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Model of the nongravitational motion for Comet 32P/Comas Solá

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Abstract. The nongravitational motion of the periodic comet Comas Solá is studied on the basis of positional observations made during nine consecutive revolutions around the Sun. Nongravitational effects in the comet motion have been examined for Sekanina's forced precession model of the rotating nucleus. We present three models which successfully link all the observed apparitions of the comet during 1926-1996. Two solutions (Models II and III) represent oblate spheroids and the third one (Model I) – a prolate spheroid (nucleus rotation around its longer axis). We have determined values of eight parameters: A, η, I, ϕ connected with the rotating comet nucleus, f_p and s describing the precession of spin-axis of the nucleus, and two constant time shifts τ_1 and τ_2. The last two parameters describe displacements of the maximum value of the known function $g(r)$ with respect to the perihelion time. The best solution was obtained assuming that between the apparitions of 1935 and of 1944 the time shift changed its value, thus τ_1 and τ_2 refer to apparitions before and after 1940 Jan. 1, respectively. Variations of angles I and ϕ with time, describing the nucleus spin-axis orientation, are presented. It appears that forced precession causes the moderate changes of the position of the rotation axis in space.

The ratio of rotational period to radius of the nucleus was found for each model. The present precession models are in agreement with sizes and periods of rotation of other cometary nuclei deduced from observations. The obtained models give some strong constraints on the physical parameters of the nucleus of comet P/Comas Solá. Assuming a prolate spheroid for the nucleus of the comet, the expected rotational period is 14 ± 4 hours for an equatorial radius of 2 km. For the same radius, the oblate Model II gives the much smaller rotational period of 2.4 ± 0.4 hours. The polar radii are 2.2 km and 1.3 km for the prolate and oblate model, respectively.

Key words: comets: individual: 32P/Comas Solá

1. Introduction

Comet 32P/Comas Solá was observed extensively during its nine appearances since its discovery in November 1926. The comet was recovered at each of its returns to the Sun several months (up to a year) before each perihelion passage and observations were carried out also a long time after perihelion. For this reason P/Comas Solá is a good candidate to study the nongravitational effects in orbital motions on the basis of positional observations. The present investigation is additionally motivated by remarkable variations of a nongravitational acceleration detected in the orbital motion of Comas Solá. Long term nongravitational effects determined by using at least three consecutive appearances (Marsden 1969, 1970) are constant or slowly varying with time for the majority of short periods comets. Comas Solá shows drastically different behaviour with rapidly varying nongravitational effects (Marsden et al. 1973; Forti 1983, 1989). Estimates of the nongravitational effects of Comas Solá taking into account also the last apparition are given in Sect. 3. It appears that close to the perihelion passage in 1944 the nongravitational deceleration of the comet unexpectedly changed into acceleration. This discontinuity could not be accounted for by any single event since neither any outburst nor a change of the comet orbit during close encounter with Jupiter has occured after 1926. The only encounter with Jupiter in May 1912 to within 0.177 AU changed the comet orbit significantly (see Table 4) – but it took place two revolutions before the comet discovery. Similar discontinuities are also visible for some other short period comets: the most spectacular examples are 16P/Brooks 2, 21P/Giacobini-Zinner and 31P/Schwassmann-Wachmann 2 (see Fig. 2 in Sekanina 1993). These objects according to Marsden & Sekanina (1971) have been qualified as *erratic comets*. Different hypotheses have been proposed to understand long-term variations in the nongravitational perturbations. According to one of the explanations an anisotropic outgassing from the nonspherical cometary figure forced the precession of the axis of the rotating nucleus. This model was applied to Comet 2P/Encke by Whipple & Sekanina (1979),(starting the series of papers on the nature of variations of nongravitational effects reported for almost all the short-period comets.) Using the observed light curves and values of the nongravitational transverse parameter A_2, Sekanina described the methods of modelling the precession of the nucleus. The forced precessing model applied by him for Comas Solá (Sekanina 1985) implies that during seven revolutions around the Sun (1926-1979) this comet precessed more rapidly than any other known comet. Two best

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Table 1. Distribution and other characteristics of the observations of 32P/Comas Solá. Numbers of residuals for first and last apparitions are decreased by taking into account normal places instead of some observations

