Astron. Astrophys. 335, 679–684 (1998)

ASTRONOMY
AND
ASTROPHYSICS

Differential rotation and meridional flow in the solar convection zone with AKA-effect

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Received 26 June 1997 / Accepted 1 April 1998

Abstract. It makes no problem to reproduce the internal rotation of the solar convection zone known by helioseismology if (artificially) the meridional flow is ignored. Its inclusion, however, strongly reduces the latitudinal differences of the angular velocity even at the surface. The amplitude of this effect, known as the 'Taylor number puzzle', depends on the unknown amplitude of the eddy viscosity. The effect in detail is described here making use of the mixing-length model of the solar convection zone by Stix & Skaley (1990). The resulting differential rotation profiles are far from the observations if indeed the free factor c_ν in the eddy viscosity definition does not exceed unity.

As one of the possibilities to escape the paradox the 'anisotropic kinetic alpha' (AKA)-effect is here involved into the computations. We have already shown that it should appear in formulations of the mean-field hydrodynamics in rotating density-stratified turbulent fluids at the same time with the MHD alpha-effect. Its rotational dependence, however, strongly differs from the latter hence it is concentrated to the upper part of the convection zone. Nevertheless, meridional flow and rotation law are so strongly influenced that the observed differential solar rotation can be reproduced even with realistic Taylor numbers.

Key words: hydrodynamics – Sun: interior – Sun: rotation – turbulence

1. Motivation

The 'observed' internal solar rotation law can be explained by means of a Reynolds stress theory in which the influence of the basic rotation on the cross correlations of the turbulent convective velocities is determined (Küker et al. 1993). It is a pure Reynolds stress theory: the meridional flow in such calculations is neglected. The reason for this ignorance is twofold. First, the observations show only a slow *poleward* flow with an amplitude of 10 m/s (Snodgrass & Daily 1996; Hathaway 1996; Hathaway et al. 1996). There is the assumption that such a slow flow only slightly modifies the results of the Reynolds stress theory. On the other hand, a rough inspection of the (observed) rotational isolines does not suggest a too strong influence of a meridional flow with one or two large cells.

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The meridional circulation tends to align the isolines of the angular momentum with the streamlines of the flow (Köhler 1969; Rüdiger 1989; Brandenburg et al. 1990). The existence of a distinct equatorial acceleration is thus indicating that a possible meridional surface drift cannot be too strong.

Moreover, for very large Taylor numbers Ta the flow system approaches more and more the Taylor-Proudman state where the isolines are two-dimensional with respect to the rotation axis, i.e. there is no variation along the coordinate (say) z. At the same time the poloidal ('meridional') circulation breaks, i.e.

$$\frac{\partial\Omega}{\partial z} = 0, \qquad \boldsymbol{u}^{\mathrm{m}} = 0. \tag{1}$$

At least, the first of these conditions is far from being realized with the Sun. The solar Taylor number, however, is large. For

$$\mathrm{Ta} = \frac{4\Omega^2 R^4}{\nu_0^2} \tag{2}$$

(Ω basic rotation rate, R solar radius) one finds with the reference value for the eddy viscosity

$$\nu_0 = c_\nu \; u_{\mathrm{T}} l_{\mathrm{corr}} \tag{3}$$

(l_{corr} correlation length, $u_{\mathrm{T}} = \sqrt{\langle \boldsymbol{u}'^2\rangle}$ turbulent rms velocity) the relation

$$\mathrm{Ta} \simeq \frac{1}{c_\nu^2} \left(\frac{2u_{\mathrm{surf}}/u_{\mathrm{T}}}{l_{\mathrm{corr}}/R} \right)^2 . \tag{4}$$

The solar surface velocity is $u_{\mathrm{surf}} \simeq 2$ km/s, c_ν is a dimensionless free parameter of order unity. With characteristic values for solar parameters, i.e. $u_{\mathrm{T}} \simeq 100$ m/s and $l_{\mathrm{corr}} \simeq 0.1R$ we find

$$\mathrm{Ta} \simeq \frac{1.6 \cdot 10^5}{c_\nu^2} . \tag{5}$$

It is usually assumed that c_ν does not exceed unity. It is a free parameter in the theory which cannot be tuned with an observation other than the large-scale flow pattern in the convection zone. On the other hand, its choice strongly influences the magnitude of the Taylor number. The latter varies between 10^5 and 10^7 if c_ν varies between 1 and 0.1. Both extrema should be considered. It must be expected that Taylor numbers like (5) produce a remarkable intensity of the meridional flow. We thus need an extra explanation why the solar meridional circulation is so slow. In Kitchatinov & Rüdiger (1995) the rotational influence on the eddy heat diffusion tensor leads to a warmer pole so that an equatorwards directed extra drift weakens the (poleward) meridional circulation.

