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ASTRONOMY AND ASTROPHYSICS

On MHD fluxes in the solar convection zone

U. Narain[1,2], **P. Agarwal**[1,2], **and R.K.Sharma**[1,2]

[1] Astrophysics Research Group, Department of Physics, Meerut College, Meerut-250001, India
[2] Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune-411007, India

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Abstract. We have estimated MHD fluxes in the solar convection zone using the best available data on turbulent (convective) velocities in combination with the formulation of Stein (1981) and suitable efficiency factors. The estimated MHD fluxes compare well with the total solar flux.

Key words: Sun: abundances – Sun: corona – Sun: magnetic fields – Sun: X-rays, gamma rays

1. Introduction

Narain & Ulmschneider (1990, 1996) have reviewed various physical mechanisms of chromospheric and coronal heating. Heating by Magnetohydrodynamic (MHD) waves is one of them. The amount of wave energy generated in stellar (solar) convection zones is one of the most important aspects of the heating by waves (Musielak 1991) although Porter et al. (1994) have a different point of view.

The generation of MHD waves by turbulent fluid motions in a homogeneous medium has been considered by a number of authors (cf. Kulsrud 1955, Osterbrock 1961, Parker 1964, Kuperus 1965, Kato 1968). Their approach has been extended to include corrections resulting from gravitational stratification and turbulent energy spectrum (Moore & Spiegel 1964, Stein 1967, 1968, 1981, Ulmschneider & Stein 1982, Bohn 1984, Musielak & Rosner 1987, 1988, Goldreich & Kumar 1988, Rosner & Musielak 1989). Unfortunately, neither theory nor observations are as yet able to define the shape of the turbulent spectrum for stratified and magnetized atmospheres correctly. Reasonable progress in this direction has been made by Musielak et al. (1994).

Recently Ulmschneider et al.(1996) investigated acoustic wave energy fluxes for late-type stars using the improved model of Musielak et al. (1994) for the solar convection zone. It seems quite worthwhile to combine the aforesaid model with the formulation of Stein (1981) to obtain MHD fluxes for the Sun without going into complicated details.

The outline of our paper is as follows: In the next section we give necessary theory. The data used and the results obtained are presented in Sect. 3. We discuss the results obtained and summarize our conclusions in Sect. 4. Throughout c.g.s. system of units is used, except that the depth in the convection zone is in km.

Send offprint requests to: U. Narain

2. Theory

Turbulent motions in the solar convection zone act as a source of waves. In the presence of magnetic fields the energy density of the turbulent motions is given by

$$\epsilon = (\rho v_t^2/2) + (\delta B^2/8\pi), \tag{1}$$

where ρ is the matter density, v_t is the turbulent (convective) velocity and δB is the turbulent magnetic field. In a strong background magnetic field, B_0, Alfvén and slow mode waves are produced by monopole emission but fast mode waves are produced by quadrupole emission (Stein 1981). In view of the above the power density in the Alfvén mode is given by

$$P_A = \eta_A \epsilon v_t^2/\ell v_A, \tag{2}$$

where ℓ is the scale length of convection or the size of the eddy producing MHD waves and η_A is an efficiency factor given by

$$\eta_A = \delta B^2/(B_0^2 + \delta B^2 + 4\pi\rho v_t^2). \tag{3}$$

The Alfvén speed, v_A, in Eq. (2), is obtained from

$$v_A = B_0/(4\pi\rho)^{0.5}. \tag{4}$$

The power density in the slow mode may be obtained by using

$$P_s = \eta_s \epsilon v_t^2/\ell v_s. \tag{5}$$

Here, v_s, the adiabatic sound speed, is given by

$$v_s = (\gamma p/\rho)^{0.5}, \tag{6}$$

where p is the pressure and γ is the ratio of specific heats at constant pressure and constant volume ($\gamma \equiv C_p/C_v$). The efficiency factor, η_s, in Eq. (5), may be estimated from

$$\eta_s = 4\pi\rho v_t^2/(B_0^2 + \delta B^2 + 4\pi\rho v_t^2). \tag{7}$$

The power density in fast mode is given by

$$P_f = \eta_A \epsilon v_t^6/\ell v_A^5 \tag{8}$$

To obtain the energy flux, F, of MHD waves due to all turbulent eddies in the convection zone one may use the formula (Osterbrock 1961)

$$F = (1/2) \int_{\Delta z} P dz, \quad (9)$$

where Δz is the thickness of the layer producing MHD waves in the convection zone.

