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Excitation of the solar p-modes by turbulent stress

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Abstract. We have revisited the problem of solar p-mode generation by turbulent stress in the outer part of the Sun, and the product $\dot{\mathrm{E}} \equiv E\Gamma$, the rate at which energy is supplied to a single solar p-mode. We assume that Γ measures energy decay rate of the mode. Away from the peak, $\dot{\mathrm{E}} \propto \nu^7$ for $\nu < 3mHz$ and $\dot{\mathrm{E}} \propto \nu^{-5}$ for $\nu > 3mHz$. The resulting oscillation power spectra can be made to agree roughly with the observations by adjusting theoretical parameters. In this paper, we correct the earlier computations by incorporating an improved description of the spatial and temporal spectrum of turbulent convection, and show how the mode energy generated in the Sun on the basis of a mixing-length model of the solar convection zone is strongly dependent on the solar model. In the present paper, we argue that the solar p-modes are primarily excited as a consequence of the emission of acoustic radiation by turbulent convective flows.

Key words: convection – Sun: interior – Sun: oscillations

1. Introduction

The solar p-mode oscillations have been investigated extensively since the oscillation modes were discovered. It is generally regarded that these modes are excited by turbulent convection in the outer part of the Sun. Goldreich & Keely (1977) and Goldreich & Kumar (1988) were the first to consider stochastic excitation as a viable driving mechanism of the solar p-mode oscillations. Although recent theoretical considerations by Osaki (1990), Goldreich & Kumar (1990), Balmforth (1992), and Goldreich et al. (1994) roughly reproduced the observed properties, many uncertainties till remain in the theoretical approaches. Goldreich et al. (1994) claim that the p-modes appear to be stochastically excited, primarily by entropy fluctuations, with fluctuations of the Reynolds stress playing a secondary role. While Osaki (1990) and Balmforth (1992) argue that the main contribution comes from the turbulent stress, and they successfully account for the excitation rates of low-frequency modes, but severely overestimate those of high-frequency modes. As a result, our understanding of these oscillations is more problematic. The uncertainties may result mainly from our poor information on the physical properties of turbulence in the outer part of the solar convection zone. Brown (1991) infers that the most significant part of the excitation occurs where the convective velocities significantly exceed their mean. Thus, the excitation should be temporally and spatially quite isolated. Furthermore, Brown et al. (1992) have looked for such events by studying frequencies above the acoustic cutoff.

In the present paper, we have recognized that these previous evaluations are not very accurate because of limitations of the simplified description of turbulence. Thus, our goal is to estimate the rate at which the fluctuations of turbulent stress supply energy to the solar p-modes. To do so, we use a more realistic model for the turbulence, an incorporation of a physically meaningful description of the spatial and temporal spectrum of the turbulent convection (Musielak et al. 1994), and new computational methods. In fact, it closely parallels the discussions of Stein (1967), Goldreich & Keeley (1977) and Balmforth (1992).

The plan of this paper is as follows, we begin our presentation with the basic physical formulation of the rate at which energy is supplied to p-modes is given in Sect. 2. In Sect. 3, we derive an expression for the excitation rate due to the fluctuations of the Reynolds stress. Then we present the results of our calculation and discuss them in Sect. 4. Finally, a few brief concluding remarks can be found in the Sect. 5.

2. Basic physical formulation

The equation describing the rate of variation in pulsating kinetic energy can be derived from mass and momentum equations. The momentum equation is augmented by the fluctuations of Reynolds stress. They are written below in the Cowling approximation as

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,, \quad (1)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p' - \rho' g = -\nabla \cdot (\rho \mathbf{u}\mathbf{u})\ , \quad (2)$$

where $\mathbf{g}$ is the gravitational acceleration, ρ' and p' are the Eulerian density and pressure. The pulsating velocity is written as $\mathbf{v} \equiv \frac{\partial \xi}{\partial t}$, where ξ is the displacement vector and $\mathbf{u}$ is the convective velocity. By making use of the scalar product of the