Perihelion time	Observation interval	Number of obs.	Number of res.	Mean res.
1927 3 22	1926 11 4 – 1927 5 31	149	181	2.″13
1935 8 6	1935 8 12 – 1936 7 16	50	84	1.″19
1944 4 11	1943 10 21 – 1944 6 14	35	58	1.″50
1952 12 10	1951 7 7 – 1953 7 4	49	87	1.″04
1961 7 4	1960 6 29 – 1962 5 4	35	70	0.″72
1969 3 29	1968 10 27 – 1970 7 1	43	86	1.″34
1978 9 24	1977 9 11 – 1979 4 26	38	72	1.″76
1987 9 18	1986 7 28 – 1988 5 19	35	67	0.″80
1996 9 18	1995 8 1 – 1996 3 10	148	175	0.″88
Total	1926 11 4 – 1996 3 10	582	880	1.″41

fitting models required a large oblateness of the nucleus, 0.52 and 0.57, respectively. Sekanina emphasized that shortly after the 1952 perihelion passage the precession model gives "intolerably large perturbation" in the spin-axis obliquity: it shifted rapidly by about 90°. Therefore, Sekanina returned to Comas Solá (Sekanina 1993) with a modified model, additionally assuming discrete-source outgassing from the cometary nucleus. He ascertained that the comet behaviour can be explained by outgassing from a single small active area and – something that makes this model particularly attractive – with a modest precession rate. However, this reasonable model was obtained only for returns to the Sun between 1944-79, and the two preceding apparitions in 1927 and 1935 do not fit to this model well. He concluded that after the 1927 perihelion or before the perihelion in 1935 an episodic outgassing event took place. This also indicates that active regions – if they exist – could be short-lived sources in comparison to the whole observational interval.

Since the analysis made by Sekanina, new, rich, good-quality data have become available from the later two returns to the Sun: the observations for the 1987 apparition and recent preperihelion observations carried out before March 1996. We show (Sect. 5) that it is possible to link all the Comas Solá apparitions on the basis of a forced precession model with physically reasonable parameters. This implies that astrometric data can provide unique information on the position of the spin axis of the comet nucleus during the cometary orbital motion. The analysis presented is a continuation of investigations of the nature of nongravitational effects detected in the orbital motions of comets undertaken by Sitarski and collaborators (e.g. Sitarski 1992, 1994; Królikowska & Sitarski 1996).

2. Observational material

We have collected all the available astrometric observations of the periodic comet Comas Solá. Our whole observational material contains 582 observations between 1926 November 4 and 1996 March 10. All the relevant characteristics of the data are presented in Table 1. The observations were selected according to mathematically objective selection criteria (Bielicki & Sitarski 1991) for each apparition separately. Many of the oldest measurements were published by observers as *comet minus star* distances in equatorial coordinates. The identification of comparison stars in modern star catalogues allows us to refine the apparent comet positions. We refer these comparison stars to the Positions and Proper Motions (PPM) Star Catalogue using the algorithm proposed by Bielicki et al. (1984). For Comet Comas Solá, 104 observations from the 1927 apparition and 8 measurements from the 1944 apparition were published in the form of *comet minus star*. For the first apparition, the positions of comparison stars for 80 observations were found in the PPM Star Catalogue (Gabryszewski 1996). This means that about 80% of comet positions of the *comet minus star* type were improved. The procedure significantly reduced the mean residual from 4.″67 to 2.″61. Using the same procedure for the 1944 apparition, among eight observations the five positions of stars were recalculated, thereby decreasing the mean residuals from 1.″78 to 1.″50. Since the observations are distributed highly nonuniformly over the apparitions, we created a number of normal places (replacing more than two observations of the same day by one average comet position) for two overpopulated returns of the comet. This yields 181 residuals rather than 260, for the 1926 apparition, and 175, rather than 289, for the 1996 pre-perihelion passage. The numbers of residuals obtained by this procedure are listed in column 4 of Table 1. After such data processing, the mean residuals have been calculated for each apparition separately (their values are listed in the last column of Table 1), and the mean *a priori* residual (Bielicki & Sitarski 1991) of 1.″41 was then obtained.

3. Variations in the nongravitational motion of comet Comas Solá

To estimate the nongravitational force acting on the rotating cometary nucleus with sublimating surface we used the widely-known Marsden method which assumes that this force has a constant direction in orbital coordinates and its value depends on the comet's distance r to the Sun:

$$F_i = A_i \cdot g(r), \quad A_i = \mathrm{const} \ \ \mathrm{for} \ \ i = 1, 2, 3, \tag{1}$$

where F_1, F_2 determine the radial (directed outward from the Sun) and transverse nongravitational force components in the orbital plane, and F_3 corresponds to the component perpendicular to the orbit plane. An analytical form of the function $g(r)$ has been deduced (Marsden et al. 1973) from studies based on the isothermal model of cometary nuclei. For an individual comet the nongravitational parameters A_i could be determined from positional observations. Their values can be calculated by using the observations from at least three consecutive perihelion passages.

It was reported earlier (Sekanina 1985, 1993) that P/Comas Solá is a member of a group of comets with rapidly varying

Table 2. Nongravitational parameters A_1, A_2, A_3 determined as constant values by linking three and four consecutive apparitions. The mean residual RMS and the number of observations used for orbit improvement are given in the last two columns.