The scale length of the convection, ℓ, is not precisely known. Following Musielak et al. (1994), we take

$$\ell = \alpha H, \quad (10)$$

where $\alpha (= 1.0, 1.5, 2.0)$ is mixing length parameter and local pressure scale height H, is given by

$$H = kT/\mu g m_H. \quad (11)$$

Here k is the Boltzmann constant, T is the temperature in K, g is acceleration due to gravity, μ is the mean molecular weight and m_H is the mass of the hydrogen atom. $\alpha = 5.0$ corresponds to solar granules (Lee 1993).

In order to have an idea about the equipartition turbulent magnetic field, b_t, following expression may be used:

$$b_t = 2 v_t (\pi \rho)^{0.5}. \quad (12)$$

3. Data and results

The pressure, mass density, temperature and mean molecular weight data are taken from Stix (1989). The data for turbulent (convective) velocity and γ are procured from Musielak et al. (1994). g is given constant value $2.74 \times 10^4 cm s^{-2}$.

The background magnetic field values are from Zwaan & Harvey (1994). According to them the equipartition field strength is around $500G$, the stronger magnetic field of $1000G$ and $3000G$ are associated with the network elements and Sunspots, respectively.

The Alfvén, fast and slow mode fluxes are calculated using Eqs. (1) through (11) for different turbulent magnetic fields, δB, and the mixing length parameter, α, at $B_0 = 500G$.

Figs. 1, 2 exhibit matter density and temperature as a function of depth, z, in the convection zone, respectively. We adopt Stix's data. The other data are exhibited for the sake of comparison only.

Fig. 3 exhibits turbulent equipartition field strength, b_t, as a function of depth, z, in the convection zone whereas Fig. 4 exhibits turbulent kinetic energy density ($\rho v_t^2/2$) and the turbulent magnetic energy density ($\delta B^2/8\pi$) as a function of depth,z.

In Figs. 5 and 6 we exhibit power density as a function of depth in the convection zone for Alfvén, fast and slow mode waves for $B_0 = 500G$ and $\delta B = 100G, 400G$, respectively.

Fig. 7 exhibits MHD wave fluxes as a function of turbulent magnetic field strength, δB, at $B_0 = 500G$. Individual contributions of Alfvén, slow and fast modes are also shown there. For the sake of comparison the total solar flux is also exhibited.

In Fig. 8 we exhibit total MHD fluxes for $\alpha = 1.0, 1.5$ and 2.0 at $B_0 = 500G$.

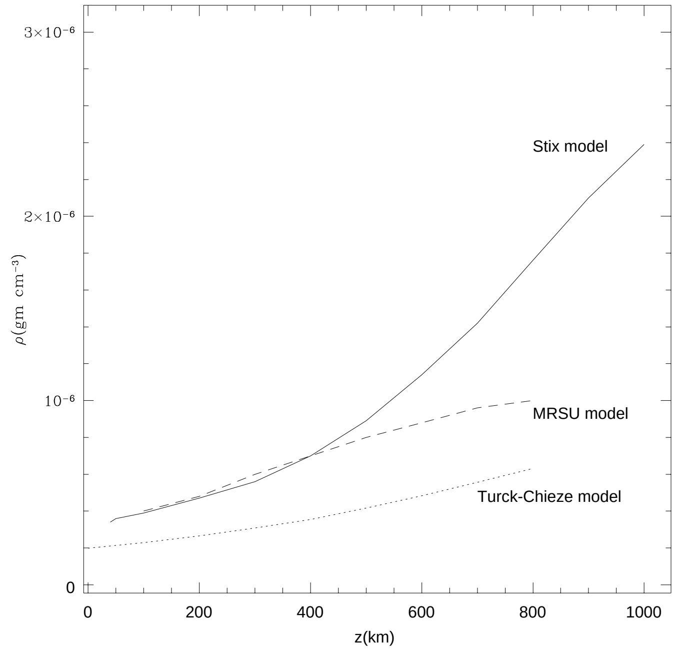

Fig. 1. Matter density, ρ, as a function of depth (z) in the convection zone. Solid line → Stix (1989), dashed line → Musielak et al. (1994), dotted line → Turck-Chiéze et al. (1988).