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complex conjugate transpose for the velocity vector $\mathbf{v}^*$, Eq. (2) and mass equation, then we find (Yan Li 1993)

$$\begin{aligned}&\rho\mathbf{v}^*\cdot\frac{\partial v}{\partial t}=-\nabla\cdot p'\mathbf{v}^*-\delta p\frac{\partial}{\partial t}\left(\frac{\delta\rho}{\rho}\right)^*-\mathbf{v}^*\cdot\nabla\cdot(\rho\mathbf{uu})\\&+i\sigma^*\left[(\xi\cdot\nabla p)\left(\frac{\delta\rho}{\rho}\right)^*+(\xi^*\cdot\nabla p)\frac{\delta\rho}{\rho}\right]\\&-\frac{i\sigma^*}{\rho}(\xi\cdot\nabla\rho)(\xi^*\nabla p),\end{aligned}\tag{3}$$

where the Eulerian perturbation and Lagrangian perturbation of the variables relative to their basic state values are denoted by the prime and the symbol δ, respectively.

We combine the real part and the imaginary part of Eq. (3) , and obtain an equation for further investigation

$$\begin{aligned}&2\sigma_R\sigma_I[(\xi\cdot\nabla p)(\tfrac{\delta\rho}{\rho})^*+(\xi^*\cdot\nabla p)\tfrac{\delta\rho}{\rho}-\tfrac{1}{\rho}(\xi\cdot\nabla\rho)(\xi^*\cdot\nabla p)]\\&=2\sigma_R\sigma_I Re[\nabla\cdot p^{'}\xi^*+\delta p(\tfrac{\delta\rho}{\rho})^*+\xi^*\cdot\nabla\cdot(\rho\mathbf{uu})]\\&+(\sigma_I^2-\sigma_R^2)Im[\nabla\cdot p^{'}\xi^*+\delta p(\tfrac{\delta\rho}{\rho})^*+\xi^*\cdot\nabla\cdot(\rho\mathbf{uu})],\end{aligned}\tag{4}$$

where the real and imaginary parts of the indicated quantity are denoted by symbols Re and Im, respectively. The σ_R and σ_I are the real and imaginary parts of the eigenfrequency σ, respectively.

The complex conjugate of Eq. (3) now reads

$$\begin{aligned}&\rho\mathbf{v}\cdot\frac{\partial\mathbf{v}^*}{\partial t}=-\nabla\cdot p'^*\mathbf{v}-\delta p^*\frac{\partial}{\partial t}(\tfrac{\delta\rho}{\rho})-\mathbf{v}\cdot\nabla\cdot(\rho\mathbf{uu})^*\\&+i\sigma[(\xi\cdot\nabla p)(\tfrac{\delta\rho}{\rho})^*+(\xi^*\cdot\nabla p)\tfrac{\delta\rho}{\rho}-\tfrac{1}{\rho}(\xi\cdot\nabla\rho)(\xi^*\cdot\nabla p)].\end{aligned}\tag{5}$$

Neglecting the terms involving $\mathbf{v}$ on the right-hand side of Eqs. (3) and (5), which do not contribute to the pulsation excitation because volume integrations are zero over the convection zone, and higher order coupling terms which represent the convective and pulsating motions for solar 5- minute oscillations, then we obtain the equation that describes the rate of variation in pulsating kinetic energy by making use of Eq. (4)

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho\mathrm{v}^2\right)=\frac{\sigma\sigma^*}{2\sigma_R}Im\left[\delta p\left(\frac{\delta\rho}{\rho}\right)^*-\xi^*\cdot\nabla\cdot(\rho\mathbf{uu})\right],\tag{6}$$

The right-hand side of Eq. (6) accounts for the source terms for the excitation of the solar p-modes by the entropy fluctuations and the pertuations of the turbulent Reynolds stress.

In this paper, we assume that only the Reynolds stresses provide energy to the solar p-modes and neglect the first term of entropy fluctuation in Eq. (6). This corresponds to the assumptions of Moore & Spiegel (1964), Stein (1967), Goldreich & Keeley (1977) and Balmforth (1992). Then, the equation that governs the solar p-mode oscillations is written, by integrating over space, as

$$\begin{aligned}&\frac{\partial}{\partial t}\int\left(\tfrac{1}{2}\rho\mathrm{v}^2\right)d^3r=-\frac{\sigma^2}{2\sigma_R}Im\int\xi^*\cdot\nabla\cdot(\rho\mathbf{uu})\,d^3r\\&=-\frac{\sigma^2}{2\sigma_R}Im\int\rho\mathbf{uu}:\nabla\xi^*d^3r.\end{aligned}\tag{7}$$

3. The excitation of normal modes

3.1. Mean modal amplitudes

The displacement, $\xi(\mathbf{r},t)$, is expanded in terms of the normal modes, $\xi_\alpha(\mathbf{r})$, as

$$\xi(\mathbf{r},t)=Re\left[\sum_\alpha A_\alpha(t)\,\xi_\alpha(\mathbf{r})\,e^{(-i\sigma_\alpha t)}\right],\tag{8}$$

where $A_\alpha(t)$ is the instantaneous amplitude, determined by the balance between damping and driving.

