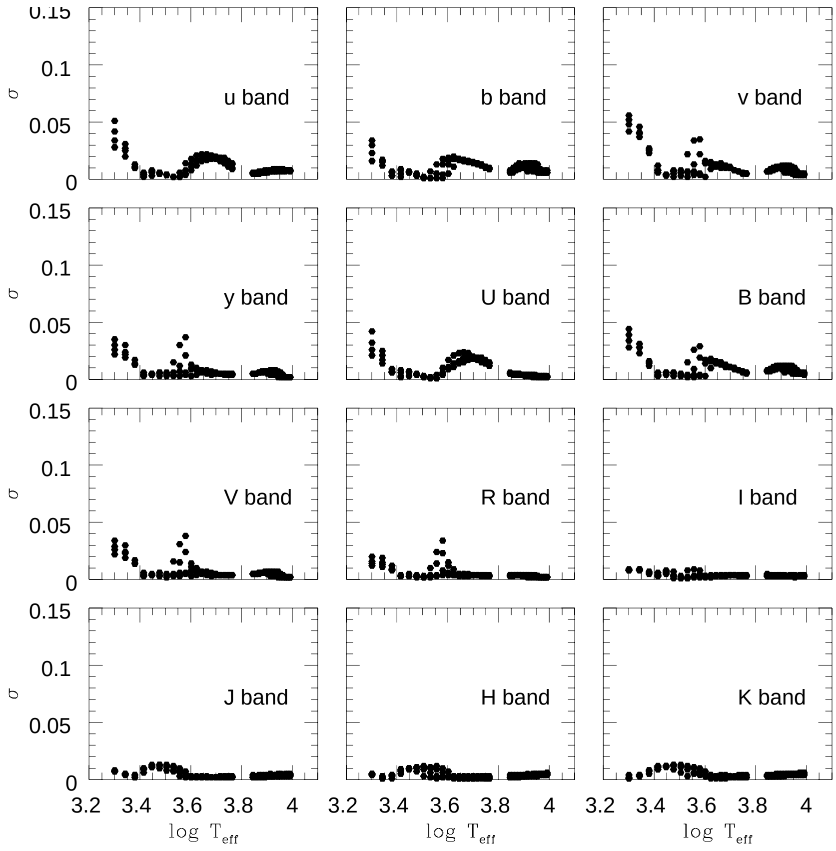

Fig. 3. σ of root-square fitting as a function of effective temperatures for the 12 photometric bands.

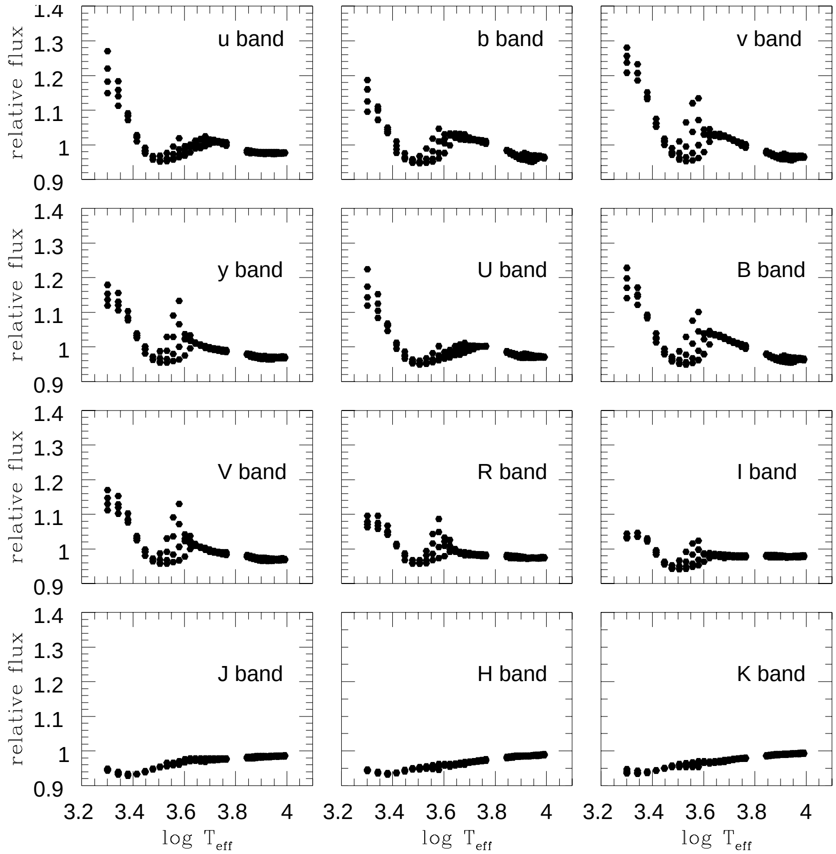

Fig. 4. The relative flux $F_B/F_{B,limb}$ versus $\log T_{eff}$ for the 12 photometric bands. Linear case.