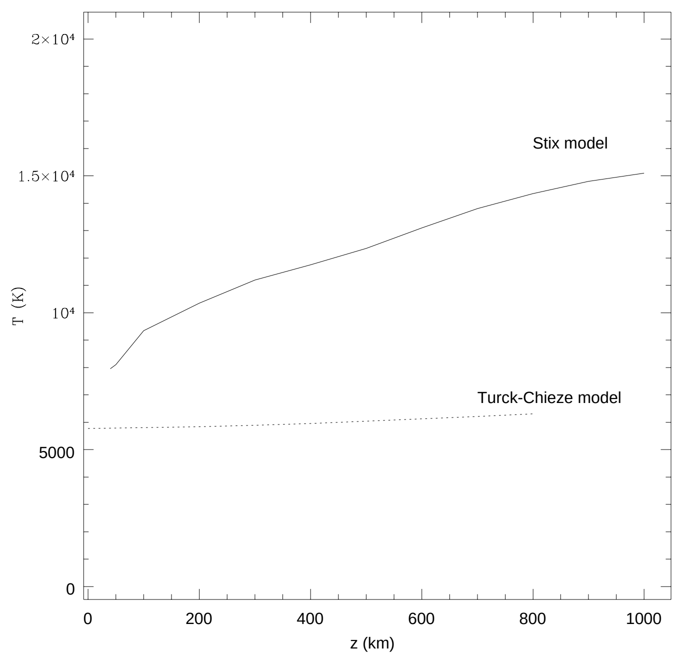

Fig. 2. Temperature, T, as a function of depth (z) in the convection zone. Solid line → Stix (1989), dotted line → Turck-Chiéze et al. (1988).

4. Discussion and conclusion

Fig."1 exhibits matter density with depth in the convection zone for three sets of data (Stix 1989, Musielak et al. 1994, Turck-Chiéze et al. 1988). The data of Musielak et al. (MRSU) agree with those of Stix (1989) up to $z = 450 km$ and diverge after that. The third set of data is always smaller than the other two

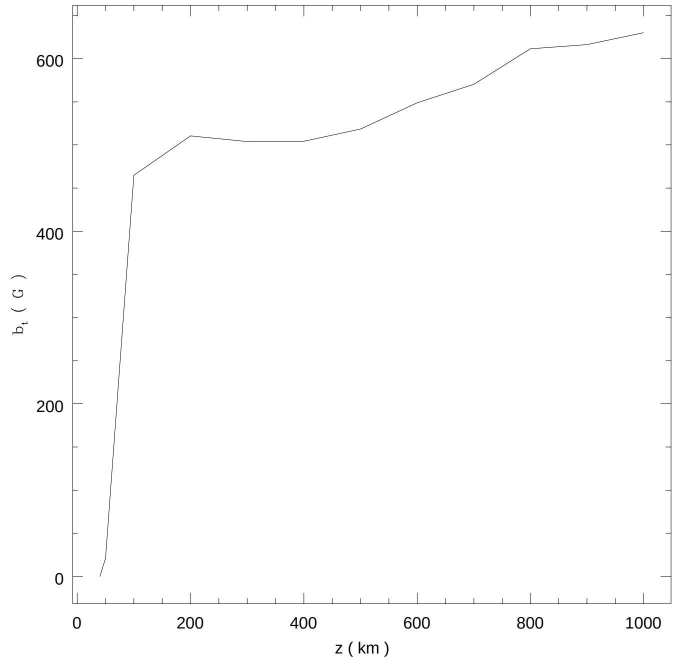

Fig. 3. Turbulent equipartition field strength (b_t) as a function of depth (z) in the convection zone.