Next, expressing the pulsating kinetic energy in terms of the normal modes and integrating it over volume, we have(e.g.,Kumar& Goldreich 1987)

$$\int\frac{1}{2}\rho\mathrm{v}^2d^3r=\sum_\alpha E_\alpha\,,\tag{9}$$

The mode energy E_α is related to the mode amplitude A_α by $E_\alpha=|A_\alpha|^2$. To evaluate the time evolution of the amplitudes, we substitute Eqs. (8) and (9) into Eq. (7), and consider that $\frac{\partial}{\partial t}=i\sigma$. Noticing that the imaginary part of the eigenfrequency, σ_I, is much smaller than the real part, σ_R, we have (e.g. Balmforth 1992, Goldreich and Keeley 1977)

$$\frac{\partial A_\alpha}{\partial t}=\frac{i\sigma_\alpha}{2}\int d^3r\rho\mathbf{uu}:\nabla\xi_\alpha e^{i\alpha_\alpha t}\,,\tag{10}$$

We restrict ourselves in this paper to radial displacements, because the stochastic excitation of modes is confined to the upper layers of the convection zone, where the correlation times of the energy-bearing eddies match the periods of the modes. In this region, the displacement vectors of the acoustic modes are nearly radial and vary most rapidly in the radial direction. Then we have

$$\nabla\xi_\alpha^*\approx\frac{\partial\xi_\alpha^*}{\partial r}Y_{\ell m}\mathbf{e}_r\mathbf{e}_r\,,\tag{11}$$

where $\mathbf{e}_r$ is the unit radial vector.

It is important to realize that the significant effects of the turbulent dynamical stresses is an averaged Reynolds stress $<\rho\mathbf{uu}>$ contributing to the hydrostatic support and linear perturbations affecting pulsation dynamics. The expression for A_α cannot be directly evaluted, because the Reynolds tensor is a fluctuating vector field. The most that can be done is to determine the statistical properties of A_α. The expectation value of $|A_\alpha|^2$, denoted by $<|A_\alpha|^2>$ is the quantity of greatest interest to us. Substituting Eq. (11) into Eq. (10), and integrating over time, we obtain mean modal amplitude equation as following as

$$\begin{aligned}&\left\langle|A_\alpha|^2\right\rangle=\frac{\sigma_\alpha^2}{4}\int_{-\infty}^t dt'\int_{-\infty}^t dt''\int_{r_b}^{R_\odot}d^3r'\int_{r_b}^{R_\odot}d^3r''\\&\rho(r')\frac{\partial\xi_\alpha^*}{\partial r}(r')\,\rho(r'')\,R_{rr}(r',t')\,R_{rr}(r'',t'')\\&\frac{\partial\xi_\alpha^*}{\partial r}(r'')\,e^{i\sigma_\alpha(t'-t'')}.\end{aligned}\tag{12}$$

where r_b is the radius interior to the base of the envelope, and the mean quantity $R_{rr}(r,t)\equiv<\mathbf{uu}:\mathbf{e}_r\mathbf{e}_r>=<u_ru_r>$ is the radial component of correlation $R(r,t)$ between the instantaneous velocity at two different turbulent source points. Here r and t are the space and time intervals the two points, respectively (e.g. Musielak et al 1994). $R_{rr}(r',t')R_{rr}(r'',t'')$ involves a fourth-order convective velocity correlations which are reduced here to second-order correlations. We discuss this problem in detail below.