Appearances	A_1		A_2 in units of 10^{-8} AU/day^2		A_3		Mean res	Number of obs.
1926 1935 1944	0.82348	±0.08777	+0.00326	±0.00150	−0.05386	±0.04492	2.″02	172
1926 1935 1935 1944	0.79926	±0.04970	+0.02029	±0.00035	−0.09863	±0.03564	2.″53	220
1935 1943 1953	0.68774	±0.05517	+0.04183	±0.00106	−0.14276	±0.03552	1.″48	125
1935 1943 1953 1962	0.56857	±0.06528	+0.16811	±0.00054	−0.18803	±0.05458	3.″35	160
1943 1952 1962	0.74371	±0.05064	−0.01586	±0.00110	−0.12971	±0.02723	1.″33	116
1943 1952 1962 1970	1.49500	±0.06098	−0.04553	±0.00048	−0.15500	±0.04178	2.″74	159
1951 1961 1970	0.69140	±0.04576	−0.07287	±0.00098	−0.21126	±0.02793	1.″33	126
1951 1961 1970 1979	0.87923	±0.03538	−0.08256	±0.00107	−0.26441	±0.02922	2.″48	162
1960 1969 1979	0.71045	±0.04805	−0.12527	±0.00219	−0.20197	±0.02974	1.″52	114
1960 1969 1979 1988	0.62540	±0.03646	−0.13445	±0.00063	−0.12358	±0.02141	1.″51	149
1968 1978 1988	0.60474	±0.10407	−0.13581	±0.00098	+0.03729	±0.04856	1.″49	114
1968 1978 1988 1996	0.45868	±0.05106	−0.12729	±0.00054	+0.08103	±0.04572	1.″89	205
1977 1987 1996	0.87175	±0.07195	−0.11248	±0.00141	−0.08982	±0.05400	1.″41	162

nongravitational parameters. This means that linkage of all the apparitions of the comet with constant nongravitational parameters is impossible. Sekanina (1993) concludes, on the basis of nongravitational parameters A_1 and A_2 as estimated by Forti (1983, 1989) and Marsden et al. (1973), that this comet shows some systematic trend in its motion. To examine variations of the nongravitational parameters with time we linked each three consecutive apparitions of the comet. For such a time interval, the constant values of A_1, A_2 and A_3 were calculated along with six corrections to the orbital elements from observational equations by an iterative least squares method. Our present evaluations of the normal component, A_3, for Comas Solá are published for the first time. The results are summarized in Table 2. Temporal variations of A_i, $i = 1, 2, 3$ are also plotted in Fig. 1 with the corresponding parameters taken from the literature: F1 – Forti (1983), F2 – Forti (1989), and MSY – Marsden et al. (1973). For comparison, values of A_3 calculated using four consecutive apparitions are plotted as open circles at the top of Fig. 1. Since the errors for A_2 are relatively small we do not show them on the graph. Note that values of A_2 determined by MSY, F1, F2 and ourselves are quite similar. The monotonic changes of A_2 from a deceleration in the comet's motion before 1943 to an anomalously large acceleration after 1960 were confirmed. Only two of 43 periodic comets analyzed by Sekanina (1993), P/Brooks 2 and P/Schwassmann-Wachmann 2, have more negative values of A_2 than does Comas Solá. Combining the three last apparitions suggests that A_2 is beginning to increase (see Fig. 1). One can see that our values of the radial nongravitational parameter A_1 calculated for the appearances of 1935-53, 1943-62 and 1951-70 are similar to the MSY values (denoted with crosses on the Fig. 1) and strongly differ from the F1 results (open squares). However, for the next two time intervals, i.e. for 1960-1979 and 1960-1988, ours and Forti's (F1, F2) results are in better agreement than those mentioned above. Generally, our calculations based on the most complete data sets suggest that the radial component A_1 is moderately variable during the whole period (oscillating between 0.60 and 0.87 in units of 10^{-8} AU/day^2). It is important to emphasize that in some cases the normal component value is comparable to (or even greater than) the transverse one. It is distinctly visible for the revolutions between 1951 and 1979.

4. Forced precession model of the rotating oblate nucleus

Variations of the nongravitational acceleration of the orbital motion of some periodic comets have been interpreted as a result of the precession of the rotating cometary nucleus due to an anisotropic outgassing from the surface. In the frame of such a treatment Marsden's nongravitational parameters $A_i, i = 1, 2, 3$ referred to the orbital coordinates can be expressed by the angular parameters η, I and ϕ of the rotating nucleus:

$$A_i = A \cdot C_i\left(\eta, I, v + \phi\right), \quad i = 1, 2, 3, \tag{2}$$

where $A = (A_1^2 + A_2^2 + A_3^2)^{1/2}$ and C_i are the time-dependent direction cosines. Here, the preliminary assumption of all A_i being constant is replaced by the more realistic A=const. The time dependence of C_i is given by the true anomaly of the comet,