There is thus reason enough to rediscuss the role of the meridional circulation and its interchange with the differential rotation (cf. Rüdiger et al. 1998). Simplifying here we shall only deal with adiabatic stratifications but as an innovation the AKA-effect is tested in its ability to influence the large-scale flow system. Its existence has been established in papers by Frisch et al. (1987), Khomenko et al. (1991) and Kitchatinov et al. (1994b), where in particular the close relationship of the AKA-effect to the MHD alpha is underlined (cf. Schüssler 1984; Gvaramadze et al. 1989; Sulem et al. 1989; Moffatt & Tsinober 1992). In linear approximation density stratification $\boldsymbol{G} = \nabla \log \rho$ and the global rotation Ω can be combined in two ways:

$$\alpha_{ij} := G_i \Omega_j, \qquad \Gamma_{ijk} := \epsilon_{ikl} G_l \Omega_j. \tag{6}$$

The first tensor is a pseudo-tensor but not the second one. As their construction is very similar there is no doubt that they appear simultaneously. The first one leads to the well-known α-effect in the electrodynamics of turbulent media $\langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle = \alpha \cdot \bar{\boldsymbol{B}} + \ldots$ while the second one appears in the turbulent hydrodynamics as the connection between Reynolds stress and mean flow, i.e.

$$Q_{ij} = \Gamma_{ijk} \ \bar{u}_k + \ldots \tag{7}$$

with the correlation tensor

$$Q_{ij} = \langle u_i'(\boldsymbol{x}, t) u_j'(\boldsymbol{x}, t) \rangle. \tag{8}$$

This fact should have important consequences. If indeed the α-effect produces dynamo-generated large-scale magnetic fields then at the same time the AKA-effect should produce an extra mean flow pattern with consequences for the dynamo. Moreover, as the AKA-effect may generate a vortex system at the convection zone surface (with azimuthal number $m = 1$, see v. Rekowski & Kitchatinov 1998) one should observe magnetic fields always together with non-axisymmetric flow patterns at the surfaces of cool stars. At least for the Sun it is not yet observed. There are only a few reports about vortex structures in the mesogranulation pattern (cf. Brandt et al. 1988) so that indeed one could question the above argumentation[1]. In Pipin et al. (1996) it is shown that both effects exhibit quite a different behavior for fast rotation. The AKA-effect decreases for fast rotation but not – as known – the MHD α-effect. This difference is considered as the explanation of the missing of vortex structure for stars (like the Sun) with Coriolis numbers,

$$\Omega^* = 2 \tau_{\rm corr} \, \Omega, \tag{9}$$

exceeding unity with $\tau_{\rm corr} \simeq l_{\rm corr}/u_{\rm T}$ as the turnover time of the convection eddies (cf. Gough 1977; Durney & Latour 1978; Stix 1989; Canuto 1991). Another complication for the vortex structure forming process is the existence of differential rotation. The preference of a $m = 1$ hydro-mode has been found by v. Rekowski & Kitchatinov (1998) for a sphere in solid-body rotation. Differential rotation may easily complicate the excitation conditions for non-axisymmetric flow patterns so that no self-excitation would be possible.

[1] A more general argument against the AKA-effect is its non-Galilean invariance (Roberts & Soward 1975)

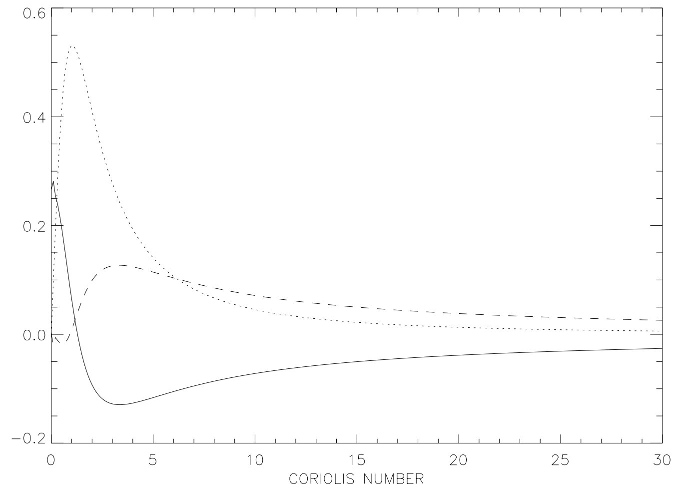

Fig. 1. The influence of the basic rotation on the Λ-effect coefficients $V^{(0)}$ (solid) and $V^{(1)} = H^{(1)}$ (dashed) after Kitchatinov & Rüdiger (1992) and on the AKA-effect (dotted) after Pipin et al. (1996)

