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Waves in structured media: non-radial solar p modes

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Abstract. For non-radial solar p modes we calculate corrections to the eigenfrequencies arising from the sound speed and velocity inhomogeneity of the convection zone. We use a simple periodic model, and obtain the frequency corrections by solving a Hill determinant. The frequency shifts are significant, and in most cases negative; they increase in magnitude with increasing frequency and harmonic degree l. The dependence on degree scales with the mode inertia. For large degree l this trend reverses.

For very large degree coupling occurs between the p modes and "vibrational" modes that only exist because of the horizontal structure in sound speed and velocity.

Key words: convection – Sun: granulation – Sun: oscillations

1. Introduction

The interaction of the solar convection with the global oscillations predominantly takes place in a rather shallow region just beneath the surface of the Sun. In this region the convection reaches its largest Mach number (although still smaller than unity). It appears that some of the discrepancies between calculated and measured oscillation frequencies have their origin in that region. Moreover, frequency changes arising from near-surface effects follow a simple scaling for modes of different harmonic degree l: Multiplied with the normalized inertia of the modes, such changes are nearly independent of l, and vary only with frequency (Christensen-Dalsgaard 1986, Christensen-Dalsgaard & Berthomieu 1991, Christensen-Dalsgaard & Thompson 1997). We shall see below that the frequency shifts calculated in the present contribution for the non-radial p modes indeed obey such a scaling.

Ideally the solution of the eigenvalue problem should be based on a solar model which includes not only the dependence of the sound velocity on depth. We should also have a full description of the turbulence in the convection zone (spectrum, amplitude of temperature and velocity fluctuations) and a theory of p modes in a turbulent convective atmosphere. Our current solar model is based on the mixing-length theory, which only provides the depth profile of the sound velocity, the mixing length, and mean amplitudes of the temperature and velocity fluctuations. It does not provide the turbulence spectrum, nor the horizontal scale of the turbulent elements.

On the other hand it now appears that the effect of turbulence on the p-mode frequencies is of the same order as the discrepancy between the frequencies based on solar models and the observed frequencies. Hence any further attempt to adjust theoretical and observed frequencies should include the effects of turbulence. But here we face two difficulties: (1) the solar model is not complete because of the approximations made in the mixing-length theory, and (2) the theory of p modes in a convective atmosphere is not complete. At present we can only use the information available from the current convection model, namely the mean sound velocity, the mean fluctuation of the temperature and the velocity, and the mixing length. Unfortunately, mixing-length theory does not provide a horizontal scale of the convective elements. We therefore introduce a parameter, namely the ratio of the horizontal scale to the mixing length.

Brown (1984) first considered the effect of velocity fluctuations on the solar p modes. In a more recent example, Zhugzhda & Stix (1994) investigated the effect of sound speed and velocity fluctuations on the propagation of acoustic waves. For the radial p modes they obtained a negative frequency correction that increases with the order n of the mode; the sign as well as the magnitude and the frequency dependence of this correction appeared appropriate to improve the theoretically calculated eigenfrequencies.

In the present paper we extend the investigation of Zhugzhda & Stix (1994) to the case of non-radial oscillations. For this purpose we employ a model of the sound speed and vertical velocity inhomogeneities, recently proposed by Zhugzhda (1997, 1998), with a harmonic horizontal dependence of those quantities. Strictly such a model corresponds to stationary convection with a regular pattern of cells, e.g. Benard cells. In the case of turbulent convection the more or less regular structure of the inhomogeneity is related to the governing convective cell size, which we identify with the mixing length used in the theory of the convection zone. In the special case of vertical wave propagation, i.e. for radial solar p modes, we shall compare the new model with the earlier model of Zhugzhda & Stix (1994), where the inhomogeneities were represented by a periodic sequence of

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vertical layers, with constant sound speed and velocity within each layer.

In the case of radial oscillations we have two versions of the structured model, the earlier layer model and the present harmonic model, both under the local conditions at each depth in the solar convection zone. In the general case of non-radial oscillations we have only the harmonic model; with this model we proceed in the same way. At each level the temperature and velocity fluctuations are taken from a mixing-length model of the solar convection zone.

The eigenfrequencies will be calculated by means of asymptotic expressions. Although such approximate frequencies differ from the exact eigenfrequencies, we may assume that the frequency *changes* will be sufficiently reliable, since they are obtained as differences between results of the structured and unstructured models, both calculated in precisely the same manner. Moreover, the asymptotic expressions permit an easier physical interpretation of the results.