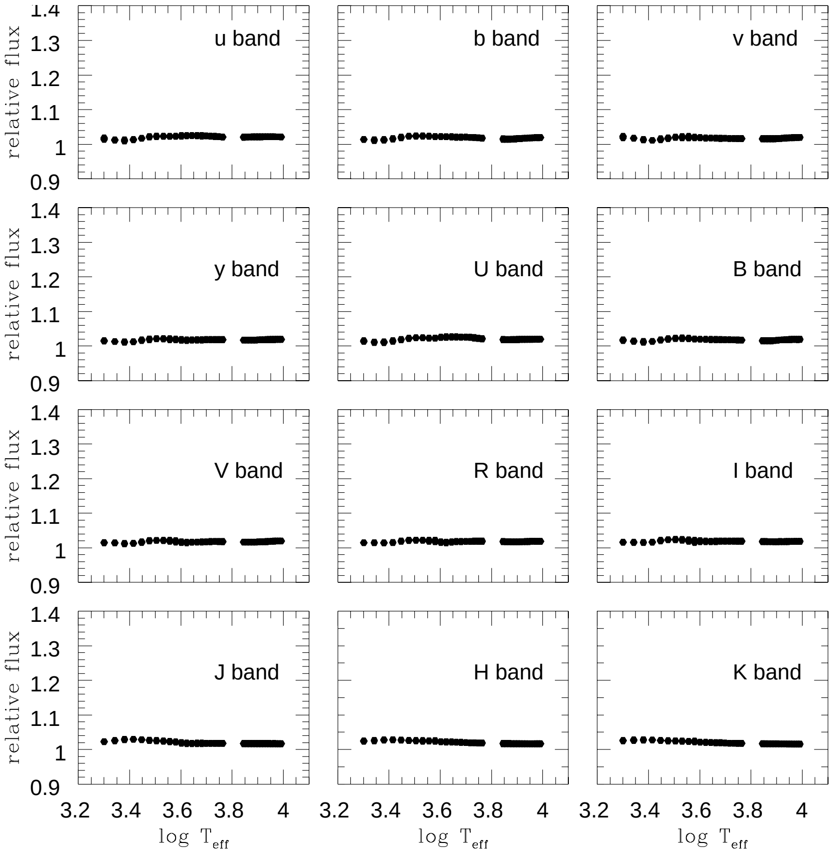

Fig. 5. The relative flux $F_B/F_{B,limb}$ versus $\log T_{eff}$ for the 12 photometric bands. Quadratic case.