The computation of the hydrodynamic flow system under the simultaneous influence of the AKA-effect and the Λ-effect is reported in the present paper. A rotating density-stratified convection zone cannot rotate rigidly. As we have shown by means of the Λ-effect but under neglecting meridional flow the theoretical rotation law in the convection zone well approaches the known results of helioseismology. The 30% pole-equator difference of the angular velocity at the surface, however, is strongly reduced by the action of meridional flow. What we demonstrate in the following is whether the AKA-effect – which is mainly active in the surface layers of the convection zone – is able to reduce the smoothing action of the meridional flow.

2. The turbulence model

A stratified turbulence can never rotate rigidly. The reason is that even in the case of uniform rotation – in contrast to any form of viscosity – the turbulence transports angular momentum. The formal description of this phenomenon is given by the non-diffusive part Q_{ij}^{Λ} of the correlation tensor (8), i.e. for the turbulent fluxes of angular momentum

$$\begin{aligned} Q_{r\phi} &= \ldots + \nu_0 \left(V^{(0)} + V^{(1)} \sin^2 \theta \right) \Omega \, \sin\theta, \\ Q_{\theta\phi} &= \ldots + \nu_0 H^{(1)} \sin^2 \theta \, \Omega \, \cos\theta, \end{aligned} \tag{10}$$

where the coefficients $V^{(0)}, V^{(1)}$ and $H^{(1)}$ are functions of the Coriolis number. The dotted parts in Eqs. (10) are denoting the contributions of the eddy viscosity tensor Q_{ij}^{ν} which in the simplest (axisymmetric) case yields

$$Q_{r\phi} \;=\; -\nu_{\rm T}\; r\; \frac{\partial\Omega}{\partial r}\;\sin\theta + \dots ,$$
$$Q_{\theta\phi} \;=\; -\nu_{\rm T}\; \frac{\partial\Omega}{\partial\theta}\;\sin\theta + \dots . \qquad (11)$$

In Kitchatinov et al. (1994a) the structure of the eddy viscosity tensor under the influence of a global rotation is derived within the framework of a quasilinear theory (cf. Roberts & Soward 1975). The tensor becomes highly anisotropic, making the theory more complicated. In the present paper we apply only the Ω-dependence of the isotropic part of the viscosity tensor, i.e. the coefficient of the deformation tensor, replacing (3) by

$$\nu_{\rm T} = \nu_0\;\phi_1(\Omega^*) \qquad (12)$$

with

$$\phi_1 = \frac{15}{128\Omega^{*4}}\left(-21 - 7\Omega^{*2} + \frac{8\Omega^{*4}}{1+\Omega^{*2}} + \right.$$
$$\left. + \frac{\Omega^{*4} + 14\Omega^{*2} + 21}{\Omega^*}\arctan\Omega^*\right) \qquad (13)$$

(cf. Fig. 2, dashed line). In Kitchatinov & Rüdiger (1992) one finds the representation for the functions V and H.

We adopt the approximation

$$\tau_{\rm corr}^2\; u_{\rm T}^2 \;\simeq\; l_{\rm corr}^2 \;\simeq\; \left(\frac{\alpha_{\rm MLT}}{\gamma}\right)^2 H_\rho^2 \qquad (14)$$

with $\alpha_{\rm MLT}$ from the usual mixing-length relation

$$l_{\rm corr} = \alpha_{\rm MLT} H_p \qquad (15)$$

between the mixing-length, $l_{\rm corr}$, and the pressure scale-height, H_p. γ is the adiabatic index,

$$H_\rho = |\boldsymbol{G}|^{-1} \qquad (16)$$

is the density scale-height. The expressions for $\mathcal{I}_0$ and $\mathcal{I}_1$ describe the rotation-rate dependence of the Λ-effect. In Fig. 1 the profiles of the functions $V^{(0)}$ and $V^{(1)} = H^{(1)}$ are given. Küker et al. (1993), under application of a model of the solar convection zone by Stix & Skaley (1990) were able to reproduce the main features of the 'observed' internal rotation of the Sun's outer envelope. They neglected, however, the angular momentum transport of meridional flows and magnetic fields.

One has to include the meridional flow into the theory. To this end one needs the knowledge of the influence of rotation on the turbulent heat transport. If this is known from an established turbulence theory, then one can compute the complete fields of pressure, circulation and differential rotation.