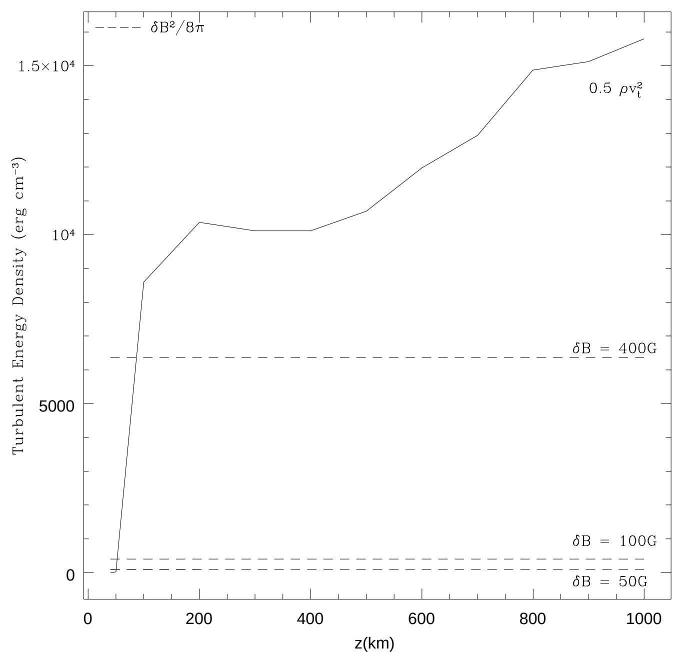

Fig. 4. Turbulent kinetic energy density ($\rho v_t^2/2$) and the magnetic energy density ($\delta B^2/8\pi$) as a function of depth (z).

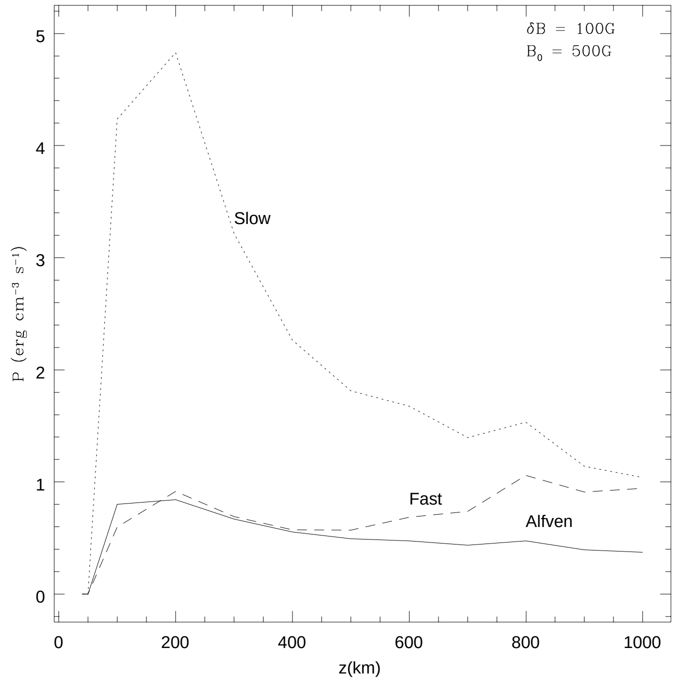

Fig. 5. Power density as a function of depth (z) in the convection zone for MHD waves for $B_0 = 500G$ and $\delta B = 100G$.