2. The harmonic model

Following Zhugzhda (1998) we describe the horizontal structure at each level by simple harmonic functions. Let $2d$ be the horizontal period; then the vertical component of the velocity is

$$V(x) = \hat{c}(V_m + 2\Delta \cos \pi x/d) \, , \tag{1}$$

and the square of the sound velocity (which we use in place of the temperature) is

$$c^2(x) = \hat{c}^2(1 + 2\delta \cos \pi x/d) \, . \tag{2}$$

Eqs. (1) and (2) define a one-dimensional model for stationary convection (or for turbulent convection that is slow relative to the acoustic time scale). This is our equilibrium model which will be disturbed by acoustic waves of small amplitude. The rms value $\hat{c}$ of the sound speed generally depends on depth. We shall solve the wave propagation problem for the case $\hat{c} =$ const., but, in the spirit of a local theory, choose a value of $\hat{c}$ that corresponds to the local conditions.

The mean vertical velocity V_m is determined from the requirement of a zero net mass flow: since there is no pressure variation in the equilibrium model we may write

$$\int_{-d}^{d} V\rho \, dx \propto \int_{-d}^{d} V/c^2 \, dx = 0 \, .$$

The evaluation of the integral yields

$$V_m = \frac{\Delta}{\delta}\left(1 - \sqrt{1 - 4\delta^2}\right) \simeq 2\Delta\delta \, , \tag{3}$$

where the last approximation holds for small δ.

Let $\boldsymbol{v}(x, z, t)$ denote the velocity field of the acoustic perturbation. Zhugzhda (1998) considers perturbations $\propto \exp i(\omega t - k_z z)$, introduces the variable

$$y = \mathrm{div}\mathbf{v}/(\omega - k_z V) \, , \tag{4}$$

and derives for y the linearized wave equation

$$\frac{d}{dx}\left[\frac{c^2}{V_{ph} - V}\frac{dy}{dx}\right] + k_z^2\left[V_{ph} - V - \frac{c^2}{V_{ph} - V}\right]y = 0 \, , \tag{5}$$

where $V_{ph} = \omega/k_z(\omega)$ is a vertical phase velocity that depends on the wave frequency.

The wave equation has solutions with the same horizontal period, $2d$, as the periodically structured equilibrium model, and also solutions with twice that period. In the present paper we restrict ourselves to the former type of solution. Except for the exponential factor depending on z and t, this solution may be written in the form

$$y = \exp(ik_x x) \sum_{-\infty}^{\infty} C_{2m} \exp\left(\frac{im\pi x}{d}\right) \, . \tag{6}$$

Expression (6) is substituted into the wave equation. As the trigonometric functions are orthogonal, one obtains an infinite set of recurrence relations for the coefficients C_m. These recurrence relations are linear and homogeneous, and their determinant, an infinite *Hill determinant*, must therefore be zero. This constitutes the dispersion relation for the acoustic waves; it has been solved by Zhugzhda (1998) for a number of illustrative cases. For a given equilibrium model, and for any given frequency ω and horizontal wave number k_x, this dispersion relation defines the vertical wave number $k_z(\omega)$ and phase velocity $\omega/k_z(\omega)$.

The diverse terms of the solution (6) are coupled together. The coupling constants are the parameters δ and Δ in Eqs. (1) and (2). If the coupling is weak, then a small number of terms will approximate the infinite sum (6) with sufficient precision. For the results reported in this paper the sum was truncated after $m = \pm 3$, so that the Hill determinant was 7×7. Truncation after $m = \pm 2$ (determinant 5×5), led to nearly identical results.

3. Comparison with the layer model

In the layer model of Zhugzhda & Stix (1994) piecewise constant functions $c^2(x)$ and $V(x)$ had been used, viz.

$$\begin{aligned} c^2 &= c_1^2, \quad V = V_1 \qquad \text{in layers } 1,\ 3,\ 5 \ldots, \text{and} \\ c^2 &= c_2^2, \quad V = V_2 \qquad \text{in layers } 2,\ 4,\ 6 \ldots . \end{aligned}$$

For a comparison with the present harmonic model we choose a sequence of layers that have all the same thickness, d, i.e. the parameter λ of the layer model is 0.5. The horizontal period is $2d$, as in the harmonic model. In addition, we make a Fourier decomposition of the above piecewise constant functions, and identify the first Fourier coefficient with the amplitude of the harmonics in (1) and (2). We obtain

$$\delta = \frac{2(c_1^2 - c_2^2)}{\pi(c_1^2 + c_2^2)} \qquad \text{and} \qquad \Delta = \frac{V_1 - V_2}{\pi\sqrt{(c_1^2 + c_2^2)/2}} \, . \tag{7}$$

Finally, since only vertical wave propagation had been considered in the layer model, we set $k_x = 0$ for the comparison.