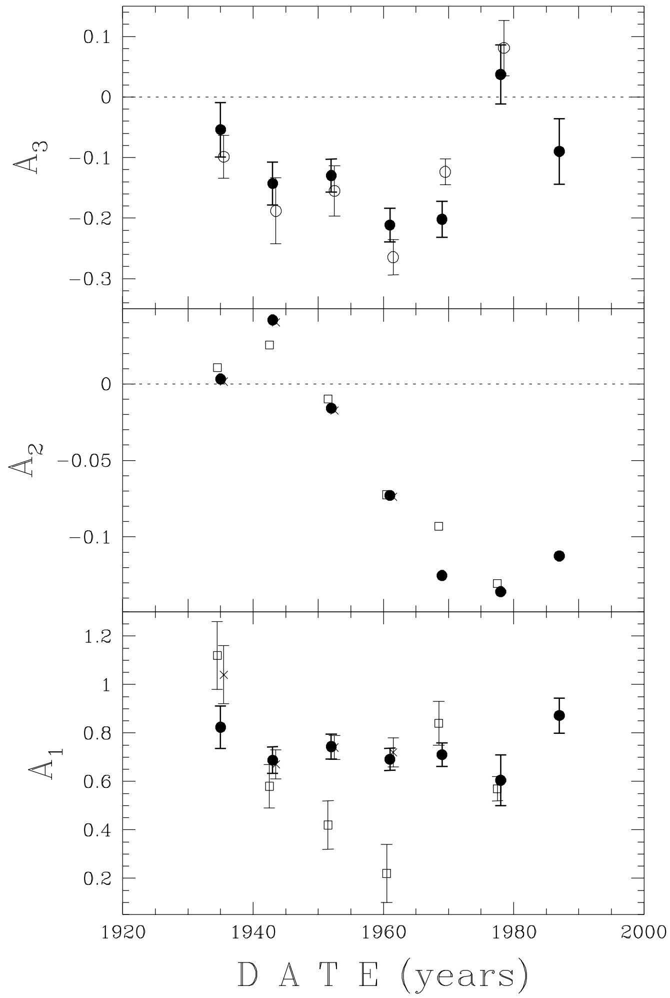

Fig. 1. Variations of the nongravitational parameters A_1, A_2, A_3 of 32P/Comas Solá versus time. The crosses denote the values of A_1 and A_2 from the orbital solutions calculated by Marsden et al. (MSY), open squares – by Forti (F1 or F2), and full circles – the present calculations of A_1, A_2 and A_3 (see Table 2). All parameters were determined by linkage of three consecutive apparitions; thus corresponding symbols refer to the central perihelion passages. For comparison, values of A_3 derived for four consecutive apearances (see Table 2) are given as open circles. Error bars are shown for A_1 and A_3; for A_2 they are omitted because they are comparable to the symbol sizes.

v, which varies with time due to the comet's orbital motion, and also by variations in (equatorial) obliquity relative to the orbit plane, I, and cometocentric solar longitude at perihelion, ϕ, both due to precessional motion of the spin axis. The lag angle, η, measures the displacement of the direction of outgassing averaged over the nucleus surface from the direction to the Sun. In our analysis we assume that η is a time-independent unknown parameter. In the present study we assume that the angles I and ϕ are functions of time as a result of the forced precession. Formulae for changes of the spin-axis orientation of the cometary nucleus due to the forced precession resulting from the asymmetric gas ejection have been derived by Sekanina (1984). The formulae for the time-dependence of I and ϕ and for the direction cosines C_i were adopted by one of authors (S. Szutowicz) to use them in the orbital computations:

Table 3. Angular parameters and orbital elements for forced precession models linking nine apparitions during November 1926 – March 1996. Solutions are given for the Epoch: 1926 Nov. 1.0 ET, and orbital elements ω, Ω, i are referred to Equinox J2000.0. Calculations were based on 880 observational equations obtained from 460 positional observations. Time shifts τ_1 and τ_2 (in days) represent the displacements of the maxima of the function $g(r)$ with respect to perihelion before and after 1940 Jan. 1, respectively. A is in units of 10^{-8} AU/day^2 and the precession factor f_p – in units of 10^7 day/AU.

	Model I prolate nucleus	Model II oblate nucleus	Model III
A	+0.87828 ± 0.01268	+1.0548 ± 0.05120	+1.2063 ± 0.0595
η	17°.54 ± 0°.81	10°.75 ± 0°.53	5°.85 ± 0°.45
I_0	49°.40 ± 2°.25	68°.91 ± 0°.20	74°.02 ± 0°.27
ϕ_0	87°.98 ± 3°.70	279°.25 ± 1°.68	287°.37 ± 1°.61
f_p	−0.18581 ± 0.00711	+0.10401 ± 0.01139	+0.08631 ± 0.00835
s	−0.10521 ± 0.02408	+0.35389 ± 0.01282	+0.35566 ± 0.01537
τ_1	−55.051 ± 3.791	−6.738 ± 1.822	−12.518 ± 1.348
τ_2	+8.1306	+19.601	+8.1306
T	1927 03 22.19964	1927 03 22.19523	1927 03 22.19628
q	1.77244000	1.77246254	1.77245800
e	0.57495849	0.57495190	0.57495280
ω	38°.50840	38°.50741	38°.50782
Ω	66°.60471	66°.60307	66°.60328
i	13°.76520	13°.76577	13°.76576
Res	2″.05	2″.11	2″.12