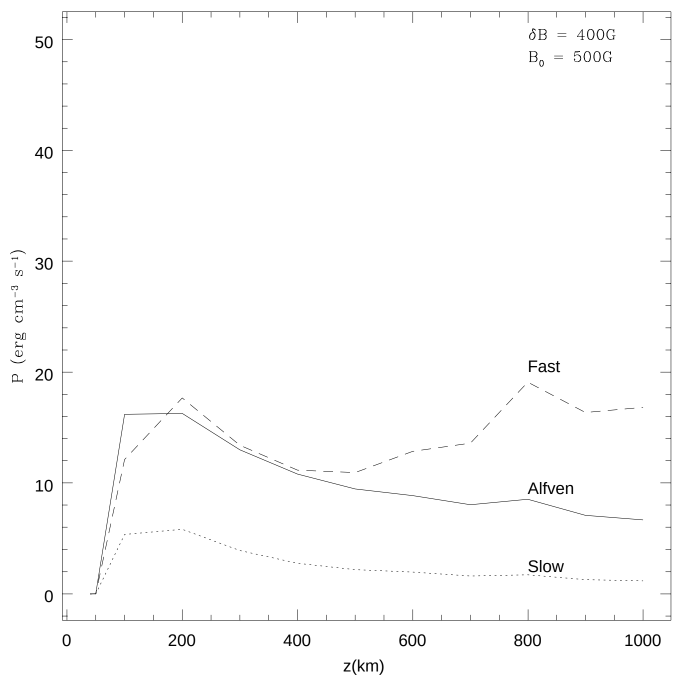

Fig. 6. Power density as a function of depth (z) in the convection zone for MHD waves for $B_0 = 500G$ and $\delta B = 400G$

mentioned above. Although we have adopted the set of data given by Stix (1989) but MRSU data could be equally good and no significant changes are likely to occur.

Fig.”2 shows the variation of temperature with depth in the convection zone as given by two sources (Stix 1989, Turck-Chiéze et al. 1988). Both sets show increasing tendency with depth but their rates of increase are different. The data used by us (Stix 1989) show increase at a faster rate than those of Turck-Chiéze et al. (1988).

It is clear from Fig. 3 that the equipartition turbulent field strength varies from about $500G$ to $600G$ in the depth range $100-900km$. Zwaan & Harvey (1994) have quoted an average value of $500G$ for the convection zone. Thus the mass density

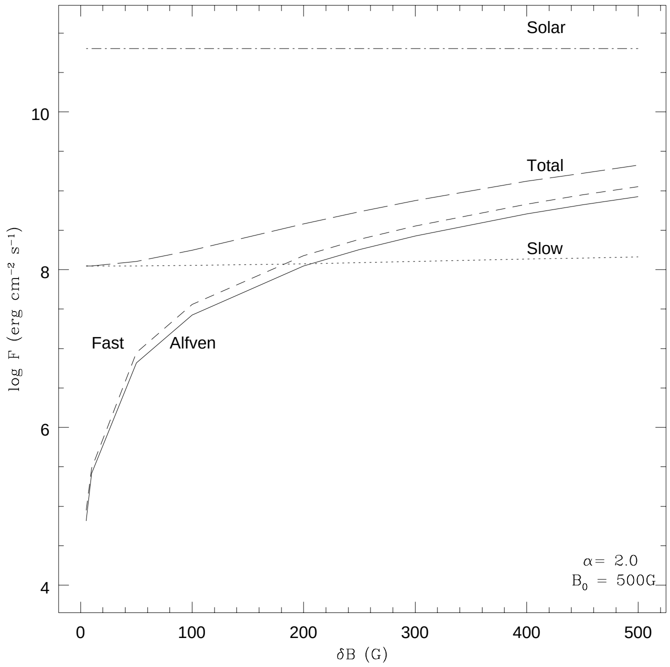

Fig. 7. MHD wave fluxes as a function of turbulent magnetic field strength, δB, at $B_0 = 500G$.

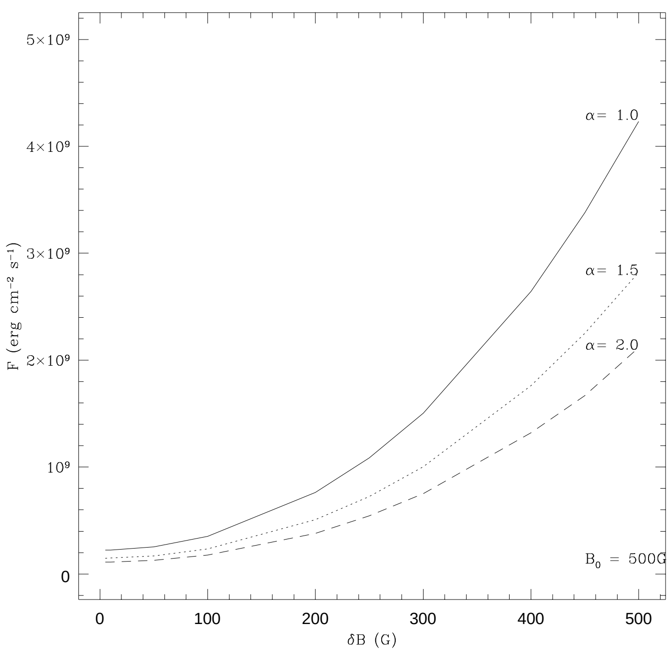

Fig. 8. Total MHD wave fluxes as a function of turbulent magnetic field strength, δB, for $\alpha = 1.0, 1.5$ and 2.0.

and the turbulent velocity data seem to be consistent with the above observation.