But even then the theory is not complete. It might not be true that the mean flow enters the correlation tensor only via its gradients but also via the flow itself:

$$Q_{ij} \;=\; Q_{ij}^\Lambda + Q_{ij}^\Gamma + Q_{ij}^\nu$$
$$\;=\; \Lambda_{ijk}\;\Omega_k + \Gamma_{ijk}\;\bar{u}_k - \mathcal{N}_{ijkl}\;\bar{u}_{k,l}. \qquad (17)$$

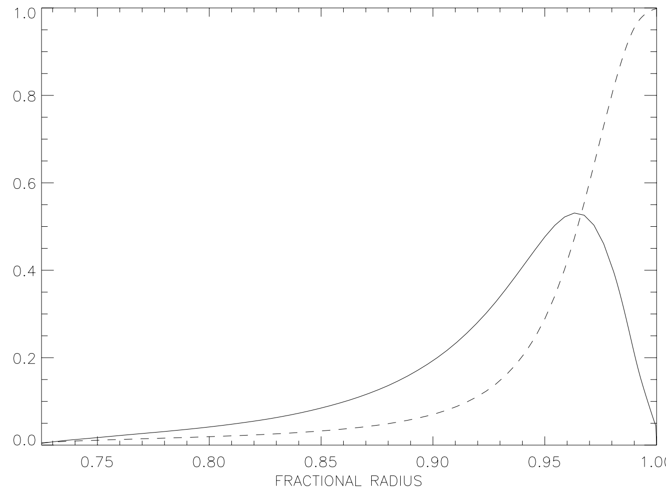

Fig. 2. The depth-dependence of the functions F_1 (AKA, solid) and ϕ_1 (eddy viscosity, dashed)

In Kitchatinov et al. (1994b) it has been demonstrated that in rotating and stratified turbulent fluids there is a part of the Γ-tensor which is due to the basic rotation, i.e.

$$\Gamma_{ijk} = \Gamma_1\left(\epsilon_{ikp}\;\Omega_j^\circ + \epsilon_{jkp}\;\Omega_i^\circ + \delta_{ik}\;\epsilon_{jlp}\;\Omega_l^\circ + \right.$$
$$\left. + \delta_{jk}\;\epsilon_{ilp}\;\Omega_l^\circ\right) G_p \qquad (18)$$

with $\boldsymbol{\Omega}^\circ$ as the unit vector parallel to the rotation axis, $\boldsymbol{g}^\circ$ given the radial unit vector. It directly results in correlations such as

$$Q_{r\theta} \;=\; \dots + \Gamma_1\;\cos\theta\;\bar{u}_\phi\;\frac{d\log\rho}{dr},$$

$$Q_{\theta\theta} \;=\; \dots - 2\Gamma_1\;\sin\theta\;\bar{u}_\phi\;\frac{d\log\rho}{dr}, \qquad (19)$$

where the dependence of Γ_1 on the basic rotation rate is given in Pipin et al. (1996) as

$$\Gamma_1 = \frac{1}{4}\;\nu_0\;F_1 \qquad (20)$$

with

$$F_1 = \frac{15}{4\Omega^{*3}}\left(\frac{3+\Omega^{*2}}{\Omega^*}\arctan\Omega^* - 3\right), \qquad (21)$$

i.e. $F_1 \sim \Omega^*$ for slow rotation (Fig. 2). Obviously, the Γ_1-effect is strongly related to the MHD α-effect which has the same structure and also exists only in rotating stratified turbulences.

3. Basic equations

We solve the stationary Reynolds equation for a turbulent flow in the inertial reference frame, i.e.

$$(\bar{\boldsymbol{u}}\nabla)\,\bar{\boldsymbol{u}} + \frac{1}{\rho}\,{\rm Div}(\rho Q) + \frac{\nabla p}{\rho} = \boldsymbol{g}. \qquad (22)$$

The tensor divergence ${\rm Div}(\rho Q)$ is a vector with the components $\partial(\rho Q_{ij})/\partial x_j$. $\boldsymbol{g}$ denotes the acceleration of gravity which is here assumed uniform. Magnetic fields and deviations from axisymmetry are not considered. Anelastic fluids are only considered, i.e.

$$\mathrm{div}(\rho\bar{\boldsymbol{u}}) = 0. \tag{23}$$

This adopted, the azimuthal component of (22) is

$$\frac{\partial}{\partial x_j}\left(\rho\, r\sin\theta\left(\bar{u}_j\bar{u}_\phi + Q_{\phi j}\right)\right) = 0. \tag{24}$$