3.2. The correlation function and turbulence

Consider the turbulent convective velocity $\mathbf{u}$, at two different points in space and time at $\mathbf{x}$ with time τ and at $\mathbf{x}+\mathbf{r}$ with $\tau+t$. The correlation tensor R_{ij} is then defind as

$$R_{ij}(\mathbf{r},t) \equiv \langle u_i(\mathbf{x},\tau)\, u_j(\mathbf{x}+\mathbf{r},t+\tau)\rangle \tag{13}$$

where the average is taken over all points $\mathbf{x}$ of a large volume which can be considered infinite and over all times τ which are long compared to all other time scales and thus can also be considered infinite. We can use the Fourier transform R_{ij} and introduce the spectral tensor

$$\phi_{ij}(\mathbf{k},\sigma) \equiv \frac{1}{(2\pi)^4}\int d^3r \int d\tau R_{ij}(\mathbf{r},t)\, e^{-i(\mathbf{k}\cdot\mathbf{r}-\sigma t)} \tag{14}$$

and its inverse transform

$$R_{ij}(\mathbf{r},t) = \int d^3k \int d\sigma \phi_{ij}(\mathbf{k},\sigma)\, e^{-i(\mathbf{k}\cdot\mathbf{r}-\sigma t)} \tag{15}$$

We now assume that the turbulence is isotropic and time-symmetric, which means that $R_{ij}(\mathbf{r},t) = R_{ij}(r,t)$ and $\phi_{ij}(\mathbf{k},\sigma) = \phi_{ij}(k,\sigma)$, where $r = |\mathbf{r}|$ and $k = |\mathbf{k}|$, and that $R_{ij}(r,t) = R_{ij}(r,-t)$ and $\phi_{ij}(k,\sigma) = \phi_{ij}(k,-\sigma)$.

In order to introduce the three-dimensional turbulent energy spectrum $E(k,\sigma)$, we assume that the turbulence is isotropic, homogeneous and incompressible, and ϕ_{ij} can be expressed in terms of the turbulent energy spectrum, $E(k,\sigma)$, by (e.g., Batchelor 1960)

$$\phi_{ij}(\mathbf{k},\sigma) = \frac{E(k,\sigma)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \tag{16}$$

We substitute Eq. (16) into Eq. (15), take $d^3k = k^2 \sin\theta dk d\theta d\phi$, and perform analytically the angle integration over θ and ϕ.

The result is

$$R_{ij}(r,t) = \int_0^\infty d\sigma \cos\sigma t \int_0^\infty dk E(k,\sigma) \left[\delta_{ij}\left(\frac{\sin kr}{kr} + \frac{\cos kr}{k^2r^2} - \frac{\sin kr}{k^3r^3}\right) - \frac{r_i r_j}{r^2}\left(\frac{\sin kr}{kr} + 3\frac{\cos kr}{k^2r^2} - 3\frac{\sin kr}{k^3r^3}\right)\right] \tag{17}$$

A simplification should be helpful to further investigation. As mentioned above, we note and that the stochastic excitation of p-modes is confined to the upper layers of the convection zone, and we may assume that $\mathbf{r}$ is limited to the z-direction ($r_x = r_y = 0$). As a result, we bave

$$R_{xx}(r,t) = R_{yy}(r,t) = \int_0^\infty d\sigma \cos\sigma t \int_0^\infty dk E(k,\sigma)\left(\frac{\sin kr}{kr} + \frac{\cos kr}{k^2r^2} - \frac{\sin kr}{k^3r^3}\right) \tag{18}$$

and

$$R_{zz} = 2\int_0^\infty d\sigma \cos\sigma t \int_0^\infty dk E(k,\sigma)\left(\frac{\sin kr}{k^3r^3} - \frac{\cos kr}{k^2r^2}\right) \tag{19}$$

The problem of prescribing the turbulent energy spectrum is difficult because at present there is no generally accepted theory of turbulence describing the motions within either the solar convection zone or a highly stratified and turbulently convecting fluid. The turbulent energy spectrum $E(k,\sigma)$ can be formally factored into a spatial and temporal part (Stein 1967)

$$E(k,\sigma) = E(k)\,\Delta(\sigma/ku_k)\ , \tag{20}$$

where the spectra are normalized by the requirement that

$$\int_0^\infty E(k)\, dk = \frac{3}{2}u_0^2\,, \tag{21}$$

and the mean convective velocity of the eddy with wavenumber k follows from the relation

$$u_k = \left[\int_k^{2k} E(k')\, dk'\right]^{\frac{1}{2}}\ . \tag{22}$$

Musielak et al.(1994) considered three distinct possible spectra and showed that the resulting wave energy fluxes were not very sensitive to the shape of the spectrum for $k < k_0$. We employ the extended Kolmogorov form for the spatial component of the turbulent energy spectrum, which could be used to account for the small wavenumber contributions.