Figs. 1 and 2 demonstrate that the two models yield nearly the same result. For parameter values corresponding to the near-surface region of the Sun, namely $c_1 = 12$ km/s, $c_2 = 8$ km/s, and $V_1 = 2$ km/s, Fig. 1 shows the vertical phase velocities of the upward and downward propagating acoustic waves. The

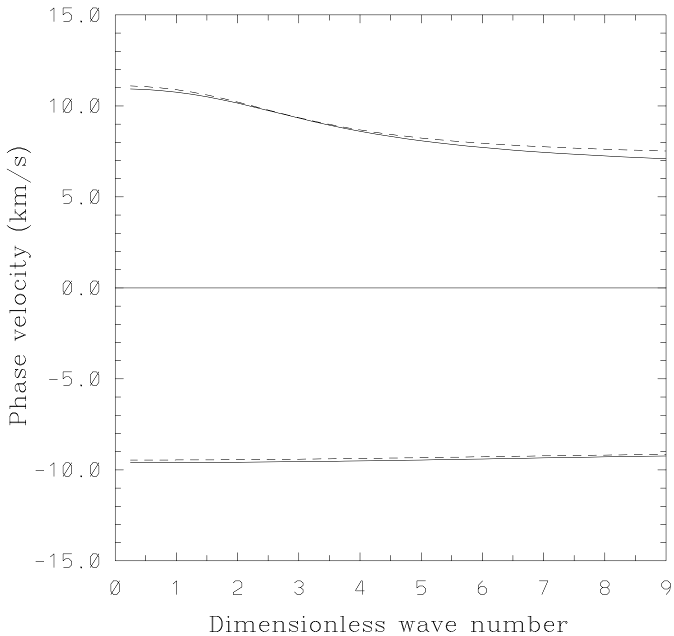

Fig. 1. Phase velocity of vertically propagating acoustic waves, for constant coefficients. Layer model (*dashed*), for the velocity values specified below Eq. (7); harmonic model (*solid*), for $\delta = 0.245$, $\Delta = 0.09$, as determined by (7)

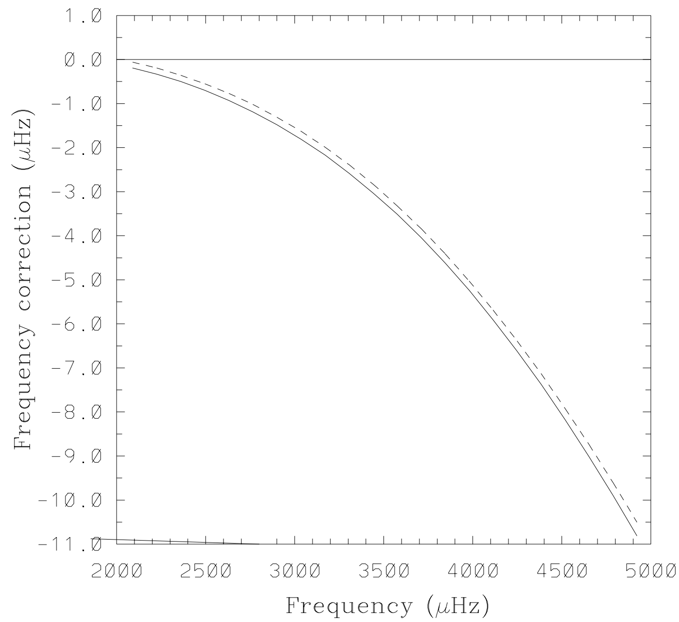

Fig. 2. Frequency correction for radial solar p modes, for coefficients derived from a mixing-length model of the convection zone. Layer model (*dashed*), harmonic model (*solid*)

ensuing frequency corrections for solar p modes are calculated as in Zhugzhda & Stix (1994): The condition