$$
\begin{aligned}
I(t) &= I_0 + \int_0^t \mathrm{d}t \cdot \dot{\varphi} \cos(\alpha+\eta), \\
\phi(t) &= \phi_0 - \int_0^t \mathrm{d}t \cdot \dot{\varphi} \sin(\alpha+\eta) / \sin I, \\
\dot{\varphi} &= A f_p g(r)(2-s) \sin\psi \cos\psi \left(1 - S\sin^2\psi\right)^{-3/2} \cdot B \\
& \quad \sin\psi = \sin I \sin\lambda, \\
& \quad S = s(2-s), \quad S_1 = S(2-S), \\
& \quad B = (1 - S_1 \sin^2\psi)^{1/2} \\
& \quad \tan\alpha = \cos I \tan\lambda, \ \ \lambda = v + \phi, \\
& \quad I_0 = I(t_0), \ \ \phi_0 = \phi(t_0), \ \ t_0 \text{ is the} \\
& \qquad \text{starting epoch of integration,} \\
C_1 &= [\cos\eta + (1 - S - \cos\eta)\sin^2 I \sin^2\lambda]/B, \\
C_2 &= [\sin\eta\cos I + (1 - S - \cos\eta)\sin^2 I \sin\lambda\cos\lambda]/B \\
C_3 &= [\sin\eta\cos\lambda - (1 - S - \cos\eta)\cos I \sin\lambda]\sin I/B
\end{aligned} \tag{3}
$$

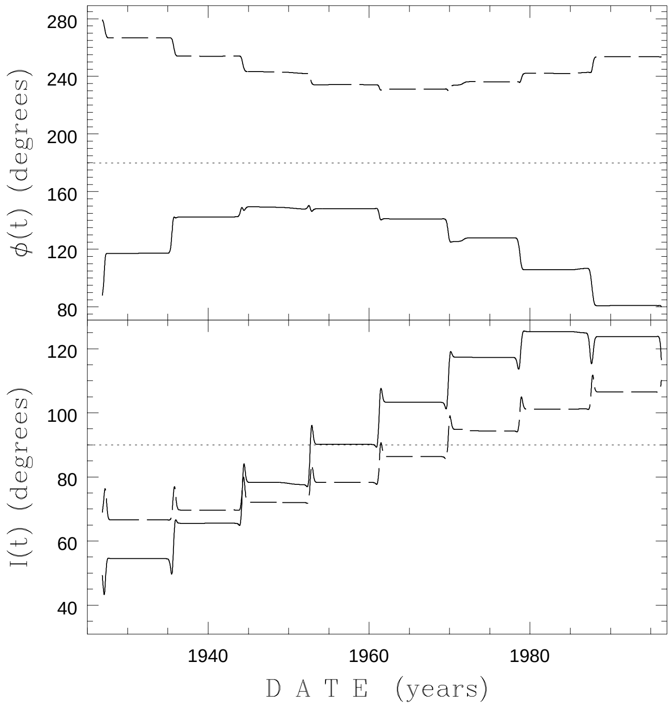

Fig. 2. Variation of I (bottom) and ϕ (top) with time for 32P/Comas Solá due to the spin axis forced precession of the comet nucleus. Solid curves represent the model with a prolate nucleus (Model I), and dashed curves the model with an oblate nucleus (Model II).