$$E(k) = \begin{cases} 0 & 0 < k < 0.2k_0 \\ \frac{u_0^2}{k_0}\left(\frac{k}{k_0}\right) & 0.2k_0 \le k < k_0 \\ \frac{u_0^2}{k_0}\left(\frac{k}{k_0}\right)^{-5/3} & k_0 \le k \le k_d. \end{cases} \tag{23}$$

The factor $a = 0.785$ is determined by the normalization condition Eq. (21), and k_d represents the scale at which viscous effects become important. u_0 is the turbulence velocity scale and $k_0 \equiv 2\pi/\ell_0$, where ℓ_0 is the turbulent dimension.

It has been suggested that the temporal spectrum can best be represented by the so-called modified Gaussian frequency factor given by

$$\Delta(\sigma/ku_k) = \frac{4}{\sqrt{\pi}}\frac{\sigma^2}{|ku_k|^2} e^{-\left(\frac{\sigma}{ku_k}\right)^2} \tag{24}$$

Fig. 1 and Fig. 2 give the spatial spectrum and temporal factor of the turbulent convection.

From Eq. (20), we have integrals of the type

$$\int_0^\infty d\sigma \cos\sigma t \int_0^\infty dk E(k,\sigma) = E(k)\int_0^\infty d\sigma \cos\sigma t \,\triangle\left(\frac{\sigma}{ku_k}\right)\ , \tag{25}$$

Using Eq. (24) and taking $\alpha = ku_k$, we get

$$\int_0^\infty d\sigma \cos\sigma t \,\triangle\left(\frac{\sigma}{ku_k}\right) = \frac{4}{\sqrt{\pi}}\int_0^\infty \frac{d\sigma}{\alpha} \cos\sigma t \frac{\sigma^2}{\alpha^2} e^{-\sigma^2/\alpha^2} = \left(1 - \frac{\alpha^2 t^2}{2}\right) e^{-\alpha^2 t^2/4}\ , \tag{26}$$

Making use of Eqs. (19) and (26), and noticing that $\mathbf{r}$ is limited to the z-direction ($r_x = r_y = 0$), we then obtain

$$R_{rr}(r',t')\, R_{rr}(r'',t'') = 4\int_0^\infty dk_1 \int_0^\infty dk_2 E(k_1)\, E(k_2) \left(\frac{\sin k_1 r'}{k_1{}^3 r'^3} - \frac{\cos k_1 r'}{k_1^2 r'^2}\right) \backslash \left(\frac{\sin k_2 r''}{k_2^3 r''^3} - \frac{\cos k_2 r''}{k_2^2 r''^2}\right) \left(1 - \frac{\alpha_1^2 t'^2}{2}\right)\left(1 - \frac{\alpha_2^2 t''^2}{2}\right)\exp\left(\frac{-\alpha_1^2 t'^2 - \alpha_2^2 t''^2}{4}\right) \tag{27}$$

where $\alpha_1 \equiv k_1 u_{k_1}$ and $\alpha_2 \equiv k_2 u_{k_2}$.

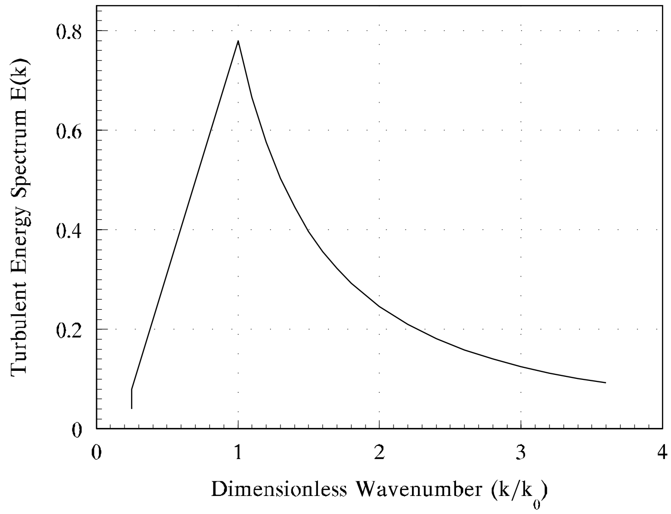

Fig. 1. Spatial part of turbulent energy spectrum $E(k)$ in units of u_0^2/k_0 as a function k/k_0.