$$\oint k_z dz = 2n\pi \tag{8}$$

is solved for the frequency ν, both in the case where k_z is taken from the dispersion relation of the structured model (applied locally at each depth in the convection zone) and in the reference case where $k_z = 2\pi\nu/c(r)$. The difference of the two results is the frequency correction $\Delta\nu$ for overtone number n. This correction is shown in Fig. 2 for the two structured models. As in Zhugzhda & Stix (1994), we have related the horizontal period of the model to the local pressure scale height through $d = f_S \alpha H_P$, with a size parameter $f_S = 2.5$. In the same manner as in that paper we have also obtained the local fluctuation amplitudes δ and Δ from the mixing-length results ΔT_{mod} and V_{mod} of a standard solar model (Stix & Kiefer 1997), cf. Eq. (7) above and Eqs. (55)–(57) of Zhugzhda & Stix (1994):

$$\delta = \frac{2}{\pi}\frac{\Delta T_{mod}}{T_{mod}} , \qquad \Delta = \frac{2}{\pi}\frac{V_{mod}}{\hat{c}} . \tag{9}$$

4. Frequencies of non-radial p modes

Non-radial solar p modes correspond to acoustic waves propagating at a finite inclination to the vertical direction. In the present model this means $k_x \neq 0$; for given degree l we obtain k_x as a function of depth,

$$k_x = \frac{\sqrt{l(l+1)}}{r} . \tag{10}$$

As for the radial pulsations, we calculate the eigenfrequencies by means of the asymptotic formula (8), except that we now add a constant phase shift $2\pi\hat{\alpha}$ on the right which accounts for the fact that the reflection does not occur at rigid walls but at adjacent evanescent regions (cf. Christensen-Dalsgaard et al. 1985):

$$\oint k_z dz = 2\pi(n + \hat{\alpha}) . \tag{11}$$

We take values between 1 and 1.5 for the constant $\hat{\alpha}$. The results reported below do not greatly depend on this choice; this is plausible because both the reference and the corrected frequencies are calculated from (11), and we are only interested in their difference. The integral extends from the solar surface to the lower turning point and back to the surface. For modes with turning points within the convection zone the depths obtained for k_{z+} and k_{z-} slightly differ from each other, and from the turning point of the unstructured reference case. But these differences are small, and for the actual calculations we take the turning point of the reference case. According to test calculations this has no significant effect on the results.

In the reference case of the unstructured model the vertical wave number is

$$k_{z\pm} = \pm\sqrt{\left(\frac{2\pi\nu}{c}\right)^2 - k_x^2} , \tag{12}$$

where k_{z+} and k_{z-}, respectively, must be used for the upward and downward parts of the loop integral (11). In the structured model, $k_{z+}(\nu)$ and $k_{z-}(\nu)$ are obtained from setting the Hill determinant to zero. This is done at each depth, with the appropriate local values of the variables δ, Δ, d, and k_x substituted.

Fig. 3 shows the frequency correction for p modes of small and intermediate degree l. For small degree, $l \lesssim 10$, the result

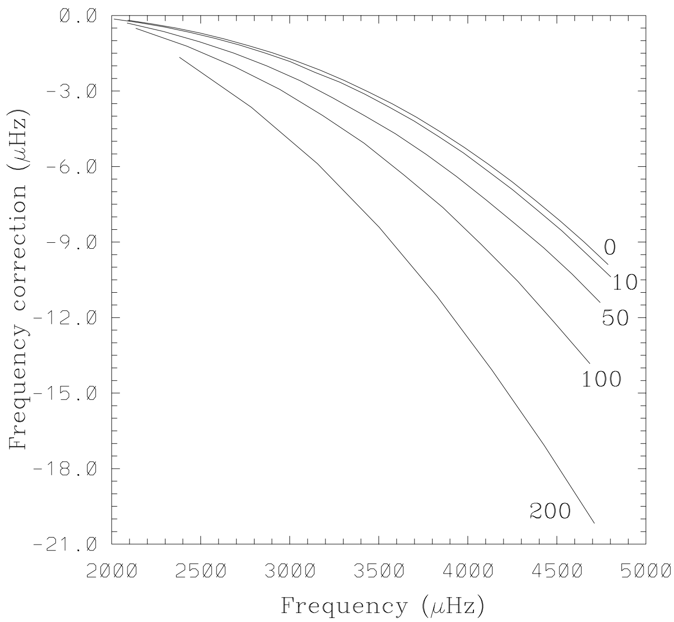

Fig. 3. Frequency correction for solar p modes, calculated with the harmonic model. The label is the degree l of the mode; the size parameter is $f_S = 2.5$.

is rather close to the radial case, $l = 0$. These modes have their inner reflection well below the convection zone, and propagate almost vertically within that zone, and in particular so within the shallow layer where the convective structure has a noticeable effect.