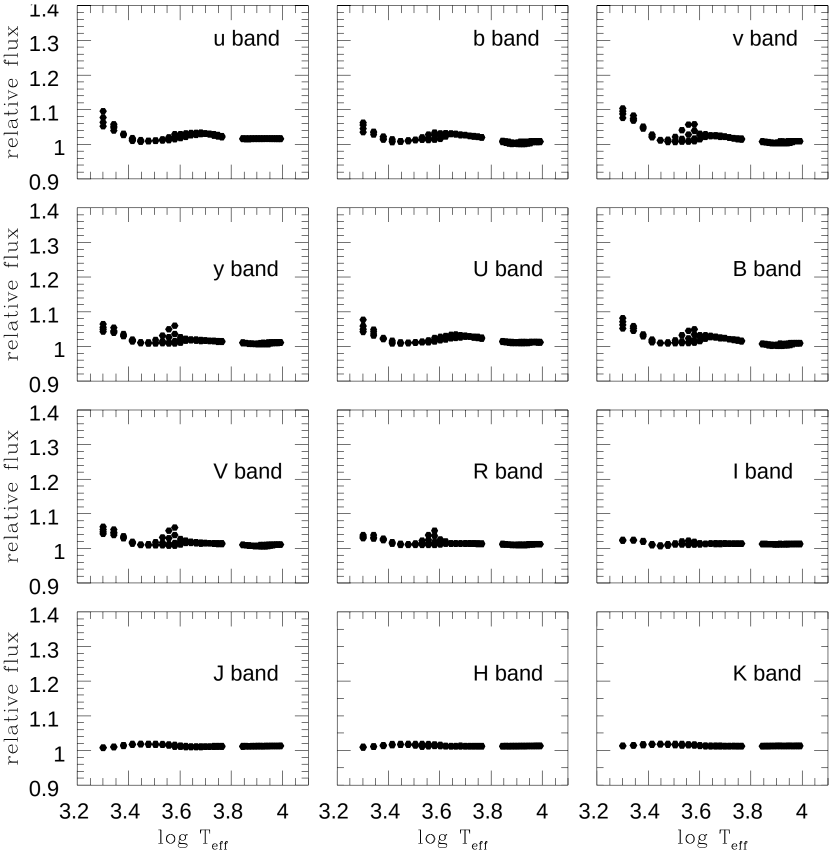

Fig. 6. The relative flux $F_B/F_{B,limb}$ versus $\log T_{eff}$ for the 12 photometric bands. Root-square case.

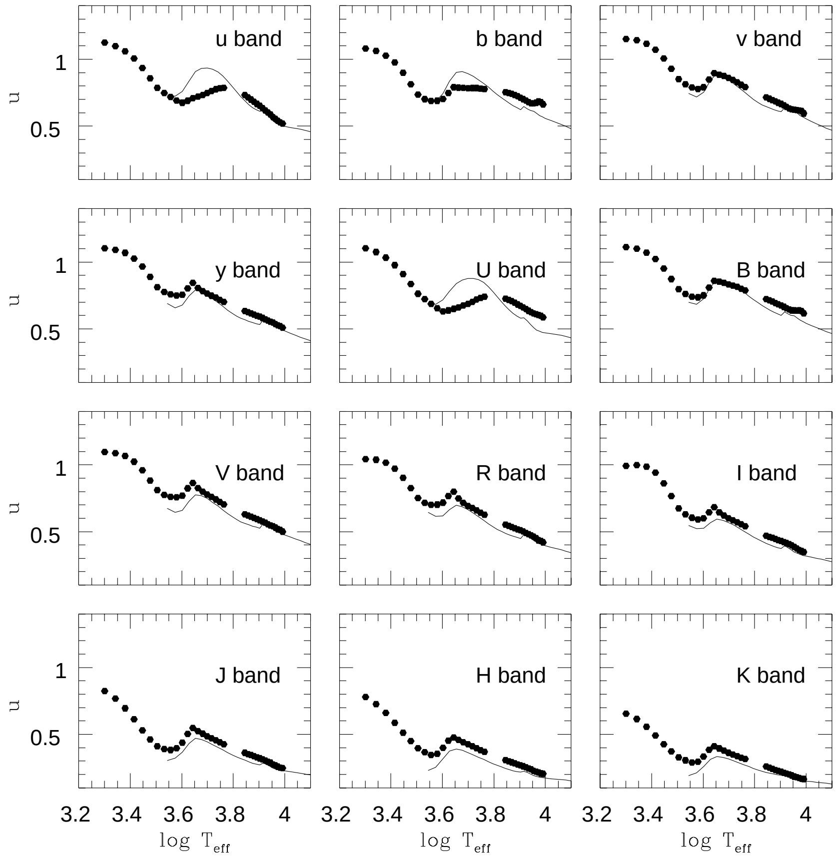

Fig. 7. The linear limb-darkening u computed following PHOENIX intensities (full hexagons) and those computed following ATLAS results (full lines) versus effective temperatures for log g = 5.0.