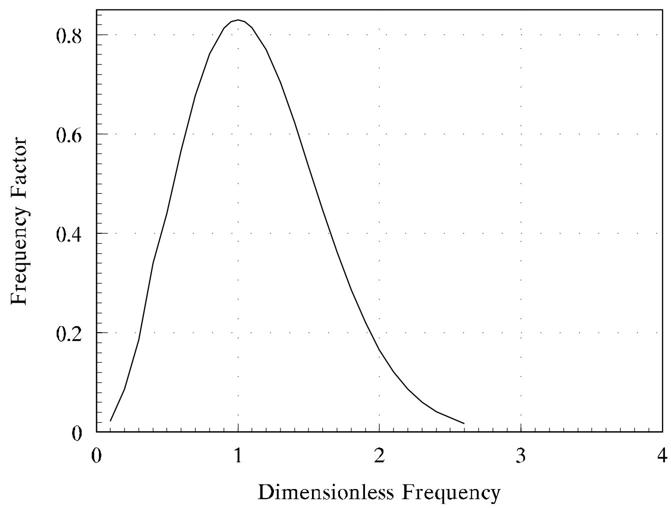

Fig. 2. Turbulent frequency factor $\Delta(\sigma/ku_k)$ as a function k/k_0.

4. An expression for the mode energy

Now consider the integration over t', t'' and r'' in the expression for $< |A_\alpha(t)|^2 >$ of Eq. (12). Substituting Eq. (22) into (12), we have

$$\int dt \exp(i\sigma t)\left(1-\frac{\alpha^2 t^2}{2}\right)\exp\left(\frac{\alpha^2 t^2}{4}\right) = \left(1+\frac{\alpha^2}{2}\frac{d^2}{d\sigma^2}\right) I(\sigma) \tag{28}$$

where

$$I(\sigma) \equiv \int dt \exp(i\sigma t)\exp\left(\frac{-\alpha^2 t^2}{4}\right) = \frac{2\sqrt{\pi}}{\alpha}\exp\left(\frac{-\sigma^2}{\alpha^2}\right) \tag{29}$$

Making use of Eqs. (28) and (29), we obtain the term that involves only the temporal factors in Eq. (12) given by the function

$$G(k_1,k_2,\sigma) = 16\pi\frac{\sigma^4}{\alpha_1^3\alpha_2^3}\exp\left(\frac{-\sigma^2}{\alpha_1^2+\alpha_2^2}\right) \tag{30}$$

Substituting Eqs. (27) and (30) into (12), we have

$$E_\alpha = \int_0^\infty dk_1 \int_0^\infty dk_2 \, E(k_1)\,E(k_2)\,G(k_1,k_2,\sigma)\,I(k_1,k_2) \tag{31}$$

with

$$I(k_1,k_2) = \int_{r_b}^{R_\odot} d^3r' \int_{r_b}^{R_\odot} d^3r'' \rho(r')\frac{\partial\xi_\alpha^*}{\partial r}(r')\,\rho(r'')\,\frac{\partial\xi_\alpha^*}{\partial r}(r'')\left(\frac{\sin k_1 r'}{k1^3 r'^3}-\frac{\cos k_1 r'}{k_1^2 r'^2}\right)\left(\frac{\sin k_2 r''}{k_2^3 r''^3}-\frac{\cos k_2 r''}{k_2^2 r''^2}\right) \tag{32}$$

where $E_\alpha \equiv \frac{1}{2} < |A_\alpha|^2 >$ is the expectation value of the energy in mode α.

5. Results and discussion

Before we discuss the numerical results of Eqs. (31) and (32), we briefly describe the solar mixing-length model used in the calculations.

5.1. Modelling of convection zone

Since acoustic waves can be generated in the outermost layers of the Sun, the solar structure computation can be limited to model a sequence of plane-parallel envelopes. Therefore, the model of the convective envelope of the Sun was obtained by integrating the stellar equations for solar radius, luminosity, and mass inward from optical depth $\tau = 10^{-4}$. For comparison between ours and previous results, we use the equilibrium model data from the solar model of Christensen-Dalsgaard (1982). It is characterized by the mixing-length parameter $\alpha = \frac{\ell_0}{H_p}$ which is the ratio of the characteristic convective mixing length ℓ_0 to the pressure scale height H_p. Then, the adiabatic eigenfunction of solar p-modes are obtained by integrating numerically the equations that govern the linear nonradial adiabatic oscillations (Unno et al. 1989). In this paper, we adopt Christensen-Dalsgaard's model (1982) with mixing-length parameter $\alpha = 1.7$.