For larger degree the corrections become markedly more negative. At this point we may check our results for the scaling with harmonic degree. Taking the inertia ratios Q_{nl} from Fig. 13 of Christensen-Dalsgaard & Berthomieu (1991) we find that Q_{nl} times the calculated frequency shift indeed depends only on frequency, but not on l, within the accuracy that can be obtained graphically. This is to be expected for an effect that originates in a layer close to the surface. Nevertheless, in order to illustrate the l-dependence of the frequency shift, we show the unscaled results in Fig. 3.

The absolute magnitude of the frequency shift, as well as its increase with frequency, appears to be welcome in view of the discrepancy between the observed p-mode frequencies (e.g. Libbrecht et al. 1990) and the theoretical eigenfrequencies of standard solar models. Our own solar models, cf. Stix & Kiefer (1997), have such eigenfrequencies; other authors find similar results, e.g. Noels et al. (1984), Cox et al. (1989), Turck-Chièze & Lopes (1993), or Morel et al. (1997). Although these models have been calculated with different input such as the form of the equation of state and the opacity table, they all show a small frequency excess that increases with increasing frequency, and has a dependence on l in accord with the mode inertia scaling.

5. Mode coupling

The calculation of the frequency corrections shown in Fig. 3 is straightforward as long as the degree, l, of the eigenmode is not too large. A complication arises however when l becomes so large that the horizontal wavelength of the oscillation is of the same order as the size of the convective eddies, that is, as the period $2d$ of our model. The critical condition is $k_x = \pi/2d$ or, for large l, $r\pi = 2lf_S\alpha H_P$. The depth where this condition is satisfied strongly depends on the degree l, as Fig. 4 illustrates for two values of f_S. The larger l, the smaller is the critical depth. Hence, for larger l the critical depth lies closer to the shallow surface layer within which the fluctuation amplitudes δ and Δ reach appreciable values.

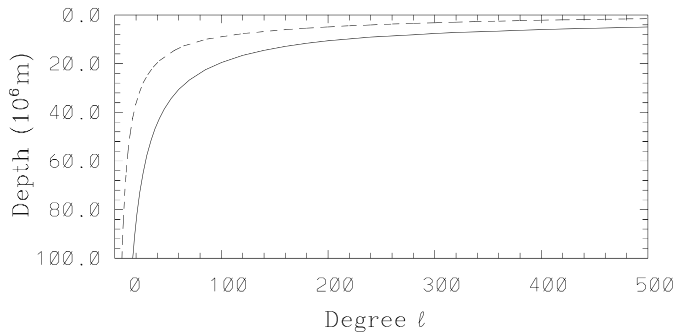

Fig. 4. The depth below the solar surface where $k_x = \pi/2d$, for $f_S = 1$ (*solid*) and $f_S = 2.5$ (*dashed*)

The interesting phenomenon that occurs at the critical depth is *mode coupling*. The Hill determinant has as many solutions as rows, of which only one corresponds to the acoustic mode that is our main concern here. The other solutions correspond to *vibrational modes*, cf. Zhugzhda (1998). At the depth where $k_x = \pi/2d$ the acoustic mode attains the same phase velocity as one of the vibrational modes. In the case of horizontal inhomogeneity, i.e. $\delta \neq 0$ and/or $\Delta \neq 0$, the modes couple together.

In order to see the mode coupling we consider the smallest non-trivial Hill determinant, with three rows. Restricting the attention to the case $\Delta = 0$, where only a sound speed variation is present, we find the dispersion relation

$$\left(\omega^2 - \omega_L^2 - k_z^2\hat{c}^2\right)\left[\omega^2 - (\omega_L + 2\omega_c)^2 - k_z^2\hat{c}^2\right] \times \left[\omega^2 - (\omega_L - 2\omega_c)^2 - k_z^2\hat{c}^2\right] = 2\delta^2\Bigg[\left(\omega^2 - \omega_L^2 - 4\omega_c^2 - k_z^2\hat{c}^2\right) \times \left(\left(\omega_L^2 + k_z^2\hat{c}^2\right)^2 + 4\omega_L^2\omega_c^2\right) + 16\omega_L^2\omega_c^2\left(\omega_L^2 + k_z^2\hat{c}^2\right)\Bigg], \quad (13)$$

where

$$\omega_L = k_x\hat{c}$$

is the Lamb frequency, corresponding to horizontal propagation, and

$$\omega_c = \pi\hat{c}/2d$$

is an eigenfrequency of the convective eddy, resp. of our horizontal layer structure. Let us introduce the dimensionless vari-