Our energy equation will be very simple. The turbulence is considered to be so intense that the medium is isentropic, i.e.

$$S = C_v \log\left(p\rho^{-\gamma}\right) = \mathrm{const.}, \tag{25}$$

hence there is a direct relation between density and pressure (C_v is the specific heat). If this is true the pressure term in (22) can be written as a gradient which disappears after application of *curling*:

$$\mathrm{rot}_\phi\left(\left(\bar{\boldsymbol{u}}\nabla\right)\bar{\boldsymbol{u}} + \frac{1}{\rho}\,\mathrm{Div}(\rho Q)\right) = 0. \tag{26}$$

It remains to fix the flow field by

$$\bar{u}_r = \frac{1}{\rho\, r^2\sin\theta}\,\frac{\partial A}{\partial\theta}, \qquad \bar{u}_\theta = -\frac{1}{\rho\, r\sin\theta}\,\frac{\partial A}{\partial r},$$

$$\bar{u}_\phi = \Omega_0\, r\sin\theta + u_\phi. \tag{27}$$

For normalization the relations

$$r = R\,x,\ \ \rho = \rho_*\,\hat{\rho},\ \ A = \rho_* R\nu_0\,\hat{A},\ \ u_\phi = R\Omega_0\,\hat{u}_\phi \tag{28}$$

and

$$\bar{u}_r = \frac{\nu_0}{R}\hat{\bar{u}}_r,\ \ \bar{u}_\Theta = \frac{\nu_0}{R}\hat{\bar{u}}_\Theta,\ \ \bar{u}_\phi = R\Omega_0\,\hat{\bar{u}}_\phi \tag{29}$$

are used. The density stratification is then represented by $H_\rho = R\,\hat{H}$. As the conservation law of the angular momentum we get from (24) the dimensionless equation

$$\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\hat{\rho}\hat{Q}_{r\phi}\right) + \frac{1}{x\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\hat{\rho}\hat{Q}_{\theta\phi}\right) + \frac{\hat{\rho}}{x}\hat{Q}_{r\phi} + \\ + \frac{\rho\cot\theta}{x}\hat{Q}_{\theta\phi} - \frac{2\cot\theta}{x}\frac{\partial\hat{A}}{\partial x} + \frac{2}{x^2}\frac{\partial\hat{A}}{\partial\theta} = 0. \tag{30}$$

Here also the turbulent fluxes of angular momentum are written in dimensionless form in accord with

$$Q_{i\phi} = \nu_0\,\Omega_0\,\hat{Q}_{i\phi} \qquad i = r,\theta. \tag{31}$$

Eq. (30) is linearized by means of the assumption of mild differential rotation,

$$|u_\phi| \ll \Omega_0\, R. \tag{32}$$

The meridional Eq. (26) is formulated under the same condition. Hence the 'centrifugal force' is reformulated as

$$\bar{u}_\phi^2 \simeq r^2\sin^2\theta\,\Omega_0^2 + 2r\sin\theta\,\Omega_0\,u_\phi. \tag{33}$$

The result is

$$\frac{1}{x}\frac{\partial}{\partial x}\left[\frac{1}{\hat{\rho}x}\frac{\partial}{\partial x}\left(x^2\hat{\rho}\hat{Q}_{r\theta}\right) + \frac{1}{\hat{\rho}\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\hat{\rho}\hat{Q}_{\theta\theta}\right) + \right. \\ \left. + \hat{Q}_{r\theta} - \cot\theta\hat{Q}_{\phi\phi}\right] - \frac{1}{x}\frac{\partial}{\partial\theta}\left[\frac{1}{\hat{\rho}x^2}\frac{\partial}{\partial x}\left(x^2\hat{\rho}\hat{Q}_{rr}\right) + \right. \\ \left. + \frac{1}{\hat{\rho}x\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\hat{\rho}\hat{Q}_{r\theta}\right) - \frac{1}{x}\hat{Q}_{\theta\theta} - \frac{1}{x}\hat{Q}_{\phi\phi}\right] - \\ - \frac{\cos\theta\ \mathrm{Ta}}{2x}\frac{\partial}{\partial x}\left(x\hat{u}_\phi\right) + \frac{\mathrm{Ta}}{2x}\frac{\partial}{\partial\theta}\left(\sin\theta\hat{u}_\phi\right) = 0. \tag{34}$$