5.2. The frequency dependence of the excitation rate

In computations, we identify the turbulence velocity scale u_0 in Eq. (21) with the convective velocity of the solar model, u_0 =v. The average mode energy is calculated numerically according to Eqs. (20), (30), (31) and (32) by using the extended Kolmogorov spectrum $E(k)$ and the the modified Gaussian turbulent frequency factor $\triangle(\sigma/ku_k)$.

If pulsations are intrinsically damped and excited by turbulent stress, the line width Γ of the principal peaks in a power spectrum of the oscillations provides direct measures of the modal damping rates (Christensen- Dalsgaard, Gough & Libbrecht 1989). The power going into each mode is given by $\dot{E}$=$E\Gamma$. It is formed by multiplying the average values of E_α given in Eq. (31) by Γ obtained from observations (Libbrecht 1988). $E\Gamma(\nu)$ is plotted against cyclic frequency, $\nu = \sigma_\alpha/2\pi$ in Fig. 3.

Fig. 3 shows the examples of average modal excitation rates computed from the expression given in Eq. (31) and measured

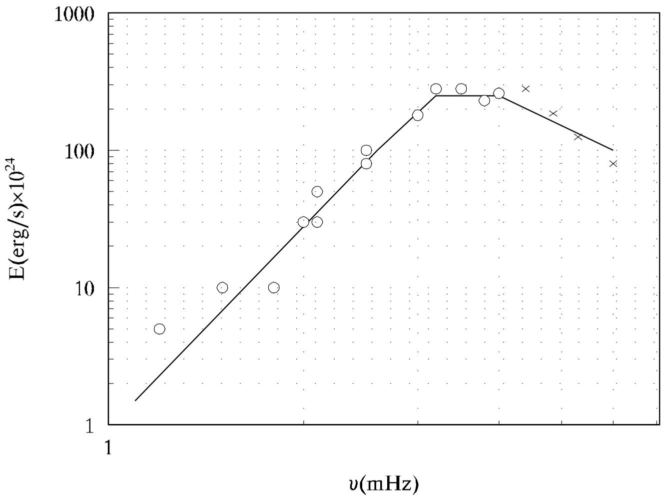

Fig. 3. The frequency dependence of the excitation rate $E\Gamma$, where mode average mode energy E is obtained by numerically integrating the expression in Eq. (31), and Γ is the oscillation line width. The solid line shows theoretical excitation rates. The circles show measured values (Libbrech 1988; Libbrecht & Woodard 1991). The crosses at high frequency come from a separate analysis of $\ell = 60$ data.

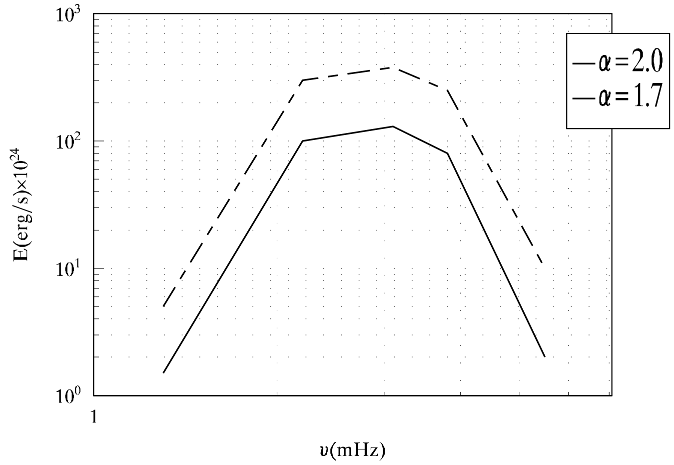

Fig. 4. Average mode energy E_α as a function of cyclic frequency υ for different solar convective zone models.

values (Libbrecht 1988; Libbrecht & Woodard 1991). Both $E\Gamma$ are similar in magnitude and shape, and peak at about $\nu = 3.1mHz$. Away from the peak, $\dot{\mathrm{E}}\propto \nu^7$ for $\nu < 3mHz$ and $\dot{\mathrm{E}}\propto \nu^{-5}$ for $\nu > 3mHz$. It is evident that the observationally determined rates are comparable with those calculated by using different mixing-length parameters on the hypothesis that the modes are stochastically excited by the fluctuations of Reynolds stress.