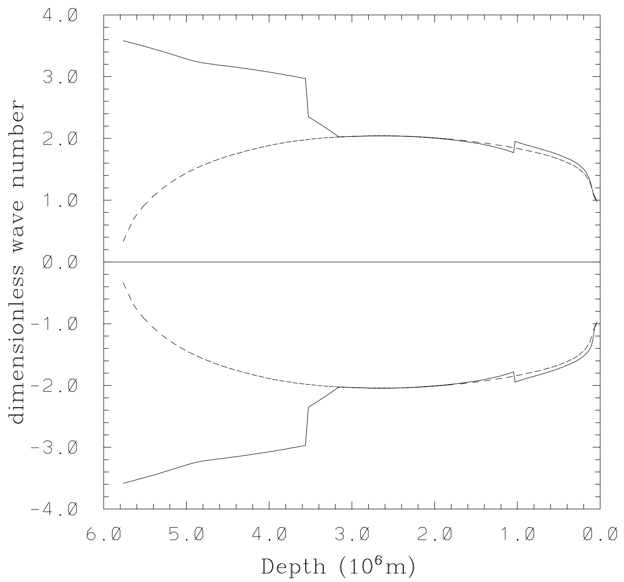

Fig. 5. Dimensionless vertical wave numbers $(2d/\pi)k_{z\pm}$ of the upwards and downwards propagating acoustic waves, for the mode p2, degree $l = 600$. At a depth of $\approx 3.2 \cdot 10^6$m the solution of the Hill determinant (*solid*) switches over from the acoustic to a vibrational mode. In our calculation we have avoided such transitions; instead, the calculation follows the acoustic mode, which runs close to the acoustic mode of the unstructured reference case (*dashed*).

ables

$$\Omega = \omega/\omega_L, \quad \Omega_c = \omega_c/\omega_L, \quad K = k_z/k_x \ . \tag{14}$$

The three solutions of (13) are then, for small δ,

$$\Omega^2 \approx 1 + K^2 - 2\delta^2 \left[1 - \frac{(1-K^2)^2}{4(1-\Omega_c^2)}\right] \ , \tag{15}$$

$$\Omega^2 \approx (1+2\Omega_c)^2 + K^2 + \frac{\delta^2}{4\Omega_c}\frac{(1+K^2+2\Omega_c)^2}{1+\Omega_c} \ , \tag{16}$$

$$\Omega^2 \approx (1-2\Omega_c)^2 + K^2 - \frac{\delta^2}{4\Omega_c}\frac{(1+K^2-2\Omega_c)^2}{1-\Omega_c} \ . \tag{17}$$

The first is the acoustic mode, the second and third are fast and slow vibrational modes.

The critical depth is now determined by $\Omega_c = 1$. Expressions (15) and (17) make clear that this is indeed the critical condition; in fact these approximations become invalid near $\Omega_c = 1$. For $\delta = 0$ the acoustic and the slow vibrational modes coincide when $\Omega_c = 1$. For $\delta \neq 0$ a crossover of these two modes occurs at $K = 1$. It should be kept in mind that Ω_c decreases as one goes from the solar surface into the convection zone.

Further vibrational modes appear when more than three rows of the Hill determinant are taken into account. This implies further mode 'crossings'. These 'crossings' occur at depths where $k_x = n\pi/2d$, i.e. where the horizontal period of our model is an integer multiple of one-half of the horizontal wave length of the oscillation. Strictly speaking there is no real mode crossing at these points; instead, the crossing is 'avoided', and the acoustic mode connects to one of the vibrational modes while a vibrational mode turns into the acoustic mode. Although in principle there may thus be an ambiguity about the nature of the modes, in practice there is no confusion about the acoustic mode, which always lies very close to the acoustic mode of the unstructured model. In the numerical calculation of $k_{z\pm}$, however, the acoustic mode could be lost in the vicinity of a 'crossing'. An example is shown in Fig. 5, where the two vertical wave numbers required for the up and down parts of the integral (11) are drawn as functions of depth. Two 'crossings' occur at depths of $\approx 1.1 \cdot 10^6$m and $\approx 3.2 \cdot 10^6$m, respectively. Obviously in this illustrative case, the calculation turned away from the acoustic mode into a vibrational mode at the second crossing. However, both δ and Δ are already rather small at the depth in question; nothing dramatic should therefore happen at that depth.