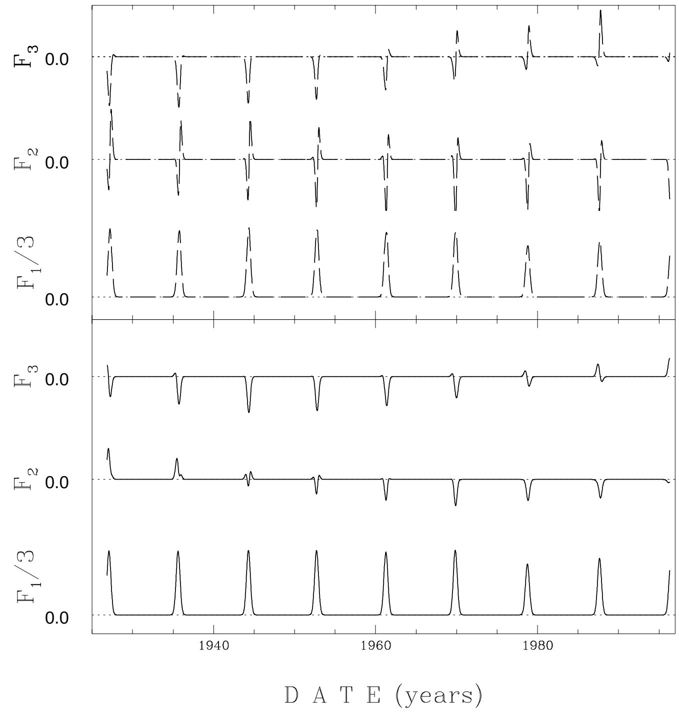

Fig. 3. Components $F_i = A_i \cdot g\,[r(t-\tau)]$ for $i = 1, 2, 3$ of the nongravitational force $\boldsymbol{F}$ as functions of time for 32P/Comas Solá. The lower panel shows curves for Model I, the upper panel for Model II.

One can see that variations of I and ϕ also depend on the oblateness of the nucleus, s, and the precession factor, f_p. The latter is related to the torque factor introduced by Sekanina (1984) by $f_p = s \cdot f_{tor}$. Putting $s = 0$ in Eq. 0(3), relations for C_i can be transformed to those derived by Sekanina (1981) for the spherically symmetric model of the rotating nucleus.

The values of six unknown parameters: $A, \eta, I_0, \phi_0,\ f_p$, and s could be determined simultaneously with six corrections to the orbital elements from the observational equations by an iterative least squares process. In practice, preliminary values of A, η, I_0, and ϕ_0 usually can be estimated from the constant nongravitational parameters A_i determined earlier by combining several apparitions of the comet; starting values of f_p and s are taken to be zero.

5. Linkage of all the apparitions

After extensive numerical calculations we found that linkage of all the apparitions is possible only with some additional assumptions about asymmetry in cometary activity during the comet's passage through perihelion. This is in agreement with the observed asymmetries in the light curves. For this reason we present here asymmetric acceleration models for the forced precession of the spin-axis of a rotating cometary nucleus. Following Sekanina's (1988) idea we replaced the symmetric function $g(r)$ in Eq. (1) by the function $g(r'),\ r' = r(t-\tau)$, where the constant value of τ represents a time shift of the maximum value of $g(r)$ with respect to the perihelion time. The best solution was obtained when we assumed that between the apparitions in 1935 and 1944 the time shift, τ, changed its value. This coincides with the moment of transition from deceleration to acceleration visible in the transverse nongravitational parameter A_2 (see Fig. 1). Therefore, for the models presented we set the discontinuity of τ at the beginning of the year 1940 (close to aphelion) as

$$g\,[r(t-\tau)] \quad \text{where } \tau = \begin{cases} \tau_1 & \text{for } t < 1940.0 \\ \tau_2 & \text{for } t \geq 1940.0 \end{cases} \tag{4}$$

Finally, we had to determine values of the eight unknown parameters: $A, \eta, I_0, \phi_0,\ f_p,\ s,\ \tau_1$, and τ_2 with six corrections to the orbital elements from the observational equations by a least-squares iterative process. Since we were not able to obtain any solution for both shifts parameters τ_1 and τ_2 simultaneously, we applied the following procedure. We calculate 13 parameters ($A, \eta, I_0, \phi_0,\ f_p,\ s,\ \tau_2$ and six corrections to the orbital elements) by integrating the equation of motion from the year 1996 backward to the 1935 apparition (or 1944 apparition[1]). Then, starting from the derived values for the beginning of the observational data (1 Nov. 1926), we determine the 13 parameters (including τ_1 among them) keeping τ_2 fixed using the whole observational material up to 1996. It turns out that this process is convergent and we were able to obtain a fully consistent forced precession model with assumed asymmetry of cometary activity. In this way we found two solutions (Models I and II) for a

[1] Due to numerical difficulties the prolate model was obtained including the 1935 apparition while the oblate Model II was obtained without it.

Table 4. The evolution of the orbit of Comet 32P/Comas Solá according to Model I; perihelion distance q is in AU, period P is in years, angular parameters ω, Ω, i are referred to J2000.0. The apparitions which have been observed and used for the orbit improvement are denoted by an asterisk.