Here the Taylor number Ta couples the centrifugal terms in (34) to the contributions of the turbulence-originated Reynolds stress. In contrast to (31) the latter are normalized with ν_0^2/R^2. The Γ_1-effect is normalized with $\Gamma_1 = \Omega_0\, R^2\,\hat{\Gamma}_1$.
The differential rotation u_ϕ and the stream function A are the unknowns of our equation system (30) and (34). The basic rotation Ω_0 is prescribed so that the system becomes inhomogeneous with the details given in Rüdiger (1989). Stress-free boundary conditions are imposed, i.e.

$$\hat{A} = 0, \qquad \hat{Q}_{r\theta} = 0, \qquad \hat{Q}_{r\phi} = 0, \tag{35}$$

and $\hat{u}_\phi = 0$ at the equator. Like in Küker et al. (1993) we have used the convection zone model by Stix & Skaley (1990) to specify the radial stratification of the parameters. Density and turbulence intensity as well as the mixing-length are taken from the convection zone model. Besides the basic solar rotation rate Ω_0, the only free parameters are the mixing-length parameter α_{MLT} and the factor c_ν in the eddy viscosity expression. As an extra parameter to tune the AKA-effect the normalized amplitude Γ^* has been introduced with $\Gamma^* = 1$ as the standard case.

4. Results

In the sense of an experiment we start with a Taylor number artificially decreased by 3 orders of magnitude. This can easily be done using large values for c_ν. Then the meridional flow is expected to be small as well as its influence on the differential rotation. The rotation law solution of Küker et al. (1993) is reproduced – close to the empirical knowledge (Fig. 3). The pole is decelerated by about 30% with respect to the equator. The model does not exactly fit the observations deep in the convection zone, i.e. our lower boundary (stress-free) condition is not very realistic (Rüdiger & Kitchatinov 1997). However, the agreement between theory and observation is high in the upper layers of the SCZ. Also the isocontours of the angular velocity given in the bottom line of Fig. 3 comply with the known plots by Libbrecht (1988) and Thompson et al. (1996).

For more realistic values of the parameter c_ν this situation is *not* preserved (Fig. 4). The meridional flow grows to considerable values (19 m/s) even at the bottom of the SCZ. There it flows towards the equator. The differential rotation at the surface, however, proves to be destroyed. This finding naturally agrees with the content of the 'Taylor number puzzle' stressed by Brandenburg et al. (1990).

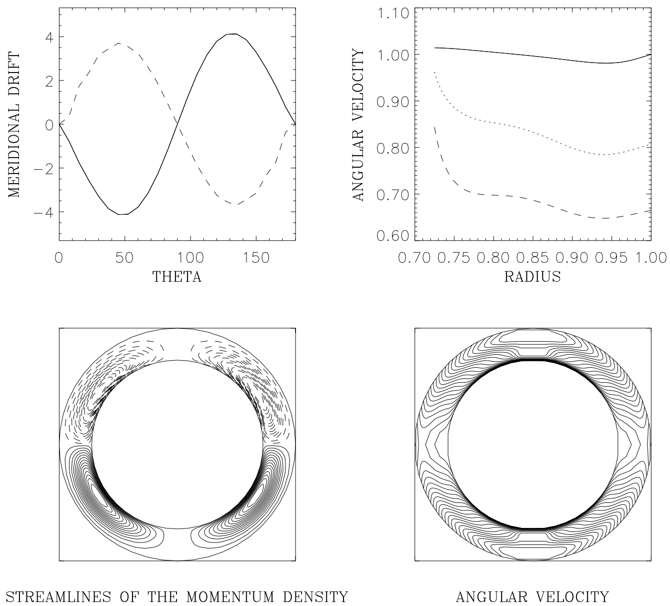

Fig. 3. The flow pattern in the solar convection zone (SCZ). Here as an experiment the meridional flow is suppressed with an artificially high eddy viscosity: $c_\nu = 30$. TOP LEFT: Normalized meridional flow $\hat{\hat{u}}_\Theta$ at the top (solid) and the bottom (dashed) of the SCZ in units of 10/7 m/s, TOP RIGHT: convection zone rotation law at the equator (solid), in mid-latitudes (45°, dotted) and at the poles (dashed). BOTTOM: isoline representations of meridional flow and angular velocity. It is $\Gamma^* = 0$, $\alpha_{\rm MLT} = 2.5$