Table 1. Frequency corrections, in μHz, for high-degree p modes.

l	200	400	600	800	1000	1200	1400
p 1	-0.2	-3.3	-7.0	-7.7	-4.9	0.6	5.2
p 2	-1.7	-8.7	-14.9	-14.3	-8.1	1.1	8.4
p 3	-3.6	-14.5	-24.2	-21.5	-11.6	0.4	9.8
p 4	-5.9	-20.6	-33.1	-28.9			

If care is taken that a run-away such as shown in Fig. 5 does not occur, we may determine the frequency corrections for acoustic modes of large degree. A few results for the ridges p1 to p4 in the (l, ν)-plane are shown in Table 1. For the calculation of these results we have always selected the mode in closest vicinity to the reference mode (12) of the unstructured model.

Fig. 5 also helps to understand the results listed in Table 1. It is striking that the increase of the (unscaled) frequency corrections with increasing degree does *not* continue for very large l. Instead, the corrections become smaller and finally even reverse in sign. The reason for this behaviour is that the layer where $k_x < \pi/2d$ and, therefore, a negative contribution is obtained shrinks with increasing l. On the other hand, the layer beneath the critical level, which yields a positive contribution moves upwards, closer to the region where the fluctuation amplitudes δ and Δ are significant, and thus becomes more effective. For $l = 1400$ the maxima of these amplitudes fall already into the layer with positive contribution. The change in sign of the contribution to the frequency corrections is a consequence of the mode crossing. It is clearly visible in the example of Fig. 5: Within the first $\approx 10^6$ m, the larger wave number corresponds to a retarded phase velocity, below that level the smaller wave number means enhanced phase velocity.

6. Discussion

In this paper we have extended our previous study of the effect of convection on the solar eigenfrequencies to the case of non-radial p modes. We should like to point out here that most of this effect arises from the velocity fluctuation, while only a small fraction comes from the sound speed fluctuation, as already anticipated by Brown (1984). One reason for this difference is that $\delta \propto \nabla - \nabla'$, $\Delta \propto (\nabla - \nabla')^{1/2}$ and, therefore, δ decreases with depth much faster that Δ (∇' is the mean gradient $d\ln T/d\ln P$ of the convective eddies, e.g. Stix 1989). Another reason is the apparent stronger coupling of the harmonic term in the sum (6) by the velocity fluctuation as compared to the coupling by the sound speed fluctuation. Formally this can be seen by inspection of the Hill determinant (Zhugzhda 1998): The second and third off-diagonals have non-zero elements only if $\Delta \neq 0$.

It is worth emphasizing that the combined effect of the sound speed and velocity fluctuations leads to a marked asymmetry between the upwards and downwards propagating acoustic waves. As already explained in our earlier contribution, the two effects reinforce each other for the upward wave, but oppose each other for the downward wave. In a model with many vertical columns or layers the picture of a mean wave velocity $c/(1 - v^2/c^2)$ (Brown 1984, Rosenthal 1997) is therefore generally insufficient. It would apply only in a single stream where one only needs to average the inverse up and down wave velocities.

Since the velocity fluctuation dominates our calculated frequency correction, and since the largest velocity amplitude occurs just below the solar surface, we may ask whether perhaps overshoot into the stable solar atmosphere further enhances the effect. We have checked this question, extending the region with $\Delta \neq 0$ by up to two pressure scale heights above the maximum of the convective velocity, with a constant gradient. This much overshoot seems to exist on the Sun according to spectroscopic results (Nesis & Mattig 1989, Komm et al. 1991) and to a recent calculation of M. Kiefer (1997, private communication). We find that the effect on the frequency corrections is very small for small and intermediate degree, but becomes substantial for large degree, $l \gtrsim 1000$, say. For large-l modes the corrections become more negative (or less positive, cf. Table 1). The resonance cavity of these modes is rather shallow, hence the addition of the overshoot layer matters more; it extends the uppermost part of the cavity which makes a negative contribution to the frequency correction, as demonstrated in the preceeding section.