Fig."4 shows that the turbulent kinetic energy density increases from $8 \times 10^3 ergcm^{-3}$ at $z = 100km$ to $3 \times 10^4 ergcm^{-3}$ at $z = 1000km$, almost monotonically. Part of this energy is converted into MHD wave energy. The turbulent magnetic energy is quite insignificant for $\delta B = 50G, 100G$. At $\delta B = 400G$ it is about $6 \times 10^3 ergcm^{-3}$ and is comparable to the turbulent kinetic energy at $z = 100km$. For $z < 100km$ the turbulent magnetic energy dominates the kinetic energy when $\delta B = 400G$. The second set of expressions of Stein (1981) may be used in this region but the first set of relations should be used for $z \geq 100km$. It is not clear whether such strong turbulent magnetic fields ($\delta B = 400G$) exist in the convection zone or not (Lee 1993).

We have used efficiency factor η_A for magnetic waves (phase speed v_A) and η_s for acoustic waves (phase speed v_s). The efficiency, in general, may be defined as the ratio of the energy converted to acoustic or magnetic waves to the total energy of the eddies producing MHD waves. Since the energy converted to acoustic or magnetic waves is not known, we have approximated the efficiencies by Eqs. (3) and (7) by assuming that η_s should be equal to the ratio of the turbulent kinetic energy density ($\rho v_t^2/2$) to the total energy density (which includes the energy density due to the background magnetic field also) for acoustic waves and η_A should be equal to the ratio of turbulent magnetic energy density ($\delta B^2/8\pi$) to the total energy density. This approximation seems quite reasonable for η_s because the turbulent kinetic energy densities are known to reasonable accuracy. Unless the turbulent magnetic fields as a function of depth in the convection zone are known, η_A cannot be determined more precisely.

It is clear from Fig. 5 that slow MHD modes are produced predominantly in the 100 - 400 km zone. The Alfvén and fast mode waves are produced almost uniformly from $z = 100km$ to $z = 1000km$. This scenario emerges when turbulent motions dominate the turbulent magnetic field ($\delta B = 100G$). If the turbulent magnetic field is stronger ($\delta B = 400G$) (cf. Fig. 6) the fast and Alfvén modes are produced prominently. The production of slow modes is almost uniform from $z = 100km$ to $z = 1000km$.

In Fig. 7 the Alfvén fast and slow mode and total MHD fluxes are shown to vary with the turbulent magnetic field strength, δB. As expected, the slow mode flux increases quite slowly. The Alfvén and fast mode fluxes increase appreciably with δB. Whereas the slow mode flux remains of the order of $10^8 ergcm^{-2}s^{-1}$, the Alfvén and fast mode fluxes increase from $10^5 ergcm^{-2}s^{-1}$ at $\delta B = 5G$ to $10^9 ergcm^{-2}s^{-1}$ for $\delta B \geq 400G$ in presence of background magnetic field $B_0 = 500G$. Lee (1993) also obtains Alfvén fluxes of the same order. All these fluxes are smaller than the total solar flux ($6.39 \times 10^{10} ergcm^{-2}s^{-1}$).

Fig."8 shows that the total MHD fluxes increase with turbulent magnetic field, as expected but they decrease with the mixing length parameter, α. The later behaviour implies that smaller ($\alpha = 1.0$) eddies are more efficient in producing MHD waves than larger ($\alpha = 2.0$) ones when turbulent magnetic fields are comparable to equipartition field strength. For smaller turbulent magnetic fields the dependence on α is not appreciable.

It may be concluded that the present simple approach leads to reasonable MHD fluxes in the convection zone. Accurate turbulent magnetic fields as a function of depth should make this approach more useful.

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