On the basis of the above result, we conclude that for realistic turbulence models both the shape and the magnitude of the mean modal amplitudes are relatively well-determined. Furthermore, we have apparently solved the problem with the high-frequency behavior found in previous calculations (e.g. Balmforth 1992). The main reason for this is that despite the small amount of information we have concerning the exact nature of the turbulence, the spatial and temporal turbulent energy spectra are apparently sufficient to reasonably determine mean modal amplitudes.

5.3. Dependence of the results on the solar model

We now investigate the dependence of our results on the solar mixing-length models.

Fig. 4 shows examples of average mode energy spectra computed from the expression given in Eq. (31) for different mixing-length parameters $\alpha = 1.7$ and $\alpha = 2.0$, by using the extended Kolmogorov spectrum and the modified Gaussian frequency factor. It is seen that the energy spectra of the two models are rather similar in shape but not in magnitude, and that the energy of the modes increases with increasing mixing-length parameters. This is because the convective velocity increases with increasing mixing-length parameter α. In a complete solar model, the value of α is determined by calibrating the model to have the correct radius. However, the mixing-length description is unlikely to be a good representation of turbulence in the convection zone; thus the dependence of α within this description can be regarded as a measure of the sensitivity of our results to the properties of turbulence.

6. Conclusions

The principal limitations of our calculations are reviewed below. The most important modification is the improved treatment of the spatial and temporal behavior of the turbulence.

At low frequency, the form of excitation rate is comparable to the results obtained by Balmforth (1992). The high frequency behavior is also well explained by us. It is essential that we adopted the modified Kolmogorov spectra for the spatial component of the turbulent energy spectrum, which has the virtue of being reasonably realistic. The temporal component of the full spectrum consists of a turbulent frequency factor, which has a broad maximum at frequency $\nu = ku_k$, goes to zero at $\nu = 0$ as well as at $\nu = \infty$, and is symmetric in the vicinity of ku_k.

According to our calulations, the modelling of acoustic emission processes indicates that turbulent stresses are primarily responsible for exciting pulsations. The physical process is therefore equivalent to Lighthill's (1952) original noise-generation mechanism.

However, our calculations still contain many uncertainties, for example, the α-dependence of the excitation rates. Further and extensive studies are required.

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References

Balmforth N. J., 1992, MNRAS 255, 639
Batchelor G. K., 1960, The Theory of homogeneous turbulence, Cambridge University Press, Cambridge p. 49
Brown T. M., 1991, ApJ 371, 393

Brown T. M., Bogdan T. J., Lites B. W., Thomas J. H., 1992, ApJ 394, L65
Christensen-Dalsgaard J., 1982, MNRAS 199, 735
Christensen-Dalsgaard J., Gough D. O., Libbrecht K. G., 1989, ApJ 341, L103
Goldreich P., Keeley D. A., 1977, ApJ 212, 243
Goldreich P., Kumar P., 1988, ApJ 326, 462
Goldreich P., Kumar P., 1990, ApJ 363, 694
Goldreich P., Murray N., Kumar P., 1994, ApJ 424, 466
Kumar P., Goldreich P., 1989, ApJ 342, 558
Li Y., 1993, A&A 276, 367
Lighthill M. J., 1952, Proc. R. Soc. Lond, A211, 564
Libbreich K. G., 1988, ApJ 334, 510
Libbreich K. G., Woodard M. F., 1991, Science 253, 152
Moore D. W., Spiegel E. A., 1964, ApJ 139, 48
Musielak Z. E., Rosner R., Stein R. F., Ulmschneider P., 1994, ApJ 432, 474
Osaki Y., 1990, in Progress of Seismology of the Sun and Stars, ed. Y. Osaki and H. S
Shibahashi, Springer-Verlag, Berlin p.75
Stein R. F., 1967, Sol. Phys. 2, 385
Unno W., Osaki Y., Ando H. et al., 1979, Nonradial Oscillation, University of Tokyo Press, Tokyo p. 168