Rüdiger et al. (1997) used the mean-field concept in order to investigate the influence of turbulence on the p-mode frequencies. Although their approach is rather different, some results are similar. Their "redshift", originating from the negative density-temperature correlation, parallels our negative frequency corrections arising from the sound speed fluctuation. They also consider a mean flow as a consequence of mass conservation, as we do, cf. Eq. (3); in both approaches the effect is rather small. The main difference to the present treatment concerns the *velocity* fluctuation. Whereas in our study the local velocity produces a Doppler effect that in most cases yields a net frequency *decrease*, or "redshift", of the global eigenmodes, Rüdiger et al. find a "blueshift" originating from the turbulence pressure term of the Reynolds stress tensor. It should be kept in mind however that the turbulence pressure can be fully assessed only in conjunction with the solar equilibrium model itself.

An open question is to what extent the mode crossing described in Sect. 5 is relevant to the non-radial solar p modes. Of course the solar convection is not a regular pattern like a crystal, and there is no single "grid constant" such as the horizontal period $2d$ of our model. Nevertheless, at each depth the convective flow and temperature structure have a typical scale, and some interesting interaction should take place when the horizontal wave length of a p mode is of the same order. In our model we found a coupling of the acoustic mode to the vibrational modes that owe their existence to the horizontal structure. We wonder whether a similar coupling to structure-dependent wave modes, or *scattering*, occurs on the Sun.

The frequency corrections calculated in this contribution depend on our model and on the special choice of the model parameters. In particular the choice of f_S is crucial. Since we do not know the best value of this parameter we may only guess that $f_S = 2.5$ is a plausible choice. In any case, the sign, the order of magnitude, and the frequency dependence of the corrections appear to be right. In a complete assessment of the discrepancy between calculated and measured solar eigenfrequencies such corrections should be taken into account, in addition to the corrections arising from improvements of the equilibrium model of the Sun. One may even hope that the present theory can be developed in a way that allows to determine parameters like f_S that are otherwise unknown, and so to make a first step towards a seismology of convection.

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References

Brown, T. M., 1984, Science 226, 687
Christensen-Dalsgaard, J., 1986, in Seismology of the Sun and the Distant Stars, (Gough, D. O., Ed.), Reidel, Dordrecht
Christensen-Dalsgaard, J., Berthomieu, G., 1991, in Solar Interior and Atmosphere, (Cox, A. N., Livingston, W. C., Matthews, M. S., Eds.), Univ. Arizona Press, Tucson, p. 401
Christensen-Dalsgaard, J., Duvall, T. L., Jr., Gough, D. O., Harvey, J. W., Rhodes, E. J., Jr., 1985, Nature, 315, 378
Christensen-Dalsgaard, J., Thompson, M. J., 1997, MNRAS 284, 527
Cox, A. N., Guzik, J. A., Kidman, R. B., 1989, ApJ 342, 1187
Komm, R., Mattig, W., Nesis, A., 1991, A&A 243, 251
Libbrecht, K. G., Woodard, M. F., Kaufman, J. M., 1990, ApJ Suppl. 74, 1129
Morel, P., Provost, J., Berthomieu, G., 1997, A&A 333, 444
Nesis, A., Mattig, W., 1989, A&A 221, 130
Noels, A., Scuflaire, R., Gabriel, M., 1984, A&A 130, 389
Rosenthal, C. S., 1997, in Solar convection and oscillations and their relationship, (Pijpers, F. P., Christensen-Dalsgaard, J., Rosenthal., C. S., Eds.), Aarhus, p. 145
Rüdiger, G., Hasler, K.-H., Kitchatinov, L. L., 1997, Astron. Nachr. 318, 173
Stix, M., 1989, The Sun, Springer Berlin Heidelberg
Stix, M., Kiefer, M., 1997, in Solar convection and oscillations and their relationship, (Pijpers, F. P., Christensen-Dalsgaard, J., Rosenthal., C. S., Eds.), Aarhus, p. 69
Turck-Chièze, S., Lopes, I., 1993, ApJ 408, 347
Zhugzhda, Y. D., 1997, in Solar convection and oscillations and their relationship, (Pijpers, F. P., Christensen-Dalsgaard, J., Rosenthal., C. S., Eds.), Aarhus, p. 181
Zhugzhda, Y. D., 1998, A&A 332, 314
Zhugzhda, Y. D., Stix, M., 1994, A&A 291, 310