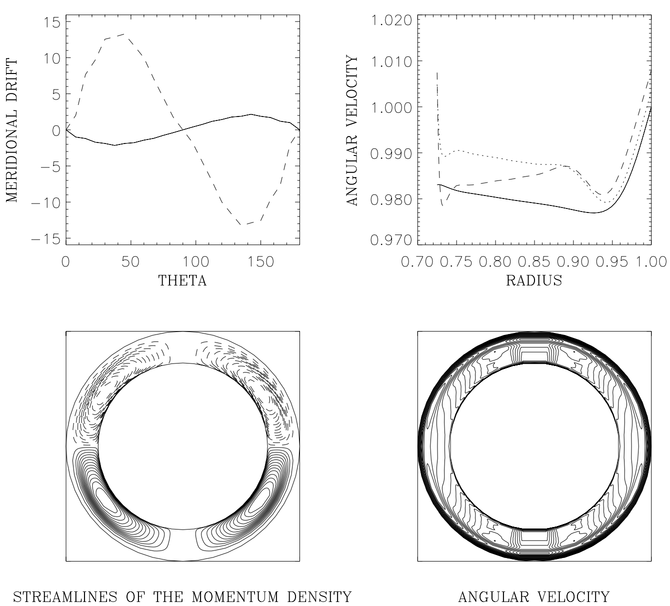

Fig. 4. The same as in Fig. 3 but for the viscosity factor $c_\nu = 1$ and $\Gamma^* = 0$

One possible solution of the Taylor number puzzle was presented by Kitchatinov & Rüdiger (1995) using the rotation-induced anisotropy of the thermal eddy conductivity tensor. In the following the question is answered whether the AKA-effect can act in the same direction.

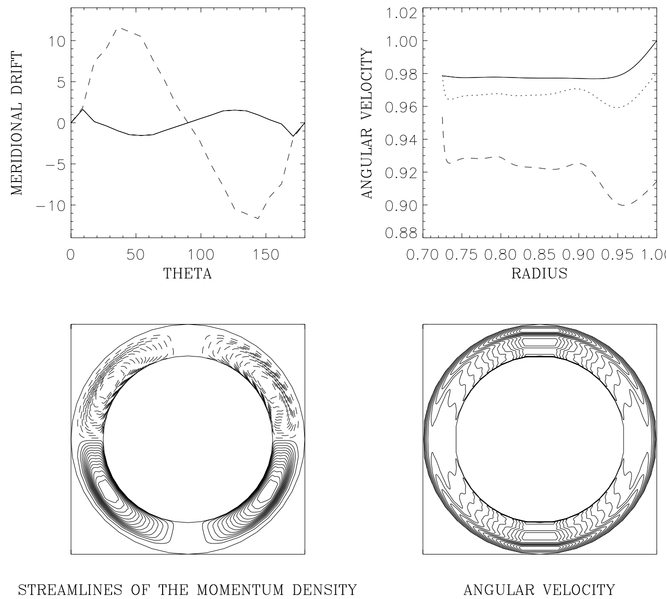

Fig. 5. The same as in Fig. 3 but for $\Gamma^* = 1$, $F_1 = 0.53$ (maximal value) and the viscosity factor $c_\nu = 1$

As shown by Fig. 5 the answer is Yes. Without AKA-effect in Fig. 4 the differential rotation is very small and the isolines are of Taylor-Proudman type. With AKA-effect in Fig. 5 the pole-equator difference at the surface and the Ω-isolines change their shape towards the structure presented in the flow-free case of Fig. 3. The result is that indeed with inclusion of the AKA-effect in our formulation a consistent picture of the solar differential rotation can be formed. The meridional circulation is organized in one dominant cell which in the northern hemisphere flows counterclockwise. While the meridional drift at the surface remains weak, it proves to be much stronger (with values of 15 m/s) at the bottom of the convection zone. There the material flows towards the equator.

However, in Fig. 5 a simplified model for the AKA-effect is used as its Ω^*-dependence was ignored. The maximum of the function F_1 ($\approx$ 0.53) has been used throughout the bulk of the SCZ. If the depth-dependence of the function F_1 (due to our turbulence model) is taken into account, i.e. the AKA-effect is concentrated in the surface layers (cf. Fig. 2), then the differential rotation at the surface is again reduced to 6% with accelerated equator (Fig. 6). The effect is still too small but it goes in the right direction. One can however increase the pole-equator difference to 25 % by increasing the amplitude of Γ by a factor of only 2. This is due to the appearance of a second cell of meridional circulation with more than 20 m/s. As it flows equatorwards the differential rotation is increased but only for the outer layer of the convection zone.

Acknowledgements. The authors are grateful to the Deutsche Forschungsgemeinschaft for the financial support under the grant Ru 488/6.