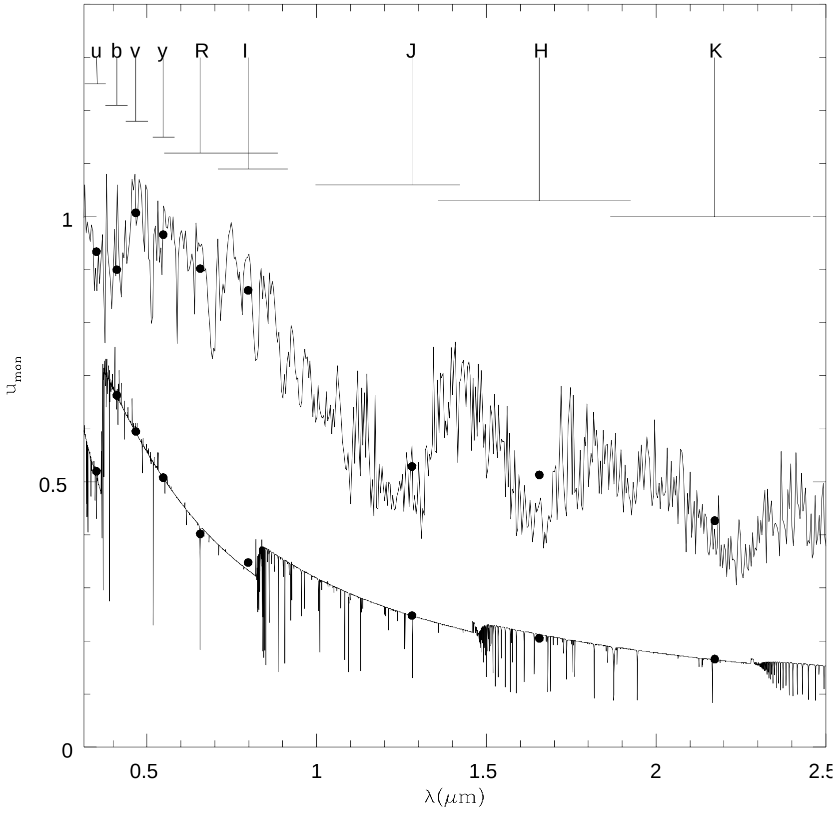

Fig. 8. The monochromatic linear limb-darkening for T_{eff} = 2800 and 9800 K (log g = 5.0). The Strömgren u v b y and R I J H K filters are shown for orientation. Full circles indicate bandpass values. For sake of clarity only 680 points were used to represent the cooler model.

coefficients for which the transmission is maximum agrees reasonably well with those computed using filters. For the model with T_{eff}= 2800 K, due to the presence of numerous lines, the situation is more complicated and it is not advisable to use monochromatic values centered in the maximum filter transmission to represent the bandpass values.

The tables 1-3 are organized as following: Table 1 contains the linear coefficients u and Table 2 the two coefficients of quadratic fit a and b. The root-square coefficients c and d are tabulated in Table 3. All coefficients are given as a function of log g, T_{eff} and of the filters.

As a final remark we would like to stress the importance of the non-linearity of the limb-darkening laws shown in this paper. For higher effective temperatures we have already pointed out this characteristic. For very low effective temperatures we have shown that the non-linearity is even more drastic. Astrophysicists in general, and users of synthetic light curves modeling codes in particular, should take this fact into account . A substantial effort was made to extend the limb-darkening calculations to 12 photometric bands covering a wide range of effective temperatures (2000 K up to 50000 K) and surface gravities. Unfortunately, many people still use the linear approximation. We urge such users to adopt the non-linear coefficients given the results presented here.

Acknowledgements. I acknowledge Dr. Peter H. Hauschildt for providing the data of the model atmospheres. Dr. M. A. L. Valverde is particularly acknowledged for the comments. The Spanish DGYCIT (PB94-0007 and PB96-0840) are gratefully acknowledged for support during the development of this work.

References

Alcock, C. et al. 1997, ApJ, 491, 436
Allard, F., Hauschildt, P. H. 1995, ApJ, 445, 433
Allard, F., Hauschildt, P. H., Alexander, D. R., Starrfield, S. 1997, ARA&A, 35, 137
Allen, C.W. 1976, ”Astrophysical Quantities”, 3rd. Edition, Athlone Press, London
Alonso, A, Arribas, S., Martínez-Roger, C. 1994, A&AS, 107, 365
Baron, E., Hauschildt, P. H., Branch, D. 1994, ApJ, 4326, 334
Bessel, M. S. 1990, PASP, 102, 1181
Claret, A., Giménez, A. 1990, A&A, 230, 412
Claret, A., Giménez, A. 1992, A&A, 256, 572
Claret, A., Díaz-Cordovés, J., Giménez, A. 1995, A&AS, 114, 247
Díaz-Cordovés, J., Giménez, A. 1992, A&A, 259, 227
Díaz-Cordovés, J., Claret, A., Giménez, A. 1995, A&AS, 110, 329
Hauschildt, P. H., Baron, E., Allard, F. 1997a, ApJ, 483, 390
Hauschildt, P. H., Allard, F., Alexander, D. R., Baron, E. 1997b, ApJ, 488, 428
Klinglesmith, D.A., Sobieski, S. 1970, AJ, 75, 175
Kurucz, R.L. 1993 in ”Light Curve Modeling of Eclipsing Binary Stars”, edited by E.F. Milone, Springer-Verlag, New York, p. 93
Manduca, A., Bell, R.A., Gustafsson, B. 1977, A&A, 61, 809
Mould, J. R. 1976, ApJ 210, 402
Peletier, R. 1994, private communication
Van Hamme, W. 1993, AJ, 106, 2096
Wade, R.A., Ruciński, S.M. 1985, A&AS, 60, 471