	T (ET)	q	e	P	ω	Ω	i	Epoch
	1900 Oct. 22.3041	2.165361	0.514183	9.41	44°.7376	69°.1997	18°.6926	1900 Oct. 28
	1910 Mar. 9.4074	2.152201	0.514962	9.35	44°.8225	69°.1183	18°.7073	1910 Feb. 18
	1918 Sep. 17.4082	1.770155	0.575475	8.51	38°.3539	66°.6941	13°.7814	1918 Sep. 4
*	1927 Mar. 22.1963	1.772426	0.575061	8.52	38°.5085	66°.6042	13°.7649	1927 Mar. 21
*	1935 Oct. 6.5866	1.777111	0.574458	8.53	38°.8149	66°.3803	13°.7239	1935 Oct. 5
*	1944 Apr. 11.5035	1.766549	0.575763	8.50	38°.8851	66°.3979	13°.7331	1944 Apr. 20
*	1952 Sep. 10.7853	1.766921	0.576971	8.54	39°.9711	63°.5859	13°.4692	1952 Sep. 25
*	1961 Apr. 4.0435	1.777188	0.576099	8.58	40°.0435	63°.5199	13°.4435	1961 Apr. 11
*	1969 Oct. 29.0646	1.768773	0.576924	8.55	40°.0901	63°.4259	13°.4503	1969 Oct. 26
*	1978 Sep. 24.2236	1.869866	0.565816	8.94	42°.8652	63°.0983	12°.9606	1978 Sep. 9
*	1987 Aug. 18.6832	1.830265	0.569815	8.78	45°.5432	61°.0540	12°.9556	1987 Sep. 2
*	1996 June 10.4724	1.846364	0.567794	8.83	45°.7651	60°.8695	12°.9173	1996 June 6
	2005 Apr. 1.3498	1.833012	0.569249	8.78	45°.8249	60°.7918	12°.9286	2005 Apr. 20

prolate ($s < 0$) and oblate ($s > 0$) nucleus, respectively. We also found a second solution for the oblate nucleus (hereafter Model III) keeping τ_2 fixed at the value obtained for the prolate nucleus model. The characteristics of these three models are presented in Table 3. (The negative value of the oblateness s in Model I indicates that the cometary nucleus is a slightly prolate spheroid rotating around its longer axis.) In Model I the ratio of equatorial to polar radius is equal to $R_a/R_b = 9/10$. The remaining two solutions represent the oblate model of the nucleus. For all three solutions we obtained a negative time shift τ_1 and a positive time shift τ_2. This implies that in these models the light curves at the 1927 and 1935 apparitions peak before perihelion, while the maxima of the light curves during 1943-96 are shifted towards the postperihelion time. Thus, in the first two apparitions the comet reached its maximum of activity before the perihelion passage and in the next seven apparitions afterwards. However, only in Model I is the derived time shift τ_1 large. The absolute values of τ_1 for the remaining models and the values of τ_2 for all the models are smaller than 20 days. Unfortunately, the rather poor quality of the light curves observed at the apparitions of 1927-88 (Kamel 1992; and Fig. 1 in Sekanina's paper 1985) do not allow us to verify these solutions.

The motion of the cometary rotation axis represented by angles I and ϕ is given in Fig. 2 which shows the variations of I and ϕ for Model I (solid curve) and Model II (dashed curves). The calculated obliquity, I, increased with time (as is shown on the lower panel of Fig. 2). The initial values of the obliquity in the two oblate models are very similar to those derived by Sekanina (1985). In our models, the variations of ϕ presented in the upper part of Fig. 2 appear to be moderate (the behaviour of ϕ for Model III is very similar to that given for Model II). During all nine revolutions around the Sun the configuration of the poles did not change. Of the two types of results, Model I represents a solution according to which the northern pole of rotation was turned toward the Sun at the 1927 perihelion passage, while Models II and III represent the solutions favoring the southern

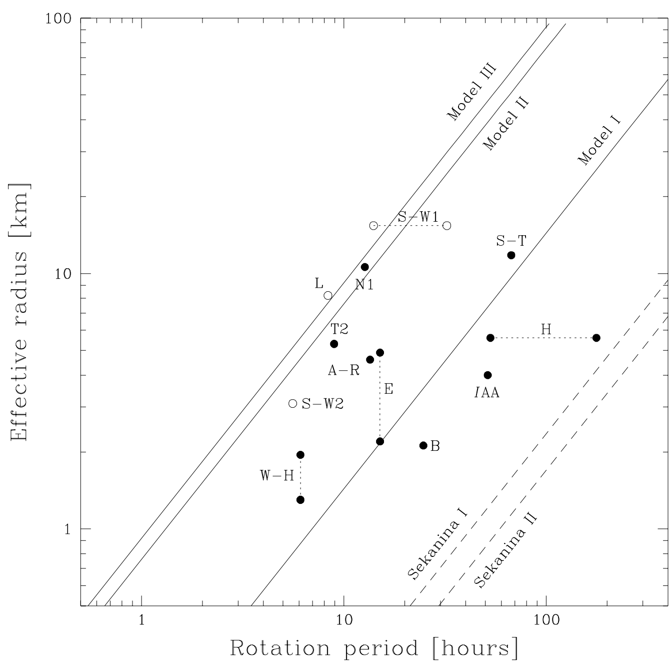

Fig. 4. The logarithm of the nucleus radius (i.e. effective radius, see Table 8) vs. the logarithm of the rotational period for a sample of well-observed cometary nuclei. The plotted nuclei are H – Halley, S-T – Swift-Tuttle, W-H – Wilson-Harrington, E – Encke, A-R – Arend-Rigaux, T2 – Tempel 2, N1 – Neujmin 1, S-W1 – Schwassmann-Wachmann 1, S-W2 – Schwassmann-Wachmann 2, L – Levy, IAA – *IRAS*-Araki-Alcock, B – Borrelly. Open circles denote the upper limits of the nuclear radius. Linear relationships resulting from Models I–III are plotted by solid straight lines. Analogous relationships obtained from Sekanina solutions are shown by dashed lines.