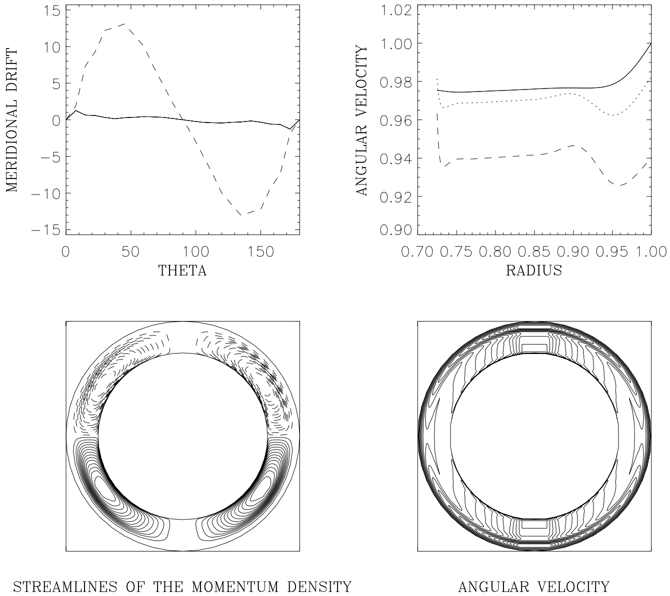

Fig. 6. The same as in Fig. 3 but for the viscosity factor $c_\nu = 1$ and $\Gamma^* = 1$

Appendix

The normalized components $\hat{Q}_{ij}$ of the (symmetric) correlation tensor Q_{ij} are

$$\hat{Q}_{rr} = -2\phi_1 \frac{\partial}{\partial x}\left(\frac{1}{\hat{\rho}x^2 \sin\theta}\frac{\partial \hat{A}}{\partial \theta}\right),$$

$$\hat{Q}_{r\theta} = \phi_1 \frac{\partial}{\partial x}\left(\frac{1}{\hat{\rho}x \sin\theta}\frac{\partial \hat{A}}{\partial x}\right) - \frac{\phi_1}{x}\frac{\partial}{\partial \theta}\left(\frac{1}{\hat{\rho}x^2 \sin\theta}\frac{\partial \hat{A}}{\partial \theta}\right) - $$
$$- \frac{\phi_1}{\hat{\rho}x^2 \sin\theta}\frac{\partial \hat{A}}{\partial x} - \frac{\hat{\Gamma}_1}{4\hat{H}}\,\mathrm{Ta}\;\;\cos\theta \hat{u}_\phi,$$

$$\hat{Q}_{r\phi} = -\phi_1 \frac{\partial \hat{u}_\phi}{\partial x} + \frac{\phi_1}{x}\hat{u}_\phi + \frac{1}{x}\left(V_0 + V_1 \sin^2\theta\right)\hat{\hat{u}}_\phi -$$
$$- \frac{\hat{\Gamma}_1}{\hat{H}}\frac{\cos\theta}{\hat{\rho}x\sin\theta}\frac{\partial \hat{A}}{\partial x} - \frac{\hat{\Gamma}_1}{\hat{H}}\frac{1}{\hat{\rho}x^2}\frac{\partial \hat{A}}{\partial \theta},$$

$$\hat{Q}_{\theta\theta} = \frac{2\phi_1}{x}\frac{\partial}{\partial \theta}\left(\frac{1}{\hat{\rho}x \sin\theta}\frac{\partial \hat{A}}{\partial x}\right) - \frac{2\phi_1}{\hat{\rho}x^3 \sin\theta}\frac{\partial \hat{A}}{\partial \theta} +$$
$$+ \frac{\hat{\Gamma}_1}{2\hat{H}}\,\mathrm{Ta}\;\;\sin\theta \hat{u}_\phi,$$

$$\hat{Q}_{\theta\phi} = \phi_1\,\frac{\cot\theta}{x}\hat{u}_\phi + \frac{\cos\theta\sin\theta}{x}H_1\hat{\hat{u}}_\phi - \frac{\phi_1}{x}\frac{\partial \hat{u}_\phi}{\partial \theta} +$$
$$+ \frac{2\hat{\Gamma}_1}{\hat{H}\hat{\rho}x}\frac{\partial \hat{A}}{\partial x},$$

$$\hat{Q}_{\phi\phi} = \frac{2\cos\theta\;\phi_1}{\hat{\rho}x^2\sin^2\theta}\frac{\partial \hat{A}}{\partial x} - \frac{2\phi_1}{\hat{\rho}x^3 \sin\theta}\frac{\partial \hat{A}}{\partial \theta} - \frac{\hat{\Gamma}_1}{2\hat{H}}\,\mathrm{Ta}\;\sin\theta \hat{u}_\phi.$$

They have to be inserted into Eqs. (30) and (34) yielding the inhomogeneous equation system which is solved with the boundary conditions (35).

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