Table 5. Linear relationships between rotational period and equatorial radius for the nucleus precession models I–III (Table 5a) and for two models (Table 5b) obtained by Sekanina (1985).

Present investigations (**a**)

	Model I with prolate nucleus	Model II with oblate nucleus	Model III with oblate nucleus
f_{tor} (day/AU)	$(1.766 \pm 0.472) \times 10^7$	$(0.294 \pm 0.043) \times 10^7$	$(0.243 \pm 0.034) \times 10^7$
P (hours)	$(7.12 \pm 1.90) \cdot R_a$	$(1.19 \pm 0.17) \cdot R_a$	$(0.98 \pm 0.14) \cdot R_a$
Equatorial radius:			
R_a =1 km			
P (hours)	7.1 ± 1.9	1.19 ± 0.17	0.98 ± 0.14
R_a =2 km			
P (hours)	14.2 ± 3.8	2.38 ± 0.34	1.96 ± 0.28
R_a =5 km			
P (hours)	36 ± 10	5.93 ± 0.86	4.89 ± 0.68

pole. At this initial position of the sunlit pole our oblate models are similar to Sekanina's Solution I.

Fig. 3 shows qualitative variations of the nongravitational force components acting on the comet during its succesive returns to the Sun. One can see that the changes of F_1 for both types of model are very similar while the variations of F_2 and F_3 are significantly different.

6. Evolution of the orbit and conclusions

We have obtained three satisfactory numerical models for the nongravitational motion of 32P/Comas Solá applying Sekanina's forced precession model for the rotating comet nucleus. Using Model I, we investigated the evolution of the comet orbit during 1900–2005.The comet's equations of motion have been integrated by the recurrent power series (Sitarski 1984) including all the planetary perturbations. The results are given in Table 4.

Our precession models for the nongravitational motion of the 32P/Comas Solá impose some constraints on the physical properties of the nucleus, namely its size and rotational period, which makes these models additionally attractive and useful. Using our three solutions we can calculate a value of the Sekanina torque factor $f_{tor} = f_p/s$ for 32P/Comas Solá. Sekanina has shown that the torque factor is related to the rotational period P and the equatorial radius R_a of the cometary nucleus by the formula: $f_{tor} = 5P/4\pi R_a$. This implies that using parameters derived in our models we were able to calculate a value for the P/R_a ratio. The relevant values of f_{tor} and the linear equations of P and R_a are given in Table 5a. Sekanina derived about 1 km for the equatorial radius of Comas Solá from Solution I and II

Table 5. (continued)

Sekanina's models (**b**)

	Solution I with oblate nucleus	Solution II with oblate nucleus
Polar radius (km)	0.48	0.49
Equatorial radius (km)	0.99	1.15
Rotation period (hours)	35.3	55.9
s	0.52	0.57
f_{tor} (day/AU)	8.9×10^7	12.1×10^7
P (hours)	$35.88 \cdot R_a$	$48.79 \cdot R_a$

(see Table 5b). In Table 5a we list the relevant values of the rotational period implied by our models for some values of the equatorial radius.

Fortunately, several relatively well-determined sizes and periods of rotation of cometary nuclei are now available from observations. We updated the data collected by Rickman (1991) and Campins et al. (1995) mainly by CCD observations at large heliocentric distances. There are also direct detections of two cometary nuclei with the Hubble Space Telescope. The radii and rotation periods obtained from observations of individual cometary nuclei are plotted in Fig. 4.
There relationships between effective radii and rotation periods resulting from our models are also shown together with the anal-

ogous curves (Table 5b) obtained from two solutions given by Sekanina (1985). It seems that our models are in agreement with the whole observational material. Assuming that Comas Solá lies within the region occupied by comets with well-determined sizes and rotation periods, our models give some constraints on the physical parameters of the nucleus of Comas Solá. Assuming a value for the equatorial radius and using our prolate model we obtain different values of the rotational period as compared to the oblate models. For example, if we assume that Comas Solá rotates around its longer axis, the expected rotational period is 14 ± 4 hours for $R_a = 2$ km. The oblate Model II gives a much smaller rotational period of 2.4 ± 0.4 hours. The polar radii are 2.2 km and 1.3 km for the prolate and oblate model, respectively. We believe that future observations of 32P/Comas Solá will make it possible to verify our models.

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